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OBSERVATIONAL STUDIES OF THE ATMOSPHERIC GENERAL CIRCULATION

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ABSTRACT

Observational research results dealing with the general circulation of the atmosphere, obtained from investigations carried on at M' I' during the past seventeen years are summarized for more convenient reference, primarily with a view to the study of analogous behavior in certain respects of other cosmical fluid systems such as the solar atmosphere in particular. A total of seventy six selected papers are included in the volume, together with a preface and foreword by the compilers. A list of project reports issued during the period is also appended.

PREFACE

Economy of effort in scientific research demands that progress in each branch of study be shared with others involving similar or partially similar problems. The benefits of such cross-bracing have long been noted and many historic instances can be cited. It has come to the attention of the editors of this volume, as well as to many of our associates, that some of the characteristic mechanisms of the earth's atmospheric general circulation repeat themselves in other cosmic fluid masses. That this is so can no longer seriously be doubted. Evidence for it is being given not only by meteorologists but also by astronomers, in the case of the solar atmosphere, the galaxies and other fluid bodies.

Whereas our colleagues in astronomy have been most generous in making available their findings in numerous books, journals and more common printed media of communication, the meteorologists have yet to make good a reciprocal offer of findings and newly gotten information in their own field. We hope that the material here printed will at least in a small way be helpful in this respect.

This volume contains a selection of papers dealing with observational research concerning the general circulation of the atmosphere and related topics. The bulk of this work was performed at the Massachusetts Institute of Technology during the past seventeen years under sponsorship of the U. S. Air Force. Some of the work concerning the stratosphere was also supported by the U. S. Atomic Energy Commission during the later portions of this period. The observational study of a laboratory model (see p. 193) was made at the Hydrodynamics Laboratory of the University of Chicago.

In view of the character of the present compilation, it may be regarded as fulfilling in some measure the purposes of a companion collection to, "Selected Papers on the Theory of Thermal Convection with Special Applications to the Earth's Planetary Atmosphere", by B. Saltzman, Dover Publications, 1962. Whereas the Dover publication presents theoretical approaches to the subject, the present work aims to depict the dynamic processes actually present from observational evidence.

It has in the past been our custom to issue a series of MIT reports entitled "Studies of the Atmospheric General Circulation". Since these were printed in limited editions only, the earlier ones, particularly the

first dated May 1954, quickly became unavailable. Numerous requests for them have been addressed to us through the years. It is our hope that the existing need thus evinced will hereby be largely answered.

Victor P. Starr
Massachusetts Institute of Technology
June, 1966

NOTE

Due to the relationship of this volume to the Dover Publication mentioned above, we have been fortunate in securing the expert assistance of Dr. Barry Saltzman of the Travelers Research Center, Inc., in the compilation of the present collection.

V. P. S.

Foreword

Since this compilation deals with a continuing program of observational research on the basic nature of the general circulation of the earth's atmosphere, it seems fitting to begin with a few remarks concerning what we believe to be the guiding aims and rationale for the work involved.

We are probably not oversimplifying the matter by saying that the observational studies reprinted in this collection have been motivated by two main goals: the achievement of an ever more complete global statistical description of the atmosphere, in terms of the distribution in time and space of all its properties (e. g., motion, temperature, pressure, content of water and other components); and the achievement of that "understanding" of these statistics which can only come through demonstrations of their consistency with, and deducibility from, the fundamental physical laws and the special boundary and state conditions which define the earth-sea-atmosphere system. Since we mean by "statistical description" not only the average conditions, but also the variabilities and bounds measured by the higher moments of the probability distributions, these goals imply a study of the complete three-dimensional "climate" of the atmosphere. What distinguishes this from the study of "weather" is the concern for ensembles of instantaneous spatial distributions rather than for the details of particular instantaneous distributions.

The role of observational studies in achieving the first goal is quite obvious, but in the achievement of the second goal, as well, observational studies must play an important and direct part. Their importance in this second context is basically due to the fact that the atmosphere, as a gaseous fluid, easily redistributes its properties through its own motion, with the result that the final statistical distribution depends critically on transport processes. These processes are in fact the sine qua non of all dynamical activity in the atmosphere, since, in a manner of speaking, all motions owe their origin to a need for transporting heat or momentum in order to relieve unstable arrangements of the fluid with regard to its temperature or motion structure. A knowledge of the modes by which these transports are effected, and of their role in the budgets of conserved quantities such as mass, momentum, energy, and potential vorticity, are, accordingly, vital ingredients in a dynamical description of the atmosphere aimed at giving "understanding."

Moreover, as is well known, in order to go forward with a deductive theory for a complicated system like the atmosphere, with its many

degrees of freedom, it is first necessary to use observational knowledge to specialize the general mathematical representation of the fundamental laws for the particular scales or frequencies under investigation. It has simply never proved feasible to write and solve equations which simultaneously govern phenomena over the whole spectrum of spatial and temporal variation. The specialization procedure implicit in all theoretical studies is, therefore, to isolate from the full series expansions of the atmospheric variables some band of scales or frequencies in which we are interested and either to neglect, specify, or parameterize in terms of the retained structure, the effects of the remaining structure, thereby closing the problem mathematically. It is only later, by comparison of the solutions with observations, that justification for the specializations can be claimed. In fact, it is just this process of ab initio specialization of known laws and a posteriori verification of the consequences that constitutes 'theoretical study' in the sense that the term is generally used in meteorology and the other applied geophysical sciences. The continuing goal is to learn what effects can be legitimately neglected from consideration in a given problem, and, also, to reduce the amount of information initially specified in favor of either broadening the spectrum of phenomena under simultaneous investigation or applying meaningful parameterization laws. In systems where there is little interaction between different portions of the spectrum (e.g., linear systems) the problem of parameterization hardly arises at all. However, in a fluid system like the atmosphere, which is capable of internal transport, a basic nonlinearity is introduce which links the properties of one scale with the properties of others, and a knowledge of this linkage is essential in order to close theoretical models in a physically realistic way.

It follows that for an understanding of the behavior of the atmosphere, it is necessary to compute not only averages and variances of the meteorological variables, but also the transports, or co-variances of the variables with the field of motion, on all scales in time and space. In addition, other quantities which enter into the fundamental equations, such as diabatic heating due to radiation and latent heat release, must also be measured and parameterized for a successful framing of the complete theoretical problem.

In general, then, the long-range observational goals for a study of our planetary atmosphere seem to require the measurement of all of the atmospheric variables at an increasingly dense network of observing points and at increasingly smaller time intervals, for a record of many years, and the derivation, from these measurements, of the averages in time and space, the variances and co-variances, and, in turn, the other higher order functions involved in the budgets of energy and potential vorticity.

The raw material for this program is the space-time continuum of values of the motion, the temperature, the pressure or geopotential, the mixing ratios of water vapor and other substances. As prerequisites for the development of the statistics of these raw data, it is first necessary to make decisions regarding the manner of resolution of this apparently complex atmospheric space-time continuum. In making this decision, we have traditionally been guided by the desire to quantify meaningfully the statistical effects of significant physical features (such as cyclones) while at the same time provide theoretical frames of reference which permit the application of established concepts from theoretical hydrodynamics.

The main developments in the study of the planetary circulation over the past two decades, observational contributions to which are included in this collection, have rested largely on the adoption of modes of resolution which successfully fulfill these two aims. The most important of these have been the following:

1. <u>The resolution of the complete distribution into a mean zonal state defined by the average along latitude circles (i.e., the ZONAL, or axially-SYMMETRIC component), and the departures from this average which represents the EDDY or axially-ASYMMETRIC components [e.g., 1, 8]</u>*. This resolution strongly commends itself by an examination of hemispheric weather charts which show a primary symmetric variation of temperature from the equator to the pole, with secondary variations in the form of tongues of cold and warm air associated with meanders in symmetric westerly and easterly currents. The predominately zonal character of the distribution is the expected physical result of the main external influences, namely solar heating and rotation, both of which are themselves axially symmetric. In view of the wave-like appearance of these eddies in the free atmosphere, and of the usefulness of pure wave forms for theoretical purposes, it has been of great benefit to further resolve the departures into Fourier components [e.g., 7, 16, 41, 42]. In this way, it has been possible to make distinctions and measure the interaction between different portions of the wave spectrum ranging from ultra-long waves to the cyclones and, also, to apply much of the formalism and rigorous thinking developed in the classical study of hydrodynamical turbulence to the planetary circulation (with some markedly different results, however [e.g., 33, 36, 38, 46]).

2. <u>The resolution of the complete distribution at any instant into a STATIONARY, or TIME-AVERAGE, component and a TRANSIENT component which is the departure from this average [e.g., 10, 15,]</u>. This

*Numbers refer to the list of papers in the Table of Contents.

resolution serves to isolate, most clearly, what we mean by 'climate,' both as regards the average values of the properties and their variabilities. Again many of the concepts and formalisms of turbulence theory can be drawn upon. A natural extension of this resolution is a further analysis of the transient departures in terms of a frequency spectrum. In this way a separation, susceptible to rigorous treatment, can be made between variations having period of years (climatic fluctuations), seasons, weeks (the 'index cycle'), days (the cyclones), hours (fronts, squall lines, etc.), minutes and seconds (thermal and mechanical turbulence), and so forth. As in the case of the spatial Fourier breakdown described above, a main interest here is to study the nonlinear interactions and parameterizations between the different frequency scales.

3. The resolution of physical processes into those of a purely inertial nature in which energy sources and sinks are disregarded (i. e., the BAROTROPIC mode), and those in which energy sources and sinks are of paramount importance (i. e., the BAROCLINIC modes involving the primary convective motions in a heated, viscous, rotating fluid [e. g., 22, 24]). In the atmosphere both of these processes occur simultaneously, of course, but the artifice of the distinction has proved to be invaluable conceptually. This resolution is effected by averaging in the vertical.

When the statistics generated by these various resolutions are summarized in convenient tabular, graphical, or mapped forms, we achieve a formal statistical description which, together with interpretive analysis in terms of 'Lagrangian' systems and processes, gives a complete description of the atmospheric climate.

Up to the present time, most of the studies directed toward this description and its understanding have been applied to the scales represented by existing daily Northern Hemisphere synoptic charts, mainly for the mid-troposphere and lower stratosphere, and mainly for extratropical latitudes. Regarding the future, then, in addition to the need for refinements of these measurements, a more complete description must await an extension to the boundary layer of the atmosphere, the high atmosphere, tropical latitudes, the entire Southern Hemisphere, and, in all regions, an extension to smaller scales than has hitherto been possible. Active work is currently under way in most of these areas [e. g., 55, 59, 60] . A conclusion which is continually emerging from these extensions is that, just as it is impossible to close off physically one portion of the time spectrum from the remaining portions, it is also impossible to close off a portion of the spatial spectrum, as would be implied by treating one region of the atmosphere in isolation of the rest. The reason in both cases is the same; i. e., the transfer properties of the fluid atmosphere. As one example, it has become apparent from the momentum

budget studies that the existence of the easterlies in the tropics is inseparably related to the existence of the westerlies in middle latitudes and hence cannot be explained as an independent tropical feature.

Also concerning the future, it would appear that the problem of parameterizing synoptic and subsynoptic variables in terms of macrovariables in geophysical fluid systems has not yet been given nearly as much attention as will be necessary. It seems likely that this problem will demand a combination of purely statistical studies with theoretical studies. Powerful statistical methods, such as multiple discriminant analysis, offer the possibility for extracting useful empirical relationships which, hopefully, will find an acceptable theoretical foundation.

As we contemplate the scope of the problems posed by future programs such as these, we are forced to recognize that the data-processing tasks that lie ahead are at least an order of magnitude greater than those which have already been completed. If, however, many of these tasks now seem straightforward, it is perhaps a tribute to the successful gropings of many meteorologists, mainly during the last two decades, to formulate physically sound frameworks about which to rally the systematic observation of our planetary atmosphere.

Barry Saltzman

The Travelers Research Center, Inc.
Hartford, Connecticut

April, 1966

I. THE MOMENTUM BUDGET

A. The Planetary Circulation

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AN ESSAY ON THE GENERAL CIRCULATION OF THE EARTH'S ATMOSPHERE

By Victor P. Starr

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(Manuscript received 15 December 1947)

ABSTRACT

In this discussion of the general circulation the course of the normal transfer of absolute angular momentum from the belts of easterlies near the equator to the belts of surface westerlies in middle latitudes is studied. It is suggested that the horizontal transfer is brought about by the large-scale troughs and ridges in the mid-troposphere, which are adapted to perform this function by their departure from sinusoidal form. Also, the shapes of the subtropical circulations are found to be such as to produce a transport of angular momentum poleward. It is proposed that the downward flow of angular momentum in the westerly belts is effected by the presence of surface cyclones of the Bjerknes type in these regions. The upward flow in the easterly belts is assumed to be effected through some analogous mechanism, although the details are not clear owing to the scarcity of proper observational data.

The various processes which take place in the atmosphere and result in large-scale air motions are so extremely complex in their operation that up to the present time no rational theory approaching any degree of completeness has been devised to explain what we may call the general circulation. It is even doubtful whether all the physical processes which may have an ultimate influence on air motions have as yet been ascertained. It has, however, been realized for many years that there exist certain requirements which impose restrictions on these large-scale motions and impress upon them several characteristics which can be verified observationally. The purpose of this paper is to enquire further into the nature and consequences of some of these restrictions in the light of the added wealth of observational evidence which has been accumulated during the past several years, and to point out further possibilities for research.

In order to base the discussion on principles whose validity cannot be open to question and which are still not of a trivial nature, we shall first consider the fact that a dynamical system such as the atmosphere (or a portion of it) cannot change its absolute angular momentum about a given axis except through the addition or abstraction of angular momentum about that axis from or by external agencies. Considering the whole atmosphere, the only significant external interaction is with the earth's surface, almost entirely through friction. This general fact has been used as a basis for discussing the general circulation by Jeffreys [3] in a paper to which more reference will be made later. In reality, this essay may be construed as a further extension of the approach to the problem initiated by Jeffreys.

Before embarking upon the examination of details we shall review the simple qualitative picture of the conditions which normally prevail in the atmosphere from the standpoint of the distribution and transfer of absolute angular momentum about the earth's polar axis. In the regions between roughly 30°N and 30°S latitude there are present winds having a component from the east which cover most of this zonal belt. These so-called trade winds are probably the most steady and extensive air motions at the earth's surface. Since they are located in a region where the distance from the axis of the earth is large, the frictional effects at the surface are such as to produce a relatively large eastward (positive) torque upon the atmosphere. We thus have in this region a continuous and intense flow of absolute angular momentum from the earth to the atmosphere. This angular momentum can be removed only through the exertion of a negative torque by the earth upon the atmosphere at latitudes farther to the north and farther to the south mostly through friction at the surface in the regions of the prevailing westerlies. There must thus exist a horizontal flow of absolute angular momentum away from the equator in middle latitudes, diminishing poleward, however, as the surface frictional effects of the westerlies come into play. It is therefore a matter of great importance to study the details of the mechanism whereby this transfer is effected. Although we have made a tacit assumption that a steady average state is present so that no progressive accumulation or deficiency of angular momentum takes place, this limitation does not have to be imposed when relatively short periods of time are under consideration. We may also note at this point that a poleward flow of angular momentum does not necessarily imply a transfer of mechanical energy in the same direction.

In the vicinity of the poles, the north pole especially, there is often present a mass of cold air near the ground in which the wind has a component from the east. If such anticyclonic conditions persist for an appreciable length of time, it should be expected that an equatorward flow of angular momentum would set in from this region to the belt of westerlies. On rare occasions in the northern hemisphere this anticyclonic condition may become so exaggerated that for practical purposes the belt of westerlies vanishes temporarily at the surface. It is to be expected that during periods such as these the normal regime of transfer of absolute angular momentum is profoundly altered.

Since the zonal velocities relative to the earth which are present in actual wind systems are small as compared to the linear eastward velocity of the earth's surface itself, except in the vicinity of the poles, it follows that the distribution of absolute angular momentum in the atmosphere does not differ very much on a percentual basis from that corresponding to solid rotation at the angular speed of the earth. It is therefore apparent that there must normally exist a large gradient of absolute angular momentum northward and southward from the equatorial regions.

Although it is true that the combined system composed of the earth and the atmosphere cannot alter its total absolute angular momentum except for extremely slow secular changes resulting from tidal action as was pointed out by Darwin and others, still there is no reason to expect that the partition of angular momentum in the composite system should remain constant when seasonal and other short time-intervals are considered. Because of the great contrast in the moments of inertia of the two components, short-period anomalies of this kind represent major anomalies in the behavior of the wind systems, but, on the other hand, imply practically undetectible inequalities in the rate of the earth's rotation. From the standpoint of availability of observational material in regard to the motions of the atmosphere, we cannot, at the present time, follow these changes when the atmosphere is considered *in toto*. However, when restriction is made to the northern hemisphere alone, some approach to the problem could be made with existing data, and deductions might in this way result concerning the partition of atmospheric angular momentum between the hemispheres.

In view of the fact that about one-half of the mass of the atmosphere is found already below an elevation of five kilometers above sea level, it is possible to look upon the atmosphere as a two-dimensional film of approximately spherical form in many aspects of our study. This fact combined with the fact that it is not possible to deal with the actual exact values of the atmospheric angular momentum, but rather only with the variations in it and the processes tending to produce them, enables us to discuss the problem in a relatively simple fashion with sufficient accuracy for our purpose. Focusing attention on the portion of the atmosphere north of a particular latitude circle, we may think of its total absolute angular momentum as being the sum of two quantities. The first is the angular momentum due to the air motions relative to the earth's surface, while the second is due to the rotation of the same air about the polar axis with the constant angular speed of the earth itself. The second quantity depends not only upon the total mass of air north of the chosen latitude as expressed essentially by the surface pressure, but also upon the radial distribution of this mass with respect to distance from the polar axis. With the approximation as to the vertical extent of the atmosphere mentioned earlier, this radial distance is expressed by the latitude. Also, we may observe that the second quantity is far larger than the first, although this fact is not of direct concern to us. Rather, it is interesting to note that as far as synoptic and seasonal variations in the total absolute angular momentum are concerned, calculation shows that the contributions from the two quantities are of rougly the same order of magnitude. Furthermore, changes in the mass distribution are of greater significance in this sense, if they occur at relatively low latitudes.

We may next consider the processes which tend to alter the absolute angular momentum of this portion of the atmosphere. In the first place there can exist a horizontal transfer of such angular momentum across the selected latitude. Secondly, the interaction of the air with the earth's surface largely through friction can produce a flow of this angular momentum either from the atmosphere to the earth or *vice versa*. Taking the first of these transfer processes, it will be assumed that the air has negligible viscosity, so that there is no truly frictional interaction across the latitude circle. The actual horizontal transfer may also be thought of as being the result of two effects, one due to the advection of air having a positive or negative angular momentum relative to the earth across the latitude line, and the other due to the advection of angular momentum corresponding to the angular velocity of the earth's rotation. In order to secure a net contribution from the former effect, it is not necessary that there be a net transport of mass. Such a transport of mass is, however, necessary in order to secure a contribution from the latter effect, because the angular momentum per unit mass due to the arth's rotation is constant along the latitude so that a mere exchange of air produces no net result. Except for short-period variations and slow seasonal fluctuations, the latitudinal mass distribution of the atmosphere is constant, so that in all probability the significant north–south transport of absolute angular momentum is accomplished through the exchange of relative angular momentum.

The frictional interaction at the earth's surface is more or less proportional to the square of the surface wind speed, although the value of the constant of proportionality varies greatly with the local character of the surface. The tangential stress produced upon the earth is in the direction of the surface wind, and conversely, the earth's surface exerts a stress upon the air in a direction opposite to that of the surface wind. Corresponding to this stress there exists a flow of absolute angular momentum whose intensity depends upon the magnitude of the eastward component of the stress on the atmosphere and upon the distance from the polar axis. In addition to the frictional interaction, it is possible that at times differences of atmospheric pressure between the eastern and western sides of mountain ranges, especially those of great north–south extent, may produce significant torques upon the atmosphere. Thus the Rockies in North America and the Himalayas in Asia might produce sensible effects of this kind in the northern hemisphere, although, for simplicity in the first instance, we shall not take this phenomenon into account in what follows.

Returning to our principal task, namely the examination of the transfer of absolute angular momentum within the atmosphere, we have seen that this flow is brought about by appropriate exchange processes involving air motions relative to the earth's surface. Thus, if a latitude is selected in the belt of surface westerlies of the northern hemisphere, where normally the transfer is toward the north, the northward-moving individual masses of air should at the same time have a larger eastward component of motion than do the southward-moving ones. In agreement with the conclusion reached by Jeffreys, it seems reasonable to suppose that the necessary positive correlation between northward and eastward velocity components is brought about by the associated upper structures of the cyclones and anticyclones present, which thus form an integral part of the mechanism of the general circulation and constitute the individual eddies that bring about a turbulent transfer of angular momentum poleward on a grand scale. As contrasted with other studies of turbulence, we have in this case rather well-defined information concerning the structure of the eddies themselves, and hence we should be able to obtain a better grasp of the nature of the turbulent process in question.[1] To this end we shall attempt to analyze certain characteristic pictures of flow patterns which resemble those found on synoptic maps and which at the same time exhibit the necessary correlation between the horizontal velocity components.

The general character of the flow patterns observed in middle latitudes a short distance above the surface is a westerly motion on which are superposed troughs in the general vicinity of cyclonic disturbances, so that frequently the appearance of the streamlines at a fixed level resembles that shown in fig. 1. The important

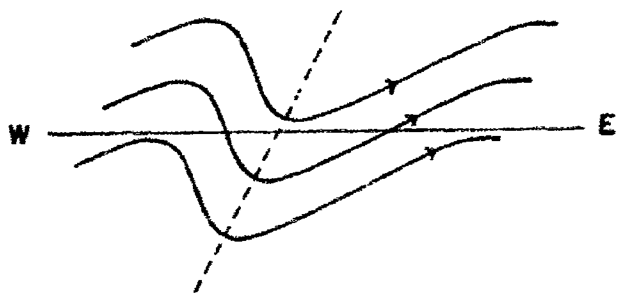

FIG. 1. Schematic picture of horizontal streamlines in a typical middle-latitude flow pattern in the upper westerlies.

feature is the departure from symmetrical sinusoidal form and the associated northeast–southwest tilt of the trough line. It is apparent that this characteristic produces the necessary velocity correlation, since the southward motions are associated with smaller eastward components than are the northward motions. Often the departure from sinusoidal form and tilt of the trough line are small at higher latitudes near the northern border of the surface westerly belt where the northward transfer of angular momentum is small, and become more and more pronounced at lower latitudes where a large transport of angular momentum is normally present. It thus seems that this typical atmospheric flow pattern, so common on meteorological maps, is a necessary automatic adjustment to provide for a poleward transfer of atmospheric angular momentum.

Examination of the rather rare cases when the belt of westerly winds at the surface is weak or absent has left the writer with the impression that many of the troughs present on the upper-level maps of the northern hemisphere for such periods display only small tilt or even a tilt from northwest to southeast. Also, during more normal periods with, however, rather well developed easterly surface winds near the pole, troughs extending to high latitudes seem to possess a reverse tilt north of the belt of surface westerlies as would be dictated by the probable southward flow of angular momentum at these high latitudes during such periods.

If the general scheme outlined above is correct, it becomes a matter of considerable interest to determine at what levels in the atmosphere the bulk of the poleward transport of angular momentum takes place. From preliminary statistical studies of daily hemispherical data it seems that at 45°N latitude the transport during winter is pronounced and almost always directed toward the north at an elevation of 10,000 feet above sea level. Further information must await much more extensive statistical study, although in the

[1] A legitimate question may be raised concerning the use of the word turbulence in the present connection. We shall however continued to use it because of the lack of a better term.

opinion of the writer, this transport is brought about mainly through the action of the large-scale upper troughs and ridges in the main body of the troposphere, at least within the confines of the surface westerly belt in each hemisphere.

Because of the basic importance of the transport process depicted by the streamline pattern sketched in fig. 1, it is instructive to examine it from other points of view than the one which has already been given. If a transport of angular momentum takes place across latitude circles in the general vicinity, it follows that such a transport must also exist across any other fixed curve oriented in a more or less west–east sense but not necessarily extending along a parallel of latitude. It is thus possible to speak of the instantaneous flow of angular momentum across a fixed curve which coincides with one of the streamlines in the figure. This flow is in general due to the torque exerted by the pressure forces acting across the curve, and also to any exchange of air masses of differing angular momentum across the curve. In the present instance no mass exchange takes place at the level under consideration, since the curve is a streamline, so that only the torque produced by pressure forces is operative. In order that a torque be exerted by the air to the south on the air to the north, it is essential that the pressure be greater on the west side of the southward bulge of the streamline than it is on the east side. It then follows that the horizontal streamline cannot at the same time be a line along which the pressure is constant.

Viewing the matter still otherwise, we may consider the instantaneous poleward flow of angular momentum across a fixed curve which coincides with an isobar in the given horizontal surface. Such isobars have more or less the same shape as the streamlines. In this case the pressure forces can produce no net torque, so that the transport must be due to an exchange of air masses of differing angular momentum across the curve. Qualitatively, the relation of the air motions to the isobar shown in fig. 2 could accomplish the transfer. A rela-

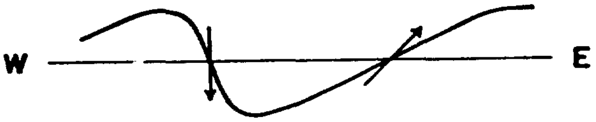

FIG. 2. Schematic picture showing the northward transport of angular momentum across an isobar. Arrows indicate the direction of air motions.

tion of the winds to the isobars similar to this has been noted from observational data by Houghton and Austin [2], and further statistical studies of the matter are at present in progress at the Massachusetts Institute of Technology.

One may ask finally how angular momentum is transported from the easterly belts at low latitudes to the belts of westerlies farther to the north and to the south. The important fact here is that the zone of the trade winds in each hemisphere is not continuous, but together with the equatorward portion of the westerlies forms a few large anticyclonic systems in the subtropics. Again using the northern hemisphere as an illustration, these anticyclonic circulations have horizontal streamlines of approximately the shape shown in fig. 3. It is evident that here also we have the neces-

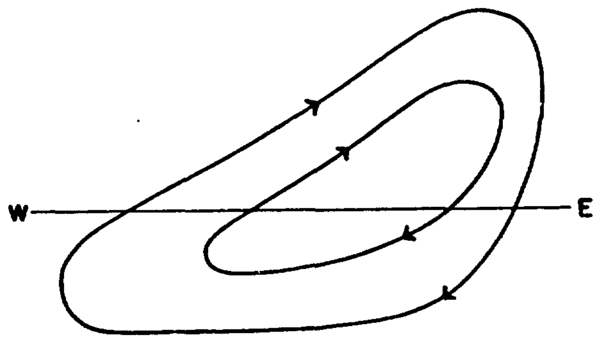

FIG. 3. Schematic picture of horizontal streamlines in a subtropical flow pattern.

sary correlation between the velocity components to accomplish the transfer, and the other facts as regards the pressure distribution in relation to the streamlines must follow a scheme similar to that which has been discussed in the case of the westerlies. We may then say that the separation of the high-pressure belts of the subtropics into individual cells is a necessary automatic adjustment in the atmosphere which provides for a poleward transfer of absolute angular momentum from the low-latitude easterlies.

We come now to the consideration of questions relating to the vertical transport of absolute angular momentum within the atmosphere. In view of the fact that at least a large part of the horizontal transport is probably accomplished at moderate elevations above the surface, it is obvious that there must exist some mechanism whereby angular momentum is communicated upward to these levels in the trade-wind zones, and also there must exist a mechanism for its downward transfer in the vicinity of the surface westerlies. Because the vertical dimension of the atmosphere is so small compared to the horizontal ones, it would be simple to invoke the virtual viscosity of the air due to small-scale turbulence in order to effect these vertical exchanges. On the other hand, but little is known concerning the efficacy of such friction above the surface turbulence layer, so that it is possible that friction in the free air, to the extent that it is present, may work together with other modes of angular momentum transfer in the vertical.

We have observed that in the case of horizontal transfer it is necessary that there exist a certain organization of the north–south air motions in reference to

the absolute angular momentum of the air masses taking part in these motions. Similarly, we might expect to find an organization of the vertical air motions with respect to absolute angular momentum in the case of vertical transfer. We have noted also that in spite of the presence of air motions relative to the earth, there exists a strong poleward gradient of absolute angular momentum in the atmosphere. In the case of the downward transfer within the belts of surface westerlies, generally speaking, the upward branches of vertical circulations should occur closer to the poles than the descending branches of the same circulations. The magnitude of organized vertical velocities in the atmosphere is exceedingly small, so that it is difficult to secure measurements of such motions and consequently our information concerning them is still rather sketchy. We may nevertheless make the indirect inference that relatively vigorous upward motions take place over those regions where active precipitation is observed, since precipitation of sensible intensity requires an adiabatic cooling of the air produced by upward motions of air particles to levels of lower pressure. The areas of precipitation and therefore of ascending motion in middle latitudes are found more or less on the northeastern and the northern sides of the surface cyclones, using the northern hemisphere for purposes of illustration. It is commonly agreed that descending motions are to be found on the southwestern sides of these disturbances, in the relatively cold air which normally sweeps around to the south of the cyclone center. It thus would appear that the typical Bjerknes cyclones [1], as distinguished from the large troughs at upper levels, perform the function of turbulence units in the downward transport of absolute angular momentum from the upper levels in middle latitudes. We should therefore expect that the frequency of occurrence of Bjerknes cyclones should be greater on the eastern sides of the large troughs at upper levels than on the western sides, because these are the regions where vigorous northward transport of angular momentum takes place. This condition is common on meteorological maps.

The meteorological processes which take place in the trade-wind belts have not as yet been subjected to the same detailed scrutiny as those of middle latitudes. We are therefore at a disadvantage in attempting to trace the course of the upward flow of absolute angular momentum in these regions. It has nevertheless been observed that certain synoptic disturbances called easterly waves are present on the equatorward sides of the subtropical high-pressure cells. It is not inconceivable that these waves are accompanied by organized vertical motions of the type necessary to provide for an upward flux of angular momentum. The probability is, however, that the rather sporadic occurrence of tropical hurricanes in these regions cannot account for the normal upward transfer. Although the normal poleward gradient of absolute angular momentum is small in the trade-wind region, still its presence would favor the location of the upward branches of vertical circulations nearer to the equator than the descending branches. Such a distribution of vertical motions is not in conflict with the general climatological characteristics of the tropics and subtropics as indicated by the meridional distribution of rainfall within the subtropical high-pressure cells, the precipitation being more abundant nearer to the equator. On the present hypothesis the general selective effects of the phenomena of middle latitudes and of the trade-wind regions upon the location of descending motions evidently conspire to produce relatively arid zones in the intervening belts.

The reader has doubtless observed many gaps and shortcomings in the rough picture of the atmospheric motions which has been sketched, perhaps necessarily with a rather broad stroke. For example, questions relating to the sources and transfer of energy have not been touched upon, nor have the associated heat-transfer processes been treated. Likewise it is not immediately clear what role is played by frontal discontinuities in the scheme, and the characteristic phenomena of the atmospheric tropopause have not been related to the mechanics of the system. Nevertheless it is a matter of interest that a few pieces of the puzzle which the general circulation presents can be made to fit together, although the problem of why these pieces have the precise shapes they do is a far more profound and difficult subject. Efforts to deal with this latter question have recently been made by Rossby [4] and others [5].

Analytical representations of the concepts introduced have not been given, since it is felt that the first step in the treatment of a subject such as the present one is the formulation of a physical picture. Furthermore, the reader who is mathematically inclined can easily supply such representations where they are of obvious application.

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A STUDY OF THE FLOW OF ANGULAR MOMENTUM IN THE ATMOSPHERE

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ABSTRACT

A study is made of the flow of angular momentum in the atmosphere for the month of January 1946. The results generally confirm the pattern proposed by Starr on theoretical grounds. Angular momentum is transferred from the earth to the atmosphere in regions of surface easterly winds (chiefly the tropical and subtropical easterlies), transported upward, then horizontally poleward, and finally downward, being removed in regions of surface westerly winds. The torques due to surface friction are found to be of the same order of magnitude as those due to differentials of pressure across mountain ranges. During a period of the length of this study or less, it is found that the change and transport of angular momentum due to shifts of mass are of the same order of magnitude as the change and transport of relative angular momentum. If one accepts the method used for estimating the surface torques, there appears to be an excess of transfer of angular momentum to the atmosphere in the northern hemisphere. From a study of the normal January pressure profile, it would appear that this excess represents a flow of angular momentum to the southern hemisphere, where it is needed to balance accounts.

1. Introduction

One of the earlier mentions of the importance of angular momentum in any consideration of the general circulation was by Jeffreys [7]. In this paper, Jeffreys shows that, by considering the amount of angular momentum transported across a given latitude circle and the net loss of angular momentum by frictional torque north of this latitude, it is impossible to have a zonally symmetric distribution of wind and pressure if surface friction is present. He concluded that there must exist large-scale air streams extending through a major part of the troposphere with a strong meridional component of motion (of the same order of magnitude as the zonal component) and that the cyclones and anticyclones are a necessary part of the general circulation rather than being merely oscillations about a possible zonally symmetric steady state. It should be remembered that these conclusions were reached despite the absence of the extensive amount of upper-air data which is now available. Jeffreys' conclusions were discussed at some length in further papers by himself and others [3; 4; 8; 13], but without significant change in the conclusions presented above.

The importance of the angular-momentum concept in studies of the general circulation has been re-emphasized in a recent paper by Starr [11]. In his paper it is pointed out that, inasmuch as the earth and atmosphere may be considered as practically an isolated system, there must be a flow of angular momentum from the earth to the atmosphere in regions of surface easterly winds (the most important of these regions being those of the tropical easterlies or "trade" winds) and a reverse flow in regions of surface westerly winds (particularly in the prevailing westerlies of the temperate zones). There must then exist, on the average, a poleward flow of angular momentum. There must also exist an upward transport of angular momentum over the easterlies and a downward transport over the westerlies.

As over long periods there is no progressive net change in the distribution of atmospheric mass over the earth, the significant long-term meridional transport of angular momentum is accomplished by the meridional interchange of air masses with differing relative angular momentum. Starr has suggested that this interchange is effected principally by the upper-air trough and ridge systems with axes tilted from northeast to southwest. The transfer of angular momentum between the earth and the atmosphere[3] is effected by surface friction and by differentials of atmospheric pressure across mountain ranges.

The importance of considering the flow of angular momentum in any study of the general circulation should be obvious. Although even a complete knowledge of the angular-momentum transfer in the atmosphere cannot by itself furnish a solution of the problem of the general circulation, any proposed scheme for this circulation should include a means for securing the angular-momentum flows which are observed. For this reason, it is essential that all possible

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[3] Hereafter often referred to as the generation and removal (or similar terms) of angular momentum in the atmosphere.

observational knowledge concerning these processes be obtained. It is hoped that the study reported here represents a beginning in the accomplishment of such an aim.

At the suggestion of Prof. Starr it was decided to investigate quantitatively and as extensively as possible the generation and transport of angular momentum in the atmosphere for a period of one month. It was planned that this investigation should serve as a pilot project for similar later studies covering more extensive periods. The results reported here should therefore not be considered as final until confirmed by further studies.

2. Theoretical considerations

Since the force of gravity can exert no torque about the earth's axis, we may write the equation of zonal motion without approximation in the form

$$dM/dt = -\rho^{-1}r(\partial p/\partial x - D_u), \tag{1}$$

which states that the absolute angular momentum of an individual unit mass of air increases at a rate equal to the external torques exerted upon it by the pressure force and by friction. (See the table of symbols at the end of this article.)

Equation (1) may be rewritten with the aid of the following considerations:

$$\begin{aligned}\rho\, dM/dt &= \rho\, \partial M/\partial t + \rho c \cdot \nabla M \\ &= \frac{\partial \rho M}{\partial t} + \nabla \cdot \rho M c - M\left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho c\right),\end{aligned} \tag{2}$$

where c is the absolute vector particle velocity. The quantity in parentheses in (2) vanishes identically because the general equation of continuity of mass states that this expression is zero. Accordingly (1) becomes

$$\frac{\partial \rho M}{\partial t} = -\nabla \cdot \rho M c - \frac{\partial p r}{\partial x} + r D_u, \tag{3}$$

still without approximation. This equation states that the rate at which absolute angular momentum is increasing at a point fixed in space is equal to the negative divergence (convergence) of the transport of angular momentum plus the rate at which it is imparted to the air at the point by the external torques due to pressure and friction.

Noting that ρM is the angular momentum per unit volume, we may form a volume integral of (3) over a portion of the atmosphere bounded by the earth's surface and extending from the pole to a given latitude ϕ. The upper boundary is assumed to be the "top" of the atmosphere. The term on the left-hand side gives an integral which is the rate of increase of the total angular momentum within the region, while all the terms on the right give integrals which are expressible in terms of quantities measured at the boundaries of the space. Let us designate an element of volume by dV, and an element of the surface of this volume by dS. Further, let $d\sigma$ represent the projection of dS on the meridional plane passing through a given point and τ_x the eastward frictional stress acting at the boundary on the air inside. We may then write

$$\frac{\partial}{\partial t}\int \rho M\, dV = \int \rho M c_n\, dS + \int p r\, d\sigma + \int r \tau_x\, dS, \tag{4}$$

which is still rigorous and could have been written immediately from physical considerations.

In the first term on the right c_n is the inward component of the velocity and the quantity as a whole represents the rate at which angular momentum is brought into the region by air motions across the boundary. The only significant air motions across the boundary take place at the vertical surface at latitude ϕ. For all practical purposes we may assume that $r = R\cos\phi$, in all the discussion which follows, so that

$$M = uR\cos\phi + \omega R^2 \cos^2\phi. \tag{5}$$

At this southern boundary $c_n = v$, and we may without sensible error say that $dS = dx\, dz$. The first term on the right side of (4) may then be written as the sum of two terms, namely,

$$R\cos\phi \iint \rho u v\, dx\, dz, \tag{6}$$

expressing the rate of the northward advection of *relative* angular momentum, and the term

$$\omega R^2 \cos^2\phi \iint \rho v\, dx\, dz, \tag{7}$$

expressing the rate of northward advection of angular momentum due to the earth's rotation (hereafter referred to as ***ω-angular momentum***). We note that the expression (7) cannot contribute unless there is a net flow of mass across the latitude circle.

The second integral on the right of (4) can give a contribution only because the lower boundary is not a smooth spherical one, but has imperfections in the form of mountain ranges at whose sides $d\sigma$ does not vanish. The measurement of this effect is discussed below.

The third term on the right side of (4) may be written in the form given because friction represents a mode of exchange of momentum and can give a net contribution only when the frictional interaction is present with the surroundings at the boundary of the region. The main effect of this nature results from the interaction at the earth's surface north of the latitude ϕ. Small-scale eddy friction appears to be far too small to contribute significantly to the flow of angular

momentum across the vertical at latitude ϕ. We are thus left with the quantities (6) and (7) to account for the large meridional transfer of angular momentum for the maintenance of the general circulation.

In the long-run average the angular momentum of the atmosphere is constant and the left-hand side of (4) is zero. For shorter periods, however, this is not the case. By using (5) it follows that

$$\frac{\partial}{\partial t}\int \rho M\, dV = \frac{\partial}{\partial t}\int \rho R u \cos\phi\, dV + \omega R^2 \frac{\partial}{\partial t}\int \rho \cos^2\phi\, dV. \quad (8)$$

Here the first term on the right is the rate of increase of relative angular momentum, while the second term is the rate of increase of the angular momentum due to the earth's rotation. This latter quantity (ω-angular momentum) can be changed only by net shifts of mass from one latitude belt to another. Such shifts of mass are measured in terms of surface pressure changes.

By use of (5), (6), (7), and (8), (4) may now be written as

$$\frac{\partial}{\partial t}\int \rho R u \cos\phi\, dV + \omega R^2 \frac{\partial}{\partial t}\int \rho \cos^2\phi\, dV$$
$$= R\cos\phi \int\!\!\int \rho u v\, dx\, dz + \omega R^2 \cos^2\phi \int\!\!\int \rho v\, dx\, dz + \int p r\, d\sigma + \int r \tau_x\, dS. \quad (9)$$

This investigation consists of an attempt to evaluate the six terms in (9) from actual atmospheric data.

3. General procedure

The month of January 1946 was chosen for this investigation, mainly because of the availability of the necessary data for this month and the fact that a general inspection of the maps for this period did not reveal any too outstanding abnormalities. It was found later that this month has a somewhat higher than normal zonal index (surface westerlies, 700-mb westerlies, and surface subtropical easterlies). What relation this may have to the results reported here must await investigation of other periods with differing indices.

The data for sea level and the 500-mb level were obtained from the Northern Hemisphere Historical Weather Maps [1]. The 700-mb level data were obtained from photostats of northern-hemisphere charts analyzed by the U. S. Air Force. On several days, only data for the western half of the northern hemisphere were available at 700 mb. There was one map per day at each level: 0400 GCT at 700 mb and 500 mb, 1230 GCT at sea level. In this study, the difference between the time of the maps at sea level and that at higher levels was neglected. In general, complete data were available from 80°N to 30°N at the upper levels and to 10°N at sea level. Computations for the upper levels south of 30°N were estimated from the available but incomplete data there.

Throughout this study, it was necessary to assume that the actual wind was sufficiently well approximated by the geostrophic wind, due to the lack of adequate actual wind data. Machta [9] has studied the validity of this assumption as it applies to the transport of relative angular momentum through computations based on a theoretical model of a trough with its axis tilted with respect to the meridians. This model suggests that fair agreement might be expected between the geostrophic and actual transport of relative angular momentum, aside from the effect of meridional circulations.

Lorenz[4] has also studied the validity of the assumption, using geostrophic deviation data gathered by Machta;[5] unfortunately but necessarily these data were limited to the United States. His results indicate that the geostrophic transport of relative angular momentum is of the same order of magnitude as the actual transport, but the geostrophic assumption gives transports away from the equator which are somewhat less than the actual transport.

Due to the fact that it was necessary to use geostrophic winds and that (as will be discussed later) the density was taken as constant for each level and latitude, it was not possible to compute directly the net transport of mass (and therefore of ω-angular momentum) across latitude circles.[6] The method of computation actually used for these quantities will be discussed later.

The surface frictional torque was computed from the sea level geostrophic wind, assuming the surface wind to be in the same direction as and 0.6 as great as the sea-level geostrophic wind. The transport and change of relative angular momentum as computed from the sea-level geostrophic winds were assumed to be representative of conditions at the geostrophic wind level. The geostrophic mean zonal winds were assumed equal to the actual mean zonal winds.

Due to the fact that neither the transport of relative angular momentum nor the surface frictional torque are linear functions of wind velocity, it was necessary to compute these quantities individually

[4] E. Lorenz, "Investigation of the general circulation of the atmosphere," Report no. 2, Contract W28–099 ac-406, between Watson Laboratories, AMC, and the Massachusetts Institute of Technology, 1948.

[5] L. Machta, "A study of the observed deviations from the geostrophic wind," Unpublished Sc.D. thesis, Massachusetts Institute of Technology, 1948 (and additional data not included in thesis).

[6] The use of the geostrophic wind assumption leads to a zero net transport of mass; *i.e.*, any contributions from circulation cells in a meridional cross section representing the mean condition around the earth are automatically neglected.

point by point and day by day, and then to sum or average as needed. Other quantities (change in relative angular momentum, transport and change of ω-angular momentum, torque due to mountains) are linear functions and could be calculated from pressure profiles or (in the case of the torque due to mountains) mean maps.

The winds were calculated on the basis of pressure (or contour height) data recorded for each 5 degrees of latitude and longitude within the limits of the analysis. The wind components at a given latitude and longitude were obtained from the pressures (contour heights) 5 degrees north and south (or east and west) of the given point. The wind components at each point were assumed to be representative within longitudes $2\frac{1}{2}$ degrees east and west of the point in computing the transport of relative angular momentum; for computing the surface frictional torque, they were assumed to be representative of the area $2\frac{1}{2}$ degrees east and west, and 5 degrees north and south, of the point.

The densities at each latitude and level were taken as the normal for January at that latitude and level. The 700-mb densities were computed from the normal contour and temperature map.[7] The 500-mb densities were extrapolated (assuming a moist adiabatic lapse rate) from the normal 20,000-ft pressure and temperature map [12]. The surface densities were computed on the basis of normal surface temperatures given by Haurwitz and Austin [5]; they were extrapolated to 900 mb to obtain the densities at the geostrophic wind level. The use of the normal, constant density introduces two sources of error: first, from the difference between the normal January densities and the mean January 1946 densities; second, from the fact that a correlation between density and wind direction would be expected. A preliminary investigation of the second point has indicated that it produces an error in the relative angular-momentum transport whose magnitude is of the order of 10 per cent.

4. Transport of relative angular momentum

This is the determination of the term,

$$R \cos \phi \int\int \rho uv \, dx \, dz,$$

in (9). In actual practice, the quantity

$$\int_0^{2\pi} \rho r^2 uv \, d\lambda, \qquad (10)$$

which gives the transport of relative angular momentum per unit time and per unit height across a given latitude, was determined at certain latitudes and levels.

For a constant-pressure level, under the procedure set forth above, the transport is given by

$$-\frac{9\rho g^2 \cos \phi}{4\pi\omega^2 \sin^2 \phi} \Sigma(\Delta z)_y (\Delta z)_x, \qquad (11)$$

where the summation is made for 72 points, each separated by 5 degrees of longitude, completely around the parallel of latitude. For a constant-level surface, this transport is equivalent to

$$-\frac{9 \cos \phi}{4\pi\omega^2 \rho \sin^2 \phi} \Sigma(\Delta p)_y (\Delta p)_x. \qquad (12)$$

In integrating the transport (and also the change) of relative angular momentum through height, it has been assumed that the transport at the geostrophic wind level is representative from the surface (assumed to be at sea level) to 1.5 km; that at 700 mb, from 1.5 km to 4.5 km; and that at 500 mb, from 4.5 km to 7.5 km. In the absence of data above 500 mb, no attempt was made to estimate the vertical distribution of the transport and change of relative angular momentum above 7.5 km. In integrating through time, it has been assumed that a quantity computed from a map is representative of the period from 12 hours before to 12 hours after map time.

Fig. 1 shows the net total transport of relative angular momentum in the three layers from map time of 1 January 1946 to map time of 31 January 1946.

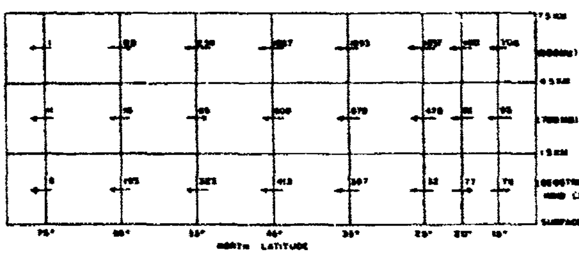

FIG. 1. Net amounts of relative angular momentum (in units of 10^{29} g cm^2 sec^{-1}) transported horizontally by geostrophic motion across entire latitude circles during January 1946 for the indicated horizontal layers.

It will be noted that there is generally a poleward transport of relative angular momentum and that the transport generally increases with height south of 50°N. The transport increases with increasing latitude up to 35°N, presumably due to the addition of angular momentum to the atmosphere in the zone of surface easterly winds. It decreases north of 35°N, presumably due to the removal of angular momentum in the surface westerlies. There is some evidence, particularly in the highest layer shown, at 65°N, of a comparatively minor flow of angular momentum generated in the polar easterlies, southward to the zone of the surface westerlies.

[7] U. S. Weather Bureau, "Normal 700-mb charts," Extended Forecasting Section, Washington, D. C. (unpublished photostats).

5. Change of relative angular momentum

The relative angular momentum of a given horizontal layer of air of unit thickness between two latitudes is

$$\int_0^{2\pi} \int_{\phi_1}^{\phi_2} \rho u r^2 R \, d\phi \, d\lambda. \tag{13}$$

For a constant-pressure level, this becomes

$$-\omega^{-1} 18 R^3 \rho g \overline{(\Delta z)}_y \left[\cos\phi + \ln\tan\tfrac{1}{2}\phi\right]_{\phi_1}^{\phi_2}, \tag{14}$$

and for a constant-height level

$$-\omega^{-1} 18 R^3 \overline{(\Delta p)}_y \left[\cos\phi + \ln\tan\tfrac{1}{2}\phi\right]_{\phi_1}^{\phi_2}. \tag{15}$$

In this study, it is chiefly the change in relative angular momentum over a period of time that is of interest. This corresponds to the term,

$$\frac{\partial}{\partial t} \int \rho R u \cos\phi \, dV,$$

in (9). The changes in the relative angular momentum within the three layers from the first to the last day of January 1946 are given in table 1. In general there

TABLE 1. Changes in relative angular momentum from map time of 1 January 1946, to map time of 31 January 1946 (in units of 10^{29} g cm^2 sec^{-1}).

Latitude belt	Levels 0–1.5 km	1.5–4.5 km	4.5–7.5 km
75–65	− 10	+ 16	− 7
65–55	+ 4	+ 38	+ 28
55–45	+101	+107	+174
45–35	− 38	+146	+ 10
35–25	−186	−331	−203
25–20	−123	− 9	0
20–15	− 78	−194	+112

was, during the month, an increase of relative angular momentum in the temperate westerlies and a decrease in the zone south of 35°N.

6. Change and transport of ω-angular momentum

This is the determination of the terms

$$\omega R^2 \frac{\partial}{\partial t} \int \rho \cos^2\phi \, dV,$$

and

$$\omega R^2 \cos^2\phi \int\int \rho v \, dx \, dz,$$

in (9) (the integrated effect from 1 January to 31 January 1946 being considered here). The first is the change of ω-angular momentum and is computed through computing the changes of mass within certain latitude belts in the three layers: surface to 10,000 ft, 10,000 ft to 18,000 ft, and 18,000 ft to infinity.[8] The second is the transport of ω-angular momentum which can be computed directly from the transport of mass. However, as was noted earlier, the use of the geostrophic assumption prevents any direct computation of the mass transport. The transport of mass has therefore been computed from the changes of mass and continuity considerations, starting at the north pole and assuming no net vertical transport of mass between layers within a latitude belt,[9] inasmuch as all net transport of mass into a polar cap must take place through the latitude circle at its southern boundary.

The change in mass within a latitude belt is computed from the changes in the pressures within that belt and is converted to change in ω-angular momentum by multiplying by

$$\frac{1}{\Delta\phi} \int_{\phi_1}^{\phi_2} \omega r^2 \, d\phi = (R^2\omega/\Delta\phi)\left[\tfrac{1}{2}\phi + \tfrac{1}{4}\sin 2\phi\right]_{\phi_1}^{\phi_2}. \tag{16}$$

The transport of mass across a latitude circle is converted to transport of ω-angular momentum by multiplying by $r^2\omega$. Fig. 2 illustrates the net change and transport of ω-angular momentum from map time of 1 January 1946, to map time of 31 January 1946.

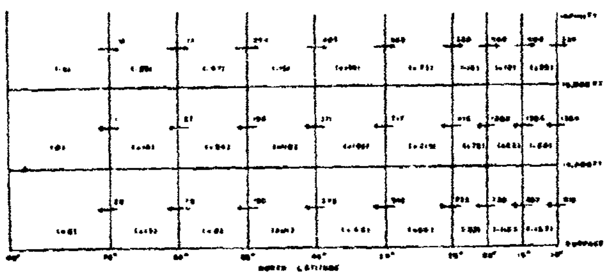

FIG. 2. Net horizontal transport across entire latitude circles during January 1946 (arrows) and change (figures centered in blocks in parentheses) of angular momentum due to the earth's rotation (ω-angular momentum) from map time 1 January 1946 to map time 31 January 1946, assuming no net vertical transport of mass (in units of 10^{29} g cm^2 sec^{-1}).

7. Surface frictional torque

The surface frictional torque corresponds to the term

$$\int r \tau_x \, dS,$$

in (9); its contribution per unit time within a latitude belt is [2]

$$\int_0^{2\pi} \int_{\phi_1}^{\phi_2} \kappa \rho u (u^2 + v^2)^{\frac{1}{2}} r^2 R \, d\phi \, d\lambda. \tag{17}$$

[8] The 700- and 500-mb contour heights were converted to pressures at 10,000 ft and 18,000 ft respectively.

[9] This assumption of no vertical transport of mass is admittedly questionable, but it is necessary to obtain any estimate of the vertical distribution of the transport of ω-angular momentum. It does not affect the total of the transport from the surface to infinity; the vertical distribution of this transport is in error to the extent that this assumption is in error.

Under the previously stated procedure, this is equivalent to

$$\frac{9\kappa R}{4\omega^2\pi\rho}[\phi + \cot\phi]_{\phi_1}^{\phi_2} \Sigma(\Delta p)_y \left[(\Delta p)_y^2 + \frac{(\Delta p)_x^2}{\cos^2\phi} \right]^{\frac{1}{2}}. \quad (18)$$

The determination of the surface frictional torque is perhaps the most questionable of any of the procedures used in this study. To begin with, the entire subject of the stresses exerted by a moving fluid on its boundary has not been satisfactorily determined and the relation used in this study (essentially that the force is equal to $\kappa\rho c^2$, where c is here the relative speed of the fluid) is not necessarily the best one. The value of κ, the coefficient of skin friction, varies with the type and topography of the surface, probably with wind speed, and very probably with other factors. It is hoped that the value used here (0.003) is a reasonable approximation of a satisfactory mean value. In addition, the values of u and v in the computation have been taken as 0.6 of the sea-level geostrophic wind with no correction for the commonly observed change in direction between the surface and the geostrophic wind level.

In both the surface frictional torque and the torques due to differentials of pressure across mountain ranges, the sign convention has been chosen so that a minus sign indicates transfer of angular momentum from the earth to the atmosphere and vice versa. This arbitrary convention was so chosen because the sign of the surface frictional torque is then the same as the sign of the eastward component of the surface wind velocity.

The values of the surface frictional torques within certain latitude belts in the period from 1 January 1946 to 31 January 1946, are given in table 2.

TABLE 2. Integrated effect of surface frictional torque from map time of 1 January 1946 to map time of 31 January 1946 (in units of 10^{23} g cm^2 sec^{-1}).

Latitude belt	Torque
80–75	− 86
75–70	− 158
70–65	− 122
65–60	− 188
60–55	+ 606
55–50	+ 875
50–45	+ 1251
45–40	+ 1842
40–35	+ 138
35–30	+ 205
30–25	− 1530
25–20	− 2470
20–15	− 4770
15–10	−10190

8. Torques due to differentials of pressure across mountain ranges

This corresponds to the term

$$\int pr\, d\sigma$$

in (9) and was computed by White [14]. The reader is referred to his report elsewhere in this issue for a discussion of the procedure used. The time-integrated values of the mountain torques are given in table 3.

TABLE 3. Integrated effect of torques due to differentials of pressure across mountain ranges from map time of 1 January 1946 to map time of 31 January 1946 (in units of 10^{23} g cm^2 sec^{-1}).

Latitude belt	Torque
65–60	− 411
60–55	− 221
55–50	+ 191
50–45	+ 870
45–40	+1550
40–35	+ 965
35–30	+ 194
30–25	− 681

The procedures used in all the calculations given above have many sources of error. In addition to those mentioned previously, perhaps one of the most serious is the smoothing of the pressure (and contour height) patterns, both in the original analyses and in the subsequent manipulations. It is felt that, in general, the values computed are correct at least as to direction and order of magnitude. It is probably not possible to improve the accuracy of the procedure in any major degree with the data available at the present time.

9. Conclusions

This study has confirmed the picture of the generation and transport of angular momentum which was proposed by Starr from theoretical considerations. This can be seen by reference to the various tables and diagrams which have been previously mentioned. Angular momentum is generated in the subtropical easterlies, is transported northward, and is lost to the earth in the prevailing westerlies. The polar easterlies act as a secondary, but rather minor, source of angular momentum. Inasmuch as the transport of ω-angular momentum can have no progressive net effect over long periods of time, fig. 1 perhaps represents, in a very general way, the long term horizontal transport of angular momentum in the atmosphere.

Starr stated without detailed discussion that the interchange of angular momentum between the earth and the atmosphere might be effected by the differentials of pressure across mountain ranges as well as by surface friction. This has turned out to be the actual existing condition (*cf.*, tables 2 and 3); the two processes are found to be of the same order of magnitude. Furthermore, for the month as a whole, the two effects have generally the same direction at the same latitude. White has found that the torque due to the mountains during January 1946 closely approximates the normal condition for the month of January.

Although over long periods of time, there can be no progressive net change or transport of ω-angular

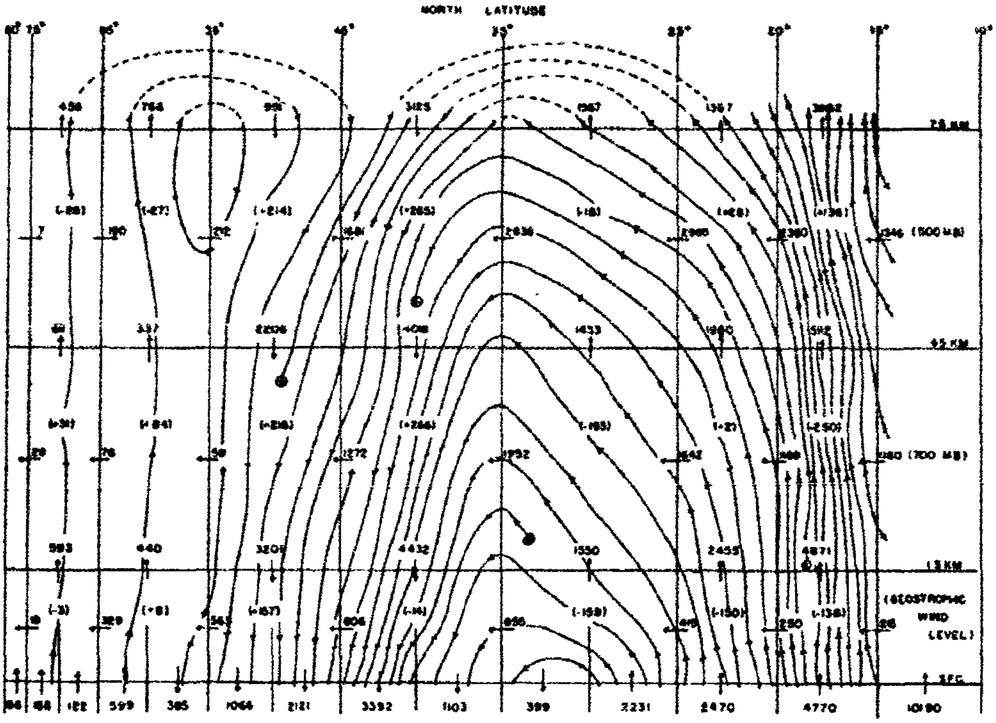

FIG. 3. Net total generation and transport of absolute angular momentum from map time 1 January 1946 to map time 31 January 1946 (in units of 10^{23} g cm^2 sec^{-1}). Small arrows indicate total flow of angular momentum between adjacent vertical or horizontal boundaries. Values in parentheses indicate total change of angular momentum within block. Each streamline indicates net flow of approximately 500 $\times$ 10^{23} g cm^2 sec^{-1} of angular momentum. Dotted streamlines indicate flow of relative angular momentum at undetermined heights above 7.5 km.

momentum, it has been found that during January 1946 the ω-angular momentum and the relative angular momentum terms were in general of the same order of magnitude, considering both individual days and the entire month. On some days, the ω-angular momentum term was larger than the relative angular momentum term. It would be expected that there should be a normal change and transport of ω-angular momentum for the month of January; this could be determined if normal maps for January 1 and January 31 were available. How closely January 1946 resembles this normal is not known.[10]

It is extremely desirable to construct an integrated picture of the contributions from all the terms of the angular momentum equation (9). In constructing this picture, it was found advisable to take account of the fact, previously mentioned, that the actual poleward transport of relative angular momentum is somewhat greater than the geostrophic transport. It has been assumed for this purpose, somewhat arbitrarily, that the actual transport is 1.5 times the geostrophic transport.

Such a picture is given in fig. 3, which illustrates the total generation and transport of absolute angular momentum during January 1946. All transport of ω-angular momentum is shown as occurring below 7.5 km; therefore, all the transport above this level (the vertical distribution of which is at present indeterminate) is in the form of relative angular momentum. The vertical transport of angular momentum was obtained from continuity considerations, beginning with the lowest layer.

It is possible, from this figure, to present the picture envisioned by Starr in somewhat greater detail. Not only does the horizontal transport of angular momentum increase in general up to the limit of the data, but, furthermore, apparently about one-half as much angular momentum is transported northward across 35°N above 7.5 km as below. The southward transport of the angular momentum generated in the polar easterlies would appear to occur chiefly above 7.5 km. The vertical transport of the angular momentum is concentrated mainly over the regions of generation and loss.

Inspection of fig. 3 reveals that apparently considerably more angular momentum was generated than can be accounted for by loss in the westerlies and increases in the angular momentum of the atmosphere (even if reasonably large increases are assumed to occur above 7.5 km). This excess of generation is even more pronounced when it is realized that the region between 10°N and the equator (not included on the

[10] Later preliminary investigations using normal maps for December, January, and February have indicated that the change and transport of ω-angular momentum during January 1946 was considerably larger than that which is normal for January.

figure due to lack of data) can also be assumed to be a region of generation. It would appear that the only area available to act as a sink for this excess momentum would be the southern hemisphere. A semiquantitative investigation of the plausibility of this assumption was made, using a mean January surface pressure profile for the years 1910-1934 extending from 70°N to 60°S. (This profile was prepared by the Extended Forecasting Project at Massachusetts Institute of Technology from data in [10].) The surface frictional force was approximated from the mean zonal wind.[11] As no estimate was possible of the torque due to the mountains in the southern hemisphere, the effect of the mountains was neglected throughout.

Although the results obtained probably have only a qualitative validity, they do appear to indicate that more angular momentum is generated in the northern hemisphere, particularly in the tropical and subtropical latitudes, than is dissipated in the northern-hemisphere temperate latitudes. The reverse is true in the southern hemisphere, due chiefly to the great surface intensity of the temperate latitude circumpolar vortex. There would appear to be, then, in the mean, an appreciable net transport of angular momentum from the northern to the southern hemisphere during January. In fact, a qualitative inspection of similar profiles for other seasons, indicates that while in some seasons both hemispheres may be substantially self-sufficient as to angular momentum, at no time, in the mean, would there apparently be an appreciable net transport of angular momentum from the southern to the northern hemisphere while the reverse might apparently often be the case. Investigations by White of the normal torque due to the mountains in the northern hemisphere indicate that this factor, although changing the magnitude of the excesses and deficits, does not eliminate the excess of angular momentum generated in the northern hemisphere.

The mechanism of the transport of angular momentum across the equator is far from apparent, and it will probably require an extensive study of equatorial air currents, based on actual rather than geostrophic wind data at all levels, even to begin to understand it. The above results would, nevertheless, seem to lend support to the opinions of those who believe that no final solution of the general circulation problem can be reached without considering the joint interaction of both hemispheres. Some further studies of these points and the interesting possibilities arising from them are now under way at the Massachusetts Institute of Technology.

The interchange of angular momentum between the earth proper and the atmosphere may, at times, produce a considerable net shift of angular momentum from one member of the system to the other. Due to the great difference in the masses of the two, relatively large changes in the atmospheric angular momentum would be expected to produce only small variations in the rate of rotation of the earth. To determine the order of magnitude of this change, it was decided to determine what the effect would be if the atmosphere were to lose all its relative angular momentum to the earth and the two were to rotate as a solid. The relative angular momentum of the earth proper was estimated from data on its surface density and mass [6], assuming a linear increase in density from the surface to the center. The relative and ω-angular momentum of the atmosphere was estimated from the January 1946 data, assuming the southern hemisphere to be a mirror image of the northern hemisphere.[12] It was found that even this change in the angular momentum distribution, extreme as it is, would decrease the length of a year (*i.e.*, 365 revolutions) by only 0.8 sec. It is understood that this change would be just noticeable from astronomical observations if it were to persist for an entire year. It would therefore appear that there is little hope of determining short period variations in the atmospheric angular momentum from changes in the speed of the earth's rotation.

It is interesting for purposes of comparison to compute the mean relative angular momentum of a major part of the atmosphere of the northern hemisphere. The average value of this quantity for the month of January 1946, for the portion of the atmosphere bounded by latitudes 35°N and 75°N, the surface and 7.5 km, is 2400×10^{29} g cm^2 sec^{-1}. The amount of angular momentum removed from within these boundaries, due to the surface frictional torque and the torque due to the mountains, during the thirty days under consideration was 7188×10^{29} g cm^2 sec^{-1}. It is clear that if no angular momentum were transported through the boundaries except at the surface and if the values of the torques were to remain constant in spite of the resultant decrease in the motion, the atmosphere within this region would cease to have any net relative angular momentum after approximately ten days. The necessity for a continual poleward transport to maintain the normally observed circulation is apparent.

Acknowledgments.—The writer is indebted to Prof. V. P. Starr of the Massachusetts Institute of Technology for originally suggesting this investigation and for his help and encouragement. Dr. E. Lorenz and Mr. R. White, of the General Circulation Project at the Massachusetts Institute of Technology have

[11] The surface frictional force correlates with the mean zonal surface wind (around a latitude belt) to give a coefficient of +0.81 or better, on a daily basis.

[12] Angular momentum of solid earth = $52.4 \times 10^{[illegible]}$ g cm^2 sec^{-1}. Relative angular momentum of the atmosphere = 12.82×10^{32} g cm^2 sec^{-1}. ω-angular momentum of the atmosphere = 1.021×10^{36} g cm^2 sec^{-1}.

given continual assistance as has been indicated, in part, above.

Various data were obtained through the courtesy of Dr. H. Wexler and Mr. J. Namias of the United States Weather Bureau and Dr. H. C. Willett of the Massachusetts Institute of Technology.

Credit is due for the contribution of the writer's wife, in undertaking the tedious task of recording data read from various maps.

Table of Symbols

R = mean radius of the earth
r = distance from the earth's axis
ϕ = latitude
λ = longitude
u = eastward component of the wind velocity (along a parallel of latitude)
v = northward component of wind velocity
g = acceleration of gravity
ρ = density
ω = angular speed of rotation of the earth
x = linear distance in eastward direction (along a parallel of latitude)
z = linear distance along the vertical
M = absolute angular momentum per unit mass
D_u = eastward component of frictional force per unit volume
p = pressure
t = time
$(\Delta z)_y$ = difference in contour height across a 10-degree interval northward
$(\Delta z)_x$ = difference in contour height across a 10-degree interval eastward
$(\Delta p)_y$ = difference in pressure across a 10-degree interval northward
$(\Delta p)_x$ = difference in pressure across a 10-degree interval eastward
κ = coefficient of skin friction (assumed 0.003)
$\overline{(\Delta p)}_y$ = difference in mean pressure between two given latitudes
$\overline{(\Delta z)}_y$ = difference in the mean contour height between two given latitudes.

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SHORTER CONTRIBUTIONS

THE ROLE OF MOUNTAINS IN THE ANGULAR-MOMENTUM BALANCE OF THE ATMOSPHERE

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(Manuscript received 4 February 1949)

There exists an extensive literature on the relation of mountains to meteorological phenomena which emphasizes the important role that these stable natural barriers play in meteorology. Many of the studies deal with the thermodynamical effects on air flowing across these barriers, while others attempt to evaluate the hydrodynamical effects produced. Starr [4] recently has pointed out the importance of mountain ranges in the angular-momentum balance of the atmosphere. The mountain effect discussed here deals only with this role.

The basis of Starr's proposal is that the flow of absolute angular momentum in the atmosphere constitutes an important feature of the general circulation, and hence any complete theory of the general circulation should explain the observed exchanges of angular momentum. Since the atmosphere and earth can be considered a closed system, the only way in which the atmosphere can gain or lose angular momentum is by interaction with the earth's surface. Interaction with the earth's surface may occur in two ways; either through surface frictional interaction, or through interaction due to the torques exerted by mountain ranges.

The surface frictional effects are such as to produce eastward torques on the atmosphere in the regions of easterly winds, and westward torques in regions of westerly winds. Between the equator and about 35°N in the region of easterly trades, there is a continual flow of angular momentum into the earth's atmosphere. In midlatitudes the surface frictional interaction acts to abstract angular momentum from the atmosphere in the region of prevailing westerlies. In the weak polar easterly cell there is again a flow of angular momentum from the earth to the atmosphere. The angular-momentum exchange by surface frictional interaction depends on the direction of the wind, and its intensity depends on the strength of the surface winds and the distance from the polar axis.

On the other hand, the angular-momentum exchange between atmosphere and earth due to the torque exerted by mountain ranges is not dependent on wind direction in the same way, but is rather determined by the magnitude of the pressure difference across mountains. The mechanism of mountains in exerting a torque on the atmosphere can be considered simply as this differential pressure effect. If the pressure on the eastern side of the range is greater than the pressure on the western side, the mountain range must exert a force on the atmosphere directed to the east. The situation is reversed when the pressure is higher to the west than the east.

As a matter of convention, the mountain effect is defined as positive when the mountains act to abstract angular momentum from the atmosphere, that is, when the pressure on the west side of a mountain range is greater than the pressure on the eastern side. This is the case when mountains act to slow down the westerlies, since in those regions the winds are traveling faster in an easterly direction than the surface of the earth. The mountain effect is defined as negative when the mountains act to supply angular momentum to the atmosphere, that is, when the pressure is higher on the eastern side of the mountain than on the western side. In view of the fact that extensive amounts of pressure data are available, it was decided to study observationally the torques so produced, both on a daily and monthly normal basis.

One of the problems in the measurement of these effects from data is the construction of accurate topographic profiles of sufficient simplicity to permit easy calculation of pressure differences across mountain ranges that would be representative.[2] It was necessary to establish some basis for determining the effective heights of the principal mountain barriers of the northern hemisphere. Isolated mountain peaks extending far above the general level of the mountain chain could be of little consequence in computing the pressure difference across the range. It was assumed that only mountain ranges of broad latitudinal or longitudinal extent could be significant as barriers across which, on the average, significant differences in pressure could occur. All mountains whose extent is less than five degrees of latitude or five degrees of longitude were neglected. The validity of this assumption is borne out by the data.

By nature mountains rise to their heights irregularly, so that it was necessary to simplify their profiles considerably. Thus it was decided to break each mountain range down into a series of 1-km blocks across which the pressure differences were calculated. These simplifications lead to inaccuracies, but by no means are these errors seriously detrimental to the final results.

[1] This investigation was made possible through funds made available under terms of Contract No. W28-099 ac-406 between the U. S. Air Force and the Massachusetts Institute of Technology.

[2] Topographic profiles were determined with the aid of [2].

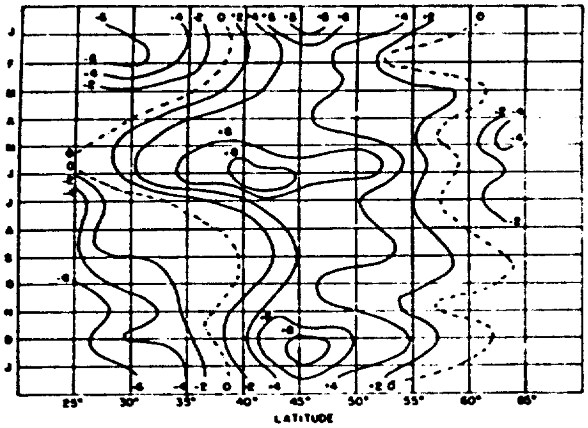

FIG. 1. Normal monthly total amounts of angular momentum transferred from the atmosphere to the earth due to mountain torques, in units of 10^{25} g cm^2 sec^{-1} per 5-degree latitude belt. (Negative values indicate transfer from earth to atmosphere.)

Within the limits of the accuracy of available pressure data the procedure seems to be sound. South of 25°N the mountain barriers are few and the data sparse. North of 65°N the only real mountain range is the Greenland ice cap, an effective barrier some 2–3 km in height. However in view of the proximity to the pole, and in view of the paucity of data, it was decided to assume the mountain effect north of 65°N to be negligible. This assumption is borne out by results of the research.

The monthly normal sea-level pressure data for the northern hemisphere were taken from figures after Shaw [3]. Data for the twelve months of the year were available for every five degrees of latitude and every ten degrees of longitude. Monthly normal 10,000-ft pressures were taken from data gathered and compiled by the U. S. Army Air Forces (1944 revised). The month of January 1946 was selected for a detailed breakdown of the mountain effect on a daily basis. Sea-level and upper-air pressure data for this month were obtained from figures after Widger [5] based on daily synoptic weather charts prepared by the U. S. Air Forces [1]. The computations described below are based entirely on these sea-level and 10,000-ft pressures.

The average pressure differences across each 1-km block of mountain range was computed. It was necessary to assume a linear decrease of pressure with height between sea level and 3 km. Some minor error is introduced here in view of the fact that sea-level pressures have been reduced artificially from station level and this reduction is not exactly "reversed" by the assumption of a constant lapse of pressure with height. In some instances when the 3-km pressures were unavailable it was assumed that the pressure difference across the mountain at that level was zero. The pressure differences were summed for each latitude, the total representing the pressure force acting along that belt of latitude around the entire hemisphere over a meridional distance of one centimeter. In order to find the torque exerted by the mountains, it was necessary to multiply this result by the cosine of the latitude and by the mean radius of the earth. It was assumed that the mountain effect obtained for a one centimeter zonal strip is representative of a zonal strip $2\frac{1}{2}$° to the north and south of the given latitude line.

The total normal monthly mountain effect was computed for each of the 12 months of the year by 5-degree latitude belts between 25°N and 65°N. The results are shown graphically in fig. 1. The magnitude and basic regularity of pattern suggests that the mountain effect, which in itself is a product of the pressure distribution around the earth, is an important part of the angular-momentum balance in the atmosphere. The maximum values of the mountain effect are found at the beginning of the summer season and the beginning of the winter season. However, there exists a significant difference in the character of these maxima. During the month of June it is most noteworthy that the mountain effect acts to abstract momentum from the atmosphere from 25°N to 60°N over almost the entire range of the northern hemisphere. Within the limits of the data, the only area where the mountains supply momentum to the atmosphere in June is north of 60°N, but the amount is insignificant.

On the other hand, the month of January shows quite a different picture. Mountains act to abstract momentum from the atmosphere only in a restricted area between 40°N and 60°N in the belt of prevailing westerlies. During January the maximum positive effect is between 45°-50°N, while in June the maximum positive effect is located between 40°-45°N. It would appear that the intensification and extension of the area of positive mountain effect into the subtropics during May and June is a normal feature of the angular-momentum balance of the atmosphere. Thus the westerlies, which are already weak in June, cannot abstract through friction the large amounts of momentum still being generated by the relatively strong subtropical easterlies. Apparently at least part of the excess is removed by mountain torques. There is, however, no comparable reverse effect during the early winter when the westerlies are strong. There is then perhaps no need for such a process. When considered in this light the mountain effect may act as an additional regulating factor in the production and consumption of angular momentum. Presumably it is able to act in this manner because it is not dependent on the zonal winds.

It is interesting to note the gradual progression of the reversal point of the mountain effect toward the tropics with the approach of spring and its retreat back to its normal winter position between 40°-50°N

during the remainder of the year. The reversal point is defined as the more southern latitude at which the mountain effect changes from negative to positive.

In general the latitude of maximum abstraction of angular momentum occurs in the region of prevailing westerlies. As one proceeds northward, angular momentum is abstracted during longer and longer periods. Thus at 35°–40°N, such momentum is abstracted only during May and June; at 40°–45°N momentum is abstracted during the entire year. This continues to latitude 55°N where the abstraction of momentum varies from month to month but is always weak. North of 60°N momentum is supplied in small amounts during the greater number of months of the year.

The daily values of the mountain effect for January 1946 were computed in the same manner as the monthly normals, and the values for this individual month were compared with normal January. There is a general correspondence between the magnitude and the distribution of the mountain effect for January 1946 and the normal January. Graphs for January 1946 and the normal January are shown in fig. 2. The latitude of the reversal point and the latitude of the maximum abstraction of momentum from the atmosphere for January 1946 are displaced about five degrees south of the normal positions.

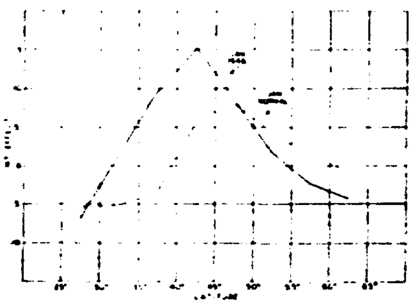

FIG. 2. Comparison between the total mountain effect for January 1946 and the normal for January (ordinate labeled in units of 10^{31} g cm^2 sec^{-1} per 5-degree latitude belt).

One of the basic aims of the research was to determine the relative importance of the mountain effect in supplying to or abstracting momentum from the atmosphere as compared with classical surface frictional effects. Values of the surface frictional interaction for January 1946 were determined by Widger [5]. The comparative values of the mountain effect and the frictional interaction are shown in table 1. It is seen that at almost every latitude the mountain effect acts in the same direction as surface friction. Further, it is apparent that the mountain effect is of the same order of magnitude as the effect of surface friction. It is to be concluded then that the mountain effect cannot be neglected in any consideration of the angular-momentum balance of the atmosphere, since in most cases it is important in both abstracting and in supplying angular momentum.

TABLE 1. Comparative values of mountain effect and surface frictional effect totaled for the month of January 1946 (in units of 10^{31} g cm^2 sec^{-1} per 5-degree latitude belt).

Latitude	Mountain effect	Surface friction
25–30	−681	−1530
30–35	+194	+205
35–40	+965	+138
40–45	+1550	+1842
45–50	+870	+1251
50–55	+191	+875
55–60	−221	+606
60–65	−411	−188

The major contributions to the mountain effect are from the two great mountain complexes of the northern hemisphere; the Rockies and the Asiatic ranges. The extent to which the two major systems dominate the total mountain effect is clearly brought out when one considers the correlation coefficients between the sum of the pressure differences across the Rockies and the Asiatic ranges and the total pressure difference summed for the hemisphere. Such coefficients were calculated for 35°, 40° and 45°N for the 31 days in January 1946. In all cases the correlations are not less than 0.85. These correlation coefficients are even more interesting in view of the fact that no significant correlation was found between the pressure difference across the Rockies and the pressure difference across the Asiatic ranges at the same latitudes.

Conclusions.—It appears that the mountain effect is a sensible factor in the angular-momentum exchange between earth and atmosphere. It is of the same order of magnitude as the surface frictional effect.

Mountains normally act to abstract angular momentum from the atmosphere in midlatitudes and to supply such momentum to the atmosphere in low latitudes. In addition, it appears that during the spring and early summer the mountains act to abstract angular momentum from the atmosphere even in low latitudes with a maximum total effect in May and June.

The major contributions to the mountain effect are from the two great mountain complexes of the northern hemisphere, the Rockies and the Asiatic ranges.

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A mechanism for the vertical transport of angular momentum in the atmosphere

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31 May 1950

The angular momentum balance of the atmosphere has been studied notably by Jeffreys[1] and more recently by several other writers. Whatever view one adopts concerning the nature of the general circulation, it is probable that a transfer of eastward angular momentum takes place from low to middle latitudes in the high tropospheric layers of the atmosphere. This necessitates an upward transport in low latitudes and a downward transport in middle latitudes.

An eddy mechanism for this vertical transport has been suggested by Starr,[2] which is associated with disturbances of the size of cyclones and anticyclones in middle latitudes. The correlation between the vertical and zonal components of the wind velocity would have to be negative through most of the troposphere in middle latitudes, where a downward transport of angular momentum is required. If there is a relation between the vertical velocities in the atmosphere and the occurrence of precipitation, it is then possible that a study might reveal a relation between the presence of precipitation and the zonal wind. Such a study was undertaken.

[1] H. Jeffreys, "On the dynamics of geostrophic winds," *Quart. J. R. meteor. Soc.*, **52**, 85–104, 1926.
[2] V. P. Starr, "An essay on the general circulation of the earth's atmosphere," *J. Meteor.*, **5**, 39–43, 1948.

Grouping the wind data according to the occurrence of precipitation or clear skies as a measure of the type of vertical velocities at the time of observation is crude. It is true that ascending velocities exist in regions where precipitation is occurring, but the range of elevation over which such vertical velocities are occurring and the strength of such ascending motions are extremely variable. In addition, a region of clear skies does not necessarily imply a region of descending velocities. Nevertheless, over a large sample of observations it is highly probable that the regions of precipitation have substantial ascending velocities through a significant portion of the lower troposphere and that

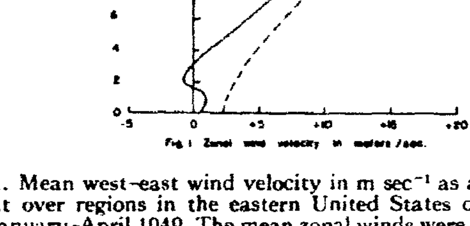

FIG. 1. Mean west–east wind velocity in m sec^{-1} as a function of height over regions in the eastern United States during the period January–April 1949. The mean zonal winds were computed from approximately 120 rawin soundings in the case of precipitation (solid line) and 300 soundings in the case of clear skies (dashed line).

TABLE 1. Analysis of winds aloft under contrasting weather conditions. The zonal component of the wind velocity in m sec^{-1} is indicated by $\bar{u}$. The values given by Student's "t" test are indicated by t, below which the significance level is given. The number of cases is denoted by n.

	2000 ft		6000 ft		10,000 ft		14,000 ft		20,000 ft	
	Ppt.	Clear	Ppt.	Clear	Ppt.	Clear	Ppt.	Clear	Ppt.	Clear
n	127	283	144	333	135	328	130	310	111	270
$\bar{u}$	−0.92	+3.05	+4.52	+6.82	+8.40	+11.77	+11.60	+14.92	+15.32	+18.51
t	6.07		3.63		4.14		3.46		2.59	
Significance level	<.001		<.001		<.001		<.001		<.010	

regions of clear skies have vertical velocities which in the mean are different and probably in the opposite direction.

A group of eight rawin stations over the eastern United States was selected for study. These included Nashville, Tenn., San Antonio, Tex., Little Rock, Ark., Bismarck, N. D., Caribou, Me., International Falls, Minn., Nantucket, Mass., and Rapid City, S. D. The zonal component of the observed wind as reported from all the rawins during the period January–April 1949 at 0300 Z was computed at the following levels: 1000, 2000, 4000, 6000, 8000, 10,000, 14,000, and 20,000 ft. Those observations when the weather was clear (cloudiness $\leq 3/10$) or precipitation of any kind was occurring at observation time were selected.

The mean zonal wind component in m sec^{-1} for each level when grouped as described above is shown in fig. 1. It is seen that smaller zonal winds are associated with precipitation than with clear skies, in the sense which might be expected if the vertical transport mechanism suggested by Starr is operative. The difference between the mean zonal wind components in the two categories was tested for statistical significance by application of Student's "t" test (see table 1.) The significance levels obtained suggest, but do not establish, the existence of a correlation between the zonal and vertical velocity components associated with eddies the size of cyclones, which is in the right sense to account for a downward transport of angular momentum in middle latitudes.

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SHORTER CONTRIBUTION

Geostrophic Departures in the Jet Stream

By VICTOR P. STARR, Massachusetts Institute of Technology[1]

(Manuscript received 2 April 1950)

Abstract

The effect of gradients in the cross-current eddy motions on the geostrophic balance is illustrated by means of an integrated model. It is suggested that gradients of the Reynolds stress associated with such eddy motions in the atmosphere and in the oceans may lead to sensible average geostrophic departures which appear as a normal inertial effect in eddying currents.

In the past many investigators have devoted attention to the study of the degree to which actual wind currents may be measured by the geostrophic approximation. Some of these studies have been directed toward an examination of effects which might be expected on rational grounds, while others are entirely empirical in nature. In view of the fact that research workers interested in the large-scale currents in the atmosphere are usually forced to resort to wind estimates obtained from pressure data, the nature of the approximations involved in this procedure is of much practical concern. The purpose of this discourse is to bring attention to one systematic effect which might be expected on a rational basis in those regions of the atmosphere where the circulations attain their greatest intensities, although lesser manifestations of the same kind might be found to exist at other levels in the atmosphere as well as in the oceans.

The effect to be discussed cannot be regarded as any new concept in the field of hydrodynamics, having long ago been recognized in the treatment of such topics as the turbulent flow of fluids in channels (see for example GOLDSTEIN 1938). However, it appears that the meteorological applications of the concepts involved should be emphasized.

Although the subject is not dependent upon the existence of any particular integrated model of flow, we shall find it convenient to introduce the basic arguments through the examination of one such specific model. In terms of cartesian coordinates in which x positive is taken eastward and y positive northward, let us study the two-dimensional flow of a fluid which satisfies the following equations of motion and continuity:

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = fv - \frac{1}{\varrho}\frac{\partial p}{\partial x} \\ &\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -fu - \frac{1}{\varrho}\frac{\partial p}{\partial y} \\ &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \end{aligned}\right\} \tag{1}$$

Here u is the eastward and v the northward particle velocity, p is pressure, t time, f the Coriolis parameter and ϱ density. We assume that ϱ is constant and uniform and that

$$f = f_0 + \beta y \tag{2}$$

where f_0 and β are constants.

The three relations (1) form a closed system of partial differential equations for the determination of the three dependent variables u, v, p in terms of the independent variables x, y, t. It has been shown

[1] This research was performed under contract with the U. S. Air Force Cambridge Research Laboratories.

by NEAMTAN (1946) that this system of simultaneous equations possesses an integral in which

$$u = U + V\cos(\alpha y + \varepsilon) - Al\sin k(x - ct)\cdot\cos ly, \tag{3}$$

$$v = Ak\cos k(x - ct)\cdot\sin ly, \tag{4}$$

provided that

$$\alpha = \sqrt{k^2 + l^2};\quad c = U - \beta/(k^2 + l^2) \tag{5}$$

and U, V, A, k, l, ε are arbitrary constants which we take as being real.

This (particular) solution is completed by writing that

$$\begin{aligned}\frac{p}{\varrho} = & -\frac{1}{2}(u^2 + v^2) - Alc\sin k(x - ct)\cdot\cos ly \\ & -\frac{1}{2}A^2\alpha^2\sin^2 k(x - ct)\cdot\sin^2 ly + \\ & + A\sin k(x - ct)\cdot\sin ly\,[f_0 + \beta y + \\ & \qquad V\alpha\sin(\alpha y + \varepsilon)] \\ & - f_0 U y - \frac{V}{\alpha}(f_0 + \beta y)\sin(\alpha y + \varepsilon) - \\ & \qquad \frac{1}{2}\beta U y^2 \\ & - V\left(\frac{\beta}{\alpha^2} - U\right)\cos(\alpha y + \varepsilon) + \\ & \qquad \frac{1}{2}V^2\cos^2(\alpha y + \varepsilon) \\ & + F(t),\end{aligned} \tag{6}$$

obtained through the integration of the first two equations of (1) with the aid of (3), (4) and (5). The last term in (6) is an arbitrary function of time alone and may ordinarily be taken to be an appropriate constant without affecting the other characteristics of the solution. In view of the form of the continuity equation contained in (1), it is possible to use a stream function ψ for the velocity components, of the form

$$\psi = -Uy - \frac{V}{\alpha}\sin(\alpha y + \varepsilon) + A\sin k(x - ct)\cdot\sin ly. \tag{7}$$

It is apparent that the motion consists of a basic zonal flow on which periodic disturbances are superposed.

Letting u_g represent the geostrophic wind toward the east, we write that

$$u_d = u - u_g, \tag{8}$$

so that u_d is the nongeostrophic part of u. Also, introducing the space average of a quantity ( ) over a wave length L in the eastward direction by the relation

$$\overline{(\ )} = \frac{1}{L}\int_L (\ )\,dx, \tag{9}$$

we have that

$$\bar{u}_d = \bar{u} - \bar{u}_g. \tag{10}$$

Either by using the geostrophic formula with (6) and comparing with (3), or more directly by substitution in the second equation of (1) from (3) and (4) and averaging, we find that

$$\bar{u}_d = -\frac{A^2k^2l}{f}\sin ly\cdot\cos ly. \tag{11}$$

It is thus seen that $\bar{u}$ contains a systematic nongeostrophic component which varies in a periodic fashion with the independent variable y alone.

Since v vanishes for $y = O$ and for $y = \pi/l$, it will tend to clearness if we visualize the flow as contained in a channel between two parallel walls at these ordinates. The departure $\bar{u}_d$ then vanishes according to (11) at the two walls and also at the center, $y = \pi/2l$. In the southern half of the channel, between $y = O$ and $y = \pi/2l$, $\bar{u}_d$ is negative while in the northern half it is positive. On the average over the whole channel the conditions are such that the flow is geostrophic.

By way of physical explanation of this phenomenon we might note that the minimum value of the Reynolds stress $-\varrho\overline{uv}$ is found at $y = \pi/2l$. This is interpreted as a net transfer of positive y-momentum from the southern half to the northern half. This drain must be made good in the lower half by an excess of the mean northward pressure gradient force over and above the requirements needed to balance meridional Coriolis forces on the fluid in this region. The reverse process takes place in the upper half.

Viewed some what differently the process may be explained physically by noting that individual particles enter the southern half with negative y-momentum and leave with positive y-momentum. They must therefore be accelerating northward on the average while in the southern half and southward while in the northern half. From this one may infer that $\bar{u}_d$ cannot be zero everywhere, unless indeed the motion is purely zonal.

It is to be noted that the results described do not

depend upon the profile of $\bar{u}$ but only upon that of $\overline{\varrho v v}$. We may, for example, place a maximum or »jet» of $\bar{u}$ in the southern half at $y = \pi/4\, l$ by taking an appropriate value U together with $V > 0$ and proper values of α, ε.

Much of what has been said follows directly if we write the second equation of (1) in the form of a continuity equation for y-momentum with the aid of the third and average so that

$$\frac{\partial \overline{\varrho v}}{\partial t} + \frac{\partial \overline{\varrho u v}}{\partial x} + \frac{\partial \overline{\varrho v v}}{\partial y} = -f \overline{\varrho u} - \frac{\partial \bar{p}}{\partial y}. \quad (12)$$

The results stated above follow immediately, since the first two terms vanish. Moreover we see that similar results are to be expected in any two dimensional model with a quasi-steady regime of turbulence or eddy motion. For all such cases the mean geostrophic deviation is determined by the cross-stream gradient of the Reynolds stress $-\overline{\varrho v v}$.

The direct application of the model described to the atmosphere would lead to the presence of a westward directed mean geostrophic departure equatorward from the latitude at which $\overline{\varrho v v}$ is a maximum in each hemisphere, and an eastward mean departure poleward. However, the effect is in actuality superposed on other sources of geostrophic departures such as the curvature of the latitude circles which produces a westward departure even in the case of purely zonal flow. Another factor is that for the actual atmosphere even in the simple cartesian equation (12) an additional vertical term must be added so that, taking z positive upward and w as the upward particle velocity, and neglecting horizontal density variations in the x-direction,

$$\bar{\varrho} \bar{u}_d = -\frac{1}{f} \left(\frac{\partial \overline{\varrho v v}}{\partial y} + \frac{\partial \overline{\varrho v w}}{\partial z} \right). \quad (13)$$

It is difficult to estimate the effect of this vertical term, although if (13) be integrated also with respect to height through the whole atmosphere, the total contribution of this added term must result from the presence of a net meridional surface stress at a given latitude. There is not much *a priori* reason to expect that such net meridional surface stresses around latitude circles are of importance.

The latitude at which $\overline{\varrho v v}$ reaches a maximum is probably displaced farther poleward in each hemisphere as compared with the latitude at which $\bar{u}$ reaches its maximum, although observational studies should be made concerning this point. Furthermore, there is no reason to suppose that this quantity tends to zero as the pole is approached. For purposes of orientation let it be supposed that $\overline{v v}$ increases by 10^7 cm^2/sec^2 for an increase of 2 000 km in y. Neglecting horizontal density variations, the first member on the right hand side of (13) then gives a contribution of -6.7 meters per second to $\bar{u}_d$, if $f = 0.75 \times 10^{-4}$ (corresponding to about 30° N latitude). This may give some notion of correct orders of magnitude in the vicinity of the jet stream. At lower levels the contribution may be one fifth or one tenth as large.

In an interesting paper published recently, Loewe and Radok (1950) have presented a meridional cross section of the atmosphere in the southern hemisphere through the jet stream. These writers make a comparison of geostrophic versus actual wind measurements in the jet. The actual winds show velocities much lower than those computed from the isobaric contours. Although many questions might arise concerning the representativeness of the data, it may also be that a part of the discrepancy is due to the manifestation of the phenomenon here described. It is to be noted that local climatological time averages such as those used by Loewe and Radok would tend to approximate space averages except for the possible presence of standing semipermanent disturbances.

Finally it should be mentioned that the considerations used here may be of some consequence in the charting of mean ocean currents by means of dynamic velocity computations, in those cases where there exists a gradient in the cross-current eddy motion. This may, for example, be significant in the case of such currents as the Gulf Stream.

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A note on the eddy transport of angular momentum

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SUMMARY

A pilot study of the horizontal eddy transport of angular momentum in the meridional direction, based on radio wind observations over the North American sector of the northern hemisphere, is presented. The results are in harmony with general frictional requirements except that the transports are rather large. The desirability of much more extensive compilations of data concerning the subject is indicated.

1.

The problem posed by the angular momentum balance of the earth-atmosphere system is probably one of the most fundamental considerations in meteorological science. In view of the fact that within the last few decades an extensive body of proper observational data has been accumulated, it is becoming possible to do more than merely speculate abstractly in this regard. As a result a number of papers, both observational and theoretical, have appeared relating to this matter. A selection of these is contained in the annotated list of references at the end of this note, although many others should be included in a more complete bibliography.

The particular question to which much attention is directed currently relates to the manner in which the necessary meridional transfer of angular momentum takes place within the atmosphere. Is this flow accomplished primarily through the agency of the so-called mean meridional circulations, or primarily through horizontal exchange processes? The nature of our approach to the subject of the general circulation depends largely upon the answer to this question.

In order to seek enlightenment, recourse must be made to data. One may try to measure either one or the other of the two processes mentioned. The writer and his colleagues working with the general circulation project at the Massachusetts Institute of Technology have been engaged in studies which aim at measuring the eddy processes. Thus Widger (1949) made a hemispheric analysis of winds computed from isobaric maps for the month of January 1946 and obtained substantial indications of the importance of the eddy processes in the lower and middle troposphere.

Owing to the fact that isobaric maps for relatively high levels in the atmosphere are of uncertain reliability and are still difficult to construct, it is a tempting matter to utilize direct radio wind measurements for the purpose at hand. However, since such wind data are not available on a hemispheric basis, great care must be exercised in treating limited amounts of observational material in order to obtain meaningful results. Specifically it is essential to sample a significant and representative sector of longitude. Also, it is necessary to use averages over appreciable intervals of time to improve reliability, because of the effects of missing reports

* This research was performed under contract with the Cambridge Research Laboratories, U.S. Air Force.

and other factors such as the higher *noise level* in actual wind data. The last item has been discussed by Charney (1947). Selectivity in favour of light winds is doubtless also a factor, especially in the higher reaches of the soundings. Finally, unlike wind data secured from isobaric maps, the radio winds are available only at stations arranged in an irregular array over the weather map, so that some appropriate scheme must be devised to approximate the momentum transports across individual latitude circles.

2.

It is by no means altogether clear how the needed calculations should be conducted to best advantage. For this reason the writer desires to enlist the advice of the readers of this *Journal*.* For purposes of orientation an admittedly rough and restricted pilot study is presented below to serve as a basis for discussion. It is probable that a more comprehensive project will be undertaken at a later date when the best mode of procedure is apparent.

A beginning in the proper direction has been made in an interesting paper by Priestley (1949), although it appears that a more elaborate plan is required. His method consists of first computing the time-average value of the product of the eastward and northward wind components, $\overline{uv}$; for each individual station at each selected pressure level. From each of these quantities the contribution due to the net meridional transport of air at the station and level, namely $\bar{u}\,\bar{v}$, is subtracted, the bars again denoting time averages. The difference $\overline{uv}-\bar{u}\,\bar{v}$ is thus related to the temporal fluctuations of the wind and would be zero under steady wind conditions. Since standing eddies, *i.e.* semi-permanent or average features of the general circulation, contribute to the meridional eddy flux of physical properties, it is necessary to include their effects also by some modification of the scheme.

We may, on the other hand, use averages both over time and over a suitable segment of a latitude circle at a given pressure level, and use the bar to denote such a space-time average. Thus $\overline{uv}$ now gives us a measure of the total momentum transfer across the segment. A portion of this total may be due to a net flow of air across the segment, $\bar{v}$. If an entire closed latitude circle were considered, $\bar{v}$ would for practical purposes represent a meridional circulation the effects of which it is desired to exclude, and therefore $\overline{uv}-\bar{u}\,\bar{v}$ would be the needed quantity. For a large but limited segment $\bar{v}$ may also include the effects of standing eddies of larger dimensions than the segment chosen. Under these circumstances the subtraction of $\bar{u}\,\bar{v}$ from $\overline{uv}$ may still eliminate effects of very large standing eddies, but the process does at least take partial account of the average features. A more detailed examination of this use of space-time averages has been given by White (1950).

3.

In the pilot study all the soundings for the hours 0300Z and 1500Z reported in the *Daily Upper Air Bulletin*, U.S. Weather Bureau (1949) at 52 stations during February 1949 were used. As indicated by Roman numerals on the map in Fig. 1, the stations were separated into six groups according to approximate 10° latitude

* The pages of the *Journal* are always open for correspondence on this topic.—EDITOR.

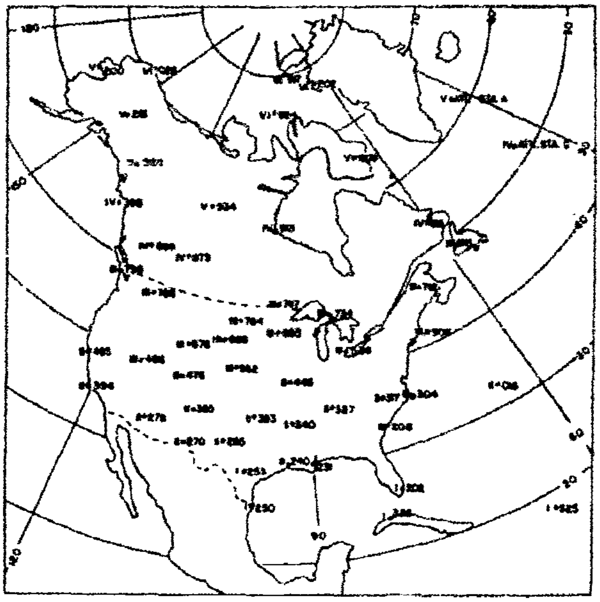

Figure 1. Map showing stations used in study. Roman numerals indicate classification according to approximate latitude belts. International station numbers entered to right of each station.

belts. The space-time average for a given latitude belt and given level was taken as the sum of all the observed values of a quantity at the several stations in the belt divided by the number n of such observations. This is rather crude, since the available observations are not uniformly distributed in space or in time.

The results are contained in Table I and Fig. 2, which are almost self-explanatory. The transports obtained may be looked upon as being given per unit mass in the vertical and per unit distance zonally, and hence may be integrated vertically with respect to pressure. These integrals were evaluated from graphs by means of a planimeter, the limits being from 100 to 1,000 mb. Furthermore, the total transports of linear momentum so obtained were converted into transports of angular momentum for a complete latitude circle in each case through the introduction of proper factors involving the earth's radius and the average latitude $\bar{\phi}$. Such integrated transports are given in cgs units in the last column of the table. The diagram in Fig. 2 gives the corresponding profiles showing the vertical distribution of these same transports of angular momentum. In effect the values in the last column of the table are proportional to the integrated areas of these profiles in the figure.

The writer's colleague Dr. W. K. Widger has made an estimate of the normal rate at which angular momentum is removed from the atmosphere through surface effects during February. For the entire polar cap north of 35° this rate appears to be about 10×10^{23} cgs units of angular momentum/sec. Upon comparison of this figure with the integrated transport for Group II, it is seen that the latter is about four times larger than the former. Various reasons may be present to

TABLE I. Analysis of radio wind soundings for February 1949

Press mb	n	$\bar{u}$ m sec^{-1}	$\bar{v}$ m sec^{-1}	$\bar{u}\,\bar{v}$ m^2 sec^{-2}	$\overline{uv}$ m^2 sec^{-2}	$\overline{uv}-\bar{u}\,\bar{v}$ m^2 sec^{-2}	Integral gm cm^2 sec^{-2}
			GROUP I $\bar{\phi}$ = 25°N.				
100	30	+ 4·57	− 2·93	−13·39	+ 2·20	+15·59	
200	104	+21·21	− 1·76	−37·33	+18·09	+55·42	
300	177	+18·20	+ 2·28	+41·50	+91·33	+49·83	
500	252	+11·20	+ 4·31	+48·27	+91·65	+43·38	+56·0 × 10^{24}
700	273	+ 3·89	+ 3·55	+13·81	+26·68	+12·87	
850	275	− 0·37	+ 2·63	− 0·97	+ 8·06	+ 9·03	
1000	234	− 3·21	+ 0·90	− 2·89	− 0·99	+ 1·90	
			GROUP II $\bar{\phi}$ = 35°N.				
100	36	+17·94	+ 2·58	+46·29	+46·69	+ 0·40	
200	148	+25·21	+ 1·42	+35·80	+85·39	+49·59	
300	314	+23·29	+ 0·66	+15·37	+68·82	+53·45	
500	612	+17·77	+ 1·08	+19·19	+49·89	+30·70	+41·5 × 10^{24}
700	720	+11·09	+ 1·35	+14·97	+30·11	+15·14	
850	693	+ 5·18	+ 1·24	+ 6·42	+17·31	+10·79	
1000	356	+ 0·22	+ 0·32	+ 0·07	+ 5·04	+ 4·97	
			GROUP III $\bar{\phi}$ = 45°N.				
100	43	+13·07	− 2·79	−36·47	−29·95	+ 6·52	
200	125	+20·00	− 0·89	−17·80	+ 6·61	+24·41	
300	225	+18·40	− 1·23	−22·63	− 4·93	+17·70	
500	448	+16·34	− 2·46	−40·20	−35·18	+ 5·02	+ 8·4 × 10^{24}
700	596	+11·35	− 1·23	−13·96	−13·89	+ 0·07	
850	553	+ 6·30	− 0·23	− 1·45	− 0·17	+ 1·28	
1000	140	+ 1·41	− 0·60	− 0·85	− 2·61	− 1·76	
			GROUP IV $\bar{\phi}$ = 55°N.				
100	17	+16·94	+ 0·88	+14·91	+11·29	− 3·62	
200	89	+16·18	− 0·44	− 7·12	−46·92	−39·80	
300	153	+17·61	+ 1·17	+20·60	− 0·92	−21·52	
500	268	+13·82	− 0·62	− 8·57	− 8·91	− 0·34	−6·2 × 10^{24}
700	285	+ 7·78	+ 0·06	+ 0·47	+ 5·47	+ 5·00	
850	289	+ 6·51	+ 0·37	+ 2·41	− 0·90	− 3·31	
1000	140	+ 3·16	− 0·66	− 2·09	− 4·71	− 2·62	
			GROUP V $\bar{\phi}$ = 63°N.				
100	6	+34·67	0·00	0·00	+54·83	+54·83	
200	92	+12·41	+ 2·65	+43·49	+ 8·08	−35·41	
300	135	+12·62	+ 1·73	+21·83	+27·30	+ 5·47	
500	206	+ 8·48	+ 0·34	+ 2·88	−10·86	−13·74	−2·0 × 10^{24}
700	228	+ 5·02	+ 0·77	+ 3·87	+ 1·78	− 2·09	
850	228	+ 3·21	+ 0·57	+ 1·83	− 5·05	− 6·88	
1000	91	− 7·14	+ 0·13	− 0·93	+ 4·18	+ 5·11	
			GROUP VI $\bar{\phi}$ = 76°N.				
100	3	+18·33	− 4·67	−85·60	−112·00	−26·50	
200	31	+ 6·58	− 0·19	− 1·25	−21·61	−20·36	
300	89	− 2·11	− 1·73	+ 3·65	+21·67	+18·02	
500	138	− 4·25	− 2·76	+11·73	+26·61	+14·88	+0·6 × 10^{24}
700	154	− 1·62	− 1·55	+ 2·51	+10·19	+ 7·68	
850	151	− 0·84	− 0·45	+ 0·38	+ 1·48	+ 1·10	
1000	101	− 1·35	− 0·28	+ 0·38	− 1·46	− 1·84	

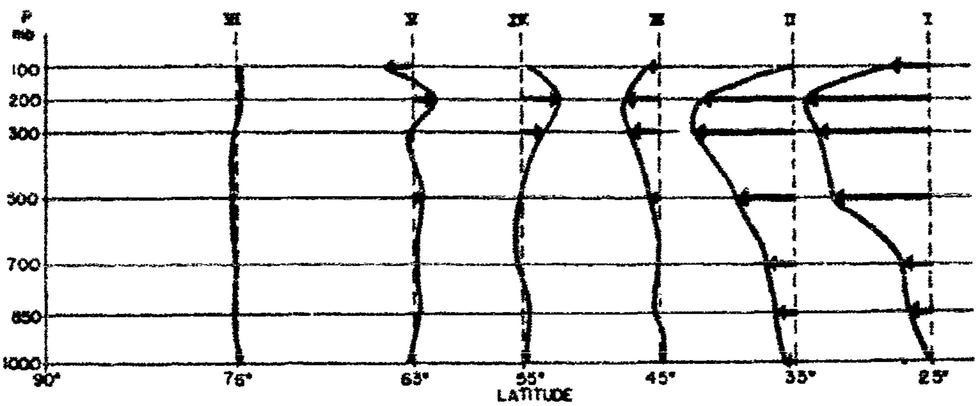

Figure 2. Profiles of angular momentum transport vectors for complete latitude circles. Transports are for layers of unit mass in the vertical. It should be noted that the profiles would be much altered if elevation were used as the vertical coordinate and the transports given for layers of unit geometrical thickness.

explain so large a discrepancy, if one were to assume that the computations presented here are correct.

(1) The North American sector of the northern hemisphere is characterized by better developed eddies than, let us say, the Eurasian sector and hence the values obtained are not representative of the hemisphere.
(2) The month of February 1949 is an abnormal February. That departures from normal as large as this could exist is easily possible.
(3) The estimates of normal surface torques used is too small.
(4) Angular momentum is transferred also by meridional cells. A direct cell in the troposphere would however, add to to the transport and make matters worse.

Of these four possibilities (1) and (3) might be the most plausible, since examination of hemispheric conditions during February 1949 does not show too striking abnormalities, and no good reason suggests itself for recourse to (4).

The table and diagram suggest the presence of very strong eddy transports at the jet-stream level as might perhaps be expected on general grounds from the violence of the winds in this region*. If any reliance can be placed upon the results at the 100 mb level, the sharp drop in the transports (except for Group V) shown here is most remarkable. It should be remembered that the effects of selectivity at this level are most severe, however. Generally speaking the meridional gradients of the transports are what should be expected from friction and other surface effects, except for the decrease from 25°N. to 35°N.

The most that can be said for the results is that they suggest the presence of strong eddy transfers of momentum, as proposed by the writer elsewhere (Starr 1948). The need for comprehensive treatment of the subject is apparent.

* NOTE ADDED IN PROOF.—The writer's colleague, Dr. H. L. Kuo, has pointed out that on hydrodynamical grounds the meridonal profile of angular momentum flow obtained for these higher levels, when considered in relation to the mean zonal wind profiles, suggest the presence of a transfer of existing kinetic energy from the large-scale perturbations to the mean zonal motion.

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An attempt to explain variations in the rate of the earth's rotation through exchanges of angular momentum with the atmosphere and oceans.

COMPUTATIONS OF THE BALANCE OF ANGULAR MOMENTUM

AND THE POLEWARD TRANSPORT OF HEAT

by Edward N. Lorenz

ABSTRACT

The geostrophic transports of angular momentum and sensible heat are investigated theoretically. At a given latitude and a given pressure level, the poleward geostrophic transport of angular momentum is found to be proportional to the square of the horizontal mass exchange across the given latitude, and a suitably defined average departure of the troughs and ridges at the given pressure level from a north-south orientation. The poleward geostrophic transport of heat is found to be proportional to the square of the horizontal mass exchange, and a suitably defined average departure of the troughs and ridges at the given latitude from a vertical orientation. The analytic expressions for the geostrophic transports of angular momentum and heat suggest an ideal procedure for computing the transports from observational data.

The various terms in the equation expressing the balance of angular momentum are computed from observational data, for the period 1 November 1945 through 28 February 1946, for various regions within the northern hemisphere. From these computations a diagram depicting the angular momentum balance in the northern hemisphere during this period is constructed. The computed northward transport of angular momentum is found to be consistent with the computed torque exerted on the atmosphere by the earth's surface, at most latitudes. A few modifications of the computed surface torque are suggested.

Computed values of the northward transport of sensible heat are presented, for the period 1 November 1945 through 28 February 1946, across various latitudes in the northern hemisphere.

PART I. GEOSTROPHIC TRANSPORT OF ANGULAR MOMENTUM AND HEAT

1. Introduction

Considerable attention is currently being given to the balance of absolute angular momentum about the earth's axis, and to the balance of total energy. Because eastward angular momentum is usually removed from the atmosphere by friction in middle latitudes, where the surface winds tend to be westerly, and is added to the atmosphere in low latitudes, where the surface winds are easterly, there must exist in the mean a flow of angular momentum in the atmosphere from low to middle latitudes. The importance of this flow of angular momentum was noted by Jeffreys (1926). In a somewhat analogous way, a flow of some form of energy from low to high latitudes in the atmosphere is necessary, because of the net heating by radiation in low latitudes and cooling by radiation in high latitudes. Various estimates of the incoming and outgoing radiation have been summarized by Haurwitz (1941).

Since there are no pronounced net long-period changes in the distribution of the mass of the atmosphere, any instantaneous local poleward flow of mass across a given latitude must be counterbalanced by an equal equatorward flow across this latitude. Since this compensating flow occurs at some other longitude, some other elevation, or some other time, there are several methods for resolving any mass flow into several terms. One method consists of resolving the instantaneous local mass flow into two types of flow — the longitudinal average flow at the particular elevation and time, and the instantaneous local departure from this average. Flow of the former type may be said to form meridional cells, while flow of the latter type may be said to form horizontal eddies.

A transport of any quantity, such as angular momentum or energy, may theoretically be accomplished by either meridional cells or horizontal eddies. The relative importance of these types of flow in accomplishing the transport of angular momentum and energy has recently been given considerable attention.

If sufficiently accurate, representative, and complete data were available, the total transport of any quantity could be computed. In the absence of complete upper wind observations, use of the geostrophic wind approximation naturally suggests itself. The geostrophic wind possesses the property that any instantaneous local mass flow accomplished by it must be counterbalanced by a mass flow at some other longitude, at the same elevation and the same time. Thus it is impossible to determine the presence of meridional cells from geostrophic wind observations. It is possible, however, that the geostrophic wind is an acceptable approximation to the actual wind, for the purpose of computing the horizontal eddy transport of certain quantities. Computations of the angular momentum transport, based upon geostrophic winds, have been made by Widger (1949), while White (1950) has presented computations of the geostrophic transport of sensible heat, the latter being one form in which energy may be transported.

It was proposed by Starr (1948) that the positive correlation between poleward mass flow and eastward velocity in middle latitudes necessary to yield a poleward transport of eastward angular momentum might be associated with horizontal streamline patterns in which the trough and ridge lines have a northeast-southwest rather than a north-south orientation. Exploiting this idea, Machta (1949) obtained solutions of the vorticity equation for plane nondivergent motion, in which the trough and ridge lines were not necessarily oriented north-south, and found a northward transport of linear momentum in those solutions where the orientation was northeast-southwest. For the spherical earth, Mintz (1949) found that under certain not unreasonable assumptions the transport of angular momentum must be northward when the troughs and ridges are oriented northeast-southwest. It is likely that the troughs and ridges in the pressure patterns have orientations resembling those in the streamline patterns, even when at individual points the wind deviates considerably

from the geostrophic wind. In a somewhat analogous way, the positive correlation between poleward mass flow and temperature necessary to yield a poleward flow of sensible heat may be associated with pressure patterns on a vertical surface at one latitude in which the troughs and ridges are displaced westward with elevation instead of being vertical, for such patterns occur when the warmer air lies east of the troughs and west of the ridges, and hence in the region of poleward flow.

2. Formulae for geostrophic transport

Although it is customary to speak of the poleward transport of a quantity at a given level, the transport occurring at exactly one level must be zero. To obtain a nonvanishing quantity, one must deal with the transport occurring within a layer. Among the layers which may be chosen for study are layers bounded by two horizontal surfaces and layers bounded by two surfaces of constant pressure. One may thus consider the transport per unit of elevation at a given elevation, or the transport per unit of pressure at a given pressure. Such quantities as the transport per unit of elevation at a given pressure are of questionable significance, however; there is no way in which the atmosphere may be broken up into a set of layers of uniform thickness centered at constant pressure surfaces.

In studying the geostrophic transports of angular momentum and sensible heat (the latter transport will be called simply the transport of heat), it is convenient to choose for independent variables the time t, longitude λ, latitude ϕ, and pressure p. The following dependent variables and constants then appear:
z = elevation, ρ = density, T = temperature, u = eastward velocity, v = northward velocity, a = earth's radius, ω = angular velocity of earth's rotation, g = acceleration of gravity, R = gas constant for air, and c_p = specific heat of air at constant pressure.

Expressions will now be developed for the geostrophic transports of absolute angular momentum and heat. Expressions for the absolute angular momentum per unit mass and the sensible heat per unit mass appear in table 1 (on page 6). Since the flow of mass per unit time across a unit area of a constant latitude surface is ρv, and since the area of a section of this surface whose horizontal extent is one unit of length and whose vertical extent is one unit of pressure is $(g\rho)^{-1}$, the total transports of angular momentum and heat across a latitude per unit time within a layer one pressure-unit deep are as given in table 1. In these expressions a bar over a quantity denotes the value of the quantity averaged over longitude.

Under the assumptions of geostrophic and hydrostatic equilibrium,

$$v = g\,(2\omega \sin\phi)^{-1}(a \cos\phi)^{-1}\,\partial z/\partial\lambda\,, \quad u = -g\,(2\omega \sin\phi)^{-1}a^{-1}\,\partial z/\partial\phi\,,$$

Table 1. Expressions associated with the geostrophic transports of angular momentum and heat.

Quantity	Absolute Angular Momentum	Sensible Heat
Quantity per unit mass, at latitude ϕ	$a^2\omega\cos^2\phi + a\cos\phi\, u$	$c_p T$
Transport of quantity across latitude ϕ at pressure p, per unit of pressure and per unit of time	$2\pi a^2 g^{-1}\cos^2\phi \times \overline{(a\omega\cos\phi + u)v}$	$2\pi a g^{-1} c_p\cos\phi \times \overline{Tv}$
Geostrophic transport of quantity across latitude ϕ at pressure p, per unit of pressure and per unit of time	$-\frac{1}{2}\pi\omega^{-2}\cos\phi\sin^{-2}\phi \times \overline{Z_\lambda Z_\phi}$	$-\pi\omega^{-1}g c_p R^{-1}(\sin\phi)^{-1} p \times \overline{Z_\lambda Z_p}$

and $T = -gR^{-1}p\,\partial Z/\partial p$. Thus the three partial derivatives of Z are measures of northward velocity, eastward velocity, and temperature. It will be convenient to use the symbols Z_λ, Z_ϕ, and Z_p in place of $\partial Z/\partial\lambda$, $\partial Z/\partial\phi$, and $\partial Z/\partial p$, since these derivatives appear frequently.

It follows that across a given latitude at a given pressure, per unit of pressure and per unit of time, the geostrophic transports of angular momentum and heat are as given in the final row of table 1. The expression for the angular momentum transport contains the latitude explicitly, while the expression for the heat transport contains both latitude and pressure explicitly. However, across one latitude at one pressure, both geostrophic transports are completely determined by longitudinal averages of products of partial derivatives of Z, i.e., by $-\overline{Z_\lambda Z_\phi}$ and $-\overline{Z_\lambda Z_p}$. One consequence of this result is that the time variations of the transports are determined by the time variations of $-\overline{Z_\lambda Z_\phi}$ and $-\overline{Z_\lambda Z_p}$. In the next two sections the quantities $-\overline{Z_\lambda Z_\phi}$ and $-\overline{Z_\lambda Z_p}$ will be investigated in detail.

3. A simple model

It is convenient to consider a simple model first. In this model the height profile along each "circle" of constant latitude and pressure is a simple sine curve containing n waves. The amplitude and phase angle of these waves may vary with latitude and pressure. The instantaneous field of elevation is

then described by the expression

$$Z(\lambda, \phi, p) = Z_0(\phi, p) + A(\phi, p) \cos n[\lambda - \varepsilon(\phi, p)] \quad . \tag{1}$$

For a given ϕ and p, minimum and maximum values of Z occur when

$$\lambda = \varepsilon(\phi, p) + k\pi/n \quad , \tag{2}$$

where k is any integer. If troughs and ridges are defined as curves composed of these minimum and maximum points, the equation for a trough or ridge on a constant pressure surface is given by (2), with ϕ variable and p constant, while the equation for a trough or ridge on a vertical surface at constant latitude is given by (2), with p variable and ϕ constant. The deviation of the troughs and ridges from a north-south orientation, expressed in units of longitude per unit of latitude, is therefore $\partial\varepsilon/\partial\phi$. A positive value of $\partial\varepsilon/\partial\phi$ indicates a northeast-southwest orientation. The deviation of the troughs and ridges from a vertical orientation on a surface of constant latitude, expressed in units of longitude per unit of pressure, is $\partial\varepsilon/\partial p$. A positive value of $\partial\varepsilon/\partial p$ indicates a westward displacement upward. It is convenient to use the symbols ε_ϕ and ε_p in place of $\partial\varepsilon/\partial\phi$ and $\partial\varepsilon/\partial p$. The quantities ε_ϕ and ε_p will be called the horizontal tilt and the vertical tilt of the troughs and ridges.

Differentiation of (1) with respect to the coordinates, and integration over longitude, shows that

$$-\overline{Z_\lambda Z_\phi} = \tfrac{1}{2} n^2 A^2 \varepsilon_\phi \quad , \tag{3}$$

$$-\overline{Z_\lambda Z_p} = \tfrac{1}{2} n^2 A^2 \varepsilon_p \quad . \tag{4}$$

These results follow from the easily established relations $\overline{\sin^2 n\lambda} = \tfrac{1}{2}$, $\overline{\sin n\lambda \cos n\lambda} = 0$. Thus across one latitude at one pressure, in this simple model, the geostrophic transports of angular momentum and heat each depend upon the trough and ridge orientation and one other factor, namely $\tfrac{1}{2} n^2 A^2$. Since this facter is always positive, the signs of the transports are completely determined by the orientation of the troughs and ridges.

The meaning of the other factor is evident from the relation

$$\overline{Z_\lambda^2} = \tfrac{1}{2} n^2 A^2 \tag{5}$$

Thus at one latitude and one elevation, the factor $\tfrac{1}{2} n^2 A^2$ is proportional to the geostrophic value of $\overline{v^2}$, which is the variance of v over longitude, and is a measure of the square of the horizontal exchange of mass across the

latitude. It closely resembles the square of the meridional index, which has been used (see Willett 1948) as a measure of the mass exchange. In addition, $\overline{v^2}$ is proportional to the kinetic energy contained in the north-south motions, which may be called simply the north-south kinetic energy.

Combination of (3) and (4) with (5) shows that

$$-\overline{Z_\lambda Z_\phi} = \overline{Z_\lambda^2}\, \varepsilon_\phi \quad , \tag{6}$$

$$-\overline{Z_\lambda Z_p} = \overline{Z_\lambda^2}\, \varepsilon_p \quad . \tag{7}$$

That is, across one latitude at one pressure, in this simple model, the geostrophic transport of angular momentum is proportional to the horizontal tilt of the troughs and ridges, and the square of the mass exchange, while the geostrophic transport of heat is proportional to the vertical tilt of the troughs and ridges, and the square of the mass exchange.

4. The general case

In general the height profile along a given "circle" is not a simple sine curve. It may, however, be expressed as a convergent Fourier series in λ if it is reasonably smooth. The instantaneous field of elevation may then be described by the expression

$$Z(\lambda,\phi,p) = Z_0(\phi,p) + \sum_{n=1}^{\infty} A_n(\phi,p) \cos n\left[\lambda - \varepsilon_n(\phi,p)\right] \quad . \tag{8}$$

For a particular n, the function $A_n \cos n(\lambda - \varepsilon_n)$ will be called the n^{th} component field of elevation. In contrast, the field of elevation described by (8) will be called the complete field of elevation. Equation (8) expresses the complete field as infinite sum of component fields, Except for the term Z_0, each component field resembles the field of elevation described by (1).

The troughs and ridges of the different component fields may have different orientations. For the n^{th} component field, the horizontal tilt is $\partial \varepsilon_n / \partial \phi$, and the vertical tilt is $\partial \varepsilon_n / \partial p$. No such simple expressions exist for the tilts of the troughs and ridges in the complete field. The equation for the troughs and ridges is

$$\sum_{n=1}^{\infty} A_n(\phi,p) \sin n\left[\lambda - \varepsilon_n(\phi,p)\right] = 0 \quad , \tag{9}$$

which has no simple solution analogous to (2) for λ in terms of ϕ and p.

Differentiation of (8) with respect to the coordinates, and integration over longitude, shows that

$$-\overline{Z_\lambda Z_\phi} = \sum_{n=1}^{\infty} \tfrac{1}{2} n^2 A_n^2\, \partial \varepsilon_n / \partial \phi \quad , \tag{10}$$

$$-\overline{z_\lambda z_p} = \sum_{n=1}^{\infty} \tfrac{1}{2} n^2 A_n^2 \; \partial \varepsilon_n / \partial p \quad , \tag{11}$$

$$\overline{z_\lambda^2} = \sum_{n=1}^{\infty} \tfrac{1}{2} n^2 A_n^2 \quad . \tag{12}$$

These results follow from the relations $\overline{\sin^2 n\lambda} = 1/2$,

$\overline{\sin n\lambda \sin m\lambda} = 0$ if $m \neq n$, $\overline{\sin n\lambda \cos m\lambda} = 0$

Thus across one latitude at one pressure, the geostrophic transports of angular momentum and heat depend upon the orientation of the troughs and ridges of each component field, and the north-south kinetic energy of each component field. Equation (12) shows that the north-south kinetic energy of the complete field is the sum of the north-south kinetic energies of the component fields.

The concept of average horizontal and vertical tilts of the troughs and ridges may now be introduced. The average horizontal tilt will be defined as the average over all values of n of the value of $\partial\varepsilon_n/\partial\phi$, weighted according to the value of $\tfrac{1}{2} n^2 A_n^2$, i.e., the average over all component fields of the horizontal tilt of the troughs and ridges of a component field, weighted according to the north-south kinetic energy of the component field. The average vertical tilt will be defined analogously. The symbols $(\varepsilon)_\phi$ and $(\varepsilon)_p$ will denote the average horizontal and vertical tilts so defined; thus

$$(\varepsilon)_\phi = \Big(\sum_{n=1}^{\infty} \tfrac{1}{2} n^2 A_n^2 \Big)^{-1} \sum_{n=1}^{\infty} \tfrac{1}{2} n^2 A_n^2 \; \partial \varepsilon_n / \partial \phi \quad , \tag{13}$$

$$(\varepsilon)_p = \Big(\sum_{n=1}^{\infty} \tfrac{1}{2} n^2 A_n^2 \Big)^{-1} \sum_{n=1}^{\infty} \tfrac{1}{2} n^2 A_n^2 \; \partial \varepsilon_n / \partial p \quad . \tag{14}$$

Unlike the symbols ε_ϕ and ε_p used in studying the simple model, $(\varepsilon)_\phi$ and $(\varepsilon)_p$ cannot be regarded as partial derivatives of a quantity (ε) ; there is no reason why $\partial(\varepsilon)_\phi/\partial p$ and $\partial(\varepsilon)_p/\partial\phi$ should be equal.

From (10), (11), (12), (13), and (14), it follows that

$$-\overline{z_\lambda z_\phi} = \overline{z_\lambda^2} \, (\varepsilon)_\phi \quad , \tag{15}$$

$$-\overline{z_\lambda z_p} = \overline{z_\lambda^2} \, (\varepsilon)_p \quad . \tag{16}$$

These equations are a generalization of (6) and (7). They show that, across one latitude at one pressure, the geostrophic transport of angular momentum is proportional to the average horizontal tilt of the troughs and ridges, and the square of the mass exchange, while the geostrophic transport of heat is proportional to the average vertical tilt of the troughs and ridges, and the square of the mass exchange.

The preceding conclusions of course depend upon the particular method of defining the average tilts. One may legitimately question how closely $(\varepsilon)_\phi$, the average horizontal tilt defined by (13), resembles the horizontal tilts of the

troughs and ridges which satisfy (9) i.e., the troughs and ridges of the complete field as distinguished from the troughs and ridges of the component fields. It is not easy to answer such a question.

In the first place, since the different troughs and ridges appearing on a synoptic map usually have different tilts, these tilts must be averaged in some way to obtain a quantity which may be compared with $(\epsilon)_\phi$. Although it may be obvious in certain cases which troughs and ridges are of major importance, and which are less important, it is not at all obvious how to assign objectively a numerical weighting factor to each one. In contrast, the weighting factors used in defining $(\epsilon)_\phi$ seem to be logically as well as objectively chosen, since those component fields having the most north-south kinetic energy, and hence contributing most strongly to the mass exchange, are presumably the most important ones. Perhaps one should be content to ask whether $(\epsilon)_\phi$ must lie between the extreme values of the horizontal tilts of the troughs and ridges of the complete field.

Examination shows that on purely theoretical grounds such a question must be answered in the negative, for one may artificially construct complete fields whose troughs and ridges all have north-south orientations, but where $(\epsilon)_\phi$ is not zero. Such a field is readily obtained by taking a field whose contour lines are simple sine curves, and deforming portions of the contours which are not too close to the troughs and ridges. But whether complete fields where $(\epsilon)_\phi$ does not lie between the extreme tilts of the troughs and ridges are actually found in the atmosphere, or whether they are purely artificial creations, is another question. Perhaps an extensive observational study of horizontal tilts is the only feasible method of answering it.

No such study has been undertaken here. Perhaps the nearest approach to such a study was made by Mintz (1949), who measured the horizontal tilts, at 35° N, of all trough and ridge lines appearing on analyzed daily 500 millibar maps for December 1945. Mintz found positive tilts in over eighty percent of the cases, and, weighting each trough and ridge equally, found a positive average horizontal tilt. At the same time, he found a northward transport of angular momentum across 35° N, implying a positive time average of $(\epsilon)_\phi$.

It does not seem unreasonable to suppose that $(\epsilon)_\phi$ usually resembles a suitable average value of the tilts in the complete field. In the absence of an entirely suitable objective method of averaging these tilts, it is possible that $(\epsilon)_\phi$ is the best obtainable measure of an average horizontal tilt of the troughs and ridges.

Similar remarks apply to a comparison of $(\epsilon)_p$ with the vertical tilts of the troughs and ridges of the complete field.

One might argue that $(\epsilon)_\phi$ and $(\epsilon)_p$ are nothing more than the quotients of quantities already known to be important, namely, the geostrophic transports of angular momentum and heat and the north-south kinetic energy. The writer feels that in view of the interpretation of $(\epsilon)_\phi$ and $(\epsilon)_p$ as average tilts,

these quantities are important in their own right, aside from their relation to the transports. If this is so, a study of the behavior of $(\varepsilon)_\phi$ and $(\varepsilon)_p$, including a study of their time variations, should be a worthwhile contribution to the study of the general circulation. Such a study, combined with a study of the behavior of the mass exchange, would also amount to an additional method for studying the transports of angular momentum and heat.

5. An ideal computation procedure

The simple expressions $-\overline{z_\lambda z_\phi}$ and $-\overline{z_\lambda z_p}$ occurring in table 1 suggest a simple procedure for computing instantaneous geostrophic transports of angular momentum and heat. In these computations, partial derivatives of z are replaced by finite differences. The necessary data consist of values of z at a three-dimensional network of points. These points are the intersections of specified sets of constant-longitude surfaces, constant-latitude surfaces, and constant-pressure surfaces.

A portion of a typical network appears in fig. 1. In this example the constant-longitude and constant-latitude surfaces are each spaced at intervals of five degrees. For reference certain intersections are designated by capital letters. Such expressions as $z(A)$ will denote such quantities as the value of z at the point designated by A in fig. 1.

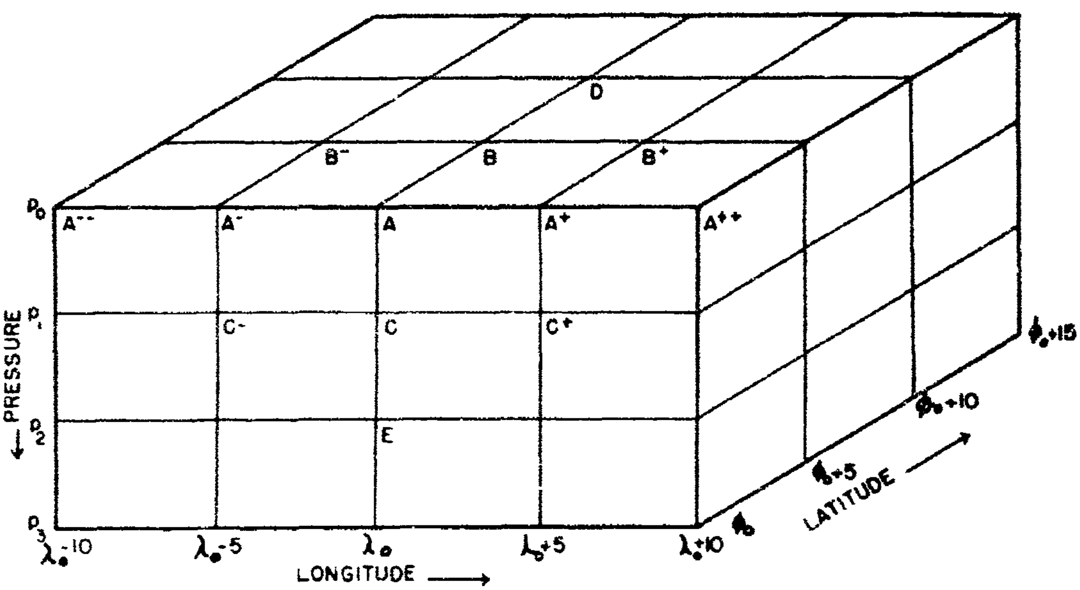

Fig. 1. A portion of a network of points used in computing the geostrophic transports of angular momentum and heat.

The quantity $v'(A) = z(A^+) - z(A^-)$ is related to the geostrophic value of v at A , or more precisely to the average geostrophic value of v between A^- and A^+, by a factor depending only upon latitude. Similarly,

the quantity $\mu''(A,B) = z(A) - z(B)$ is related to the average geostrophic value of μ between A and B by a factor depending only upon latitude, while the quantity $T''(A,C) = z(A) - z(C)$ is related to the average temperature between A and C by a factor depending upon latitude and pressure. The symbol $\sim$ between two quantities will be used to indicate that the quantities differ by a factor which is determined by the coordinates ϕ and p. Two quantities so related may be said to be measures of each other.

A summation sign Σ will always denote a sum over all longitudes; thus, for example,

$$\sum z(A) = \sum z(A^+) = \sum z(A^{++}) = \cdots$$
$$= z(A) + z(A^+) + z(A^{++}) + \cdots ,$$
$$\sum z(A)\, z(B^-) = \sum z(A^+)\, z(B) = \sum z(A^{++})\, z(B^+) = \cdots$$
$$= z(A)\, z(B^-) + z(A^+)\, z(B) + z(A^{++})\, z(B^+) + \cdots$$

The transport of heat is most conveniently measured within a layer, across one latitude. The geostrophic transport of heat across latitude ϕ_0 within the layer between pressures p_0 and p_1 may be measured by

$$-\overline{z_\lambda z_p}(A,C) \sim \sum \left[\tfrac{1}{2}\left(v'(A) + v'(C)\right) T''(A,C)\right]$$
$$= \sum \left[z(A)\, z(C^+) - z(A)\, z(C^-)\right]$$
$$= \sum z(A)\, v'(C) .$$

In an analogous way,

$$-z_\lambda z_\phi(A,B) \sim \sum \left[\tfrac{1}{2}\left(v'(A) + v'(B)\right) \mu''(A,B)\right]$$
$$= \sum \left[z(A)\, z(B^+) - z(A)\, z(B^-)\right]$$
$$= \sum z(A)\, v'(B) .$$

Unlike $-\overline{z_\lambda z_p}(A,C)$, the quantity $-\overline{z_\lambda z_\phi}(A,B)$ does not measure a transport within a layer, but rather the average northward geostrophic transport of angular momentum per unit pressure between latitudes ϕ_0 and $\phi_0 + 5$ at the single pressure p_0. It may, however, be regarded as a measure of the transport across latitude $\phi_0 + 2\tfrac{1}{2}$.

It should be noted that the square of the mass exchange across latitude ϕ_o, or the north-south kinetic energy per unit pressure at ϕ_o, may be measured by

$$\overline{Z_\lambda^2}(A) \sim \Sigma\,[v'(A)]^2$$
$$\sim \Sigma\,[z^2(A) - z(A)\,z(A^{--})]$$
$$= \Sigma\, z(A)\, v(A^-)$$

Thus, without actually calculating v' at any points of the network, one may measure $\overline{Z_\lambda^2}$, $-\overline{Z_\lambda Z_\phi}$, and $-\overline{Z_\lambda Z_p}$, each by the difference of two longitudinal sums of products of heights. Alternatively, each quantity may be measured by a single longitudinal sum of products of a height with a northward velocity v', if the values of v' are first obtained.

Since the transports of angular momentum across latitudes ϕ_o, ϕ_o+5, ϕ_o+10, ... rather than $\phi_o+2\frac{1}{2}$, $\phi_o+7\frac{1}{2}$, ... are often desired, the transport across ϕ_o+5, for example, may be measured by the sum

$$-\overline{Z_\lambda Z_\phi}(A,B) - \overline{Z_\lambda Z_\phi}(B,D) \sim \Sigma\,[z(A)v'(B) + z(B)v'(D)]$$
$$= \Sigma\,[(z(A) - z(D))\,v'(B)]$$

One might also use ten-degree rather than five-degree differences in z to measure u, say $u'(B) = z(A) - z(D)$. In this case

$$-Z_\lambda Z_\phi(B) \sim \Sigma\, v'(B)\, u'(B)$$
$$= \Sigma\,[(z(A) - z(D))\,v'(B)]$$

The two methods therefore yield the same result; each actually represents the transport across latitude ϕ_0+5 by an average transport between ϕ_0 and ϕ_0+10. Since only the former method also obtains the average transport between ϕ_0 and ϕ_0+5, the use of five-degree differences of z to measure u actually yields more information concerning the transport of angular momentum.

Analogous reasoning applies to the possibility of obtaining the transport of heat at pressure p_1 by means of an average temperature between A and E.

Since the use of five-degree differences of z to measure u and T yields more information then the use of ten-degree differences, it might seem that additional information could also be obtained by using five-degree differences of z to measure v, say $v''(A,A^+) = z(A^+) - z(A)$. But in this case,

$$-\overline{z_\lambda z_\phi}(A,A^+,B,B^+) \sim \sum \tfrac{1}{4}\left[v''(A,A^+) + v''(B,B^+)\right]\left[u''(A,B) + u''(A^+,B^+)\right]$$

$$\sim \sum \left[z(A)\,z(B^+) - z(A)\,z(B^-)\right],$$

which is the result obtained when v is measured by a ten-degree difference of z. Thus no additional information is obtained. An analogous result occurs when $-\overline{z_\lambda z_p}(A,A^+,C,C^+)$ is considered.

The ideal computation procedure is therefore as follows: One computes all sums of products of heights of the form $\sum z^2(A)$,

$\sum z(A)\,z(A^{--})$, $\sum z(A)\,z(B^+)$, $\sum z(A)\,z(B^-)$, $\sum z(A)\,z(C^+)$, and

$\sum z(A)\,z(C^-)$. One then takes the differences $\sum\left[z^2(A) - z(A)\,z(A^{--})\right]$,

$\sum\left[z(A)\,z(B^+) - z(A)\,z(B^-)\right]$, and $\sum\left[z(A)\,z(C^+) - z(A)\,z(C^-)\right]$

to represent $\overline{z_\lambda^2}$, $-\overline{z_\lambda z_\phi}$, and $-\overline{z_\lambda z_p}$. Alternatively, one may calculate v' at each point of the network, and then compute all sums of products of the form $\sum z(A)\,v'(A^-)$, $\sum z(A)\,v'(B)$ and $\sum z(A)\,v'(C)$. In no case will any labor at a later time be saved by first calculating u'' or T'' at the points of the network. The factors necessary to convert the sums of

products into the geostrophic transports of angular momentum and heat may be tabulated once and for all as functions of latitude and pressure alone.

The latter procedure, involving calculation of the values of v', probably requires less labor than any other procedure which could be based upon the same network of points. On the other hand, the sums of products of heights ($\Sigma z(A) z(B^-)$, etc.), obtained in the former procedure, may be interesting quantities in their own right. If the former procedure is chosen, it may be of interest also to obtain the sums of the form $\Sigma z(A) z(B)$, $\Sigma z(A) z(C)$, and perhaps certain others. These particular sums actually appear in expressions measuring the standard deviations of M and T. It should be noted that each sum is part of the numerator of a longitudinal correlation coefficient between heights. Possibly all sums of the form $\Sigma z(A) z(X)$, where X is any point near A in the network, are significant features of the general circulation. It would be interesting to observe how each sum varies with changes of the large-scale weather pattern.

PART II. THE BALANCE OF ANGULAR MOMENTUM DURING THE WINTER OF 1945-46

1. Computation procedure

The balance of absolute angular momentum in the atmosphère for the month of January 1946 was studied in detail by Widger (1949). Widger's results are included in an early report of the General Circulation Project (Report No. 3, contract W28-099 ac-406).

One of the principal current aims of the General Circulation Project has been the extension of Widger's study to a longer period of time, namely, the 120-day period 1 November 1945 through 28 February 1946. The desired computations have now been completed, and the results are presented here.

The writer wishes to emphasize that these results are not primarily his, nor were they obtained by any one person. Rather, they represent the combined work of persons associated at present or formerly with the General Circulation Project. The basic method for obtaining the results was proposed by Starr. The procedure for computing most of the quantities is essentially that developed and used by Widger, while the procedure for computing the mountain torque is the one developed and used by White (1949). Most of the computations (excepting January 1946) were performed by the Center of Analysis, M.I.T., largely by the use of punched-card machines. The writer's part has consisted mostly of assembling the computations.

As in the case of Widger's study, all computations explicitly involving the wind have been based upon computed geostrophic winds. As pointed out in part I, such a procedure automatically omits the effect of meridional cells. The omission of meridional cells from the computations is not to be

regarded as a proposition that meridional cells are unimportant. It merely reflects the impossibility of observing meridional cells from the available data. The consistency or inconsistency of the final results may perhaps be regarded as a measure of the necessity for assuming the existence of meridional cells to explain the balance of angular momentum.

It was almost inevitable that during the past two years, while the computations were being performed, a number of minor modifications should suggest themselves. Some of these modifications have been incorporated into the present computations. This section aims to present a brief but adequate description of the computation procedure, together with the principal numerical results. All deviations from the procedure used by Widger are discussed. Further details of the procedure are described in Report No. 3, contract W28-099 ac-406.

The absolute angular momentum contained in a unit mass of atmosphere may be expressed as

$$M = a^2 \cos^2\phi\, \omega + a \cos\phi\, u = M' + M'' \quad , \tag{17}$$

where $M' = a^2 \cos^2\phi\, \omega$ is the angular momentum due to the earth's rotation, which has been called ω-angular momentum, while the term $M'' = a \cos\phi\, u$ is the angular momentum of the motion relative to the earth's surface, which has been called relative angular momentum.

From the equation for eastward acceleration and the equation of continuity it appears that

$$\rho \frac{dM}{dt} = \frac{\partial(\rho M)}{\partial t} + \operatorname{div} \rho M \vec{c} = -\frac{\partial p}{\partial \lambda} - a \cos\phi\, \rho D, \tag{18}$$

where $\vec{c}$ is the velocity vector relative to the earth's surface, and D is the westward acceleration due to friction. If (18) is integrated over a volume V whose surface is S, there results the equation

$$\begin{aligned} &\frac{\partial}{\partial t}\int_V \rho M' \, dV + \frac{\partial}{\partial t}\int_V \rho M'' \, dV \\ &= \int_S \rho M' c_n \, dS + \int_S \rho M'' c_n \, dS - \int_V a \cos\phi\, \rho D \, dV - \int_V \frac{\partial p}{\partial \lambda} dV, \end{aligned} \tag{19}$$

where c_n is the component of $\vec{c}$ normal to S, directed into V. Equation (19) expresses a balance between the time rate of change of ω- and relative angular momentum contained in V, the transport of ω- and relative angular momentum into V, the total eastward frictional force on V, and the total eastward pressure gradient force on V.

It is convenient to choose for V a "ring-shaped" region whose boundary S consists of two vertical surfaces at latitudes ϕ_1 and ϕ_2, and two horizontal surfaces at elevations z_1 and z_2. In the event that such a

region intersects the earth's surface, however, V will be taken to be that portion of the volume occupied by atmosphere rather than solid earth. The four portions of V may conveniently be called the top, the bottom, and the north and south sides of V. The term "ring-shaped region" will always refer to a region of this sort.

Evidently the entire atmosphere may be expressed as an infinite sum of ring-shaped regions, and it may be expressed as a finite sum if $z_2 = \infty$ is allowable as an upper boundary of a region. Lack of worldwide data has limited the present study to the portion of the atmosphere north of 20° N and below 7.5 km. This portion has been divided into 36 ring-shaped regions. It is first divided into three layers: a lower layer extending from the earth's surface to 1.5 km, a middle layer from 1.5 to 4.5 km, and an upper layer from 4.5 to 7.5 km. Each layer is then divided into twelve ring-shaped regions: one bounded by 90° N and 75° N, and eleven bounded by latitudes five degrees apart, i.e., 75° N and 70° N, 70° N and 65° N, ..., 25° N and 20° N. Of course the northernmost region in each layer is actually disc-shaped rather than ring-shaped.

The angular momentum balance has been studied in each region individually, for the 120-day period 1 November 1945 through 28 February 1946. The data consist of individual sea-level pressures and 700- and 500- millibar heights, extracted from analyzed daily northern hemisphere maps, for each day of the period, at each five degrees of latitude and longitude, throughout the portion of the northern hemisphere covered by the analyses. The sea-level and 500-mb maps were taken from the Northern Hemisphere Historical Weather Map series (U. S. Air Weather Service 1945-46), while the 700-mb maps were photographic copies of maps analyzed by the U. S. Air Weather Service and the U. S. Weather Bureau.

Computed geostrophic winds at sea-level, 700 mb, and 500 mb were assumed to be representative of winds in the lower, middle, and upper layers, for the purposes of this study.

Since M', the ω- angular momentum per unit mass, depends only upon geographical position, the change and transport of ω- angular momentum are determined by the change and transport of mass. Moreover, because of continuity, the transport of mass across the boundary of a region must equal the change of mass within the region. It does not follow, however, that the transport of ω- angular momentum across the boundary equals the change of ω- angular momentum within the region, since the ratio of ω- angular momentum to mass varies with latitude. Thus, the total transport of mass into a small region is usually the small difference between two relatively large and nearly equal quantities, the inflow and outflow of mass. These usually occur at different latitudes, and must be multiplied by different latitude factors to yield the inflow and outflow of ω- angular momentum, whose difference is the transport of ω- angular momentum.

More precisely, since $\partial\rho/\partial t$ is the change of mass per unit volume, and ρc_n is the transport of mass across a unit area, the equation of continuity states that

$$\int_V \partial\rho/\partial t \; dV = \int_S \rho c_n \, dS \quad ,$$

while the change and transport of ω-angular momentum are given respectively by

$$\int_V a^2 \cos^2\phi \, \omega \, \partial\rho/\partial t \, dV \quad \text{and} \quad \int_S a^2 \cos^2\phi \, \omega \, \rho c_n \, dS \quad ,$$

which are in general not equal.

It will be convenient to use the term "cone-shaped region" to describe a region bounded by two latitudes, the earth's surface, and the top of the atmosphere. Evidently three ring-shaped regions, one in each layer, together with a ring-shaped region extending from 7.5 km to the top of the atmosphere, combine to form a cone-shaped region. Thus one may refer to the cone-shaped region which contains a given ring-shaped region.

The mass of a cone-shaped region may be measured approximately by the sea-level pressure p_o , i.e.,

$$\int \rho \, dV = \int g^{-1} p_o \, dS \quad ,$$

the latter integral extending over the bottom of the region. The mass change during a period may therefore be computed from the sea-level pressure profiles for the initial and final instants of the period. Table 2 presents the sea-level pressure profiles for 1 November 1945 and 28 February 1946, and the computed changes and transports of mass and ω-angular momentum during the period between these dates.

The ring-shaped regions under study do not extend from sea-level to infinity, but are bounded by two horizontal surfaces. The mass change in such a region, and hence the mass transport and the change of ω-angular momentum can be computed from the pressure changes on the top and bottom. It is not possible, however, to compute the transport of ω-angular momentum from pressure changes alone, since they do not reveal how much of the mass transport takes place vertically across the top and bottom, where the latitude is not constant.

Widger introduced the assumption that all the mass transport took place horizontally. He was then able to compute the horizontal transport of ω-angular momentum without difficulty. He pointed out, however, that such an assumption was not necessarily justified. In the present study no computations are made for the change and transport of ω-angular momentum, except for those already presented in table 2.

Table 2. Computation of the change and transport of mass and angular momentum, 1 November 1945 - 28 February 1946.

ϕ (degrees north)	$P_o(\phi)$ - 1000 mb, 1 November 1945	$P_o(\phi)$ - 1000 mb, 28 February 1946	change of $P_o(\phi)$	change of mass between ϕ and $\phi+5$	transport of mass across ϕ	change of ω-angular momentum between ϕ and $\phi+5$	transport of ω-angular momentum across ϕ	convergence of transport of ω-angular momentum between ϕ and $\phi+5$
	1 unit = 10^{-1} mb			1 unit = 10^{14} gm		1 unit = 10^{30} gm cm^2 sec^{-1}		
80	108	346	238					
75	040	295	255	211*	211*	2*	4	4
70	061	185	124	128	339	3	12	8
65	104	157	53	77	416	3	22	10
60	109	155	46	52	468	3	35	13
55	121	144	23	42	510	4	50	15
50	143	146	3	18	528	2	65	15
45	171	141	-30	-21	507	-3	75	10
40	197	139	-58	-74	433	-13	75	0
35	187	148	-39	-81	352	-16	71	-4
30	172	153	-19	-55	297	-12	66	-5
25	155	153	-2	-21	276	-5	67	1
20	143	149	6	4	280	1	73	6

*Change between 75° and 90° N

Table 3. Change of relative angular momentum, 1 November 1945 - 28 February 1946.

ϕ (degrees north)	Change of relative angular momentum between ϕ and $\phi+5$, 1 unit = 10^{30} gm cm^2 sec^{-1}		
	0 - 1.5 km	1.5 - 4.5 km	4.5 - 7.5 km
75	0*	0*	0*
70	-3	-3	-1
65	-3	-7	-4
60	0	-7	-8
55	-2	-3	-7
50	-3	-3	-1
45	-6	-6	-1
40	-6	5	4
35	5	11	14
30	7	14	17
25	8	22	21
20	5	-1	34

*Change between 75° N and 90° N

If the period of time under study were sufficiently long, the change of ω-angular momentum could of course be neglected, since there are no appreciable net long-period changes in the distribution of mass. The transport of ω-angular momentum into a ring-shaped region could also be neglected in the absence of any net mass inflow at one latitude accompanied by net mass outflow at another latitude, and hence in particular in the absence of mean meridional cells. Whether or not the change and transport of ω-angular momentum may properly be neglected in the study of a 120-day period, in comparison with the remaining terms of (19), will be considered in a later section.

The change of relative angular momentum may be written

$$\int_V a \cos\phi \; \partial(\rho u)/\partial t \; dV$$

Like the change of ω-angular momentum, it can be computed from data for the initial and final instants of the period. Values of $\partial(\rho u)/\partial t$, and hence of the change of relative angular momentum per centimeter of height, can be computed geostrophically from the sea-level pressure profiles. These values are then multiplied by 1.5×10^5 cm to obtain changes in the regions in the lower layer. Values of $\partial u/\partial t$, and hence of the change of relative angular momentum per unit of pressure, can be computed geostrophically from the height profiles at 700 mb and 500 mb. These values are multiplied by typical values of ρ at 700 mb and 500 mb to obtain changes per centimeter of elevation, and then by 3×10^5 to obtain changes in the regions in the middle and upper layers. The results appear in table 3.

In the computations for the middle and upper layers, $\partial(\rho u)/\partial t$ has been replaced by $\rho\, \partial u/\partial t$, so that density changes have been neglected. Since the percentage change of density is usually small compared to the percentage change of zonal wind speed, such a procedure seems justifiable. Of course, like the change and transport of ω-angular momentum, the entire change of relative angular momentum could be neglected if the period under study were sufficiently long.

The remaining terms in (19) must therefore balance each other over a long period of time, but there is no reason why each individual term should be small. Indeed, it was the observation that at individual latitudes the frictional drag of the earth's surface on the atmosphere does not drop out in the long run which led Jeffreys (1926) to consider the importance of the angular momentum balance. Computation of the remaining terms therefore necessarily requires data for the entire period under study.

The fourth term in (19) represents the transport of relative angular momentum into V. If V is a ring-shaped region, this transport consists of a horizontal transport through the sides and a vertical transport through the top and bottom. If geostrophic measurements are used, the horizontal transport into V is simply the difference of the geostrophic transports of

angular momentum across the bounding latitudes ϕ_1 and ϕ_2. An ideal method for computing this transport was described in part I. In practice the method must be modified slightly, since the layers under consideration are bounded by constant levels rather than constant pressures, and the data for the lower layer come from constant level rather than constant pressure maps. Thus, for the lower layer $-\overline{p_\lambda\, p_\phi}$ must be multiplied by ρ^{-1}, and for the other layers $\overline{z_\lambda\, z_\phi}$ must be multiplied by ρ, to obtain quantities which are measures of the transport within a layer one centimeter thick. Typical monthly normal values of ρ were used. Thus, the effect of longitudinal variations of density was neglected. An auxiliary study, described in an earlier report (final report, contract W28-099 ac-406), indicates that in middle latitudes this effect is not important.

Aside from these considerations, it must be mentioned that most of the computations had been performed before the ideal computation procedure was developed. The procedure actually used, which is the procedure developed by Widger, differs from the ideal computation procedure in that ten-degree rather than five-degree differences in z are used to measure values of u. However, it yields identical results; its only disadvantage is that it involves somewhat more labor.

At the time of Widger's measurements, preliminary investigations suggested that the geostrophic method yielded an underestimate of the transport of relative angular momentum. Widger's figures were therefore multiplied by the factor 1.5. Subsequent investigations have failed to verify the earlier results. Therefore, in the present computations, the factor 1.5 has not been included, and the figures actually represent the geostrophic transport. Values of the horizontal transport of relative angular momentum within the three layers appear in table 4.

Since the vertical transport of relative angular momentum into V involves the vertical wind speed, it cannot be computed directly from observational data. It may, however, be obtained from continuity considerations, under suitable assumptions.

The fifth term in (19) gives the total eastward torque upon V due to friction. The value of this torque depends upon how friction is defined. The wind components referred to as u, v, and w are presumably not the components of the velocities of individual molecules, but rather are velocities averaged over certain time intervals or throughout certain volumes. Such an averaging process conceals the individual molecular motions, and also conceals the eddies whose periods or sizes are smaller than the time intervals or volumes involved. Such eddies may then be regarded as turbulence, and friction may be regarded as consisting of molecular friction and turbulence.

A consideration of skin friction has led to the choice of the size of the eddies to be regarded as turbulence in the present study. A familiar formula expresses the friction force exerted on the atmosphere by a unit area of the

Table 4. Transport of relative angular momentum, 1 November 1945 - 28 February 1946

ϕ (degrees north)	Transport of relative angular momentum across latitude ϕ, 1 unit = 10^{30} gm cm^2 sec^{-1}		
	0 - 1.5 km	1.5 - 4.5 km	4.5 - 7.5 km
75	1	2	30
70	18	-3	9
65	40	-3	-20
60	33	-32	-52
55	14	-52	-75
50	-11	31	44
45	10	153	258
40	82	231	440
35	148	245	558
30	69	208	484
25	17	175	429
20	0	71	335

Table 5. Friction torque, mountain torque, and surface torque, 1 November 1945 - 28 February 1946

ϕ (degrees north)	Westward torque upon atmosphere between ϕ and ϕ+5, 1 unit = 10^{30} gm cm^2 sec^{-1}		
	friction torque	mountain torque	surface torque = friction torque+ mountain torque
75	-61*	-	-61*
70	-49	-	-49
65	-83	-81	-164
60	-116	-206	-322
55	-48	-58	-106
50	282	82	364
45	494	215	709
40	391	328	719
35	270	72	342
30	114	-145	-31
25	-334	-333	-667
20	-869	-244	-1113

*Torque between 75° N and 90° N.

earth's surface as

$$F = X \rho c^2 , \tag{20}$$

where c is the speed of the surface wind, and X is a dimensionless coefficient of skin friction. It seems desirable to review at this time the basis for this formula.

Formula (20) was originally obtained by Stanton (1913) for fluids flowing through a pipe, and its application to atmospheric flow is due to Taylor (1916). Under the assumption of steady conditions, Taylor (1915) first derived the theoretical relations

$$c/c_g = \cos \alpha - \sin \alpha , \tag{21}$$

$$H_1^2 = \frac{\mu}{\rho} \frac{1}{\omega \sin \phi} \left(\frac{3\pi}{4} + \alpha \right)^2 , \tag{22}$$

where c_g is the speed of the geostrophic wind, α is the angle between the surface wind and the geostrophic wind, H_1 is the height at which the wind direction first attains the geostrophic wind direction, and μ is the coefficient of eddy viscosity. By comparing pilot balloon observations with geostrophic winds measured from analyzed maps, Dobson (1914) obtained a set of observations of c_g, c, α, and H_1. Dobson's results enabled Taylor to verify (21), and to compute μ/ρ from (22), for various wind speeds. Later, Taylor (1916) derived the theoretical relation

$$F = 2 \mu c_g \sin \alpha \frac{\frac{3\pi}{4} + \alpha}{H_1} , \tag{23}$$

from which he computed F. From (20) he then computed X. He found no tendency for X to increase or decrease with increasing wind speed, and his values, ranging from 2.2×10^{-3} to 3.2×10^{-3}, compared favorably with Stanton's value of 4×10^{-3}. On the other hand, μ and α increased significantly with increasing wind speed.

The surface wind of Dobson's observations was actually the wind at the anemometer level of 30 meters. The observations were made in England over relatively smooth terrain. The eddies which affected his results thus may have had diameters of several meters or more, and may have resulted from such irregularities of the earth's surface as vegetation. In the present study, eddies with diameters of a few meters or less have been regarded as turbulence. With this choice, there is some basis for using formula (20) in the computations.

If only those eddies with diameters of a few meters or less are regarded as turbulence, the wind components u and v should include all larger eddies, and the horizontal transport of relative angular momentum should include the transport due to these eddies. The method of computation necessarily omits the transport due to horizontal eddies which are too small to appear on analyzed

northern hemisphere maps. It is believed, however, that most of the horizontal eddy transport of angular momentum is accomplished by the very large eddies, comparable in size with cyclones, or larger. If this assumption is correct, the omission of the horizontal transport due to somewhat smaller eddies is justified. It would appear equally justifiable to omit the horizontal transport due to eddies regarded as turbulence, i.e., to omit the torque due to friction on the north and south sides of V. The fifth term in (19) is then limited to a torque on the top and bottom of V. Only the torque due to skin friction has been computed. This torque will be called simply the friction torque.

Although much of the earth's surface is more than 1.5 km above sea level, it has been assumed for simplicity that the friction torque acts only upon the regions in the lower layer. The magnitude of the skin friction upon a unit area of the bottom of such a region has been assumed to be $\chi \rho (u^2 + v^2)$, with $\chi = 3 \times 10^{-3}$. This skin friction law has been assumed to hold over ocean and land; perhaps ocean waves are the irregularities associated with turbulence over the ocean. The eastward component of the skin friction is $-\chi \rho u (u^2 + v^2)^{1/2}$, so that the fifth term in (19) may be written

$$- \int_S a \cos\phi \; \chi \rho u (u^2 + v^2)^{1/2} \, dS ,$$

the integral extending over the portion of the earth's surface underlying V.

By a procedure analogous to the one used in computing the transport of relative angular momentum, values of $\overline{-p_\phi (p_\phi^2 + \cos^{-2}\phi \; p_\lambda^2)^{1/2}}$ were computed from the sea-level pressure values at latitudes 75, 65, 55, 45, 35, and 25° N. Ten-degree pressure differences were used as measures of p_ϕ and p_λ. These values, after multiplication by typical values of ρ^{-1}, are geostrophic measures of $\overline{\rho u (u^2 + v^2)^{1/2}}$, and hence of the frictional torque throughout a ten-degree latitude zone. Under the assumption that a typical surface wind speed is six tenths of the geostrophic wind speed, the geostrophically computed values were multiplied by 0.36. No departure of the direction of the surface wind from that of the geostrophic wind was assumed.

In Widger's computations, it was assumed that the value of $\overline{-p_\phi (p_\phi^2 + \cos^{-2}\phi \; p_\lambda^2)^{1/2}}$, computed for a latitude separating two ring-shaped regions, was representative of both adjacent regions. It is now felt that such a procedure yielded good values for the sum of the torques acting on the two regions, but that the apportionment of this sum between the two regions was not realistic. In particular, the computed torque was always numerically larger in the farther south of the regions, since the latitude factor increases southward.

It was found that the values of $\overline{-p_\phi (p_\phi^2 + \cos^{-2}\phi \; p_\lambda^2)^{1/2}}$ were very nearly proportional to the ten-degree pressure differences $-\overline{p_\phi}$, a result which is not very surprising, since the frictional torque is to a large extent

a measure of the strength of the low-level zonal flow. It was therefore felt that realistic results could be obtained by dividing the torque computed for a ten-degree zone into two parts proportional to the values of $-\cos^2\phi/\sin\phi \; \partial\bar{P}/\partial\phi$ in the two five-degree zones. Such a procedure yielded the values of the frictional torque which appear in table 5.

The final term in (19) must also vanish if V is a ring-shaped region which does not intersect the earth's surface. If, however, V is intersected by a mountain range, a difference between the surface pressures on the east and west sides of the range will result in a torque acting upon V. Such a torque will be called the mountain torque.

The method used to compute the mountain torque has been described in detail by White (1949). It seems sufficient to state here that the method involves the construction of simplified topographic profiles to represent the principal mountain ranges, and that the computations involve vertical interpolation and extrapolation from data at sea level and 700 millibars. No alterations have been made from White's method of computation.

As in the case of the friction torque, the mountain torque was assumed to act entirely upon the regions in the lower layer. Computed values of the mountain torque appear with the values of the friction torque in table 5. The sum of the friction and mountain torques will be called the surface torque; this torque also appears in table 5.

2. Diagram of the angular momentum balance

The computations presented in tables 2, 3, 4, and 5 make it possible to construct a picture of the angular momentum balance for the northern hemisphere. Such a picture is presented in the form of a diagram containing streamlines of angular momentum. The basis for such a diagram is described in this section.

The net change of ω- and relative angular momentum within any ring-shaped region must equal the net flow of angular momentum into the region, provided that the surface torque is regarded as a flow of angular momentum through the earth's surface. The surface torque constitutes the only flow through the bottom of a region in the lower layer, since no mass flows through the earth's surface. The outflow through the top of such a region can thus be determined as the net inflow through the bottom and sides, minus the net change within the region. Since the flow through the top of a region in the lower layer is also the flow through the bottom of a region in the middle layer, the same procedure determines the flow through the top of a region in the middle layer, and then through the top of a region in the upper layer.

A study of tables 2, 3, 4, and 5 shows that the change of relative angular momentum in each region in the lower layer tends to be small compared with the flow through the bottom or through a side of the region. Thus the computed vertical flow through the top would not be significantly affected if

the change of relative angular momentum were neglected. In turn, it appears that changes of relative angular momentum within regions in the middle and upper layers may be neglected in the corresponding computations.

The change of ω-angular momentum has been computed only for cone-shaped regions. The change within a cone-shaped region is evidently very small compared to the flow through the bottom of such a region. If it may be assumed that the change within a ring-shaped region is not large compared to the change within the cone-shaped region which contains the ring-shaped region, it should be proper to neglect changes of ω-angular momentum in computing the vertical flow.

Finally, although the horizontal transport of ω-angular momentum across some latitudes is moderately large, the differences between the transports across neighboring latitudes, appearing in the final column of table 2, are small. Transport of ω-angular momentum may therefore be neglected in computing vertical flow, if it is proper to assume that the net transport across the sides of a ring-shaped region is not large compared to the net transport across the sides of the cone-shaped region which contains the ring-shaped region. Such an assumption would appear reasonable if mean meridional cells are absent.

It thus appears that in studying the particular 120-day period chosen, it is permissible to neglect the three terms in (19) which would automatically have been negligible if the period had been sufficiently long. Apparently this result was not obtained simply because of the particular 120-day period selected. According to table 2, the sea-level pressure profile for 1 November 1945 was typical of a "high-index" situation, while the profile for 28 February 1946 was typical of a "low-index" situation. This index change shows that there were large net redistributions of mass during the period, and suggests that there were also large redistributions of eastward velocity. It thus seems rather unlikely that for a 120-day period selected at random, the change and transport of ω-angular momentum and the change of relative angular momentum would greatly exceed that for the particular period selected.

It might also be mentioned that if the latter half of February had been omitted from this study, the first three terms of (19) would have been greatly altered, since the index change during the latter half of February was nearly as extreme as the net index change during the 120-day period. At the same time, no major changes in the last three terms of (19) would result from omitting the latter half of February.

It therefore seems safe to state that if for any 120-day period the average values of the transport of relative angular momentum and the surface torque differ greatly from their normal values, the anomaly does not occur simply because the change and transport of ω-angular momentum and the change of relative angular momentum during this period are large. Rather, large anomalies in the transport of relative angular momentum accompany large anomalies in the surface

torque. Of course, these conclusions do not apply to shorter periods, say, periods of a week or two.

For the 120-day period under study, the vertical flow of ω-angular momentum through the bottom and top of each region has been computed, the change and transport of ω-angular momentum and the change of relative angular momentum being neglected. The results appear in table 6.

Table 6. Vertical flow of angular momentum, 1 November 1945 - 28 February 1946

ϕ (degrees north)	Upward flow of angular momentum between ϕ and ϕ+5, 1 unit = 10^{30} gm cm^2 sec^{-1} across surface	across 1.5 km	across 4.5 km	across 7.5 km
75	61	62	64	74
70	49	66	61	60
65	164	186	186	157
60	322	315	286	254
55	106	87	67	44
50	-364	-389	-306	-187
45	-709	-688	-566	-352
40	-719	-647	-569	-387
35	-342	-276	-262	-144
30	31	-48	-85	-159
25	667	615	582	527
20	1113	1096	992	898

Table 7. Stream function for flow of angular momentum, 1 November 1945 - 28 February 1946

ϕ (degrees north)	Stream function ψ at latitude ϕ and elevation z, 1 unit = 10^{30} gm cm^2 sec^{-1} $z = 0$	z = 1.5 km	z = 4.5 km	z = 7.5 km
75	-61	-62	-64	-74
70	-110	-128	-125	-134
65	-274	-314	-311	-291
60	-596	-629	-597	-545
55	-702	-716	-664	-589
50	-338	-327	-358	-402
45	371	361	208	-50
40	1090	1008	777	337
35	1432	1284	1039	481
30	1401	1332	1124	640
25	734	717	542	113
20	-379	-379	-450	-785

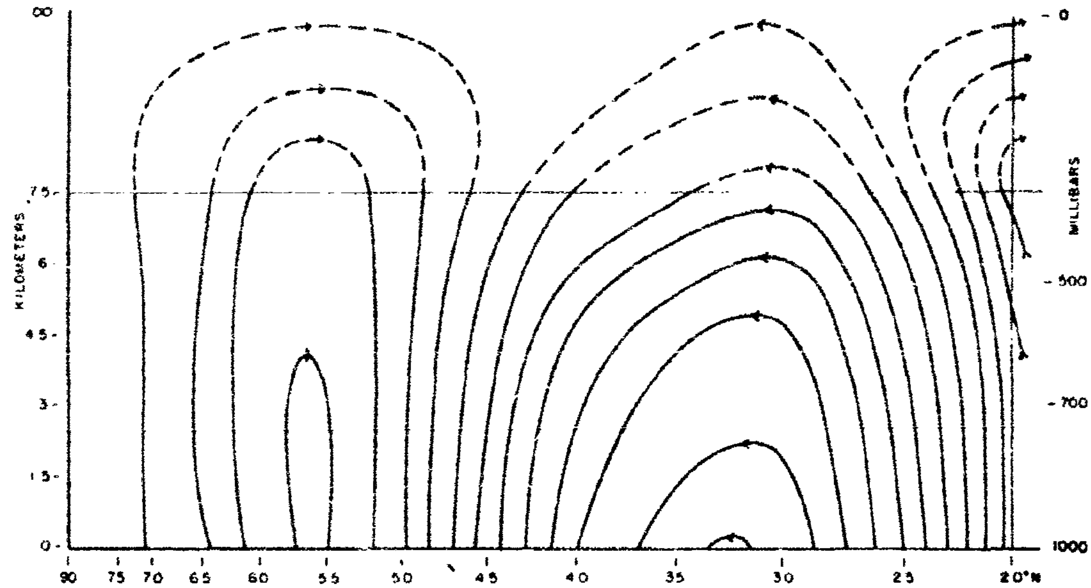

Fig. 2. Streamlines of angular momentum for the period 1 November 1945 through February 1946. Each line represents a total flow of 200 x 10^{30} gm cm^2 sec^{-1} during the period, or an average flow of approximately 2 x 10^{25} gm cm^2 sec^{-2}. Torques exerted on the atmosphere by the earth are treated as flow of angular momentum from the earth to the atmosphere.

It is now possible to construct a diagram of the momentum balance of the northern hemisphere, for the period 1 November 1945 through 28 February 1946, using the figures in tables 4 and 6. The omission of the changes of angular momentum makes a simple construction process possible, since one can then introduce a "stream function" ψ, whose isopleths are lines of flow of angular momentum. The stream function at any point simply equals the total flow of angular momentum across any simple curve connecting this point to the north pole, the stream function being positive if this flow is downward or southward. At latitude ϕ and elevation z, ψ is most conveniently computed as the flow across a line extending from the north pole to latitude ϕ at sea level, and then from sea level to elevation z at latitude ϕ. Table 7 contains the computed values of ψ.

Fig. 2 is the desired diagram of the balance of angular momentum. The streamlines in fig. 2 are isopleths of ψ. Each line represents a flow of 200 x 10^{30} gm cm^2 sec^{-1} during the 120-day period. Since 120 days = 1.0368 x 10^7 seconds, each line may also be regarded as representing an average flow of 2 x 10^{25} gm cm^2 sec^{-2}.

It seems desirable to summarize here the principal assumptions which were made to obtain this picture of the balance of angular momentum. First, and perhaps most significant, any flow of angular momentum due to mean meridional circulations has been omitted. A rather weak meridional cell could transport large amounts of ω- angular momentum, although it could yield no appreciable total transport across any latitude. A stronger meridional cell could also transport large

amounts of relative angular momentum, and could yield a significant total transport. Thus the inclusion of meridional circulations could greatly alter the appearance of fig. 2.

Next, quantities which over sufficiently long periods of time must be unimportant have been omitted. These quantities include the changes of ω-and relative angular momentum. They also include all horizontal and vertical flow of ω-angular momentum, which in the long run could result only from mean meridional circulations.

Finally, the transport of relative angular momentum has been replaced by what is essentially the geostrophic transport of geostrophic angular momentum.

Of course, other less drastic assumptions were also necessary. These assumptions were described in the preceding section. Some of them will be discussed further in the following section.

3. Discussion of the angular momentum balance

It need hardly be mentioned that fig. 2 cannot be regarded as the final word concerning the balance of angular momentum, during the winter in the northern hemisphere. In the first place, the length of the period studied was only 120 days. This period seemed to be sufficiently long to eliminate the effect of the quantities which would disappear in the long run, namely, the changes of ω-and relative angular momentum and the transport of ω-angular momentum. However, during a period of 120 days, or presumably even during a period of several years, large anomalies may be expected to exist in the transport of relative angular momentum, and in the surface torque. In the second place, certain approximations entering the computation procedure may affect the diagram as greatly as any anomalies which may be present. Some of these approximations will be discussed in this section.

Since fig. 2 consists of isopleths of the stream function ψ, all features of the diagram may be discussed as features of the field of ψ. At most points of the diagram the computed value of ψ depends upon computations of the transport of relative angular momentum and the surface torque, but at certain points some of these computations do not affect the computed value of ψ.

First, ψ must be zero over the north pole at all elevations, and at the top of the atmosphere at all latitudes, because there can be no flow of angular momentum across the north pole from one latitude to another, nor across the top of the atmosphere. The vanishing of ψ at these points is evidently independent of any computations. An obvious result is that the stream function at any point, say at latitude ϕ and elevation z, equals the total flow of angular momentum above the point northward across latitude ϕ, and also equals the total flow of angular momentum north of the point downward across elevation z.

Next, the computed values of ψ at the earth's surface are completely determined by the computations of the surface torque, and do not depend upon the

transport of relative angular momentum. In addition, they are not affected by the omission of mean meridional cells. The general features of the momentum diagram can to a large extent be inferred from the values of ψ at the earth's surface; thus it is important to determine these values as precisely as possible.

According to the computations, at sea level ψ is negative between 90°N and 48° N, with a numerical maximum near 56° N; positive between 48° N and 22° N, with a maximum near 33° N; and negative south of 22° N. The streamlines leaving the earth north of 22° N therefore remain in the northern hemisphere, and return to the earth in middle latitudes, while those leaving the earth south of 22° N presumably cross the equator and return to the earth in the middle latitudes of the southern hemisphere, there being nowhere else for them to go. As pointed out in the final report, contract W28-099 ac-406, the prevailing westerlies of the southern hemisphere are much stronger than those of the northern hemisphere at all seasons of the year, while the southern hemisphere trade winds are no stronger than those of the northern hemisphere. It is thus probable that the outflow of angular momentum (from the atmosphere to the earth) in the southern hemisphere westerly belt is not completely balanced by the inflow in the southern hemisphere easterlies, and must be balanced in part by the inflow in the northern hemisphere easterlies, the latter inflow evidently being more than sufficient to balance the outflow in the northern hemisphere westerlies. Nevertheless, it seems surprising that the latitude separating the streamlines which remain in the northern hemisphere from those which cross the equator should be as far north as 22° N. It is therefore desirable to examine more closely the assumptions made in computing the surface torque, to see whether any modifications are indicated.

The friction torque will be considered first. The assumptions $F = \kappa \rho c^2$ (formula (20))and $c/c_g = 0.6$ were used in these computations. The studies by Taylor (1915, 1916) which led to formula (20) also showed that c/c_g should vary from 0.7 for light winds to 0.6 for strong winds, and Dobson's study (1914) confirmed this conclusion. Thus one modification is suggested. However, it is likely that the computed torques would not have been greatly altered if the ratio c/c_g had been increased some ten or twenty per cent for the lighter winds.

On the other hand, Taylor's studies were based upon observations taken at only one latitude. Further examination of Taylor's work suggests the possibility of a systematic variation of c/c_g with latitude. Elimination of F, c_g, and H_1 from equations (20), (21), (22), and (23), which are due to Taylor, yields the relation

$$\sin^{1/2}\phi \, \frac{\sin\alpha}{\cos\alpha - \sin\alpha} = \frac{1}{2}\kappa c \left(\frac{\mu}{\rho}\right)^{1/2} \omega^{1/2} \quad . \qquad (24)$$

It seems reasonable to assume that μ depends upon such features as wind speed, vertical stability, and the nature of the underlying surface, rather than upon latitude. If μ/ρ does not depend explicitly upon latitude, the right hand side of (24) is independent of latitude, although it depends upon such quantities as wind speed. Thus, for a given surface wind speed, the angle α , and hence c/c_g , should vary with latitude.

If it is assumed that $c/c_g = \cos\alpha - \sin\alpha = 0.6$ when $\phi = 50°$ (approximately the latitude of Dobson's observations),

the relation $\sin^{1/2}\phi \sin\alpha (\cos\alpha - \sin\alpha)^{-1} = \frac{1}{2}$,

or equivalently

$$\cot\alpha = 1 + 2\sin^{1/2}\phi \quad , \tag{25}$$

is found to hold very closely. Thus α and c/c_g show little variation at high latitudes, but c/c_g falls off rapidly as the tropics are entered, as is shown in table 8, which is based upon equations (25) and (21). Although these results depend upon theory, rather than observations at different latitudes, it seems desirable to assume some sort of variation of c/c_g with latitude in any study involving tropical regions.

Table 8. Corresponding values of ϕ , α , and c/c_g , derived theoretically.

ϕ (deg.)	90	60	50	40	30	20	10	0
α (deg.)	18.4	19.3	20.0	21.0	22.5	24.7	28.6	45.0
c/c_g	0.63	0.61	0.60	0.57	0.54	0.49	0.40	0.00

It is evident that if the values of c/c_g appearing in Table 8, rather than the constant value $c/c_g = 0.6$, had been used in the computations, the computed friction torque in the trade wind region would have been smaller, and the latitude separating the streamlines remaining in the northern hemisphere from those crossing the equator would have appeared slightly south of 22° N.

Whatever the effect of a latitudinal variation of c/c_g may actually be, there is another phenomenon whose effect at most latitudes may be much greater. The friction torque is supposedly the torque resulting from small-scale irregularities of the earth's surface, such as ocean waves and vegetation. The mountain torque represents the torque resulting from only the largest irregularities of the earth's surface, since it was computed on the basis of pressure values taken from subjectively smoothed analyses of northern hemisphere

maps, and the major mountain ranges were represented by simplified topographic profiles. Thus neither the friction nor the mountain torque includes the effect of irregularities of intermediate size, i.e., hills and small mountains, and some of the finer details of large mountains.

Table 5 shows that at most latitudes the computed friction and mountain torques are of approximately the same magnitude. Since both small and large irregularities can give rise to appreciable torques, it seems possible that irregularities of intermediate size can also give rise to an appreciable torque. Such a torque may conveniently be called the hill torque.

Table 5 also shows that at most latitudes the friction and mountain torques act in the same direction. It thus seems not unreasonable to propose the hypothesis that at most latitudes the hill torque acts in the same direction as the friction and mountain torques, and is of the same order of magnitude. The surface torque, previously described as the sum of the friction and mountain torques, must now include the hill torque also, provided that the hill torque really exists.

If this hypothesis is true, it follows that at most latitudes the flow of angular momentum from the earth to the atmosphere, whether positive or negative, must have been underestimated in magnitude in the computations. In the absence of actual observations of the hill torque, it would be difficult to state just how large this underestimate might be. It may be justifiable to increase the computed surface torques in middle and high latitudes by a factor of 1.5, or perhaps considerably more.

Under the assumption that the hill torque is roughly proportional to the friction torque, one might be tempted to include the hill torque in the computations simply by increasing the value of χ , the coefficient of skin friction, in formula (20). A more intriguing procedure, however, consists of retaining the value $\chi = 3 \times 10^{-3}$, and redefining the surface wind speed c as the speed above the irregularities giving rise to the hill torque, i.e., above the hill tops. At this level the wind is supposed to approximate the geostrophic wind. The assumption that $c/c_g = 1.0$ rather than 0.6 would more than double the computed friction torque, and so might perhaps just include the friction torque and the hill torque. Of course this procedure requires some refinements; it would be difficult, for example, to justify it in a region devoid of hills. The procedure also cannot be expected to include the mountain torque, since, unlike the smaller hills, the major mountains extend to elevations where the geostrophic wind differs greatly from the sea-level geostrophic wind.

It seems reasonable to assume that the hill torque is most important at those latitudes where the earth's surface consists largely of land rather than ocean. The fraction of the earth's surface consisting of land is high at latitude 60° N and 50° N, but this fraction decreases rapidly southward from 50° N into the tropics. It may therefore be that, relative to the friction

torque, the hill torque is larger within the prevailing westerly belt than within the trade wind belt. Such an assumption would lead to the conclusion that the latitude separating the streamlines which cross the equator from those which remain in the northern hemisphere should be farther south than the computed latitude 22^0 N.

The following modification of the computed stream function at sea level is therefore suggested: As before, ψ is negative between 90^0 N and about 48^0 N, with a numerical maximum near 56^0 N, but this numerical maximum is larger than before; as before, ψ increases from zero near 48^0 N to a positive maximum near 33^0 N, but the maximum is larger than before; ψ then decreases to negative values at low latitudes, but crosses the zero point considerably south of 22^0 N, instead of at 22^0 N, as previously computed.

It must be emphasized that the proposed hypothesis concerning the hill torque is a mere speculation; no measurements of the hill torque have been undertaken. No account has been taken of the possibility that in hilly regions the wind at the anemometer level is greatly diminished, or altered in direction, in which case the hill torque might merely replace the friction torque instead of augmenting it. The writer feels, however, that some sort of hill torque exists, and feels that the speculations presented here are as reasonable as any other equally simple speculations which might be proposed.

As mentioned previously, the computed values of ψ at sea level together with the zero values of ψ at the top of the atmosphere, determine to a large extent the general appearance of the angular momentum diagram. They do not, of course, determine most of the details, which depend in addition upon the computed transport of relative angular momentum. The high values of ψ at sea level must decrease toward zero with elevation. Although there is no *a priori* necessity for this decrease to occur within the troposphere, one might reasonably expect a significant portion of the decrease to occur within the portion of the diagram covered by observations, i.e., one might expect a significant portion of the necessary transport of angular momentum to occur below 7.5 km. A test for consistency of the computed transport of relative angular momentum with the computed surface torque should then consist of determining whether ψ is in general numerically smaller at 7.5 km than at sea level.

Table 7 shows that according to this test the computations are consistent with each other; there are only a few latitudes (notably 20^0 N) where the magnitude of the stream function fails to decrease with elevation. At 35^0 N and 30^0 N, where ψ is largest at sea level, more than half of the necessary decrease occurs below 7.5 km.

If the suggested modification of the surface torque is introduced, while the computed transport of relative angular momentum is unaltered, consistency still prevails, i.e., ψ still decreases with elevation at most latitudes. Indeed, ψ may now decrease even at 20^0 N, if the zero point of ψ at sea level is displaced south of 20^0 N. At 35^0 N and 30^0 N, somewhat less than half of the

decrease of ψ , i.e., somewhat less than half of the transport of relative angular momentum, may now occur below 7.5 km. This modification is probably desirable, since there seems to be evidence that large transports of relative angular momentum may occur in the vicinity of the jet stream, near the tropopause (Starr 1951, Starr and White 1951, Mintz 1951, see also Report no. 5 of this project.)

It may be concluded that the computed surface torque and the computed transport of relative angular momentum combine to form a picture of the angular momentum balance of the northern hemisphere, which by and large is reasonable. A few features which seem perhaps unreasonable may be eliminated by introducing the suggested modification of the surface torque. The presence of the surface torque necessitates a horizontal transport of angular momentum, and this necessary transport is closely approximated by the computed horizontal transport of relative angular momentum, together with the transport of relative angular momentum above the level of computations which can be expected to accompany the computed transport.

PART III. THE TRANSPORT OF SENSIBLE HEAT DURING THE WINTER OF 1945-46

In Report no. 3 of this project, values were presented for the northward flow of sensible heat (enthalpy) during the winter of 1945-46. At a somewhat later date the ideal computation procedure described in part I was developed. Theoretically the ideal procedure and the procedure actually used should have yielded identical results, but a check revealed occasional large discrepancies and frequent small ones. It soon became evident that the discrepancies arose from a rounding-off process involved in the former procedure; in the ideal procedure such rounding off is unnecessary. Since the ideal procedure is relatively simple to use, it was decided to use it to recompute the transport of heat. The newly computed values may conveniently be presented at this time.

The transport of heat within the layers 1013 mb to 700 mb, and 700 mb to 500 mb, was computed. The data were the same as those used in computing the balance of angular momentum. As was the case when the transport of relative angular momentum was computed, a slight modification of the ideal procedure was necessary, since the available data included sea-level pressures rather than 1013-mb heights. The simple assumption that a one-millibar difference in sea-level pressure is equivalent to a 27.5-foot difference in 1013-mb height was introduced. The value of p at sea level then replaced the value of z at 1013 mb in the ideal computation procedure, and the factor 27.5 was introduced at the end of the computations, along with the other necessary conversion factors.

Such a procedure probably affords a fair estimate of the transport of heat between 1013 mb and 700 mb, but it does not take into account the transport within the layer of variable, and sometimes negative, thickness between sea level and 1013 mb. As pointed out in Report no. 3, there is some reason for believing that the omission of this term is not too serious.

Like the previous computations of sensible heat, the present computations were performed for the most part by the Center of Analysis, M.I.T., by means of punched-card machines. The computed values are presented in table 9. In this table, one unit equals 10^{19} calories. For the four-month period, one unit may also be regarded as representing an average flow of 10^{12} calories per second.

Table 9. Transport of sensible heat during the winter of 1945-46

Layer	ϕ	Transport of sensible heat across latitude ϕ within specified layer. 1 unit = 10^{19} calories				
		Nov 1945	Dec 1945	Jan 1946*	Feb 1946	Total*
700-500 mb	75	5	6	-	0	11**
"	65	24	23	27	15	89
"	55	49	59	45	43	196
"	45	44	45	53	37	179
"	35	19	29	28	20	96
1013-700 mb	75	-10	-1	-	10	-1**
"	65	38	46	75	58	217
"	55	116	180	168	165	629
"	45	96	209	130	173	608
"	35	50	97	35	77	259
1013-500 mb	75	-5	5	-	10	10**
"	65	62	69	102	73	306
"	55	165	239	213	208	825
"	45	140	254	183	210	787
"	35	69	126	63	97	355

* 5 days missing
** January missing

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A hemispherical study of the atmospheric angular-momentum balance

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Summary

An attempt is made to determine the meridional eddy transport of angular momentum for the northern hemisphere in the vicinity of 30°N. latitude from upper-wind observations for a period of six months. Two somewhat different methods of evaluation give essentially comparable results and indicate the presence of eddy transports which are ample to compensate for frictional losses of angular momentum farther north. No indications of the presence of a mean meridional circulation of the strength and character needed to produce a sensible northward transport of angular momentum are found.

1. Introduction

In a previous communication the first author of the present paper (Starr 1951) reported the results of a preliminary study of the eddy transport of angular momentum across various latitude circles, evaluated from upper-wind data over the North American sector of the northern hemisphere for the month of February 1949. As was therein pointed out, the subject is of such basic importance for the systematic investigation of the processes of the general circulation that no effort should be spared to explore further the mechanisms which may be operative in the atmosphere to effect the needed transfer of angular momentum. Accordingly, a second provisional investigation of greater scope in certain respects was entered upon and is presented below.

The pilot study already mentioned is deficient in many important respects. Although the lack of proper observations is one of the most serious limiting factors in any endeavour to eliminate these deficiencies, it appears that by concentrating attention on the zonal belt in the vicinity of 30°N. latitude existing data permit rough extensions in the following directions: (a) Although the network of radio wind-stations becomes sparse, the inclusion of relatively few pilot-balloon stations makes it possible to work with a latitude zone completely encircling the earth. (b) The limitation to a single latitude zone makes possible the evaluation of data over a much longer period of time without a proportional increase in the labour involved. It was thus practical to deal with mean conditions over a period of six months. (c) The consideration of a complete latitude belt renders it possible to examine the data in regard to direct indications concerning the nature of mean meridional circulations. (d) With relatively large amounts of data it was found desirable to apply statistical tests of significance to the results. Such tests probably do not furnish infallible criteria of the soundness of conclusions, but nevertheless may be considered as supporting evidence in those cases where they are fulfilled. They may thus be looked upon more as giving necessary rather than sufficient conditions for acceptance of results.

The latitude zone in the vicinity of 30°N. is of special interest for studies of the angular-momentum balance of the atmosphere. Since this zone is roughly the dividing line between the surface westerlies to the north and surface easterlies to the south, the largest meridional transports of angular momentum in the northern hemisphere should be found here, on the average.

2. Observational material

The string of 16 key stations used for the study is listed in Table I. Since reports were often missing from the key stations, it was decided that alternative stations in the vicinity of certain of the key stations should be used in order to increase the amount of observational material. These alternative stations are entered in italics below the key stations in the table.

TABLE I. List of key stations (numbered) and alternate stations (italics) used in study

Station	Latitude* (N.)	Longitude	Altitude (ft)	Type
1. Qrendi	35° 50′	14° 27′ E.	442	radio wind
2. Bahrein	26 16	50 38 E.	1	,,
3. Hyderabad	25 23	68 25 E.	95	pilot balloon
Fort Sandeman	31 21	69 27 E.	4613	,,
Peshawar	34 01	71 35 E.	1165	,,
Ambala	30 23	76 46 E.	892	,,
Bikaner	28 00	73 18 E.	735	,,
Juhdpur	26 18	73 01 E.	735	,,
Ahmedabad	23 02	72 35 E.	164	,,
4. Dibrugarh	27 28	95 55 E.	384	,,
Tezpur	26 37	92 47 E.	259	,,
Cooch Bihar	26 20	89 27 E.	157	,,
Asansol	23 41	86 59 E.	413	,,
Gaya	24 45	84 57 E.	364	,,
Calcutta	22 32	88 20 E.	20	,,
Gorakhpur	26 45	83 22 E.	253	,,
5. Tokyo	35 33	139 46 E.	37	radio wind
Kagoshima	31 34	130 33 E.	18	,,
Itazuke	33 35	130 27 E.	20	,,
6. Midway	28 13	177 21 W.	0	,,
7. Honolulu	21 20	157 55 W.	15	,,
8. Weather Ship (a)	30 00	140 00 W.	0	,,
9. Santa Maria	34 56	12 5 W.	238	,,
10. Big Spring	32 14	101 30 W.	2537	,,
El Paso	31 48	106 24 W.	3916	,,
11. New Orleans	30 00	90 16 W.	30	,,
Lake Charles	30 13	93 09 W.	32	,,
12. Miami	25 49	80 17 W.	12	,,
Tampa	27 58	82 32 W.	11	,,
13. Kindley Field	32 22	64 40 W.	16	,,
14. Weather Ship (b)	35 00	40 00 W.	0	,,
Weather Ship (c)	34 00	52 00 W.	0	,,
15. Lagens	38 45	27 05 W.	171	,,
16. North Front	36 09	05 21 W	26	,,

* Average latitude of key stations 31°

Wind reports for the levels 2, 6, 10, 14, 20, 25, 30, 35, 40, 45, and 50 thousand ft were taken from the data tabulations of the *Daily Series Synoptic Weather Maps* prepared by the U.S. Weather Bureau, in cooperation with the Army, Navy and Air Force for the period from 1 Feb. 1949 to 31 July 1949 for the hour 0300 GMT for each day. The frequency of available observations at the various levels is given in Table II.

TABLE II. PERCENTAGE OF TOTAL OBSERVATIONS WHICH ARE AVAILABLE BY STATION AT EACH LEVEL

	Elevation in thousands of ft										
Station	2	6	10	14	20	25	30	35	40	45	50
Qrendi	3·4	3·7	3·2	3·7	3·2	3·4	4·3	1·5	2·2	1·2	1.6
Bahrein	4·1	4·2	3·6	4·1	3·6	3·5	4·4	0·7	5·6	0·6	7·6
Hyderabad	8·0	0·0	7·7	0·0	7·3	4·8	2·2	0·2	0·0	0·0	0.0
Dibrugarh	8·0	0·0	6·2	0·0	3·4	2·2	1·2	0·0	0·0	0·0	0·0
Tokyo	8·2	8·7	7·8	9·2	8·7	9·9	10·9	13·1	11·4	13·0	10·8
Midway	6·2	5·9	4·5	5·1	4·3	3·8	3·8	4·7	3·9	4·8	4·7
Honolulu	7·4	8·0	7·0	8·6	7·8	8·1	8·7	11·0	10·5	12·6	13·2
Weather Ship (a)	7·1	7·3	6·3	7·4	6·5	6·7	6·7	7·3	6·2	7·1	6·3
Santa Maria	8·2	8·9	7·8	9·5	8.7	9·6	9·9	12·4	13·3	15·2	16·3
Big Spring	0·0	8·9	7·7	9·4	8·2	8·8	8·5	9·9	9·9	11.2	10.6
New Orleans	8·0	8·9	7·6	9·0	8·2	8·1	7·8	9·4	8·4	7·8	5·8
Miami	8·3	9·0	7·9	9·4	8·6	9·5	9·7	12·3	11·9	13·1	11.0
Kindley Field	6·1	6·4	5·6	4·3	3·8	2·8	2·3	1·7	1·0	0·9	0.3
Weather Ship (b)	5·8	7·0	6·2	7·4	6·6	6·8	7·2	8·9	8·7	7·8	7.6
Lagens	6·1	6·6	5·6	6·5	5·7	5·7	5·6	5·9	4·0	3·6	1.0
North Front	5·0	6·3	5·3	6·4	5·3	6·1	7·1	1·1	3·1	1·2	3.4

Surface winds were excluded from the study, since they can be too easily rendered unrepresentative by local orographic influences. No attempt was made to correct or exclude any original data where doubtful even though in some cases such procedures might have improved the results. This course was followed in order to maintain objectivity in the computations. Each wind observation was resolved into the eastward component u and northward component v, tabulated in m sec^{-1}. From these quantities the transports of angular momentum were computed by methods discussed presently.

3. General considerations

The total meridional transport of absolute angular momentum across a latitude circle may be thought of as the sum of the transport of relative angular momentum and the transport of angular momentum due to the earth's rotation. Since there is no net meridional transport of mass in the atmosphere in the long run, the transports of angular momentum must be brought about by exchange processes. Furthermore, since the angular momentum due to the earth's rotation per unit mass is essentially constant with respect to elevation in the atmosphere at a given latitude, the net effect of all exchange processes must be negligible as far as this component is concerned. It therefore follows that we may omit this contribution and deal only with the transport of relative angular momentum.

Also, owing to the fact that the distance from the earth's axis varies percentually only very slightly with elevation in the atmosphere at the given latitude, the transport of relative angular momentum is given by the transport of relative linear momentum except for a constant factor. For these reasons the transport of relative linear momentum will be considered first.

The total transport of relative linear momentum τ across a given complete latitude circle at a given level over a time interval t_1 may be expressed as

$$\tau = \int_0^{t_1} \oint \rho uv dx dt \simeq \rho^* \int_0^{t_1} \oint uv dx dt = \rho^* \overline{[uv]} L t_1 \quad . \quad . \quad (1)$$

where ρ is the density, ρ^* is an appropriate average density, dx is an element of linear distance and the square brackets denote a space average over the length L of a complete latitude circle and the bar denotes a time average over the interval t_1. In (1) the assumption is made that at a constant level the effects of density variations may be neglected. The feasibility of making this assumption is considered in a subsequent section. The quantities u, v may be separated into the mean motion $[u]$, $[v]$ and the deviations from the means u', v' so that

$$u = [u] + u' \quad ; \quad v = [v] + v' \quad . \quad . \quad . \quad (2)$$

in which the square brackets again denote instantaneous space averages. It follows that

$$[uv] = [u]\,[v] + [u'v'] \quad . \quad . \quad . \quad . \quad (3)$$

If Eq. (3) is integrated over the given time interval t_1, and averaged, one obtains

$$\overline{[uv]} = \overline{[u]\,[v]} + \overline{[u'v']} \quad . \quad . \quad . \quad . \quad (4)$$

where the bars again denote time averages. In a manner analogous to that used in writing (2) it is possible to write expressions for $[u]$, $[v]$ of the form

$$[u] = \overline{[u]} + [u]' \quad ; \quad [v] = \overline{[v]} + [v]' \quad . \quad . \quad (5)$$

where $[u]'$, $[v]'$ denote deviations of the space means from the space-time averages. It follows from (1), (4) and (5) that

$$\frac{\tau}{\rho^* L t_1} \simeq \overline{[uv]} = \overline{[u]}\,\overline{[v]} + \overline{[u]'\,[v]'} + \overline{[u'v']} \quad . \quad . \quad (6)$$

since the quantities $\overline{[u]'}$, $\overline{[v]'}$ vanish.

Within the limits set by the assumption concerning the horizontal uniformity and constancy of the density, Eq. (6) may be given a simple interpretation. Since $\overline{[v]}$ is a measure of the net meridional flow of air during the given time interval, the first term on the right measures the contribution of the *mean* meridional circulation to the momentum transport. This will be called the *transport of the first species*. Such net air flow over any extended period generally implies a return flow at some other level although the effects of net meridional mass shifts are also included in the transport of this type.

Since $[v]$ is the instantaneous meridional circulation (which may or may not be compensated for at other levels), it is possible that the temporal fluctuations of $[v]$

are correlated with $[u]$ and thus produce a contribution to the transport. This may happen even if $\overline{[v]}$ is zero, *i.e.*, if the instantaneous meridional circulation is merely an oscillation producing no net transport of air during the interval t_1. This action is represented by the second term on the right, and may be considered as a certain type of eddy transport. This will be called the *transport of the second species.* For more or less obvious reasons it would be of considerable interest to know whether this process is of importance in the atmosphere.

Finally, since $[u'v']$ is dependent on the instantaneous spatial correlation of u and v along the length of the latitude circle, the last term gives the eddy transport due to horizontal-exchange processes during the interval t_1. This may be named the *transport of the third species.*

In order to ascertain the relative importance of the three terms, one may note that the first, $\overline{[u]}\,\overline{[v]}$, is, at least in principle, obtainable directly from observed values of u and v. The second $\overline{[u]'\,[v]'}$ is obtainable from (4) and (6) being equal to $\overline{[u]\,[v]} - \overline{[u]}\,\overline{[v]}$. The third is given by (4) directly as $\overline{[uv]} - \overline{[u]\,[v]}$. Thus the basic quantities to be computed are $\overline{[u]}$, $\overline{[v]}$, $\overline{[u]\,[v]}$ and $\overline{[uv]}$.

4. Computation of transports

As an approximation the instantaneous space averages $[u]$, $[v]$, $[uv]$ were taken to be the arithmetic means of the values for the several stations reporting at a given level on a given day. Days on which only one station reported an observation were excluded. The maximum number of observations on a given date at a given level was sixteen, this being the total number of observation points.

The time means represented by the basic quantities $\overline{[u]}$, $\overline{[v]}$, $\overline{[u]\,[v]}$ and $\overline{[uv]}$ were then formed by taking the arithmetic means of the daily quantities for the number n of days for which data were present. The results are tabulated in columns 1 to 9 in Table III.

In order to secure some measure of statistical reliability of the time averages, confidence limits were calculated and are included for various quantities in the body of the table. These limits are defined as twice the standard error and indicate approximately the 95 per cent. confidence level. The formula used is

$$\sigma_{\bar{y}} = \frac{\sigma(y)}{\sqrt{n}} \quad . \quad . \quad . \quad . \quad (7)$$

where $\sigma_{\bar{y}}$ is the standard error of the mean of any quantity y, $\sigma(y)$ is the sample standard deviation of y and n is the number of cases in the sample (number of days).

The space-time means $\overline{[u]}$, $\overline{[v]}$, $\overline{[uv]}$ represent averages over the number of days involved in each case of the daily space averages, $[u]$, $[v]$, $[uv]$ respectively, as has already been explained. As such these space-time averages are formed by an equal weighting of the daily values. Owing to the fact that the daily values $[u]$, $[v]$, $[uv]$ are based upon a number of observations which may vary from 2 to 16, the equal weighting introduces a certain amount of artificiality in the procedure. It is therefore of interest to compare the results already mentioned with the results

TABLE III. Numerical analysis of data. The levels are given in thousands of ft. All velocities are in m sec^{-1}. Internal consistency of the figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u]\,[v]}$	$\overline{[uv]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u]'[v]'}$	$\overline{[u'v']}$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
50	+ 14·0	− 0·07 ± 0·76	+ 3 ± 19	+ 30 ± 34	− 1	+ 4	+ 27 ± 21	149	+ 12·3	− 0·26	+ 21	+ 24	+ 0·14	604
45	+ 18·0	− 0·45 ± 1·02	− 6 ± 27	+ 43 ± 46	− 9	+ 3	+ 48 ± 24	162	+ 16·6	− 0·10	+ 49	+ 51	+ 0·23	764
40	+ 18·8	− 0·25 ± 0·93	− 4 ± 24	+ 30 ± 31	− 5	+ 1	+ 34 ± 16	178	+ 17·0	+ 0·14	+ 32	+ 30	+ 0·12	1032
35	+ 17·2	− 0·19 ± 0.75	0 ± 19	+ 40 ± 29	− 3	+ 4	+ 39 ± 19	180	+ 16·0	− 0·12	+ 38	+ 40	+ 0·17	1177
30	+ 14·2	+ 0·11 ± 0·57	− 1 ± 13	+ 33 ± 15	+ 2	− 3	+ 35 ± 11	181	+ 13·2	+ 0·02	+ 34	+ 33	+ 0·18	1628
25	+ 11·1	− 0·12 ± 0·86	− 3 ± 7	+ 17 ± 10	− 1	− 2	+ 19 ± 7	181	+ 10·9	− 0·03	+ 17	+ 18	+ 0·14	1798
20	+ 8.8	+ 0·20 ± 0·34	+ 1 ± 4	+ 17 ± 7	+ 2	− 1	+ 17 ± 5	181	+ 8·3	+ 0·11	+ 15	+ 15	+ 0·15	2038
14	+ 6·1	+ 0·29 ± 0·29	+ 3 ± 2	+ 12 ± 4	+ 2	+ 1	+ 9 ± 4	181	+ 6·0	+ 0·19	+ 10	+ 9	+ 0·14	1888
10	+ 3·4	+ 0·19 ± 0·20	+ 1 ± 1	+ 6 ± 2	+ 1	0	+ 6 ± 2	181	+ 3·3	+ 0·19	+ 6	+ 5	+ 0·09	2286
6	+ 1·3	+ 0·86 ± 0·35	+ 1 ± 1	+ 4 ± 2	+ 1	0	+ 3 ± 2	181	+ 1·3	+ 0·68	+ 4	+ 3	+ 0·08	2008
2	− 0·3	+ 0·03 ± 0·22	0 ± 0	+ 3 ± 2	0	0	+ 3 ± 1	181	− 0·3	+ 0·03	+ 4	+ 4	+ 0·11	2172
Integral (10^7 cgs units)				+ 14·2	− 0·1	+ 0·2	+ 14·1				+ 13·9	+ 13·7		Sum 17,395

of another method of estimating the space-time averages which is free from the particular objection mentioned but which may contain artificialities of a different nature. The alternative procedure used consists of summing all the individual observations at a given level for the entire six-month period and dividing this sum by the total number of observations N (column 15). The space-time averages so computed are indicated in Table III by curly brackets, *e.g.*, $\{u\}$. Thus the quantities in columns 10, 11, 12, are to be compared with columns 2, 3, 4 respectively.*

According to Eq. (6) the quantity $\overline{[uv]} - \overline{[u]}\,\overline{[v]}$ gives the sum of the transports of the second and third species, *i.e.*, the sum of the quantities in columns 7 and 8 in the table. The analogous transport $\{uv\} - \{u\}\,\{v\}$ evaluated by the alternative method is given in column 13. It is to be noted that the two methods of evaluating the space-time averages would become identical in those cases where the number of individual observations is the same from day to day (a circumstance not present in the data used).

According to the definitions of the quantities involved, it follows that

$$\{uv\} - \{u\}\,\{v\} = r(u, v) \cdot \sigma(u) \cdot \sigma(v) \qquad . \qquad . \qquad . \qquad (8)$$

where $r(u, v)$ is the coefficient of linear correlation between u and v, and $\sigma(u)$, $\sigma(v)$ denote the standard deviations of these wind components. Eq. (8) may be used to determine r for the N observations. For the sake of general interest the several values of r are listed in column 14 of Table III.

Vertical integrals of various quantities weighted by the density are given at the foot of the table. These were computed as follows: giving the hydrostatic relation we may write that

$$\int_{z_1}^{z_2} \rho(\;)dz = -\frac{1}{g}\int_{p_1}^{p_2}(\;)dp \qquad . \qquad . \qquad . \qquad . \qquad (9)$$

where z is height, g is the (constant) acceleration of gravity, p is pressure. The right-hand side of (9) was evaluated by numerical integration using the standard-atmosphere pressures corresponding to the several levels used. The effect of this use of the standard atmosphere is to make ρ^* in Eq. (1) equal to the standard-atmosphere density. Following a method due to Priestley (1949) the integrals obtained give the transports per unit length of the latitude circle from 2,000 ft to 50,000 ft.

5. Discussion of numerical results

The results obtained contain a number of points which are of first concern in studies of the general circulation. Among these are the following:

(a) Upon comparison of the transports of the third species given in column 8 of Table III with columns 6 and 7, the figures obtained show about one order of magnitude difference. Moreover, the figures for the third species are all positive with confidence limits which in every instance exclude zero by a substantial margin.

* This alternative method is the one used previously in the North American pilot study. In view of the fact that in the pilot study only a segment of a latitude circle was sampled it was not feasible to make a distinction between the transport of the second and third species.

The figures for the other two species on the other hand suggest that their contribution is sufficiently small so that the analysis cannot give reliable estimates (or perhaps even give the correct sign). It is thus strongly suggested that horizontal-exchange processes are the dominant mechanism for the poleward transport of momentum from the easterlies to the westerlies.

(b) The sums of the items in column 7 with those in column 8 are roughly comparable to column 13 showing that somewhat different computing methods give essentially the same results. The same applies to columns 5 and 12 except perhaps at 50,000 ft. Since the transports of the third species appear to be dominant, the values in columns 5, 8, 12, 13, should all be comparable, which is roughly the case.

(c) The transports of the third species increase with altitude, showing a probably spurious irregularity at 40,000 ft, then decrease at 50,000 ft. The general shape of the profile is similar to that obtained by Starr (1951) from North American data for 35°N. latitude during winter. It is possible that a seasonal vertical shift in the level of maximum transport exists. In that case the present data approximate a mean from winter to summer. Individual monthly values are not included, since their reliability was not deemed sufficient to warrant their detailed study. It may however be noted that of the 66 of such monthly values calculated 64 were positive in sign, one was zero and one was negative (May, at 50,000 ft).

(d) Although many questions arise, as discussed in a subsequent section, the figures for $\overline{[v]}$ in column 3 would seem to show that such mean meridional circulations as may be present are so small that the analysis lacks sufficient statistical resolving power to detect them. The values themselves are remarkably small, being in most cases not statistically different from zero. The possible exceptions are the small positive values at low levels downward from 20,000 ft where the necessary condition set by the confidence limits borders on fulfilment but is surpassed only at 6,000 ft. For reference later the values of $\overline{[v]}$ calculated for the colder months, Feb., Mar. and Apr. taken together, become larger and fulfil the confidence limits (*i.e.*, zero is excluded) for the 6,000, 10,000, and 14,000 ft levels. Furthermore, it is worthy of note that although the analogous quantity $\{v\}$ given in column 11 generally has still smaller (numerical) values except at 50,000 ft, at lower levels the values are again consistently positive, being in closer agreement with $\overline{[v]}$ than elsewhere.

(e) The vertical integrals at the foot of the table reflect what has been said concerning the values for the individual levels. Thus the integral for the transport of the third species in column 8 far overshadows the integrals for the first and second species in columns 6 and 7, making it about equal to the integral for the total in column 5. The alternative method gives integrals (columns 12 and 13) which are comparable to those for the first method. If the value of the integral for column 8 is multiplied by the length of the latitude circle and by the distance from the earth's axis at 31°N. latitude so as to obtain the transport of angular momentum, one obtains 26×10^{25} CGS units for this quantity. An estimate for the polar cap north of 31°N. of the *normal* average drain of angular momentum due to surface effects for the six-month period made by Widger (1949) and White (1949) is 15×10^{25} CGS units. As noted by Starr (1951) these determinations of surface torques probably represent a substantial underestimate.

6. Critical remarks

The computations outlined above involve many questions as to the validity of the procedures used. The more important of these are the following.

(a) The location of the observation points might be biased so as to sample unduly conditions to the east or to the west of troughs (or ridges) in the streamline patterns. If this effect were marked, the expectation would be that the net meridional velocities $\overline{[v]}$ and $\{v\}$ should be large. These quantities on the whole turn out to be surprisingly small, suggesting a rather random sampling with respect to the troughs and ridges. The extension of the averaging over changing seasonal patterns doubtless aids in the achievement of such desired randomness.

(b) The geographic sampling with respect to longitude leaves much to be desired especially over southern Asia and the Pacific. Little can be done about this deficiency since only a few stations are available. However, the effects of sparse data become most marked only at the higher levels as shown in Table II.

(c) The variations in latitude of the stations introduce an undesirable element. This is especially true of Honolulu and Lagens. The seriousness of this factor cannot be immediately appraised except to say that the results should perhaps be looked upon as an average over a small belt of latitude as well as over longitude and time.

(d) The radio wind soundings are more likely to reach high elevations under relatively light-wind conditions. This effect is perhaps more marked than is commonly realized, as pointed out by Lorenz (1950). Evidence of the presence of this factor is found in that the several values of $\overline{[u]}$ are consistently higher than the corresponding values of $\{u\}$ except at 2,000 and 6,000 ft. This is to be explained by noting that evidently those days on which the upper westerlies were strong gave relatively few soundings penetrating to high levels. In calculating $\overline{[u]}$ such days were given the same weighting as other days. On the other hand, in calculating $\{u\}$ the individual soundings were weighted equally, so that the effects of the few soundings showing abnormally high winds were minimized. The presence of of this type of selectivity probably brings about an underestimate of the momentum transports. Probably the slightly smaller transports secured by the second method are likewise a reflection of this factor.

The use of the two pilot-balloon observing points in southern Asia probably increases the selectivity effects, since these soundings are even more subject to such shortcomings.

(e) The values of $\overline{[v]}$ and $\{v\}$ may not actually represent a mass flux, *i.e.*, $\overline{[\rho v]}$ and $\{\rho v\}$ might be substantially zero even though the former quantities have non-zero values. This can happen only if ρ and v are properly correlated. Such correlations seem to exist actually, although from order-of-magnitude considerations it would be difficult to interpret the high values of $\overline{[v]}$ and $\{v\}$ at the 6,000 ft level as representing a volume but not a mass transport. On the other hand, these particular values may be over-estimates since reports from the two pilot-balloon stations in southern Asia were not available for this level, as indicated in Table II. That a contribution to $\overline{[v]}$ and $\{v\}$ might be present from a density correlation is rendered more plausible by the fact that these velocities were more definitely positive and larger in magnitude below 20,000 ft during the colder months Feb.,

Mar. and Apr., as mentioned earlier. It is during these months that differential advection effects might be expected to be more intense.

(f) As indicated earlier a constant standard-atmosphere density was used as an approximation in Eq. (1). In order to secure a measure of the error thus introduced, a limited study was made over the North American sector in the vicinity of 35° N. latitude at the 20,000 ft level for Feb. 1949. Over this region thermodynamic soundings were available for each wind observation used so that the eddy transport could be evaluated both with and without the approximation mentioned, on otherwise identical data. It was found that the difference amounted to about one per cent. Further checks at other levels should be made concerning this point.

(g) It may seem paradoxical that although the confidence limits are poorly satisfied by the items $\overline{[uv]}$ and $\overline{[u][v]}$ in Table III, they are very well satisfied by the differences of these quantities $\overline{[u'v']}$ in column 8. This can happen only if the daily values of $[uv]$ and $[u][v]$ are correlated, which evidently is the case, thus perhaps implying a compensation of errors when the difference is taken.

(h) The contributions to the transport due to the residual mass of the atmosphere above 50,000 and below 2,000 ft have not been evaluated. Doubtless the contribution from the layer below 2,000 ft is small on the basis of the fact that the wind velocities become smaller here. The situation above 50,000 ft needs more study. With existing data some indications may be obtained up to, let us say, 60,000 ft although a much longer period of time is necessary to secure statistical reliability. It would nevertheless appear from casual examination of higher-level data that the eddy transports continue to decrease above the jet-stream level.

(i) The six-month period examined represents conditions during a period of warming of the northern hemisphere, *i.e.*, conditions from winter to summer. Some reservation must be made concerning various details as to their representativeness for longer periods. It is hoped that a study similar to the present one will be available at a later date covering a comparable period from summer to winter.

7. Meteorological implications

Although the results obtained in this study need to be checked by further extensive research involving other and longer periods of time as well as a better network of reporting stations, it is nevertheless very interesting to discuss the import of the tentative indications obtained upon scientific thought concerning the general circulation, assuming that there exists a rough correspondence between the true facts and the picture of conditions as here derived from observations. On this understanding the following statements may be made.

In the first place the existence of the strong transports of the third species reveals a very characteristic and deep-seated feature of the general circulation. This bears out the contention of Jeffreys (1926) communicated in the pages of this Journal some twenty-five years earlier and more recently re-examined by Starr (1948), Bjerknes (1948), Van Mieghem (1949) and Mintz (1949). The contention has already received considerable corroboration through the studies of Widger (1949) and also through the pilot study made by Starr (1951).

Since these transports of the third species appear to be ample in magnitude to compensate for frictional and other surface losses farther to the north, it is difficult

to escape the conclusion that *ipso facto* the other modes of angular momentum transfer are of relatively minor importance. This is indicated, as it should be, by the relative insignificance of the transports involving mean and instantaneous hemispherical meridional circulations, *i.e.*, the transports of the first and second species. This circumstance together with the lack of evidence for the presence of a direct mean meridional circulation of the order of 1 m sec^{-1} (which is required) would render untenable certain suggestions of Palmén (1950) concerning a driving mechanism for the general circulation based on large-scale vertical convection between the more tropical and more polar regions.

On the other hand these circumstances would suggest that theories for the mechanism of the general circulation should follow along lines such as those proposed by Rossby (1947) or by Kuo (1950) in so far as these theories specify horizontal or quasi-horizontal exchange processes as being the dominant factor in the maintenance of the zonal westerlies. These contentions have received other support from the experimental work of Fultz (1948)

As already mentioned the slight northward values of [illegible] and [illegible] may represent a volume rather than a mass transport If this is true it would support the proposal made by Starr (1949, 1951) that the kinetic energy balance of the atmosphere is capable of fulfilment in terms of horizontal exchange processes without mean meridional circulations involving mass flux across latitude circles at individual levels

As a general comment upon the type of meteorological research exemplified by the present study, it is appropriate to point out that sufficient demonstrations of the validity of conclusions based on the examination of data are in general difficult to attain. It is rather through the amassing of supporting evidence from many studies carried through by various investigators using independent data and differing methods (where permissible) that a certain measure of confidence is ultimately established. The writers therefore hope that this discussion will encourage further work of this general nature.

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Note on the seasonal variation of the meridional flux of angular momentum

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SUMMARY

A marked seasonal variation of the poleward angular-momentum flux for the northern hemisphere in the vicinity of 30°N. latitude is obtained from an analysis of upper-wind soundings for a period of one year. The angular-momentum flux appears to be about twice as great during the winter as during the summer. Indications of a mean meridional circulation of the strength and character necessary to provide a substantial flux of angular momentum are lacking. It is suggested that the horizontal eddy flux of angular momentum across 30°N. latitude is sufficient to account for the drain of this momentum in middle latitudes.

1. INTRODUCTION

In view of the fact that the prevailing westerlies in the middle latitudes of the northern hemisphere undergo a pronounced variation in strength from winter to summer at low levels, it follows that the rate of removal of angular momentum through surface torques within this belt also undergoes seasonal variations. Since the atmosphere does not store sufficiently large amounts of angular momentum, these variations must be reflected in corresponding fluctuations in the rate at which angular momentum is fed into this zonal belt, primarily across the southern boundary of the surface westerlies in the general vicinity of 30°N. latitude. One may then inquire whether these variations in the flux of angular momentum are revealed by actual measurements from upper-wind studies. Successful investigation of this question might serve two purposes (among other possible uses). In the first place, added information concerning the large-scale processes in the atmosphere might be obtained. In the second place, a reasonable result of such an endeavour would lend greater support for the validity of the techniques used to measure the flux involved. The aim of this note is to describe the results of an attempt to secure measurements of the type discussed.

2. PROCEDURE AND RESULTS

On a previous occasion the present writers (Starr and White 1951) reported the results of an endeavour to measure the flux of angular momentum across the latitude circle in the vicinity of 30°N. latitude from upper-wind data for the six-month period from February to July 1949, using a network of 16 observation points encircling the hemisphere. The present study is based upon an extension of these computations so as to cover a total period of one year, *i.e.*, February 1949 to January 1950 inclusive.

In order to avoid repetition, only a brief statement will be made here of the methods used to measure the angular-momentum flux, the reader being referred to the discussion in the paper already referred to, which is henceforth designated as (A). The necessary notation used may be summarized as follows :

u eastward component of the wind.
v northward component of the wind.
$[\alpha]$ space average of a quantity α over the length of the complete latitude circle.
α' deviation of a quantity α from its space average $[\alpha]$.
$\bar{\alpha}$ time average of a quantity α. Time and space averaging processes are not necessarily commutative.
$[\alpha]'$ deviation of the space average $[\alpha]$ from the space-time average $\overline{[\alpha]}$.
$\{\alpha\}$ arithmetic mean of all individual observations of α at a given level during entire time period considered.
n number of days for which observations were available at a given level.
N number of individual observations present at a given level during entire time-period considered.
r coefficient of linear correlation for the N pairs of u and v.

In (A) it was shown that the quantity $\overline{[uv]}$ which represents the northward transport of linear momentum per unit mass at a given level may be resolved into three components, namely :

(i) Transport of the first species, $\overline{[u]}\,\overline{[v]}$. This component represents the contribution due to a net meridional mass flux $\overline{[v]}$ during the entire period at the level in question. To the extent that a non-zero value of $\overline{[v]}$ at one level is usually compensated by values of $\overline{[v]}$ having an opposite sign at other levels, this component depends upon the existence of so-called *mean meridional circulations.*

(ii) Transport of the second species, $\overline{[u]'[v]'}$. This component represents the contribution from fluctuations in $[v]$ due to its possible correlation with $[u]$ in time.

(iii) Transport of the third species, $\overline{[u'v']}$. This component represents the contribution due to a correlation between u and v along the length of the latitude circle. It does not require the presence of net air motions $[v]$ or $\overline{[v]}$.

Since one basic question involved in the subject relates to the role of mean meridional circulations, it is useful to form the quantity $\{uv\} - \{u\}\{v\}$ which is analogous to the sum of the transports of the second and third species although the mode of averaging used is slightly different as explained in (A).

Because of slight changes in the tabulations of the source material and for other reasons some changes were made in the computation procedures for the second six-month period following the period used in (A). These are :

(a) For the entire year it was found feasible to add material for the 55,000 ft level, since a fairly large sample of winds could then be subjected to analysis.

(b) For the 6, 10, 20, 30, 40, and 55 thousand-feet levels supplementary data as reported for the 850, 700, 500, 300, 200, and 100 millibar levels, respectively, were added, although preference was given to constant-height data where available.

(c) Three key stations were added to the network, bringing the total number to 19. These are given in Table I.

TABLE I. List of key stations added

Station	Latitude (N.)	Longitude (E.)	Altitude (ft)	Type
Farouk	30° 08′	31° 24′	223	radio wind
Habbaniya	33° 22′	43° 34′	144	,,
Hong Kong	22° 18′	114° 10′	—	pilot balloon

Since a large amount of observational material is needed in order to achieve satisfactory sampling, the data were separated only into summer and winter seasons. Thus the six months May to October 1949 inclusive were considered as representative of summer conditions, while the six months February, March, April, November and December 1949 together with January 1950 were considered as forming a composite winter season. The results are given in Tables II and III which are identical in form to Table III of (A). These tables give the transports of each of the three species in columns 6, 7, and 8 together with various other quantities defined in terms of the symbols already recapitulated here, and explained more fully in (A). It might be added that a number of the quantities carry confidence limits which are defined as twice the standard error.

The means for the entire year are in themselves of considerable interest. These are given in Table IV which is similar to Tables II and III save for the inclusion of the 55,000 ft level. As might be expected, the yearly values tend to approximate averages of the summer and winter figures, although it is interesting to observe other features such as the behaviour of the confidence limits.

In all three tables vertical integrals of several quantities with respect to pressure are given at the foot. It is apparent, as was the case in (A), that the largest transports are those of the third species. This is true for the integrals as well as for the values at individual levels. The three curves in Fig. 1 show the vertical profiles of this component for summer, winter and for the entire year. A breakdown of the number of observations available for the entire year by station and level is given in Table V. The percentages listed include reports from the key stations together with reports from alternative stations used in place of the key stations as explained in (A). It is to be noted that Table V differs in form from Table II in (A)*.

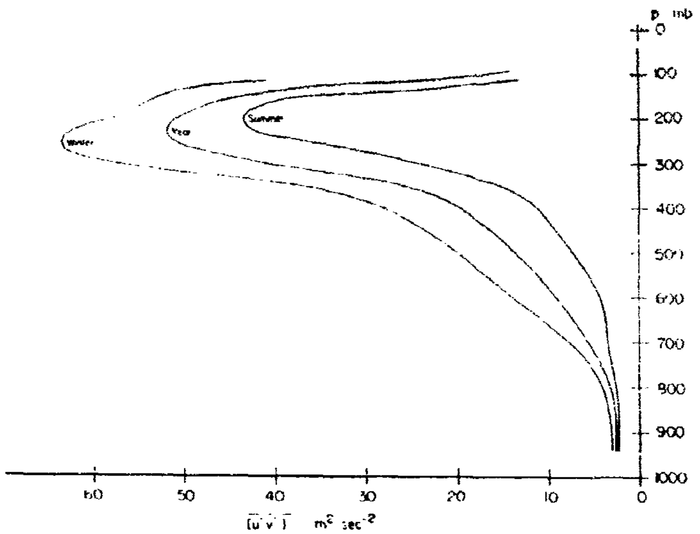

Figure 1. Vertical profiles of the eddy transport of momentum on a linear pressure scale showing the seasonal variations.

* The percentages listed in Table V are not identical in meaning to those listed in Table II of (A). Table V gives the percentage of total possible observations by station at each level, the total possible number of observations being 365 which is the number of days in a year. Table II of (A) gives percentages of actually received observations from all stations at each level as listed in Column 15 of Table III of (A).

TABLE II. Numerical analysis of data for summer. The levels are given in thousands of ft. All velocities are in m sec^{-1}. Internal consistency of the figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[uv]}$	$\overline{[u]\,[v]}$	$\overline{[u]'\,[v]'}$	$\overline{[u'v']}$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
50	+ 7·6	− 0·92 ± 0·71	− 11 ± 9	+ 3 ± 18	− 7	− 4	+ 13 ± 14	176	+ 7·5	− 0·87	+ 5	+ 11	+ 0·08	776
45	+ 11·5	− 0·47 ± 0·78	− 8 ± 12	+ 29 ± 23	− 5	− 3	+ 37 ± 16	182	+ 11·4	− 0·52	+ 34	+ 40	+ 0·22	971
40	+ 11·3	+ 0·29 ± 0·63	0 ± 9	+ 43 ± 17	+ 3	− 3	+ 43 ± 12	184	+ 11·3	+ 0·41	+ 45	+ 40	+ 0·21	1404
35	+ 9·7	− 0·39 ± 0·64	− 3 ± 8	+ 35 ± 13	− 4	− 1	+ 40 ± 10	184	+ 9·6	− 0·33	+ 34	+ 40	+ 0·25	1339
30	+ 8·1	+ 0·23 ± 0·40	+ 1 ± 4	+ 26 ± 8	+ 2	− 1	+ 25 ± 6	184	+ 8·1	+ 0·11	+ 27	+ 26	+ 0·22	1975
25	+ 6·0	− 0·02 ± 0·30	− 1 ± 2	+ 12 ± 5	0	0	+ 13 ± 4	184	+ 6·0	+ 0·03	+ 13	+ 12	+ 0·16	1918
20	+ 4·5	+ 0·15 ± 0·23	0 ± 2	+ 9 ± 4	+ 1	− 1	+ 9 ± 3	184	+ 4·5	+ 0·07	+ 9	+ 9	+ 0·15	2371
14	+ 2·8	− 0·11 ± 0·24	− 1 ± 1	+ 3 ± 3	0	− 1	+ 4 ± 2	184	+ 2·7	− 0·12	+ 4	+ 4	+ 0·09	1974
10	+ 1·6	+ 0·01 ± 0·18	0 ± 0	+ 4 ± 2	0	0	+ 4 ± 2	184	+ 1·6	+ 0·02	+ 3	+ 3	+ 0·09	2638
6	− 0·4	+ 0·18 ± 0·19	0 ± 0	+ 3 ± 1	0	0	+ 3 ± 1	184	− 0·4	+ 0·12	+ 3	+ 3	+ 0·10	2355
2	− 0·8	− 0·35 ± 0·16	+ 1 ± 0	+ 3 ± 1	0	0	+ 2 ± 1	184	− 0·8	− 0·33	+ 3	+ 3	+ 0·11	2393
Integral (10^{7} cgs units)				+ 10·3	− 0·2	− 0·6	+ 11·0				+ 10·6	+ 10·9	Sum	20,114

TABLE III. Numerical analysis of data for composite winter. The levels are given in thousands of ft. All velocities are in m sec^{-1}. Internal consistency of the figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u][v]}$	$\overline{[uv]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u]'[v]'}$	$\overline{[u'v']}$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
50	+ 18·2	− 0·63 ± 0·99	0 ± 25	+ 41 ± 36	− 11	+ 12	+ 41 ± 20	127	+ 17·7	− 0·59	+ 42	+ 53	+ 0·34	381
45	+ 22·2	− 0·84 ± 1·16	− 2 ± 32	+ 51 ± 46	− 19	+ 16	+ 53 ± 23	149	+ 21·9	− 0·51	+ 59	+ 70	+ 0·24	546
40	+ 23·7	− 0·13 ± 0·97	+ 5 ± 27	+ 62 ± 34	− 3	+ 8	+ 57 ± 18	178	+ 23·3	+ 0·39	+ 59	+ 69	+ 0·26	1046
35	+ 21·2	− 0·25 ± 0·87	+ 3 ± 21	+ 67 ± 30	− 5	+ 9	+ 63 ± 23	180	+ 21·1	− 0·20	+ 71	+ 75	+ 0·26	1030
30	+ 19·8	+ 0·10 ± 0·61	+ 2 ± 14	+ 58 ± 19	+ 2	− 1	+ 56 ± 13	181	+ 19·2	+ 0·17	+ 62	+ 58	+ 0·25	1748
25	+ 14·8	− 0·08 ± 0·51	− 1 ± 8	+ 31 ± 12	− 1	0	+ 31 ± 9	181	+ 14·9	+ 0·02	+ 33	+ 33	+ 0·20	1728
20	+ 12·6	+ 0·15 ± 0·36	+ 1 ± 5	+ 23 ± 8	+ 2	− 1	+ 22 ± 5	181	+ 12·1	+ 0·13	+ 22	+ 20	+ 0·16	2259
14	+ 8·3	+ 0·51 ± 0·31	+ 6 ± 3	+ 20 ± 5	− 4	+ 2	+ 14 ± 4	181	+ 8·2	+ 0·42	+ 19	+ 16	+ 0·17	1795
10	+ 5·1	+ 0·20 ± 0·21	+ 1 ± 1	+ 9 ± 3	+ 1	0	+ 8 ± 3	181	+ 5·1	+ 0·17	+ 9	+ 8	+ 0·13	2485
6	+ 2·6	+ 0·70 ± 0·36	+ 2 ± 1	+ 6 ± 2	+ 2	0	+ 4 ± 2	181	+ 2·6	+ 0·47	+ 6	+ 5	+ 0·10	2205
2	− 0·7	− 0·18 ± 0·25	+ 1 ± 1	+ 4 ± 2	0	0	+ 3 ± 2	181	− 0·6	− 0·17	+ 4	+ 4	+ 0·10	2136
Integral (10^7 cgs units)				+ 22·8	− 0·2	+ 1·9	+ 21·1				+ 23·3	+ 23·5	Sum 17,339	

TABLE IV. NUMERICAL ANALYSIS OF DATA FOR ENTIRE YEAR. THE LEVELS ARE GIVEN IN THOUSANDS OF FT. ALL VELOCITIES ARE IN M SEC^{-1}. INTERNAL CONSISTENCY OF THE FIGURES GIVEN IS LIMITED BY ROUNDING-OFF APPROXIMATIONS.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[uv]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u]'[v]'}$	$\overline{[u'v']}$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
55	+ 8·3	− 0·17 ± 0·59	− 1 ± 10	+ 12 ± 12	− 1	0	+ 14 ± 7	255	+ 7·6	− 0·26	+ 13	+ 13	+ 0·15	834
50	+ 12·0	− 0·80 ± 0·58	− 5 ± 12	+ 19 ± 19	− 10	+ 4	+ 25 ± 12	303	+ 10·9	− 0·78	+ 17	+ 26	+ 0·17	1157
45	+ 16·3	− 0·63 ± 0·68	− 6 ± 16	+ 39 ± 24	− 10	+ 5	+ 44 ± 13	331	+ 15·6	− 0·51	+ 38	+ 51	+ 0·22	1517
40	+ 17·4	+ 0·08 ± 0·58	+ 2 ± 14	+ 52 ± 19	+ 1	+ 1	+ 50 ± 11	362	+ 16·4	+ 0·39	+ 59	+ 52	+ 0·22	2450
35	+ 15·4	− 0·32 ± 0·53	− 1 ± 11	+ 51 ± 17	− 5	+ 4	+ 52 ± 12	364	+ 14·6	− 0·28	+ 51	+ 55	+ 0·24	2369
30	+ 13·9	+ 0·17 ± 0·36	+ 2 ± 7	+ 42 ± 10	+ 2	− 1	+ 40 ± 7	365	+ 13·3	+ 0·14	+ 43	+ 41	+ 0·23	3717
25	+ 10·3	− 0·05 ± 0·29	− 1 ± 4	+ 21 ± 6	− 1	0	+ 22 ± 5	365	+ 10·2	+ 0·02	+ 21	+ 21	+ 0·17	3646
20	+ 8·5	+ 0·15 ± 0·22	+ 1 ± 3	+ 16 ± 4	+ 1	− 1	+ 15 ± 3	365	+ 8·2	+ 0·10	+ 13	+ 14	+ 0·15	4630
14	+ 5·5	+ 0·20 ± 0·20	+ 2 ± 2	+ 12 ± 3	+ 1	+ 1	+ 9 ± 3	365	+ 3·3	+ 0·14	+ 11	+ 10	+ 0·15	3769
10	+ 3·3	+ 0·10 ± 0·14	+ 1 ± 1	+ 6 ± 2	0	0	+ 6 ± 2	365	+ 3·3	+ 0·09	+ 6	+ 5	+ 0·11	5123
6	+ 1·1	+ 0·43 ± 0·20	+ 1 ± 0	+ 4 ± 1	0	+ 1	+ 3 ± 1	365	+ 1·1	+ 0·29	+ 4	+ 4	+ 0·19	4360
2	− 0·7	− 0·27 ± 0·15	+ 1 ± 0	+ 3 ± 1	0	0	+ 3 ± 1	365	− 0·7	− 0·25	+ 3	+ 3	+ 0·10	4529
Integral (10^7 cgs units)				+ 16·8	− 0·4	+ 0·7	+ 16·5				+ 16·9	+ 17·8		Sum 38,301

TABLE V. Percentage of total possible observations at each level for each station for year

	Elevation in thousands of ft											
Station	2	6	10	14	20	25	30	35	40	45	50	55
Qrendi	36	53	55	30	46	25	50	6	26	4	8	9
Bahrein	25	56	57	21	53	18	50	2	40	1	13	14
Hyderabad	99	0	98	0	87	63	37	1	0	0	0	0
Dibrugarh	98	0	87	0	52	36	24	0	0	0	0	0
Tokyo	93	93	94	85	93	88	87	73	59	45	31	19
Midway	79	79	72	63	62	49	47	39	35	25	20	15
Honolulu	94	94	94	90	91	79	81	69	63	47	39	37
Weather Ship (a)	86	82	83	79	76	65	59	44	40	28	21	17
Santa Maria	99	99	99	99	97	93	86	79	73	59	46	35
Big Spring	---	99	99	99	93	82	77	66	60	48	35	22
New Orleans	99	98	95	93	90	80	72	64	54	41	25	9
Miami	100	99	99	99	89	93	89	83	75	58	42	24
Kindley Field	83	82	76	58	56	39	30	16	10	4	2	2
Weather Ship (b)	73	85	85	82	82	76	72	64	58	39	32	23
Lagens	78	81	79	74	75	65	62	44	37	15	6	7
North Front	51	56	55	53	50	46	53	6	18	5	12	5
Habbaniya	7	35	32	1	30	1	28	0	25	0	0	16
Hong Kong	36	23	16	7	8	1	5	0	2	0	0	0
Farouk	4	33	32	1	28	0	14	0	4	0	0	0

3. Discussion

Several points of importance may be noted concerning the general results. Among these are the following :

(a) The transports of the first species are small except possibly at high levels during winter. In general the impression is given that no well-defined seasonal variation exists in the figures for individual levels and none exists in the vertical integrals.

(b) The transports of the second species are also small except possibly at high levels during winter. None of the values is positive during summer, while with one minor exception at 20,000 ft none of the values is negative during winter. The integrals, of course, change in the same sense during the year. It may therefore be that this component possesses a seasonal variation although the magnitudes involved are small.

(c) The transports of the third species are by far the most important. They are much larger during the winter at every level, making the integral for this season about twice as large as the integral for summer. These results are also apparent from Fig. 1. The maximum transport (per unit isobaric layer) occurs at 35,000 ft during winter and at 40,000 ft during summer.

(d) In general the quantities $\{uv\} - \{u\}\{v\}$ in columns 13 agree quite well with the sum of the items in columns 7 and 8, as is also true of the corresponding integrals. This shows that somewhat different methods of handling the data yield comparable results.

(e) The confidence limits for the transports of the third species in columns 8 exclude zero in all instances as was the case in (A), except at 50,000 ft during summer.

(f) The quantities $\overline{[v]}$ and $\{v\}$ are again generally small in all three tables. The exact values cannot be taken too literally in view of the confidence limits

obtained and for other reasons discussed in (A). However, it should again be stressed that in studies of the present kind large values for these quantities would have indicated improper sampling of the hemisphere with respect to troughs and ridges in the streamline patterns, as the first possibility.

Because of the well-known lack of isotropy of the eddy processes of the general circulation from longitude to longitude, this problem of securing an adequate sampling of the hemisphere cannot be over-emphasized. Even more especially it is the *bête noire* in any attempt to secure measurements of the net meridional flow of air at given levels in the atmosphere.

As discussed in (A) there is a tendency for larger positive values of $[v]$ and $\{v\}$ to occur at lower levels during winter. The appreciable negative values obtained at high levels are interesting but selectivity and less satisfactory sampling at these levels doubtless are important factors to be considered.

Over long periods of time the net mass flux across any latitude circle must vanish. In view of this it should be expected that the vertical integrals of $[\overline{v}]$ and $\{v\}$ with respect to pressure should be small. Such integrations were performed and this condition is fulfilled.

(g) Estimates of the surface frictional torques made by Priestley (1951) reveal a required yearly mean flux of about $31{\cdot}7 \times 10^{25}$ cgs units of angular momentum/sec across latitude 30°N. In the present study the yearly mean eddy flux of angular momentum in the same units is $30{\cdot}9 \times 10^{25}$, in good agreement with Priestley's estimated requirement. Both of these quantities may however be underestimates rather than overestimates.

References

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Schemes for the study of hemispheric exchange processes

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SUMMARY

Two alternative existing schemes for the classification of eddy-exchange processes are examined. It appears that each scheme is adapted to the study of distinct physical questions concerning the atmosphere. Hemispheric wind data are used to illustrate the inherent differences of the two methods.

A relatively high positive spatial correlation is found between the eastward and northward components of the mean vector wind along the latitude circle in the vicinity of 30°N. at the level of the jet stream.

1. COMPARISON OF TWO SYSTEMS CURRENTLY IN USE

In connection with investigations concerning the angular-momentum balance of the atmosphere, two schemes have been used in the measurement of the flux of such momentum across surfaces of constant latitude. One was formulated by Priestley (1949) and the other by Starr and White (1951, 1952). Although these systems may be, and have in fact been, applied to the meridional transport of other physical properties, the descriptions which follow utilize the momentum problem as an example. The symbolism used is that of Starr and White.

A fundamental quantity in the momentum problem is the total amount of relative linear eastward momentum transferred across a complete latitude circle per unit height at a given elevation during any given time interval. It was shown (Starr and White 1951) that this quantity is principally determined by the product $\overline{[uv]}$ where u and v are the eastward and northward components of wind velocity, the square brackets denote a space average over the length L of a complete latitude circle while the bar denotes a time average over any specified time interval. The space and time averaging operations are commutative.

It was shown (Starr and White 1951, 1952) that this product can be expanded and expressed as

$$\overline{[uv]} = \overline{[u]}\,\overline{[v]} + \overline{[u]'[v]'} + \overline{[u'v']} \qquad . \quad . \quad . \quad (1)$$

where primes denote departures from the averages. It is also possible to represent this product by an expansion of the following type

$$[\overline{uv}] = [\bar{u}]\,[\bar{v}] + [\bar{u}'\,\bar{v}'] + [\overline{u'v'}] \qquad . \quad . \quad . \quad (2)$$

as shown by Priestley (1949). Both expressions (1) and (2) are equally valid, the difference being that the order of operations with respect to time and space variables is reversed in the two cases, with a resulting difference in the physical interpretation of the several terms in the two expressions.

Thus it is of interest to point out that :

(*a*) The term $\overline{[uv]}$ of Eq. (1) and $[\overline{uv}]$ of Eq. (2) both represent the flux of linear momentum across a latitude wall at a given level and over a given time interval, owing to the commutability of the brackets and the bars.

(*b*) The terms $\overline{[u]}\,\overline{[v]}$ of Eq. (1) and $[\bar{u}]\,[\bar{v}]$ of Eq. (2) depend upon the non-zero value of the mean meridional rate of air flow $\overline{[v]}$, and $[\bar{v}]$.

(*c*) The terms $\overline{[u]'[v]'}$ of Eq. (1) and $[\bar{u}'\,\bar{v}']$ of Eq. (2) are not equal. The former represents a contribution to the momentum flux which depends upon the presence of a temporal correlation between the instantaneous space means $[u]$ and $[v]$ at a given level in the atmosphere. It may be interpreted as the contribution from an instantaneous meridional net flow of air whose strength varies in time in a systematic manner with the strength of the average zonal winds. On the other hand, the latter is a contribution to the flux which is due to the presence of a spatial correlation between the time means $\bar{u}$ and $\bar{v}$ along a latitude circle. It is associated with the asymmetry of the troughs, ridges and other features of the *mean* streamline pattern.

(*d*) The terms $\overline{[u'v']}$ of Eq. (1) and $[\overline{u'v'}]$ of Eq. (2) are also not equal. The contribution of the first is due to the presence of an instantaneous spatial correlation between u and v along a latitude circle. It is dependent upon the asymmetry of the troughs and ridges and other features of the instantaneous streamline pattern, as suggested earlier by Starr (1948).

On the other hand, the second is due to the presence of a time correlation between the instantaneous values of u and v at individual points along a latitude circle. It may be associated with the temporal variations of the wind at given stations.

2. An observational example

An example of the evaluation of the terms of Eq. (1) was presented by Starr and White (1952), in which the angular-momentum flux for a period of one year (Feb. 1949 to Jan. 1950 inclusive) in the vicinity of 31°N. latitude was discussed. It was found that the third term on the right gave by far the largest contribution to the total. Various details of the computations cannot be repeated here, the reader being referred to the two previous articles by the present writers which have already been cited. In the present paper the writers have re-evaluated the same data according to the expansion given by Eq. (2).

In applying the second mode of expansion a quantity such as $[\overline{uv}]$ involved equal weighting of the several stations used. Consequently the results are rather sensitive to the inclusion of stations with only small amounts of data which may thus be rendered non-representative. In the re-evaluation it was therefore considered desirable to eliminate at each level those stations which reported less than 50 times during the year (see Table V, Starr and White 1952). Table I shows a comparison between the several terms in the two modes of expansion.* The following points may be noted concerning these results.

TABLE I. Comparison of the magnitude of the terms in the two modes of expansion. Levels are given in thousands of ft. All velocities are in m sec^{-1}. Internal consistency limited by rounding-off approximations. Vertical integrals represent flux of angular momentum across entire latitude circle

Level	Equation (1)				Equation (2)			
	$\overline{[uv]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u]'[v]'}$	$\overline{[u'v']}$	$[\overline{uv}]$	$[\bar{u}][\bar{v}]$	$[\bar{u}'\bar{v}']$	$[\overline{u'v'}]$
55	+ 12	− 1	0	+ 14	+ 18	− 1	+ 7	+ 13
50	+ 19	− 10	+ 4	+ 25	+ 12	− 13	+ 3	+ 22
45	+ 39	− 10	+ 5	+ 44	+ 35	− 14	+ 12	+ 38
40	+ 52	+ 1	+ 1	+ 50	+ 62	+ 7	+ 14	+ 41
35	+ 51	− 5	+ 4	+ 52	+ 41	− 9	+ 11	+ 39
30	+ 42	+ 2	− 1	+ 40	+ 36	+ 2	+ 10	+ 25
25	+ 21	− 1	0	+ 22	+ 20	− 1	+ 4	+ 16
20	+ 16	+ 1	− 1	+ 15	+ 14	+ 1	+ 4	+ 9
14	+ 12	+ 1	+ 1	+ 9	+ 10	0	+ 1	+ 8
10	+ 6	0	0	+ 6	+ 6	0	+ 1	+ 5
6	+ 4	0	+ 1	+ 3	+ 3	0	0	+ 3
2	+ 3	0	0	+ 3	+ 2	0	0	+ 2
Integral (10[illegible] cgs units)	+ 31·5	− 0·8	+ 1·3	+ 30·9	+ 28·6	− 1·5	+ 6·7	+ 23·6

(*a*) The terms $\overline{[uv]}$ and $[uv]$ of Eqs. (1) and (2) representing the total flux of angular momentum are similar at most levels, with better agreement in the lower levels where the data are more complete. The total integrated fluxes differ only slightly.

(*b*) The terms $\overline{[u]}\,\overline{[v]}$ and $[\bar{u}][\bar{v}]$ of Eqs. (1) and (2) appear to be in fair agreement at most levels, again being more nearly alike in the lower levels where the data are more plentiful. The integrated values of these terms are both negative and small when compared with the total flux.

* One may raise a legitimate question as to whether the exclusion of the data from stations with a small number of observations would have altered the analysis according to Eq. (1). A numerical check at 40,000 ft showed no appreciable effect of this kind, as should be expected from the form of the equation.

(*c*) On the other hand, the terms $\overline{[u]'\,[v]'}$ and $[\bar u'\,\bar v']$ of Eqs. (1) and (2) differ considerably. The former varies in sign and is small while the latter is consistently positive and of fair size except at low levels. When integrated through height the first accounts for only 4 per cent of the total flux while the second accounts for 22 per cent. Thus while it is possible to neglect the term $\overline{[u]'\,[v]'}$ of Eq. (1) in comparison with the total it is not possible to neglect the term $[\bar u'\,\bar v']$ of Eq. (2) in comparison with the total.

(*d*) The terms $\overline{[u'v']}$ of Eq. (1) and $[\overline{u'v'}]$ of Eq. (2) individually account for the major portion of the flux in their respective systems of expansion. It is quite clear from the data however that the values of the latter are generally smaller. The first when integrated through height accounts for 98 per cent of the total poleward flux of angular momentum while the second accounts for only 82 per cent.

One interesting feature of the second mode of expansion is the possibility of examining the hemispherical representativeness of the quantity $\overline{u'v'}$ computed at various stations. The vertical integral of this term was computed for each of the seven stations listed in Table II. It might be tempting to ascribe the low value at Santa Maria to selectivity or to general incompleteness of the data used. Examination of the results at individual levels shows, however, that the anomalous behaviour of the flux manifests itself markedly already at elevations where observations were received with almost perfect regularity. The low values at Honolulu and perhaps at Miami may be related to the low latitude of these stations.

TABLE II. The vertically integrated values of the quantity $\overline{u'v'}$ at the stations indicated. Values represent the flux of angular momentum across the complete latitude circle assuming that each station is representative of the hemisphere (10^{24} cgs units)

Tokyo	Midway	Honolulu	Santa Maria	Big Spring	Miami	Weather Ship (*b*) (Atlantic)
+ 57·6	+ 17·7	+ 21·5	− 00·4	+ 44·4	+ 16·0	+ 26·9

On the whole it is unlikely that the differences shown among the stations are primarily due to incomplete data or to peculiarities of the particular year studied, although some variation from year to year should no doubt be expected. It appears therefore that it is necessary to apply this mode of expansion to a large number of stations well distributed around the entire latitude circle, if the purpose of the work is to obtain a hemispherically representative sample.

The second term on the right in Eq. (2) involves a spatial correlation between $\bar u$ and $\bar v$, i.e., between the eastward and northward components of the mean vector wind, at the various stations along the latitude circle. From the definitions given one may therefore write that

$$[\bar u'\,\bar v'] = [\bar u\,\bar v] - [\bar u]\,[\bar v] = r\,(\bar u, \bar v)\,.\,\sigma\,(\bar u)\,.\,\sigma\,(\bar v) \qquad (3)$$

where $r\,(\bar u, \bar v)$ is the coefficient of linear correlation between $\bar u$ and $\bar v$, and $\sigma\,(\bar u)$, $\sigma\,(\bar v)$ denote the standard deviations of these components. Eq. (3) may be used to determine $r\,(\bar u, \bar v)$ for the number of stations used in the second mode of expansion. The several values obtained in this manner are given in Table III.

TABLE III. Coefficients of correlation in per cent between $\bar u$ and v

Elevation in 10^3 ft	2	6	10	14	20	25	30	35	40	45	50	55
$r\,(\bar u, v)$	− 3	+ 8	+ 34	+ 20	+ 64	+ 50	+ 76	+ 74	+ 75	+ 65	+ 26	+ 51
No. of Stations	16	17	19	14	18	16	18	11	14	10	9	9

It may be observed that the correlations are highest at the jet-stream levels. This would suggest that evidence of such behaviour of the mean winds should be found on mean streamline charts for upper levels, although there exist at least two difficulties which attend the actual

detection of such properties of mean maps. In the first place even though the correlations are of fair size, the standard deviations $\sigma(\bar{u})$ and $\sigma(\bar{v})$ are not very large, the latter being as high as 3 m/sec only at 40,000 ft. Maps of great accuracy in regard to small details would thus be required. In the second place it is not known at present what the importance of small mean geostrophic departures may be in this connection, so that mean isobaric maps may not exhibit this property fully. Finally it would of course be desirable to check the results given here on independent data and at other latitudes.

As a sidelight on this matter it may be noted that a theoretical study of the standing perturbations in the westerlies due to an idealized mountain, made by Bolin (1950) seems to show qualitative evidence of a correlation of the type found in the present discussion. Likewise a somewhat similar study (unpublished) made by our colleague Major P. D. Thompson, U.S.A.F., using an approximation to the actual North American mountain complex also yields results having this property.

References

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Two Years of Momentum Flux Data for 13° N

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The aim of this note is to present more statistics concerning the meridional flux of angular momentum in the tropics similar to those already published (STARR and WHITE 1952 a). A further investigation for a period of one additional year has now been completed for 13° N, in order to verify the previous conclusions with independent data. A continuous two-year period has thus been analysed and represents a concluded phase of the investigation for this latitude.

Except as indicated, the form of the accompanying tables and the computational procedures used in obtaining them are the same as the corresponding ones in the paper cited, to which the reader is referred for details not repeated here. As an experiment the station network for the second year was altered considerably and enlarged, the added new key stations being listed in Table 1. Some former key stations were also dropped, the complete new list being that given in Table 2, which

Table 1. List of added key stations (numbered) and their alternate stations (italics)

Station	Latitude (N.)	Longitude	Altitude (ft)	Type
1. Port Blair	11° 40′	92° 43′ E	262	pilot balloon
Sandoway	18 28	94 21 E	30	»
Bassein	16 46	94 46 E	13	»
Mingaladon	16 54	96 11 E	92	»
Tavoy	14 06	98 13 E	112	»
Mergui	12 26	98 36 E	66	»
Victoria Point	09 58	98 35 E	122	»
Penang	05 18	100 16 E	12	»
2. Hilo	19 44	155 04 W	33	radio wind
3. Vera Cruz	19 12	96 08 W	10	pilot balloon
Tacubaya	19 24	99 12 W	7579	»
Mexico City	19 26	99 08 W	7340	»
Cuidad Del Carmer	18 39	91 49 W	5	»
Swan Island	17 24	83 56 W	35	»
4. Albrook Field	08 58	79 33 W	21	radio wind
Plato Magdalena	09 48	74 48 W	190	»
5. Cayenne-Rochambeau	04 50	52 22 W		pilot balloon

Table 2. Percentage of total possible observations for second year

Station	Altitude in thousands of feet											
	2	6	10	14	20	25	30	35	40	45	50	55
Gao	64	65	62	44	11	0	0	0	0	0	0	0
El Fasher	92	93	92	91	84	33	27	14	10	4	1	1
Aden	75	85	73	13	71	21	64	10	58	3	3	41
Trichinopoly	100	0	100	0	88	84	75	7	0	0	0	0
Port Blair	97	86	91	41	52	24	13	2	0	0	0	0
Saigon	89	86	88	76	63	13	5	3	2	1	0	0
Clark Field	97	97	96	92	85	53	46	34	38	29	28	30
Yap	75	84	78	58	64	43	46	30	32	20	10	8
Harmon Field	88	94	93	78	89	72	86	71	83	65	54	48
Kwajalein	99	100	99	98	99	95	98	90	96	80	62	57
Johnson Island	75	91	90	70	79	57	63	45	49	31	27	16
Hilo	90	99	99	90	97	84	91	76	82	57	37	30
Vera Cruz	98	94	89	79	62	10	2	0	0	0	0	0
Albrook Field	82	93	92	77	83	61	65	47	49	24	6	1
Waller Field	100	100	100	96	97	84	91	77	84	64	47	48
Cayenne-Rochambeau	56	52	39	30	23	18	16	17	14	13	7	4
Dakar	92	91	78	41	35	18	28	16	24	12	8	8

shows the frequency distribution of the available observations for the second year alone. Due to these changes in the stations used, it is not convenient to present a frequency table for the two years combined, as was done by the writers in the case of similar studies at 31° N (STARR and WHITE 1952 b).

It is seen from Table 3 that the main results for the two year period are much the same as for the first year alone published previously. Again using parentheses to indicate vertical averaging with respect to pressure for the layer considered, $([\bar{u}])$ turns out to be -0.35 m sec^{-1} for the two years as compared with -0.50 m sec^{-1} for the first year[1]. The quantity $([\bar{v}])$ is -0.21 m sec^{-1} for the two years as compared with -0.16 m sec^{-1} for the first year, indicating most probably that a slight bias in favor of northerly wind components is still present in the data. The integrals at the foot of columns 5 and 6 in Table 3 show that about 11 per cent of the momentum transport is accomplished through mean meridional circulations for the two years as compared with about 14 per cent for the first year.

[1] This last quantity was written erroneously as $+0.50$ m sec^{-1} in STARR and WHITE (1952 a), although the associated discussion there given requires no correction.

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Table 3. Numerical analysis for two year period. The levels are given in thousands of feet. All velocities are in m sec^{-1}. Internal consistency of figures given is limited by rounding-off approximations

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[uv]}$	$\overline{[u]\,[v]}$	$\overline{[u]'\,[v]'}$	$[u'v']$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\} - \{u\}\{v\}$	r	N
55	− 0.7	− 0.53 ± 0.33	− 2 ± 3	− 2 ± 6	0	− 2	0 ± 4	548	− 0.5	− 0.58	− 1	− 2	− 0.02	1 730
50	+ 3.6	− 0.73 ± 0.36	− 4 ± 4	− 4 ± 6	− 3	− 1	0 ± 4	590	+ 3.8	− 0.85	− 3	0	0.00	1 896
45	+ 7.0	− 0.20 ± 0.39	+ 3 ± 4	+ 6 ± 7	− 1	+ 4	+ 3 ± 4	693	+ 6.8	− 0.31	+ 5	+ 7	+ 0.05	2 649
40	+ 6.7	+ 0.09 ± 0.28	+ 4 ± 3	+ 8 ± 5	+ 1	+ 3	+ 4 ± 4	724	+ 6.8	+ 0.08	+ 8	+ 7	+ 0.06	3 935
35	+ 5.5	+ 0.22 ± 0.28	+ 5 ± 3	+ 10 ± 5	+ 1	+ 4	+ 5 ± 3	719	+ 5.5	+ 0.18	+ 9	+ 8	+ 0.08	3 500
30	+ 2.4	− 0.02 ± 0.19	0 ± 1	+ 5 ± 2	0	0	+ 5 ± 2	730	+ 2.6	− 0.09	+ 5	+ 6	+ 0.08	5 236
25	0.0	− 0.38 ± 0.16	− 1 ± 1	+ 3 ± 2	0	− 1	+ 4 ± 1	728	+ 0.1	− 0.39	+ 3	+ 3	+ 0.07	4 922
20	− 1.7	− 0.08 ± 0.11	0 ± 0	+ 3 ± 1	0	0	+ 3 ± 1	730	− 1.6	− 0.12	+ 3	+ 3	+ 0.09	7 520
14	− 2.8	+ 0.13 ± 0.10	0 ± 0	+ 2 ± 1	0	0	+ 2 ± 1	730	− 2.8	+ 0.11	+ 2	+ 3	+ 0.11	6 887
10	− 3.0	− 0.09 ± 0.06	+ 1 ± 0	+ 3 ± 1	0	0	+ 2 ± 1	730	− 2.9	− 0.09	+ 3	+ 3	+ 0.12	9 279
6	− 3.6	− 0.40 ± 0.10	+ 2 ± 0	+ 4 ± 1	+ 1	+ 1	+ 2 ± 1	730	− 3.6	− 0.34	+ 4	+ 3	+ 0.12	8 842
2	− 2.6	− 1.14 ± 0.15	+ 7 ± 1	+ 9 ± 1	+ 3	+ 4	+ 2 ± 1	730	− 2.7	− 1.09	+ 9	+ 6	+ 0.18	9 303
Integral (10^7 CGS units)				+ 3.6	+ 0.4	+ 0.9	+ 2.4				+ 3.6	+ 3.4	Sum	65 699
Same for first year alone				+ 3.5	+ 0.5	+ 0.9	+ 2.2				+ 3.5	+ 3.0		27 985

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Two Years of Momentum Flux Data for 31° N

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At various times during the past two years, the authors have had occasion to report upon the results of an investigation of the angular momentum balance of the atmosphere as deduced from actual wind measurements distributed over the northern hemisphere (STARR and WHITE 1951, 1952a, 1952b). The meteorological implications of the results of these studies have been discussed in these papers. A further investigation for a period of one additional year has now been completed for 31° N, in order to verify the previous conclusions with independent data. A continuous two-year period has thus been analysed and represents a concluded phase of the investigation for this latitude.

For the second year, computations were made at the standard pressure levels. In combining the data for the first year with those of the second the 2, 6, 10, 20, 30, 40, and 55 thousand foot levels are identified with the 1000, 850, 700, 500, 300, 200, and 100 mb pressure levels respectively. For purposes of record Table I presents the final array of results, while Table II presents a frequency distribution of all available observations for the two year period. The form of these tables is identical with that of the corresponding ones given by STARR and WHITE (1952a, 1952b), to which papers the reader is referred for details of notation.

It is worthy of note that the vertical average of $[\overline{v}]$ with respect to pressure between 1000 and 100 mb is only -1.31 cm sec^{-1}.

Table I. Percentage of total possible observations at each level for each station.

Stations	Pressure levels in mb.						
	1000	850	700	500	300	200	100
Qrendi	20	57	58	52	52	33	18
Bahrein	22	63	62	61	57	47	22
Hyderabad	50	0	94	78	34	0	0
Debrugarh	80	0	87	46	17	0	0
Tokyo	91	92	97	95	85	58	24
Midway	81	79	74	66	53	42	21
Honolulu	93	88	80	73	60	46	26
Weather Ship (a)	77	84	85	82	71	59	22
Santa Maria	99	99	99	97	87	74	38
Big Spring	4	99	99	94	76	58	21
New Orleans	99	98	96	89	70	48	11
Miami	100	100	99	93	90	78	30
Kindley Field	87	87	83	68	36	13	1
Weather Ship (b)	85	79	81	74	55	40	15
Lagens	83	88	87	80	65	41	9
North Front	69	71	68	61	51	23	7
Habbaniya	25	54	52	53	49	43	29
Hong Kong	35	45	40	34	23	18	11
Farouk	32	58	58	55	38	16	2

Table II. Numerical analysis for the period Feb. 1949 to Jan. 1951 inc. The levels are given in mb. All velocities are in m sec.$^{-1}$.

Internal consistency of figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u\ v]}$	$[\overline{u}]\,[\overline{v}]$	$\overline{[u]'\,[v]'}$	$\overline{[u'\,v']}$
100	+ 8.8	—0.05 ±0.39	— 2. ± 6	+ 14 ± 8	0	— 2	+16 ±4
200	+17.6	—0.27 ±0.39	— 5 ± 9	+ 40 ± 12	— 5	0	+45 ±7
300	+14.5	0.00 ±0.24	— 1 ± 5	+ 33 ± 7	0	— 1	+35 ±5
500	+ 8.7	+0.03 ±0.14	0 ± 2	+ 14 ± 3	0	— 1	+14 ±2
700	+ 3.8	—0.01 ±0.10	0 ± 1	+ 5 ± 1	0	0	+ 5 ±1
850	+ 1.2	+0.23 ±0.12	+ 1 ± 0	+ 4 ± 1	0	+ 1	+ 3 ±1
1000	— 0.7	—0.28 ±0.09	+ 1 ± 0	+ 4 ± 1	0	0	+ 3 ±1
Integral (10⁷ CGS units)				+ 14.5	— 0.4	— 0.3	+ 15.1

	9	10	11	12	13	14	15
Level	n	$\{u\}$	$\{v\}$	$\{u\ v\}$	$\{uv\} - \{u\}\,\{v\}$	r	N
100	564	+ 7.8	+ 0.06	+ 15	+ 14	+ 0.14	2123
200	727	+ 15.9	— 0.11	+ 43	+ [illegible]4	+ 0.19	5351
300	730	+ 14.0	— 0.01	+ 33	+ 34	+ 0.19	7793
500	730	+ 8.6	— 0.01	+ 13	+ 13	+ 0.14	9912
700	730	+ 3.9	— 0.02	+ 5	+ 5	+ 0.10	10941
850	730	+ 1.2	+ 0.14	+ 4	+ 4	+ 0.09	9806
1000	730	— 0.7	— 0.30	+ 4	+ 3	+ 0.13	9043
Integral (10^7 CGS units)				+ 14.7	+ 14.8		Sum 54969

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Meridional Flux of Angular Momentum in the Tropics

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Abstract

The results of an attempt to measure the meridional flux of angular momentum across the vicinity of 13° N latitude from actual wind data are presented. It is found that the average flux for one year is directed northward but is much smaller than the corresponding flux at 31° N as obtained from a similar hemispheric study made earlier for that latitude. According to the data the flux at 13° is due mainly to horizontal exchange processes which are most vigorous at an elevation of about 35,000 feet.

1. Introduction

During the course of an extended program of research concerning the angular momentum balance of the northern hemisphere, the writers have reported elsewhere (STARR and WHITE 1951, 1952) the results of flux computations from actual-wind data in the subtropics. In the study described in these two previous papers, henceforth denoted as (A) and (B) respectively, the meridional transports were evaluated from a string of upper-wind stations encircling the earth in the vicinity of 31° N latitude for a period of one year. In the region of the subtropics and at more northerly latitudes the character of the angular momentum balance has also received observational elucidation from the geostrophic-wind studies of WIDGER (1949) MINTZ (1951), WHITE and COOLEY (1952) and LORENZ (1952), and likewise from the actual-wind studies of PRIESTLEY (1949, 1951a, b), STARR (1950) and NYBERG and SCHMACKE (1951).

However, much interest is centered in regard to the character of the transport mechanisms (vertical and meridional) which may be operative in the more tropical regions where geostrophic wind measurements are of doubtful reliability and are more difficult to obtain. For these reasons, and also because of other considerations, recourse must be made to procedures involving the use of actual-wind data in order to throw light upon the problem in this zone. In the present paper an account is given of an attempt to duplicate the studies contained in (A) and (B) for a string of upper-wind stations in the vicinity of 13° N, to the extent that this is possible with existing data. Although an endeavor was made in (B) to show the seasonal variation of the flux, no corresponding attempt was made for 13° N, due to the fact that a somewhat smaller total amount of data was available in the tropics and also for other reasons.

From general physical considerations one is led to certain over-all expectations concerning the total meridional flux across the latitude circle in the vicinity of 13° N. The more important of these are the following. (a) Since the storage capacity of the atmosphere for angular momentum is small when a period of the length of one year is considered, the total

Table 1. List of key stations (numbered) and alternate stations (italics)

Station	Latitude (N.)	Longitude	Altitude (ft)	Type
1. Gao	16° 06′	00° 03′ E	899	pilot balloon
Niamey	13 31	02 26 E	755	»
Zinder	13 48	09 00 E	1604	»
Mopti	14 30	04 12 W	906	»
Bamako	12 38	08 01 W	1086	»
2. El Fasher	13 37	25 20 E	2395	»
Kassala	15 28	36 24 E	1644	»
El Obeid	13 10	30 14 E	1887	»
Kosti	13 10	32 40 E	1253	»
Malakal	09 33	31 39 E	1276	»
Wau	07 42	28 01 E	1440	»
3. Aden Khormaksar	12 50	45 01 E	3	radio wind
Aden Scheikh Othman	12 53	44 58 E	33	pilot balloon
Riyan	14 39	49 23 E	46	»
Kamaran Island	15 20	42 37 E	20	»
4. Trichinopoly	10 49	78 42 E	256	»
Vengurla	15 55	73 40 E	—	»
Anantapur	14 41	77 37 E	1148	»
Madras	13 04	80 15 E	52	»
Mangalore	12 52	74 51 E	72	»
Fort Cochin	09 58	76 14 E	10	»
Trivandrum	08 29	76 57 E	200	»
Nagercoil	08 11	77 26 E	112	»
5. Saigon	10 49	106 39 E	33	»
Mingaladon	16 56	96 07 E	92	»
Tavoy	14 05	98 12 E	112	»
Victoria Point	09 59	98 35 E	122	»
Penang	05 18	100 16 E	13	»
Tourane	16 02	108 11 E	16	»
Pattle	16 33	111 37 E	20	»
Seno	16 40	105 00 E	502	»
Phnom Penh	11 33	104 51 E	33	»
6. Clark Field	15 10	120 34 E	644	radio wind
Laoag	18 11	120 32 E	12	pilot balloon
7. Yap	09 29	138 08 E	0	radio wind
8. Harmon Field	12 31	144 49 E	176	»
9. Kwajalein	08 43	167 44 E	10	»
Wake Island	19 15	166 30 E	0	»
10. Johnson	16 44	169 31 W	20	»
Hilo	19 44	155 04 W	33	»
11. Tehuantepec	16 20	95 14 W	75	pilot balloon
Vera Cruz	19 12	96 08 W	10	»
Ciudad del Carmen	18 39	91 50 W	5	»
12. Managua	12 08	86 12 W	178	radio wind
13. Plato Magdalena	09 48	74 48 W	190	»
Albrook Field	08 58	79 33 W	21	»
Barranquilla	10 55	74 45 W	39	»
14. Waller Field	10 36	61 12 W	0	»
San Juan	18 28	66 07 W	82	»
15. Dakar	14 40	17 26 W	131	»
St. Louis	16 01	16 30 W	10	pilot balloon
Dakar Yoff	14 41	17 25 W	62	»
Sal	16 44	22 57 W	180	»

amount transported into the polar cap north of a given latitude should be equal to the amount removed by surface torques acting on the cap. (b) Since the zone between 13° and 31° N is a region of predominant easterly winds near the surface, the northward flux at 13° N should be smaller (algebraically) than the flux at 31° N. (c) Whether or not the flux across 13° N is actually negative (directed southward) cannot be determined from general

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considerations except as noted below, since one may not assume complete symmetry of the two hemispheres about the equator in this connection. (d) Since at 31° N angular momentum is transported strongly northward by the very large disturbances in the upper westerlies, one might perhaps expect that this effect would not be suppressed completely in a distance so short as 18° to the south.

The degree to which such anticipations as those listed above are not in conflict with the numerical findings obtained, constitutes one rather general criterion for the plausibility of the results.

2. Observational material

As in the study reported in A) and (B) a string of key stations was used. These 15 stations are listed in table 1. Since reports were often missing from the key stations, alternative stations in the vicinity of the key stations were used in order to increase the amount of observational material. The alternative stations are entered in italics below the key stations in the table. Although it would have been advantageous to use the same period of time as was used for the study at 31° N, various factors made this impractical. The average latitude of the key stations is 13° N.

Wind reports for the levels 2, 6, 10, 14, 20, 25, 30, 35, 40, 45, 50, and 55 thousand feet were taken from the data tabulations of the *Daily Series Synoptic Weather Maps* prepared by the U. S. Weather Bureau, in cooperation with the Army, Navy and Air Force for the period from 1 July 1949 to 30 June 1950 for the hour 0300 GMT for each day. For the 6, 10, 20, 30, 40, and 55 thousand-foot levels supplementary data as reported for the 850, 700, 500, 300, 200, and 100 millibar levels, respectively, were added, although preference was given to constant-height data where available. No attempt was made to correct original data, although in a few cases garbled reports were omitted. The frequency of available observations at the various levels is given in table 2. This table is analogous to table 5 in (B) in that it gives the percentages of the total possible observations, i. e., 365. Each wind observation was resolved into the eastward component u and northward component v, tabulated in m sec^{-1}.

3. Computation of transports

In order to avoid repetition, only a brief statement is made here of the methods used to measure the angular-momentum flux, the reader desiring more details being referred to (A) and (B). The notation used may be summarized as follows:

Table 2. Percentage of total possible observations at each level for each station

Station	Altitude in thousands of feet											
	2	*6*	*10*	*14*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*
Gao	63	65	59	46	15	1	0	0	0	0	0	0
El Fasher	94	92	95	87	60	13	11	11	8	4	1	1
Aden	44	54	53	28	47	22	45	23	38	19	15	23
Trichinopoly	99	1	99	0	90	78	73	1	0	0	0	0
Saigon	100	95	95	75	61	11	6	3	0	0	0	0
Clark	84	86	83	75	73	52	44	32	33	23	16	11
Yap	30	25	19	12	10	4	2	1	1	1	0	0
Harmon Field	77	85	83	68	80	63	73	61	68	50	39	28
Kwajalein	92	95	92	86	93	79	82	63	70	49	38	34
Johnson	82	88	86	74	83	66	73	58	64	47	34	28
Tehuantepec	78	75	67	47	27	3	0	0	0	0	0	0
Managua	3	11	11	1	9	0	6	1	3	1	1	0
Plato Magdalena	74	84	83	70	83	63	65	50	47	28	9	0
Waller Field	93	98	97	86	93	78	88	74	81	65	50	37
Dakar	65	66	62	53	55	47	50	44	43	35	25	20

- u eastward component of the wind.
- v northward component of the wind.
- $[\alpha]$ space average of a quantity α over the length of the complete latitude circle.
- α' deviation of a quantity α from its space average $[\alpha]$.
- $\overline{\alpha}$ time average of a quantity α. Time and space averaging processes are not necessarily commutative.
- $[\alpha]'$ deviation of the space average $[\alpha]$ from the space-time average $\overline{[\alpha]}$.
- $\{\alpha\}$ arithmetic mean of all individual observations of α at a given level during entire time period considered.
- n number of days for which observations were available at a given level.
- N number of individual observations present at a given level during entire time-period considered.
- r coefficient of linear correlation for the N pairs of u and v.

In (A) it was shown that the quantity $\overline{[u\,v]}$ which represents the northward transport of

linear momentum per unit mass at a given level may be resolved into three components, namely:

(i) Transport of the first species, $\overline{[u]}\ \overline{[v]}$. This component represents the contribution due to a net meridional mass flux $\overline{[v]}$ during the entire period at the level in question. To the extent that a non-zero value of $\overline{[v]}$ at one level is usually compensated by values having an opposite sign at other levels, this component depends upon the existence of so-called mean meridional circulations.

(ii) Transport of the second species, $\overline{[u]'\,[v]'}$. This component represents the contribution from fluctuations in $[v]$ due to its possible correlation with $[u]$ in time.

(iii) Transport of the third species, $\overline{[u^{*}\,v^{*}]}$. This component represents the contribution due to a correlation between u and v along the length of the latitude circle. It does not require the presence of net air motions $[v]$ or $\overline{[v]}$.

Since one basic question involved in the subject relates to the role of mean meridional circulations, it is useful to form the quantity $\{u\,v\} - \{u\}\,\{v\}$ which is analogous to the sum of the transports of the second and third species although the mode of averaging used is slightly different as explained in (A).

The results are given in table 3 which is to be compared with table 4 of (B), both being of the same basic form. The table gives the transports of each of the three species in columns 6, 7, and 8 together with various other quantities defined in terms of the symbols already recapitulated here and explained more fully in (A). A number of the quantities carry confidence limits defined as twice the standard error indicating approximately the 95 per cent confidence level. Vertical integrals of several quantities with respect to mass (standard atmosphere pressure) are given at the foot. These figures are to be interpreted as the northward transport of linear momentum per *unit length* of the latitude circle for the layer from 2 to 55 thousand feet. For comparison the corresponding integrals for 31° N as reported in (B) are also given at the foot of table 3.

4. Discussion of numerical results

The appraisal of the numerical results obtained depends upon a simultaneous consideration of various factors which might limit the accuracy of the measurements of the flux and other computations. By and large these factors are similar to those enumerated in (A) and to some extent in (B), although certain points are worthy of especial note in view of the circumstances of the present study, specifically the following:

(a) There exists a rather serious gap in the station network extending from Johnson Island eastward to Central America. There appears to be no source of reports from this region which is known to the writers.

(b) When the twelve values of $\overline{[v]}$ given in column 3 of table 3 are averaged vertically with respect to pressure one obtains the net meridional air flow for the entire layer. This quantity turns out to be — 0.16 m sec^{-1}. Since it is quite improbable that a return flow exists below 2,000 ft., it would be necessary that an average northward flow of rather large magnitude should exist in the remainder of the atmosphere above 55,000 feet, if this figure represents actual conditions. This appears to be improbable. The most likely inference to be drawn is that it represents evidence of a slight lack of randomness of the observations with respect to the troughs and ridges of the streamline patterns, which in turn might be related to the gap in the observational network mentioned in (a) above. In any event the figure is rather excessive, the corresponding value at 31° N being only + 0.04 m sec^{-1} as obtained from data in (B).

(c) The total number of individual wind observations is 27,985 at 13° N as compared with a total of 38,301 at 31° N. This is a reflection of a poorer station network and more missing reports. This and other shortcomings such as the one discussed in (b) might be made less serious by using more than one year of data.

(d) Resort had to be made to the use of relatively more pilot balloon stations than were used at 31°. This fact contributes to a paucity of observations at high levels and may be a factor contributing to a slight lack of randomness discussed in (b).

Table 3. Numerical analysis for entire year. The levels are given in thousands of feet. All velocities are in m sec.$^{-1}$ Internal consistency of figures given is limited by rounding-off approximations

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[u]}$	$\overline{[v]}$	$\overline{[u][v]}$	$\overline{[u\,v]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u]'[v]'}$	$\overline{[u'\,v']}$	n	$\{u\}$	$\{v\}$	$\{u\,v\}$	$\{u\,v\} - \{u\}\,\{v\}$	r	N
55	+ 0.1	— 0.42 ± 0.56	— 2 ± 6	— 2 ± 11	0	— 2	0 ± 7	242	+ 0.2	— 0.38	— 1	— 2	— 0.02	664
50	+ 3.3	— 0.56 ± 0.53	— 1 ± 5	— 5 ± 8	— 2	+ 1	— 5 ± 6	279	+ 3.6	— 0.65	— 5	— 3	— 0.03	831
45	+ 6.9	+ 0.09 ± 0.60	+ 3 ± 6	0 ± 10	+ 1	+ 2	— 3 ± 6	342	+ 6.3	— 0.03	— 3	— 3	— 0.02	1174
40	+ 6.7	+ 0.24 ± 0.41	+ 4 ± 5	+ 10 ± 8	+ 2	+ 3	+ 6 ± 6	359	+ 6.4	+ 0.17	+ 11	+ 10	+ 0.08	1664
35	+ 5.3	+ 0.37 ± 0.42	+ 5 ± 5	+ 12 ± 8	+ 2	+ 3	+ 7 ± 5	355	+ 5.1	+ 0.41	+ 13	+ 11	+ 0.11	1532
30	+ 2.5	— 0.08 ± 0.28	0 ± 2	+ 6 ± 4	0	0	+ 6 ± 3	365	+ 2.4	— 0.09	+ 6	+ 6	+ 0.09	2251
25	0.0	— 0.35 ± 0.23	— 1 ± 1	+ 4 ± 2	0	— 1	+ 4 ± 2	363	0.0	— 0.32	+ 4	+ 4	+ 0.08	2113
20	— 2.0	— 0.08 ± 0.18	0 ± 1	+ 3 ± 2	0	0	+ 3 ± 2	365	— 1.9	+ 0.10	+ 3	+ 3	+ 0.10	3200
14	— 2.9	+ 0.22 ± 0.15	0 ± 1	+ 1 ± 1	— 1	+ 1	+ 1 ± 1	365	— 2.9	+ 0.22	+ 1	+ 2	+ 0.08	2950
10	— 3.1	— 0.05 ± 0.12	+ 1 ± 1	+ 3 ± 1	0	0	+ 2 ± 1	365	— 3.1	— 0.04	+ 3	+ 2	+ 0.10	3954
6	— 3.9	— 0.51 ± 0.15	+ 3 ± 1	+ 4 ± 1	+ 2	+ 1	+ 1 ± 1	365	— 4.0	— 0.42	+ 4	+ 2	+ 0.08	3718
2	— 2.4	— 1.24 ± 0.21	+ 7 ± 1	+ 7 ± 1	+ 3	+ 4	+ 1 ± 1	365	— 2.8	— 1.22	+ 7	+ 4	+ 0.12	3932
Integral (10^7 CGS units)				+ 3.5	+ 0.5	+ 0.9	+ 2.2				+ 3.5	+ 3.0	Sum	27,985
Same for 31° N				+ 16.8	— 0.4	+ 0.7	+ 16.5				+ 16.9	+ 17.8		38,301

(e) The range of latitude of the individual key stations is only 8 degrees which is much less than for the network at 31° N.

(f) A large number of alternative stations was used. This in itself should not be necessarily detrimental to the results, except to the extent that a lack of randomness might be introduced in the data.

(g) The confidence limits for the flux $\overline{[u\,v]}$ and the component $\overline{[u'\,v']}$ are smaller at 13° than at 31° N, but the values of the quantities are also smaller so that on the whole less confidence is indicated. On the other hand, the alternative method of averaging given in columns 10—14 of table 3 gives results which compare very well with the first technique.

Generally speaking the data fulfill plausible expectations such as those mentioned in the introduction. The following points may be noted:

(1) The vertical integral of the flux $\overline{[u\,v]}$ may be converted into the total flux of angular momentum by multiplying by the length of the torque arm and the length of the latitude circle. The result is $+ 8.5 \times 10^{25}$ gm cm^2 sec^{-1}, which is considerably smaller than the corresponding value at 31° N, i. e., $+ 31.5 \times 10^{25}$. The difference represents essentially the rate at which angular momentum is acquired from the surface, since probably only small contributions to the fluxes are made by the portions of the atmosphere not considered.

(2) The largest contribution to the angular momentum flow across 13° N is due to the transport of the third species, the value being $+ 5.2 \times 10^{25}$ gm cm^2 sec^{-1}. The next largest is due to the second species, the value being $+ 2.2 \times 10^{25}$. The smallest is due to the first species of transport, i. e., $+ 1.2 \times 10^{25}$. Thus about 14 per cent of the total is brought about by mean meridional circulations, according to the data.

(3) If use is made of parentheses to indicate vertical averaging with respect to standard atmosphere pressure, the quantity $(\overline{[v]})$ has a value of -0.16 m sec^{-1} as already stated. Similarly $(\overline{[u]})$ turns out to be $+ 0.50$ m sec^{-1}. Since one may write that $\overline{[u]} = (\overline{[u]}) + \overline{[u]}'$ with a similar relation involving v in place of u, it follows that

$$(\overline{[u]}\,\overline{[v]}) = (\overline{[u]})\,(\overline{[v]}) + (\overline{[u]}'\,\overline{[v]}')$$

The term on the left is a measure of the transport of the first species integrated vertically. The first term on the right represents a portion of this quantity depending on $(\overline{[v]})$ which as mentioned before may be spurious. Although this "correction" procedure is not recommended as a substitute for better observational material, it is of interest to note that in the present case the correction is of negligible magnitude compared to the last term which contains only departures from the vertical averages.

(4) In the process of calculation, individual monthly averages of $\overline{[u'\,v']}$ were obtained for the twelve levels used. Of these 144 monthly values 98 were positive in sign, whereas at 31° N 138 were positive. There appears to be a tendency for this quantity to be negative at high levels as is also shown in table 3. However, caution must be used in accepting this feature in view of the large confidence limits associated with these data.

(5) The correlation coefficients r for the N pairs of u and v given in column 14 of table 3 are smaller than the corresponding ones for 31° N except at 2,000 feet, and actually become slightly negative at the highest three levels.

5. Some meteorological implications

(a) From time to time various estimates have been made of the strength of mean meridional circulations which would be needed in order to transport the net amount of angular momentum across given latitude circles prescribed by mean surface torques, on the assumption that the entire flux is of the first species. Two difficulties attend this process. In the first place accurate estimates of the requirements due to surface torques are difficult to make. In the second place there is no objective method for specifying the functional dependence of $\overline{[v]}$ on elevation (pressure). It is of interest to substitute the total flux as measured by studies of the present kind for the surface torque requirements, and to employ a variational method to specify $\overline{[v]}$ as a function of pressure. In effect one thus seeks the *least intense* meridional circulation which could bring about the stated transport T. This procedure was suggested to the writers by Messrs. E. N. Lorenz

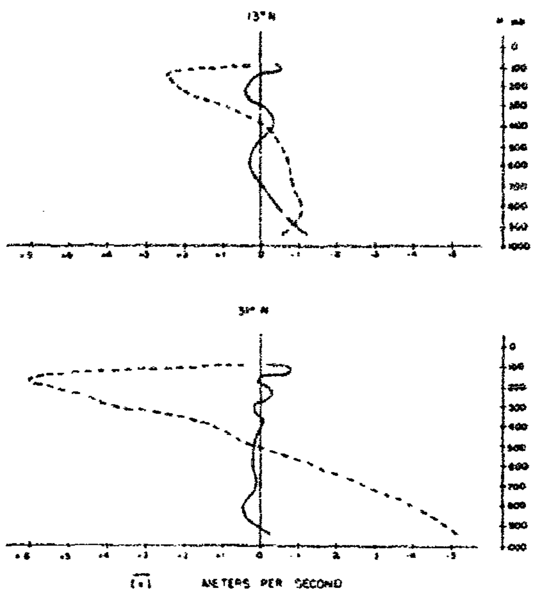

Fig. 1. Mean meridional circulations in m/sec at 13° N and at 31° N as given by data (full curves). Weakest mean meridional circulations capable of transporting total amount of angular momentum given by dashed curves.

and H. L. Kuo of the Massachusetts Institute of Technology.

The process takes the form of finding the solution of the variational problem

$$\delta \int_b^a \overline{[v]}^2 \, dp = 0$$

subject to the side conditions that

$$\int_b^a \overline{[v]} \, dp = 0; \quad \int_b^a \overline{[u]} \; \overline{[v]} \, dp = KT$$

Here standard atmosphere pressuresar e used, a and b being the values at 2,000 and 55,000 feet respectively. The dependence of $\overline{[u]}$ on p is that given by the data and K is a constant. The result is simple to state qualitatively. The "least square" cell so obtained has the same shape as the vertical profile of $\overline{[u]}$, but displaced relative to the origin in order to satisfy the first side condition which stipulates that there shall be no net transport of air. The amplitude is of such a magnitude as to satisfy the second side condition, i. e., to provide the specified momentum transport.

The profiles of $\overline{[v]}$ so obtained are given by the dashed curves in Fig. 1, together with the actually observed profiles (full curves), at 13° and 31° N. The contrast between the two sets of curves is obvious and probably cannot be ascribed to poor observational material.

It would of course be possible to impose additional side conditions upon the problem stipulating, for example, that the mean meridional circulation should transport stated amounts of energy of several forms. However, such added restrictions would not lead to less vigorous circulations in the least square sense.

(b) According to what has been said under (1) of the previous section about 23×10^{25} gm cm^2 sec^{-1} represents the average rate at which angular momentum is supplied to the zone between 13° and 31° N by surface torques. Recent estimates of such surface torques made by Priestley (1951b) from surface stresses yield a value of about 15×10^{25} gm cm^2 sec^{-1}.

The average zonal component of the surface stress for the belt as derived from the present figures is approximately 0.53 dynes cm^{-2}, which is somewhat larger than the estimate used by Priestley.

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FLOW OF ANGULAR MOMENTUM AS A PREDICTOR FOR THE ZONAL WESTERLIES

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ABSTRACT

An approximate differential equation is presented, relating the change in speed of the zonal westerly winds to the contemporary zonal wind-speed and the meridional flow of absolute angular momentum. This equation is tested statistically by means of values of the momentum flow and the zonal wind-speed, computed with the aid of the geostrophic-wind approximation, from pressure and height data extracted from analyzed northern-hemisphere maps. The momentum flow is found to be positively correlated with the contemporary zonal wind-speed, and also with the contemporary change of the zonal wind-speed, in agreement with the approximate equation. The study suggests that the momentum flow may be a useful quantity for forecasting the zonal wind-speed. It also implies that an important part of the momentum flow is accomplished by means of large-scale horizontal eddies, whose forms are not obscured by the use of subjectively analyzed maps nor by the geostrophic-wind approximation.

1. Introduction

The strength of the prevailing zonal westerly winds in middle latitudes has often been regarded as an index of the state of the general circulation of the atmosphere. Variations in the speed of the zonal westerlies have therefore been the subject of numerous investigations. Another subject which has recently received much attention is the balance of absolute angular momentum in the atmosphere. Since the speed of the westerly wind at a specified point in the atmosphere and the absolute angular momentum per unit mass at that point completely determine each other, the two subjects are closely related.

The functional relation between wind speed and angular momentum contains a latitude factor; however, within a region whose latitudinal extent is small, the average westerly wind-speed and the total angular momentum determine each other fairly closely. Changes in the total angular momentum within a region bounded by two latitudes can result only from a meridional flow of angular momentum across the vertical boundaries, or from a torque exerted by the underlying surface. The possibility of predicting changes in the strength of the zonal westerly winds on the basis of the meridional flow of angular momentum has therefore suggested itself to several investigators [4; 5; 7].

A reasonably conclusive test of the prognostic value of the meridional flow of angular momentum, which for brevity may be called simply the momentum flow, requires the use of data for a long period of time. The earlier studies [5; 7] were handicapped by the absence of more than one or two months of data. The present study has used data for the momentum flow at several latitudes and elevations over a period of four consecutive months. Less complete data for six additional months, collected for the purpose of testing relations which appeared to be significant during the first four months, have also been used. Although the results are of a preliminary nature, they strongly indicate that the momentum flow has some prognostic value. At the same time, they show that it is feasible to compute the momentum flow on a day-to-day basis from analyzed northern-hemisphere maps.

2. An approximate equation

The absolute angular momentum contained in a unit mass of atmosphere is given by the expression

$$M = \omega r^2 + ru, \tag{1}$$

where ω = earth's angular velocity, r = distance from earth's axis and u = eastward component of wind velocity. From the equation for eastward acceleration and the equation of continuity, it follows that

$$\partial(\rho M)/\partial t + \operatorname{div} \rho M\mathbf{c} + \partial p/\partial\lambda + r\rho D = 0, \tag{2}$$

where t = time, λ = longitude, p = pressure, ρ = density, $\mathbf{c}$ = wind-velocity vector and D = westward acceleration due to friction. If (2) is integrated over the entire volume V lying north of a vertical constant-latitude surface S, the result is

$$\frac{d}{dt}\int_V \rho M\, dV + \int_V \left(r\rho D + \frac{\partial p}{\partial\lambda}\right) dV = \int_S \rho M v\, dS, \tag{3}$$

where v = northward component of wind velocity.

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In (3), the first volume integral represents the total angular momentum contained in V, while the second volume integral represents the total torque exerted upon V by the earth's surface, this torque being due partly to skin friction and partly to horizontal pressure-forces exerted by topographic irregularities of the earth's surface (the mountain torque). The surface integral represents the total flow of angular momentum northward across S.

A very simple approximation to (3) is the equation

$$dU/dt + KU = a\tau, \tag{4}$$

where U is the average low-level westerly wind-speed in V, τ the total flow of angular momentum northward across S, and K and a are positive constants which depend upon the latitude of S. Equation (4) is only an approximation to (3), for a number of reasons. First, the total absolute angular momentum within a region depends not only upon the westerly wind-speed, but also upon the total mass within the region; moreover, to obtain a measure of total angular momentum, the westerly wind-speed should not be simply averaged, but should be weighted more strongly at low latitudes. Second, the low-level westerly wind is not a perfect measure of the westerly wind at higher levels; however, Willett [9] has found that the sea-level and upper-level westerly winds are highly correlated. Third, the torque exerted by skin friction is not exactly proportional to the speed of the low-level westerlies, although Widger [7] has shown that a high correlation exists. Fourth, the mountain torque has been neglected; however White [10] has shown that the mountain torque and the skin friction torque are correlated in latitude; possibly they tend also to be correlated in time, although such a relation has not yet been established.

It is evident that these approximations make (4) less accurate than (3). On the other hand, (4) is so simple that it relates the speed of the zonal westerlies to a single additional quantity, the momentum flow. With sufficient observational data, it is easy to subject (4) to statistical tests. The results of the tests may be regarded as a measure of the justifiability of the approximations used.

Some statistical properties of any quantities U and τ satisfying (4) will now be discussed. If the period of time under study is sufficiently long, the first two terms in (4) must be uncorrelated, since a contemporary correlation between U and dU/dt would imply progressive variations of U^2. The sum of two uncorrelated quantities must be positively correlated with each quantity, but not perfectly correlated with either. Significant positive contemporary correlations of τ with both U and dU/dt should therefore be present in any observational data which claims to satisfy (4), but a nearly perfect correlation between τ and either U or dU/dt is not to be expected and would, in fact, be a denial of (4). It follows as a corollary that correlations with a time lag between τ and U should be significantly positive if the time of U follows that of τ by a suitable interval, but not if the time of U precedes that of τ by a similar interval.

3. Preliminary tests

The first statistical tests were performed with the aid of data which had been previously gathered by the General Circulation Project at the Massachusetts Institute of Technology. The basic data had been extracted from analyzed northern-hemisphere maps, and consisted of individual sea-level pressures, and 700- and 500-mb heights, at each 5 deg of latitude and longitude covered by the analyses, for each day of the four-month period November 1945–February 1946. The sea-level and 500-mb maps were taken from the *Northern hemisphere historical weather maps* [1], while the 700-mb maps were photographic copies of maps analyzed by the Air Weather Service and the U. S. Weather Bureau.

From these basic data, daily values of the momentum flow across various latitudes were obtained according to the procedure developed and described by Widger [8]. Briefly, in the absence of net mass-flow across a given latitude circle at a given elevation, the momentum flow is nearly proportional to the integral, around the latitude circle, of the product of the eastward and northward components of the wind. Pressure or height differences across 10 deg intervals of latitude and longitude were used to measure these components, according to the geostrophic-wind equation. Such differences appear in the formula

$$\tau_\phi^5 = \sum_{\lambda/5=1}^{72} [h(\phi - 5, \lambda) - h(\phi + 5, \lambda)] \times [h(\phi, \lambda + 5) - h(\phi, \lambda - 5)], \tag{5}$$

where $h(\phi, \lambda)$ is the 500-mb height at latitude ϕ and longitude λ. Upon multiplication by a suitable factor, τ_ϕ^5 represents the momentum flow across latitude ϕ, within a layer near 500 mb. Analogous expressions represent the momentum flow within layers near 700 mb and sea level. The sum of the flows in these three layers, here denoted by τ_ϕ, was taken to be the total momentum flow across latitude ϕ.

Daily values of the average sea-level pressure at various latitudes were also obtained from the basic data. The average sea-level pressure P_ϕ at latitude ϕ was assumed to be the simple arithmetic average of the 72 individual pressures at that latitude. The average zonal wind-speeds in various regions were measured geostrophically by linear combinations of average pressures.

Since τ_ϕ was computed from geostrophic-wind observations, it evidently includes none of the momentum flow due to mean meridional cells, *i.e.*, due to a net

TABLE 1. Autocorrelations of daily values of pressure and momentum flow for the season November 1945–February 1946. Values are in hundredths.

Lag (days)	P_{75}	P_{65}	P_{55}	P_{45}	P_{35}	τ_{55}	τ_{45}	τ_{35}
1	87	80	72	89	87	44	48	30
2	74	57	50	76	76	11	23	08
3	64	43	36	70	69	08	18	01
4	54	37	21	69	62	14	13	−06

flow of mass at individual elevations. Instead, it includes only the flow due to horizontal eddies, *i.e.*, due to correlations in the horizontal direction between the eastward and northward wind-components. Furthermore, since the wind components used were averages over 10-deg intervals, only the flow due to large-scale horizontal eddies is included.

It was anticipated from the beginning that the most significant results, statistically, might not be those predicted by (4). It was therefore decided to make a rather exhaustive study involving the momentum flow τ_ϕ across each of the four latitudes 35, 45, 55 and 65°N, and the average sea-level pressure P_ϕ at each of the six latitudes 25, 35, 45, 55, 65 and 75°N. Daily values of each of these ten quantities were correlated with values of each other quantity, simultaneously and also with time lags of from one to ten days in each direction, for the "season" November 1945–February 1946. This procedure yielded a set of 1045 distinct correlation coefficients, including 375 of pressure with pressure, 504 of pressure with momentum flow, and 166 of momentum flow with momentum flow. Because such a procedure necessarily involved much repetitious labor, it seemed feasible to perform much of the computing by means of punched-card machines. The punched-card computations were performed by the Statistical Services Section at the Massachusetts Institute of Technology.

The most pertinent of these 1045 correlations appear in tables 1, 2 and 3. The outstanding feature of table 1 is the contrast between the high day-to-day persistence of the pressures and the low day-to-day persistence of the momentum flows. This feature will appear to be of considerable significance from the forecasting point of view.

Table 2 reveals distinct negative correlations between pressures near 70°N and pressures near 40°N. The correlation pattern is similar to patterns obtained

TABLE 2. Contemporary correlations of daily values of pressure (heading columns) with pressure (heading rows), and standard deviations of pressure, for the season November 1945–February 1946. Correlation values are in hundredths, and standard deviations are in mb.

	P_{75}	P_{65}	P_{55}	P_{45}	P_{35}
P_{75}	100	70	−20	−54	−36
P_{65}	70	100	06	−57	−50
P_{55}	−20	06	100	41	−01
P_{45}	−54	−57	41	100	65
P_{35}	−36	−50	−01	65	100
Standard deviation	6.6	4.4	2.9	3.5	2.3

by the writer [2] for correlations between five-day mean sea-level pressures during seven winter seasons. The pattern is interpreted as showing that the principal sea-level pressure variations during the season resulted from shifts of mass between two zones, one centered near 70°N and one near 40°N. The double maximum of standard deviation seems to support this idea.

If the strength of the zonal westerlies is measured by the difference between a pressure in the southern zone and a pressure in the northern zone, it is evident that strong zonal winds may be identified with a concentration of mass in the southern zone. One can, indeed, compute correlations involving zonal wind-speeds from the standard deviations and correlations in table 2. For example, if $U_{55} = P_{45} - P_{65}$, the contemporary correlations between U_{55} and P_{75}, P_{65}, P_{55}, P_{45} and P_{35} are found to be −0.71, −0.91, 0.17, 0.86 and 0.64, respectively. The previously mentioned study by the writer [2] suggested that the zonal wind-speed U_{55} was a good index for the major fluctuations of the general circulation.

Correlations of momentum flow with sea-level pressure and wind speed appear in table 3. It is perhaps a matter of opinion which of these correlations are significant, but the following features stand out. With no time lag, and also with pressure following momentum flow, the momentum flow at each latitude is correlated positively with P_{35} and P_{45}, insignificantly for the most part with P_{55}, and negatively with P_{65}

TABLE 3. Correlations of daily values of sea-level pressure and zonal wind-speed (heading columns) with momentum flow (heading rows), for the season November 1945–February 1946. Values are in hundredths. Positive lag indicates that pressure or wind speed follows momentum flow.

	Lag (days)	P_{75}	P_{65}	P_{55}	P_{45}	P_{35}	U_{55}
	−3	−12	00	09	12	21	06
	−2	−12	−02	11	15	21	09
	−1	−19	−08	15	27	27	19
	0	−33	−28	34	36	32	36
τ_{55}	1	−43	−36	14	37	36	41
	2	−41	−37	−07	29	36	38
	3	−32	−31	−07	26	34	32
	4	−31	−31	−10	33	39	36
	−3	−02	12	13	03	11	−06
	−2	−10	04	20	11	12	03
	−1	−15	−06	15	24	29	16
	0	−27	−31	13	45	41	42
τ_{45}	1	−41	−38	12	46	49	47
	2	−39	−37	04	38	47	42
	3	−29	−20	03	28	44	27
	4	−21	−18	04	32	45	27
	−3	02	07	03	00	10	−04
	−2	−03	01	07	09	10	04
	−1	−06	−10	10	13	20	13
	0	−10	−16	04	22	33	21
τ_{35}	1	−24	−23	03	27	42	28
	2	−29	−21	−04	26	38	26
	3	−30	−28	−04	22	37	29
	4	−23	−22	−02	24	32	26

and P_{35}. With pressure preceding momentum flow, the correlations are mostly insignificant. In the light of the interpretation of table 2, it appears that above-normal values of momentum flow in middle latitudes tend to be accompanied and also followed, but not preceded, by a concentration of mass in the southern zone, whence they must also tend to be accompanied by increasing mass within the southern zone. Equivalently, above-normal values of the momentum flow tend to be accompanied by strong zonal westerly winds, and also by increasing zonal westerly winds, while below-normal values of the momentum flow tend to be accompanied by weak and decreasing zonal westerly winds.

The correlations between momentum transport and wind speed in table 3 therefore have the signs predicted by (4). It must be admitted, of course, that the average wind speed U_{55} between 45 and 65°N is not the same as the average wind speed north of a given latitude, to which (4) refers. Nevertheless, the theoretical usefulness of (4) seems to be confirmed.

TABLE 4. Correlations of four-day mean values of momentum flow (heading rows) with values of sea-level pressure and zonal wind-speed on the fifth day (heading columns), for the season November 1945–February 1946. Values are in hundredths.

	P_{70}	P_{60}	P_{50}	P_{40}	P_{30}	U_{55}
τ_{55}	−55	−50	−04	46	53	55
τ_{45}	−45	−40	08	50	65	50
τ_{35}	−43	−38	−03	40	60	44

The magnitudes of the correlations seem perhaps disappointingly small, and appear at first to cast some doubt upon the practical value of (4). A closer inspection, however, shows that tables 1 and 3 together imply the existence of considerably higher correlations. For example, table 3 shows that U_{55} has a moderately high correlation with the value of τ_{55} at the same time, and also with the value of τ_{55} two days earlier. Table 1 shows that values of τ_{55} separated by two days are almost independent quantities statistically, in the sense that the correlation is near zero. When a quantity, here U_{55}, is correlated with each of two independent quantities, its correlation with some linear combination of these quantities is considerably higher. The correlation can further be increased by introducing a third independent quantity, the value of τ_{55} four days earlier. It would be possible to compute the linear combinations which give the highest possible correlations, but, to illustrate the point, it seems sufficient to exhibit the correlations between values of momentum flow averaged for four successive days and pressure and wind speed on the fifth day. The increased magnitude of these correlations, which appear in table 4, over those in table 3 is apparent. The correlations suggest that (4) may lead, after all, to some rules having forecasting value.

4. Further tests

It is evident, from the preceding section, that zonal wind-speed and momentum flow were related during the season studied. Rather than find the best relations for that season, it seems more desirable to find relations which hold during each of several seasons.

The choice of additional seasons was determined by the readily available data. The basic data in this case were made available through the kindness of the U. S. Weather Bureau—Massachusetts Institute of Technology Extended Forecasting Project. The data consisted of individual sea-level pressures and 500-mb heights, which had been extracted from analyzed northern-hemisphere maps, at each 5 deg of latitude and 10 deg of longitude covered by the analyses, for each day of the year 1949. The maps were taken from the latest northern-hemisphere weather-map series [6]. From this year, two "seasons" of three months each were chosen, one consisting of January, February and March, and the other consisting of October, November and December.

For these two seasons, only the 500-mb data were used in computing momentum flow. Daily values of the momentum flow were computed from the formula

$$\tau_\phi' = \tfrac{1}{2}\sum_{\lambda/10=1}^{36} h(\phi - 2\tfrac{1}{2}, \lambda)[h(\phi + 2\tfrac{1}{2}, \lambda + 10) - h(\phi + 2\tfrac{1}{2}, \lambda - 10)]. \quad (6)$$

In (6), the height difference in brackets is a geostrophic measure of the northward component of the wind, so that τ_ϕ' appears to be a sum of products of a height with a wind component, rather than a sum of products of wind components, like τ_ϕ^* in (5). However, it can be seen that τ_ϕ' actually is a sum of products of wind components, when it is observed that (6) may be expanded to become

$$\begin{aligned}\tau_\phi' = \tfrac{1}{2}\sum_{\lambda/10=1}^{36} &[h(\phi - 2\tfrac{1}{2}, \lambda) - h(\phi + 2\tfrac{1}{2}, \lambda)] \\ &\times [h(\phi - 2\tfrac{1}{2}, \lambda + 10) - h(\phi - 2\tfrac{1}{2}, \lambda - 10) \\ &+ h(\phi + 2\tfrac{1}{2}, \lambda + 10) - h(\phi + 2\tfrac{1}{2}, \lambda - 10)]. \quad (7)\end{aligned}$$

In (7), the eastward component of the wind at latitude ϕ is multiplied by the average of the northward components at latitudes $\phi - 2\frac{1}{2}$ and $\phi + 2\frac{1}{2}$.

Formula (6) is perhaps the simplest possible formula for computing geostrophic-momentum flow directly from tabulated data. It may easily be altered slightly, to be applicable to data tabulated for each 5 deg of longitude rather than each 10 deg. In such a form, it has been discussed in detail by the writer [3].

From the daily values of momentum flow, a set of four-day mean values,[2] overlapping every two days, was constructed. That is, if D is the first day of the

[2] Four-day means were chosen for convenience in computation. Nearly the same results should be expected from the use of five-day means, such as those used by the U. S. Weather Bureau in extended forecasting.

season for which momentum flow is measured, and D' is the last day, the first four-day period consists of D, $D + 1$, $D + 2$ and $D + 3$, the second consists of $D + 2$, $D + 3$, $D + 4$ and $D + 5$, *etc.*, and the last consists of $D' - 3$, $D' - 2$, $D' - 1$ and D'.

Daily values of the average sea-level pressure at various latitudes were also obtained for these two seasons. Four-day mean values were then determined. Zonal wind-speeds were again represented by linear combinations of pressures. In order that contemporary and lag correlations during one season might all be based upon the same number of pairs of values, average sea-level pressures were determined for four additional days at the beginning and also at the end of each season. Thus, the first four-day period for pressures and wind speeds consists of $D - 4$, $D - 3$, $D - 2$ and $D - 1$, while the last consists of $D' + 1$, $D' + 2$, $D' + 3$ and $D' + 4$.

Similar four-day averages were also determined for the season November 1945–February 1946. As with the other two seasons, only the momentum flow computed from the 500-mb maps was used.[3]

Four-day mean values of momentum flow were then correlated with values of pressure and wind speed, simultaneously and also with time lags of two and four days in each direction, for each of the three seasons. The highest correlations discovered involved the momentum flow $\tau_{52.5}$ across latitude 52.5°N and the zonal wind-speed $U_{55} = P_{45} - P_{65}$. The results are summarized in table 5. In this table, τ denotes the four-day mean value of $\tau_{52.5}$, while U denotes the four-day mean value of U_{55}. The symbols U_{--}, U_{-}, U_{+} and U_{++} also denote four-day mean values of U_{55}, occurring, respectively, four days earlier, two days earlier, two days later and four days later than the values of U. Thus, the correlation of τ with U is contemporary, while those of τ with U_{--}, U_{-}, U_{+} and U_{++} are lag correlations. The difference $U_{++} - U_{--}$ evidently represents the eight-day change of the four-day mean value of U_{55}. Thus, the correlation between τ and $U_{++} - U_{--}$ may be regarded as a contemporary correlation between τ and the rate of change of U.

[3] Actually, to make use of earlier computations, the quantity $\tau'_{52.5}$ was replaced by a nearly equal quantity, the average of τ^5_{50} and τ^5_{55} as computed by (5). It is not believed that any of the results are noticeably affected by this substitution.

Table 5. Correlations involving four-day mean values of the momentum flow across latitude 52.5°N and the sea-level zonal wind-speed between 45° and 65°N. Values are in hundredths.

Item	Quantities correlated		Nov. 45–Feb. 46	Jan.–Mar. 49	Oct.–Dec 49
1	τ	U_{--}	14	09	01
2	τ	U_{-}	35	27	30
3	τ	U	60	50	59
4	τ	U_{+}	68	62	60
5	τ	U_{++}	60	59	40
6	τ	$U_{++} - U_{--}$	41	53	36
7	τ	$U + \frac{1}{2}(U_{++} - U_{--})$	72	67	66
8	U	U_{++}	64	79	52
9	$U + \frac{1}{2}\tau$	U_{++}	70	82	53

The correlation coefficients in table 5 are in general agreement with (4). The first five items show that, as anticipated, the momentum flow is not highly correlated with the speed of the zonal westerlies if the westerlies precede the flow, but that there are fairly high correlations if the westerlies accompany or follow the flow. Thus, as shown by the sixth item, the flow is positively correlated with the change in the speed of the zonal westerlies.

In the seventh item, $U + \frac{1}{2}(U_{++} - U_{--})$ is merely a typical linear combination which has a high correlation with τ for each season. It is not necessarily the combination having the highest correlation for any particular season. Since $U_{++} - U_{--}$ represents an eight-day change, the coefficient $\frac{1}{2}$ would be in agreement with (4) if $1/K = 4$ days. These correlation coefficients may be regarded as a measure of how closely the computed values of the two sides of (4) actually balance.

The eighth item is simply an autocorrelation, which measures the persistence of the zonal wind-speed after four days. In the ninth item, $U + \frac{1}{2}\tau$ is a typical combination of U and τ which is well correlated with U_{++} for each season.[4] Evidently each correlation in item 9 is higher than the corresponding one in item 8, so that $U + \frac{1}{2}\tau$ is a quantity which in each season leads to a better-than-persistence forecast for the speed of the zonal westerlies four days later.

It frequently occurs that two meteorological quantities are highly correlated during a period of several months simply because they have similar normal seasonal trends. It might thus appear that some of the high correlations in table 5 are merely the result of seasonal variations, rather than shorter-period irregular fluctuations. A standard method for testing such an idea consists of removing the seasonal trend from the data.

The values of τ so far computed are insufficient for determination of the seasonal trend of τ. On the other hand, the seasonal trend of U can be determined with moderate precision. "Normal" values of U have been estimated by the writer [2] from monthly normal values, based upon forty years of northern-hemisphere maps. These normal values were subtracted from the observed values of U for the three seasons under study. The computations summarized in table 5 were then repeated.

Although some changes were observed for individual seasons, the average correlations for the three seasons of τ with U, and with $U_{++} - U_{--}$, were virtually unaltered. It therefore does not appear that the correlations in table 5 are the result of a relationship between the seasonal trends of τ and U.

It can hardly be claimed that the results presented in table 5 constitute a complete verification of (4).

[4] Since U and τ are dimensionally different, the coefficient $\frac{1}{2}$ in $U + \frac{1}{2}\tau$ must be dimensional. It is applicable, in this study, when $U_{55} = P_{45} - P_{65}$ is expressed in tenths of mb, and $\tau_{52.5}$ is computed from (6), with $h(\phi, \lambda)$ expressed in hundreds of ft.

For one thing, it is hard to identify the measured quantities U and τ in table 5 with the quantities U and τ in (4), since the average wind-speed between 45 and 65°N is not the same as the average wind-speed north of 52.5°N. It must be admitted that the quantities $\tau_{52.5}$ and U_{55} were chosen for presentation in table 5 because of the resulting high correlations.

Aside from this consideration, the correlations in item 7 of table 5 are far from perfect. Nevertheless, they appear to the writer to be gratifyingly high. It would be difficult to say exactly how large a correlation must be under these conditions to be "significant," but the stability of the correlations from one season to another, together with their general agreement with the theory, makes it seem highly improbable that they are merely the result of chance.

5. Conclusion

The flow of angular momentum across certain latitudes is found to be positively correlated with both contemporary and subsequent values of the zonal wind-speed at certain latitudes. The relations appear to be in general agreement with theory. Considerable further research, involving several years of additional data, will be required to determine what relations will most likely prove consistently good. When these relations are found, an additional basis for forecasting the speed of the zonal westerly winds will have been established.

Although the results so far obtained have not yet greatly improved the statistical prediction of the zonal wind-speed, they do possess far-reaching implications concerning the nature of the flow of angular momentum It should be remembered that the computations involved are based on geostrophic winds, determined from subjectively analyzed maps, by means of contour heights at points separated by several hundred kilometers, at the single level of 500 mb, once a day. Since fairly high correlations were found, it would seem that an important part of the flow of angular momentum is obtained from the computation procedure Therefore, there appears to be strong evidence in favor of the following claims:

1. An important part of the flow of angular momentum is accomplished through the medium of horizontal eddies.
2. An important part of the flow of angular momentum due to horizontal eddies is due to large-scale horizontal eddies, whose forms can be described by specifying the wind vectors at points separated by several hundred kilometers.
3. The forms of the large-scale horizontal eddies are so definite that they are not obscured by the necessarily subjective analysis of northern hemisphere maps, nor by the use of the geostrophic-wind approximation.
4. An important part of the flow of angular momentum can be deduced from measurements at one level.
5. An important part of the flow of angular momentum can be deduced from observations taken once a day.

It should be noticed that each of the above claims refers to an important part of the flow of angular momentum, and not to the entire flow Thus, the very same observations which lead to these claims also suggest methods for increasing the numerical values of the correlation coefficients obtained. Because of the sparse distribution of observing stations, particularly at certain longitudes, it hardly seems feasible at present to measure the desired flow of angular momentum on a daily basis through observed winds, rather than geostrophic winds. Neither does it seem feasible to measure the flow due to small-scale eddies, although the use of data at every 5 deg rather than every 10 deg of longitude might be desirable. On the other hand, it is altogether feasible to measure the large-scale geostrophic flow at several levels instead of one. Such a procedure may yield numerically higher correlation coefficients. Finally, the low day-to-day persistence of the flow of angular momentum suggests that, even though an important part of the flow can be deduced from obsevations taken once a day, a significant part may also be lost. Computations of the flow at 12-hr rather than 24-hr, intervals may well give numerically higher correlations than those so far obtained.

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A MULTIPLE-INDEX NOTATION FOR DESCRIBING ATMOSPHERIC TRANSPORT PROCESSES

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In a celebrated paper, Reynolds (1894) used an equation of the form

$$F = \bar{F} + F' \tag{1}$$

to express an arbitrary quantity F as the sum of its mean value $\bar{F}$ over a given region and its departure F' from its mean value. Equation (1) leads to the important equation

$$\overline{FG} = \bar{F}\bar{G} + \overline{F'G'} \tag{2}$$

for the mean value of the product of two arbitrary quantities F and G, if variations of $\bar{F}$ and $\bar{G}$ within the region are neglected.

If mean values of a quantity with respect to each of several independent variables are to be considered, the notation of Reynolds requires some amplification. It is possible to introduce several symbols, one for the mean with respect to each variable. Alternatively, it is possible to let the same symbol denote the mean with respect to any variable, and to let the position of the symbol specify the variable. The notation described in this note is based upon the latter procedure.

The notation was originally developed for treating meteorological problems involving the total flux of certain quantities across specified latitudes. It is described as it applies to such problems. It may easily be modified to apply to other problems.

At a specified latitude, the quantities involved may be regarded as functions of longitude λ, time t and pressure p. To indicate mean values of these quantities, and departures from mean values, it is convenient to attach triple subscripts to the symbols for the quantities. The first subscript refers to longitude, the second to time and the third to pressure. A subscript "1" refers to the mean value with respect to the appropriate variable; the subscript "2" refers to the departure from the mean value. A subscript "0" indicates that no averaging has been performed with respect to the particular variable. Since each of the three subscripts may take on any of three values 0, 1 and 2, an arbitrary quantity F determines a set of 27 quantities F_{ijk}. Of these quantities, those having not more than one subscript different from "0" are defined by the equations

$$F_{000} = F \tag{3}$$

$$\left.\begin{aligned} F_{100} &= \frac{1}{2\pi}\int_0^{2\pi} F\,d\lambda \\ F_{010} &= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} F\,dt \\ F_{001} &= \frac{1}{p_0}\int_0^{p_0} F\,dp \end{aligned}\right\} \tag{4}$$

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$$\left.\begin{aligned} F_{200} &= F_{000} - F_{100} \\ F_{020} &= F_{000} - F_{010} \\ F_{002} &= F_{000} - F_{001} \end{aligned}\right\} . \tag{5}$$

The remaining quantities are defined by the equation

$$F_{ijk} = ((F_{i00})_{0j0})_{00k}. \tag{6}$$

In Eq. (4), t_1 and t_2 are the limits of the time interval under consideration and p_0 is a standard pressure near sea level. It is evident that Eq. (6) is also valid when not more than one of the subscripts i, j, k is different from "0".

From Eqs. (5) and (6), it follows that

$$\left.\begin{aligned} F_{0jk} &= F_{1jk} + F_{2jk} \\ F_{i0k} &= F_{i1k} + F_{i2k} \\ F_{ij0} &= F_{ij1} + F_{ij2} \end{aligned}\right\} . \tag{7}$$

Equations (7) are analogous to Eq. (1). Corresponding equations analogous to Eq. (2), involving two arbitrary functions F and G, are

$$\left.\begin{aligned} (F_{0jk}G_{0jk})_{111} &= (F_{1jk}G_{1jk})_{111} + (F_{2jk}G_{2jk})_{111} \\ (F_{i0k}G_{i0k})_{111} &= (F_{i1k}G_{i1k})_{111} + (F_{i2k}G_{i2k})_{111} \\ (F_{ij0}G_{ij0})_{111} &= (F_{ij1}G_{ij1})_{111} + (F_{ij2}G_{ij2})_{111} \end{aligned}\right\} . \tag{8}$$

Equations (7) and (8) are useful for expanding or recombining terms in relations containing the appropriate quantities.

Repeated application of Eqs. (7) leads to the following unique expansion of F as a sum of quantities not containing the subscript "0":

$$F = F_{111} + F_{211} + F_{121} + F_{112} + F_{122} + F_{212} + F_{221} + F_{222} = \sum_{i,j,k=1}^{2} F_{ijk}. \tag{9}$$

Equation (9) may be regarded as the generalization of Eq. (1) to the case of three independent variables. The corresponding generalization of Eq. (2) for the mean value of the product of F and G is

$$(FG)_{111} = \sum_{i,j,k=1}^{2} (F_{ijk}G_{ijk})_{111}, \tag{10}$$

which follows from repeated application of Eqs. (8).

It is sometimes convenient to use less complete expansions for F and $(FG)_{111}$ than Eqs. (9) and (10). Thus Priestley (1949), and also Starr and White (1951), have expanded the total flux τ of relative angular momentum across a given latitude into the sum of three terms. The expansion used by Starr and White is not identical with that used by Priestley, and the two expansions have subsequently been compared by Starr and White (1952). Use of the multiple-index notation can further enhance the comparison.

Within the time interval between t_1 and t_2, the flux τ is given by

$$\begin{aligned} \tau &= \int_0^{p_0}\int_{t_1}^{t_2}\int_0^{2\pi} R^3 \cos^2\phi\, g^{-1} uv\, d\lambda\, dt\, dp \\ &= 2\pi R^3 \cos^2\phi\,(t_2 - t_1) g^{-1} p_0 (uv)_{111} \end{aligned} \tag{11}$$

where R is the earth's radius, ϕ is the latitude, g is the acceleration of gravity, u and v are the eastward and northward components of the wind velocity, and the flux between the standard pressure p_0 and the actual surface of the earth is omitted. It is possible to expand the flux τ into a sum of eight terms according to Eq. (10). A recombination of some of these terms leads to the less complete expansion

$$(uv)_{111} = (u_{110}v_{110})_{111} + (u_{120}v_{120})_{111} + (u_{210}v_{210})_{111} + (u_{220}v_{220})_{111}, \tag{12}$$

which is the most complete expansion in which mean values of u and v with respect to pressure are absent. A further recombination of the second and fourth terms on the right of Eq. (12) yields the expansion

$$(uv)_{111} = (u_{110}v_{110})_{111} + (u_{210}v_{210})_{111} + (u_{020}v_{020})_{111}, \tag{13}$$

which is the expansion used by Priestley (1949). A recombination of the third and fourth terms on the right of Eq. (12) yields the expansion.

$$(uv)_{111} = (u_{110}v_{110})_{111} + (u_{120}v_{120})_{111} + (u_{200}v_{200})_{111}, \tag{14}$$

which is the expansion used by Starr and White (1951).

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Harmonic analysis of the mean northern-hemisphere wind field for the year 1950

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SUMMARY

Harmonic analyses of the mean northern-hemisphere wind field for the year 1950 along latitudes 30°, 45° and 60° are presented. The seasonal variations of these spectra are investigated and the relative contributions of the various scales of mean eddies to the meridional transport of angular momentum are calculated.

1. INTRODUCTION

By examining the harmonic analysis of the mean-wind map it is possible to obtain valuable information concerning the nature of the standing eddies in the atmosphere. Of particular interest in this regard are,

(1) the determination of the wave lengths (or wave numbers) of the dominant eddies which are present on the mean map as a function of latitude,

(2) the determination of the seasonal variation of the amplitude and phase of the harmonics. This is of special interest in view of the current controversy regarding the relative importance of the heating and of mountains in giving rise to the standing eddies, and

(3) the determination of the contributions of the various scales of standing eddies to the meridional transport of angular momentum.

With a view to obtaining insight into these questions, harmonic analyses of the mean conditions for the year 1950, based on the 500-mb data presented by Buch (1954), have been performed. As a preliminary to presenting the results we shall briefly set down the basic relationships used in the study.

2. BASIC CONCEPTS

Any regularly-behaving function of longitude, $f(\lambda)$, specified along a given latitude, ϕ, and pressure surface, p, may be expressed in terms of a Fourier representation having the form,

$$f(\lambda) = [f] + \sum_{n=1}^{\infty} |F(n)| \cos n(\lambda - \epsilon(n)) \quad . \quad . \quad . \quad (1)$$

where the bracket, defined by

$$[(\ )] = \frac{1}{2\pi}\int_0^{2\pi} (\ )\, d\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

denotes the zonal average, n denotes the wave number around the latitude circle, and $|F(n)|$ and $\epsilon(n)$ are the amplitude and phase angle respectively of the nth harmonic component. These latter quantities are given explicitly by the relations,

$$|F(n)| = [\mathscr{F}_1^2(n) + \mathscr{F}_2^2(n)]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\epsilon(n) = \frac{1}{n}\tan^{-1}\left\{\frac{\mathscr{F}_2(n)}{\mathscr{F}_1(n)}\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$\mathscr{F}_1(n)$ and $\mathscr{F}_2(n)$ being the real and imaginary parts of the complex Fourier coefficient,

$$F(n) = \frac{1}{\pi}\int_0^{2\pi} f(\lambda)\, e^{-in\lambda}\, d\lambda$$
$$= \mathscr{F}_1(n) - i\mathscr{F}_2(n) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the present study we shall consider the Fourier analysis of the eastward component of the mean wind $\bar{u}$, and the northward component of the mean wind $\bar{v}$. We shall denote the complex Fourier coefficients corresponding to these quantities by

$$U(n) = \mathscr{U}_1(n) - i\,\mathscr{U}_2(n)$$

and

$$V(n) = \mathscr{V}_1(n) - i\,\mathscr{V}_2(n)$$

respectively.

The spectral function for the meridional eddy transport of relative angular momentum across latitude ϕ per unit pressure difference, due to the standing eddies, is given by the relation (cf., Van Isacker and Van Mieghem (1956))

$$T(n) = \frac{\pi a^2 \cos^2 \phi}{g} \left[\mathscr{U}_1(n)\mathscr{V}_1(n) + \mathscr{U}_2(n)\mathscr{V}_2(n)\right], \qquad (6)$$

where a is the radius of the earth and g is the acceleration of gravity.

In performing the harmonic analysis it is, of course, necessary to select a finite number of harmonics as the basis for the Fourier representation. In the present case harmonics up to wave number 12 were computed, based on information at 36 points around the hemisphere.

3. Results

As noted in the introduction, in this study we are concerned with the spectral properties of the mean 500-mb wind field for the year 1950. In particular, the Fourier resolutions along the 30°, 45° and 60° latitude circles were selected for investigation.

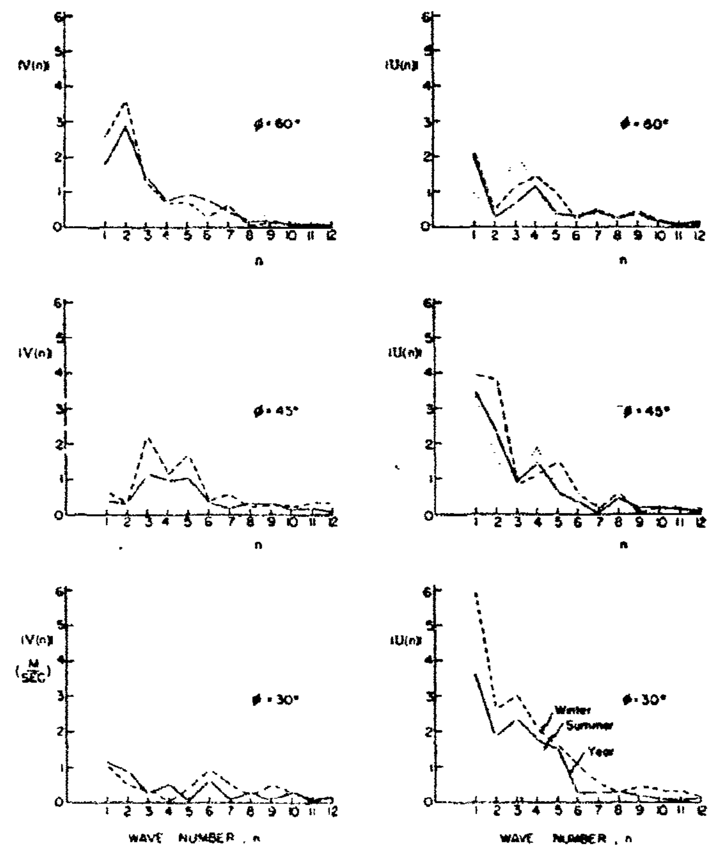

Figure 1. Amplitude spectra of meridional and zonal components of the 500-mb mean wind for the year 1950 denoted by $|V(n)|$ (first column) and $|U(n)|$ (second column), respectively. Solid line indicates *annual* mean; dashed line, *winter* mean; and dotted line, *summer* mean. Units are m/sec.

The mean-wind maps were constructed by Buch (1954) on the bases of average *actual* wind conditions reported at 81 stations in the northern hemisphere. 'Summer' and 'winter' means were obtained by dividing the year into two 6-month periods; (April-September) and (January-March, October-December) respectively. The spectra obtained for these seasons will therefore not necessarily agree with those obtained from the July and January mean maps to which reference is usually made (e.g., Scherhag 1948, Sutcliffe 1951).

The amplitude spectra, $|V|$ and $|U|$, are presented in Fig. 1. It should be noted that these are discrete 'line' spectra and, as such, have meaning only for the integral values of wave number. Continuous lines have been drawn only to aid visually in tracing the variations of the spectra.

The variations of $\bar{v}$ around the hemisphere, measured in the wave number domain by $|V(n)|$, are closely indentified with the waves appearing in the mean streamline field. On the other hand, the variations of $\bar{u}$, measured in the wave-number domain by $|U(n)|$, depend on the streakiness in the velocity of the mean zonal current around a latitude circle, associated, for example, with the meandering of the mean 'jetstream.'

The harmonics which are most prominent in the $|V|$-spectra are those of wave number 1 and 2 at 60°; wave number 3, 4 and 5 at 45°; and wave numbers 1. 2 and 6 at 30°. In general, the variations of $\bar{v}$ around the latitude circle are most pronounced at 60°, being of progressively smaller amplitude at 45° and 30°.

The amplitude spectra for the *zonal* component of the mean wind, $|U(n)|$, are shown in the second column of Fig. 1. It may be seen that most of the variation is concentrated in the longer waves with a maximum at wave number 1 at all three latitudes. This large amplitude of wave number 1 is a reflection of the fact that the mean hemispheric vortex is not symmetrical with respect to the pole, an occurrence which has been reported in detail by LaSeur (1954). The variations of zonal velocity are most pronounced at low latitudes, the reverse of the latitudinal distribution of $|V(n)|$. In general, the variation of the zonal component of the wind around a latitude circle is more pronounced than the variation of the meridional component, as evidenced by the higher amplitudes $|U(n)|$.

The spectral distributions for the winter and summer means are shown by the dashed and dotted curves respectively. For certain wave-lengths striking seasonal changes in the amplitudes are revealed. For example, $|V(5)|$ at $\phi = 45°$ is very small for the summer period whereas its value is quite high in the winter. Wave number 4 at $\phi = 30°$ shows a similar seasonal change with the maximum in this case occurring in the summer.

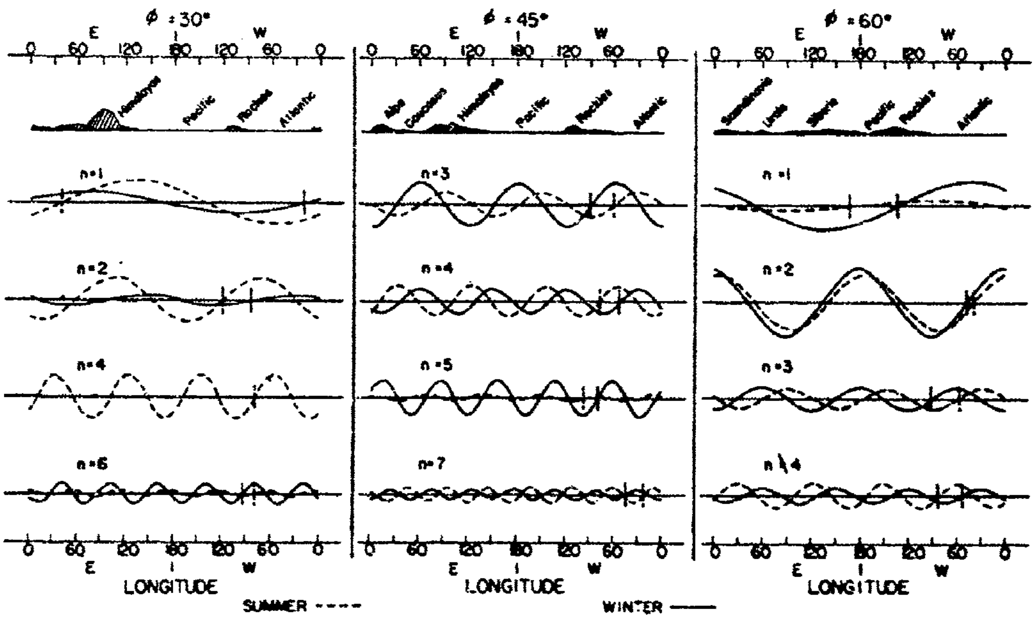

Figure 2. Schematic representation of the prominent waves which comprise the winter and summer mean fields of the meridional component of velocity. Trough-lines are located at the intersection of the abscissa and the wave progressing from a minimum to a maximum.

If the amplitude of an harmonic in the annual mean wind field lies nearly mid-way between the summer and winter values, it is implied that there is little phase shift of that Fourier component with the seasons. It may be seen that in many cases this is not true, indicating that appreciable phase differences between harmonics in the winter and summer streamline patterns actually exist. In order to investigate this phenomenon in greater detail the phase angles $\epsilon(n)$ were computed for the four harmonics in the $\bar{v}$-field having the greatest amplitude at each latitude. The results are schematically represented in Fig. 2. The waves shown (drawn to scale) are those for the variations of the meridional component of the mean velocity, so that a ' trough line ' is located at the intersection of the abscissa and the curve which progresses from a minimum to a maximum. Short vertical lines have been drawn through this intersection to facilitate an inspection of the seasonal phase shift. In many cases these phase shifts are of the order of a quarter of a wave length, a notable exception being the high amplitude harmonic, $n = 2$, at 60°N which undergoes very little change in position.

In Fig. 3 the spectra of the meridional transport of angular momentum across the three latitudes studied due to the ' standing ' eddies are presented. The net transport at 30° and 45° is northward with a southward transport occurring at 60°. At all latitudes the longer waves are most active in effecting the transport, wave number 1 being of particular importance during the winter at low latitudes.

4. Discussion

The observations reported here reveal some of the spectral properties of the mean hemispheric flow pattern for the year 1950. These properties must be related to the irregularities in the surface of the earth manifested by differences in orography and heating. The relative importance of these

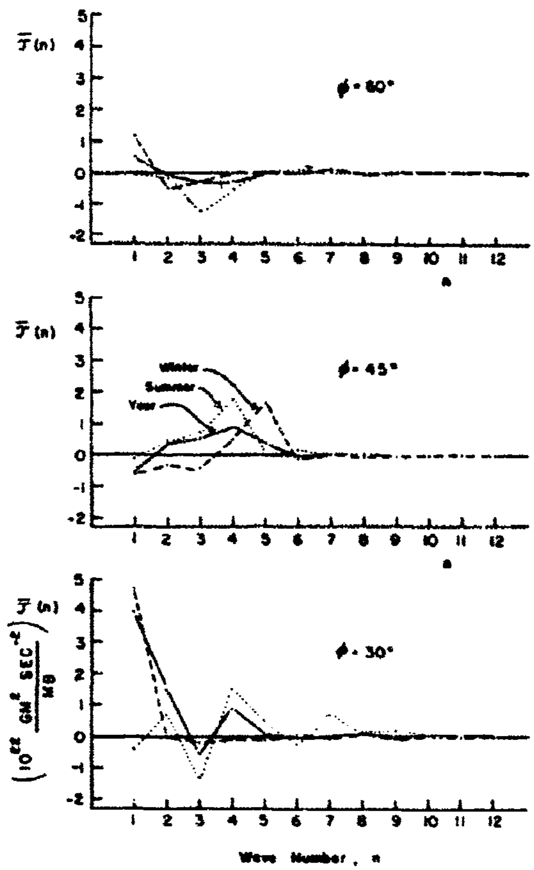

Figure 3. Spectra of the meridional transport of relative angular momentum per unit pressure difference across latitudes 30°, 45° and 60° due to the standing eddies. A positive sign indicates a northward momentum transport. Units are 10^{22} gm^2 sec^{-2} mb^{-1}.

two factors has been discussed by several writers (e.g., Charney and Eliassen 1949, Bolin 1950, Sutcliffe 1951, Smagorinsky 1953, Frenzen 1955) but, as yet, a complete theory by which one can explain observations of the type described here has not been achieved. In view of the complex nature of the spectral distribution of the mean flow and the irregular seasonal variation of the harmonics with regard to both amplitude and phase, it is likely that such a theory will involve both orography and heating in a complicated manner.

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ON THE HEMISPHERIC CORRELATIONS OF VERTICAL AND MERIDIONAL WIND COMPONENTS

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Summary — Using the values of vertical velocities computed by JENSEN (1960) and the values of the meridional component of the wind for the months of January and April 1958, the covariances of both quantities were evaluated at different levels at various latitudes. These covariances are tabulated and the terms associated with transient eddies are compared to the standing eddy terms, the relative magnitudes of both contributions being finally investigated.

Résumé — En employant les valeurs du mouvement vertical calculées par JENSEN (1960) et les valeurs des composantes méridionales des vents pour les mois de Janvier et Avril 1958, les covariances des deux quantités ont été evaluées aux différents niveaux dans différentes latitudes. Ces covariances sont tabulées et les valeurs associées aux perturbations transientes et stationnaires sont comparées afin de donner un aperçu sur l'importance relative des deux procès.

1. *Introduction* — In recent years the approach to the problem of describing and explaining the general circulation of the earth's atmosphere has been one of examining certain requirements deduced from the broad dynamic principles governing atmospheric motions. These requirements are then looked at in the critical light of observational fact.

With such a program in view it is necessary that the results of many individual studies should be systematically amassed for later interpretation. The writers wish therefore to make a particular contribution to these efforts, namely one dealing with the correlation of vertical and meridional wind components.

2. *Procedure and Results* — JENSEN (1960) made an extensive study of northern hemisphere vertical flux processes in which he introduced a finite difference equation for computing the 12-hour time-averaged vertical velocity for a layer between two isobaric surfaces using data from a single station. The laborious task of actually computing the vertical velocities was performed by the Climatic Center at Ashville N. C. and fortunately these data, through JENSEN, were available to the present writers. In addition the corresponding horizontal wind components were also provided by the Climatic Center.

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Using these data and an analysis discussed by STARR & WHITE (1952), we calculated the zonal standing eddy and the transient eddy convariances as a function of pressure layers around latitude circles. We will not enter into a detailed discussion of the statistical analysis; rather, the reader is referred to STARR & WHITE.

Briefly, using the notation of STARR & WHITE, we write:

$\bar{\alpha}$ time average of a quantity
$[\alpha]$ space average over a latitude circle
α' deviation from the time average
α'' deviation of the time average from the zonal average.

For the quantities under discussion we may write:

(1) $$[\overline{wv}] = [\bar{w}\,\bar{v}] + [\overline{w'v'}]$$

and

(2) $$[\bar{w}\,\bar{v}] = [\bar{w}][\bar{v}] + [\bar{w}''\,\bar{v}''] .$$

Combining (1) and (2) we find:

(3) $$[\overline{wv}] = [\bar{w}][\bar{v}] + [\bar{w}''\,\bar{v}''] + [\overline{w'v'}] .$$

The terms of (3) may be intepreted as follows:

$[\overline{wv}]$ either is a measure of the northward transport of vertical momentum, or is a measure of the upward flux of northward momentum through a given level and over a given time interval.

$[\bar{w}][\bar{v}]$ is the mean meridional transport effect. This term cannot be evaluated with sufficient accuracy to justify including it in our results. In effect we subtract it from both sides of eq.uation (3).

$[\bar{w}''\bar{v}'']$ is the zonal standing eddy contribution to the transfer process. It is due to the spatial correlation between the time means, w and v along a latitude circle.

$[\overline{w'v'}]$ is the transient eddy contribution, a time correlation between instantaneous values of w and v at individual points along a latitude circle.

Specifically, the data available to the writers were a tabulation of the three quantities given below, for each of approximately 100 northern hemisphere stations north of 20° N, for each of seven pressure layers, 1000-850, 850-700, 700-500, 500-300, 300-200, 200-100, and 100-50 mb, for the months of January and April 1958.

a) w monthly averaged vertical motion in cm sec^{-1}. The daily value computed from the equation derived by JENSEN (1960).

b) v the monthly averaged northward component of the horizontal wind in m sec^{-1}.

c) $\overline{w'v'}$ the time covariance between the vertical motion and northward component of the horizontal wind in cm m sec^{-2}.

A polar stereographic base map (scale approximately 1 : 40 million) was used to plot charts of the above three quantities for each month and each pressure layer. A careful analysis of the plotted data was performed and grid point values were picked off for each 10° of longitude for latitudes 20° through 50° N and for each 30° of longitude for latitudes 60° thru 80° N. Equation (2) was used to calculate the zonal standing eddy contribution. A zonal averaging of the the time covariance grid point values gave us the transient eddy contribution.

The zonal standing eddy contributions, $[\overline{w''v''}]$, are listed in Tables 1 and 2 for January and April respectively. The maximum January value is 339 cm² sec⁻² found at the 500-300 mb level at 40° N. In comparison the April value of 245 cm² sec⁻² although occurring at the same level is shifted north to the 70° latitude circle and is somewhat smaller. We could perhaps anticipate these results by considering that the maximum winds are usually found in the mid-tropospheric layers and that a general weakening and northward shift of the strongest winds to the high latitudes occurs in the spring. The dominant contribution to the net transport comes from the transient eddies, the standing eddy effects being 50 per cent smaller in almost every case. We therefore have evidence that the time varying eddies play a larger roll in this transport process than do the semipermanent standing eddies.

TABLE 1 - *Zonally averaged standing eddy term* $[\overline{w''\,v''}] = [\bar{w}\,\bar{v}] - [\bar{w}]\,[\bar{v}]$ *in* cm² sec⁻² × 10² *by pressure layer and latitude for January* 1958.

Pressure layer in mb	20	30	40	50	60	70	80
1000-850	0.052	0.512	1.638	1.863	0.373	0.189	0.326
850-700	0.108	0.340	0.892	— 0.210	0.245	— 0.268	— 0.126
700-500	— 0.187	— 0.743	— 0.087	0.933	0.500	0.386	0.383
500-300	— 0.558	— 0.971	3.390	2.726	1.146	0.192	— 0.329
300-200	1.836	— 0.248	1.857	1.318	0.598	0.820	— 0.016
200-100	— 0.876	— 0.620	0.790	1.014	0.613	0.838	0.052
100-50	— 0.074	— 0.361	— 0.424	0.598	0.745	— 0.298	0.879

TABLE 2 - *Zonally averaged standing eddy term* $[\overline{w''\,v''}] = [\bar{w}\,\bar{v}] - [\bar{w}]\,[\bar{v}]$ *in* cm² sec⁻² × 10² *by pressure layer and latitude for April* 1958.

Pressure layer in mb	20	30	40	50	60	70	80
1000-850	— 0.246	0.356	0.599	1.351	0.879	0.541	0.132
850-700	0.299	— 0.089	0.207	— 0.383	1.084	1.909	0.486
700-500	0.300	0.244	— 0.247	— 0.351	0.758	0.983	0.695
500-300	— 0.262	0.310	0.951	— 0.070	1.179	2.448	1.646
300-200	0.609	— 1.129	1.255	1.981	1.025	— 0.017	0.283
200-100	0.121	0.100	0.234	0.075	0.268	0.123	0.076
100-50	— 0.061	— 0.138	— 0.152	0.609	— 0.005	— 0.007	— 0.022

The values of the transient eddies $[\overline{w'v'}]$, are listed by month, pressure layer and latitude in Tables 3 and 4. We find consistent positive values of the covariance in the layers below 100 mb, in agreement with the well known concepts of rising northward moving air and subsiding southward moving air.

The reversal of the correlation at the 100-50 mb layer is of particular interest.

We find negative values in both months for latitudes 20° through 50° N which indirectly support the somewhat novel concepts of a northward convergence of ozone (MARTIN 1956; NEWELL 1961), and a stratospheric countergradient heat flow (STARR & WHITE 1954).

TABLE 3 - *Zonally averaged covariance of vertical motion and northward component of the horizontal wind* (transient eddy term) [w' v'], *in* $cm^2\ sec^{-2} \times 10^2$ *by pressure layer and latitude for January* 1958.

Pressure layer in mb	20	30	40	50	60	70	80
1000-850	1.095	3.661	3.500	1.972	0.917	1.500	— 0.833
850-700	0.857	3.242	3.653	2.472	0.958	1.958	0.042
700-500	0.738	3.538	4.625	3.653	1.542	2.333	1.333
500-300	2.609	11.917	9.056	3.431	3.167	7.250	1.833
300-200	2.461	6.971	8.444	4.944	4.750	3.500	0.583
200-100	2.400	2.231	1.667	2.583	3.667	1.667	1.000
100-50	— 0.353	— 0.972	— 1.028	— 1.111	1.417	1.083	1.000

TABLE 4 - *Zonally averaged covariance of vertical motion and northward component of the horizontal wind* (transient eddy term) [w' v'], *in* $cm^2\ sec^{-2} \times 10^2$ *by pressure layer and latitude for April* 1958.

Pressure layer in mb	20	30	40	50	60	70	80
1000-850	0.400	1.833	2.000	— 0.361	— 1.417	— 1.083	— 1.000
850-700	0.739	3.639	4.667	3.417	0.750	0.750	— 1.000
700-500	1.545	5.679	6.611	4.722	2.333	1.250	— 0.167
500-300	2.636	6.250	5.750	3.722	4.917	3.583	0.167
300-200	1.474	8.969	8.833	7.000	3.583	0.667	0.000
200-100	0.870	1.226	0.083	0.083	0.583	0.500	0.167
100-50	— 0.158	— 0.862	— 0.750	— 0.056	0.250	0.000	0.000

MARTIN using a small data sample collected over a limited region found subsidence occurring along southerly trajectories with a corresponding increase of ozone. Our results, indicating a subsidence with northward moving air, are substantially in agreement, reaffirming his conclusions. In addition, NEWELL's independent studies of the ozone question in this lower stratospheric layer also require negative correlations. In fact, our 100-50 mb covariance values fit neatly with his finding.

The negative covariances found likewise provide a mechanism essential for the countergradient flux of sensible heat discussed by STARR & WHITE. Air moving northward in the stratosphere is found to descend and, by adiabatic warming, increases the temperature at a particular level. This effect permits a horizontal

eddy transport of heat in a sense contrary to what we would normally expect, considering the usual southward gradient of temperature. The negative covariance between w and r in the lower statosphere thus supports and, to a degree, explains the work done by STARR & WHITE.

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Preliminary Study of Quasi-Horizontal Eddy Fluxes from Meteorological Rocket Network Data[1]

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ABSTRACT

The meridional transport of relative angular momentum by transient eddies is computed from the wind data so far available from the Meteorological Rocket Network for the 25–60 km region. The transport is northward in winter and apparently of sufficient magnitude to account for the formation of the winter polar vortex. It is suggested that the eddies arise in response to the differential heating within the region produced by radiational effects. Zonal available potential energy and its generation are calculated and compared with the kinetic energy. The transformation of eddy kinetic energy to zonal kinetic energy is of the same order of magnitude as the generation of zonal available potential energy. The importance of eddy structures as compared with mean meridional motions appears to be similar to the earlier findings for the troposphere and lower stratosphere. It is concluded that the region from 25–60 km may generate its own kinetic energy *in situ* in winter and be essentially energetically independent of the troposphere.

An alternative explanation for the high temperature of the winter polar mesosphere based upon considerations concerning the eddies is put forward.

1. Introduction

Students of the earth's planetary circulation will be aware that there is presently in progress considerable speculation concerning the circulation of the upper atmosphere, specifically the region from 30 to 100 km. In some respects the arguments are similar to those previously applied to the circulation at lower levels. It will be recalled that for a considerable number of years the prevailing view was held that the atmospheric heat engine—or that portion of it termed the troposphere—satisfied its heat budget's requirements by convective overturning in meridional planes, or mean meridional motions. Theoretical work by Jeffreys (1926) suggested that the angular momentum budget could not be satisfied by such mean meridional motions as long as surface friction were present. The budget could be balanced by quasi-horizontal northeasterly and southwesterly streams of air occurring side by side rather than one above the other as in the mean meridional motion scheme. Jeffreys pointed out that such streams are characteristic of cyclones and that cyclones are therefore an integral part of the atmospheric general scheme and not simply a perturbation on the mean zonal flow which indeed could not exist without them. These ideas were extended by Starr (1948) who showed that the positive correlation between meridional and zonal components of the wind required by Jeffreys' theory was in evidence on most weather maps in the form of the tilted troughs of middle latitudes and in the anticyclones of the subtropical region and was not simply a characteristic of cyclones. Starr proposed that the momentum transports be evaluated from actual wind data and such studies have been in progress at Massachusetts Institute of Technology since that time under his direction. A mass of observational data has been processed which clearly demonstrates the dominance of quasi-horizontal eddy motions in both the angular momentum and the heat budgets. [See for example Starr (1954) for the details of this considerable undertaking. See also Bjerknes and Mintz (1955) for a description of work at University of California at Los Angeles on these matters.] This is not to imply that mean meridional motions do not exist in the troposphere; indeed the observations show the existence of a three cell structure with southward drift in the upper troposphere of middle latitudes (Tucker, 1959; Murakami, 1960) which is presumed to compensate for the frictionally induced northward drift in the region of the surface westerlies. The observed mean meridional motions are in accord with the modern theoretical developments such as those of Eady (1950), Phillips (1954) and Kuo (1956b).

Starr used observations from the surface up to 100 mb; in 1954 no large data sample was available to extend the calculations to higher levels. Other workers made some deductions concerning the circulation of the lower stratosphere from water vapor and ozone distributions. Observations of these trace substances could apparently best be interpreted in terms of a mean meridional circulation in the stratosphere, with a northward drift from

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low to middle and high latitudes (Brewer, 1949; Dobson, 1956). The model has also been extended to account for observations of radioactive fission products by Stewart, Osmond, Crooks and Fisher (1957) and Machta (1957); a somewhat different mean cell model was also proposed for this purpose by Libby and Palmer (1960). As discussed by the present writer elsewhere (Newell 1961, 1963a) the distributions of ozone and radioactive tracers could be explained equally well on the basis of quasi-horizontal eddy transports in the stratosphere; furthermore it has proved possible to account for the main features of the ozone distribution with such a model, as first suggested by Martin (1956). One addition to the model, namely that the principal region of stratospheric-tropospheric mass exchange occurs in the vicinity of the middle latitude baroclinic zone and concomitant jet stream (see, e.g., Danielsen, 1960; Staley, 1960), allows the two main features of the distribution of long-term fission products in the troposphere (the middle latitude maxima and spring maxima) to be explained. The information on unique tracers, particularly tungsten 185, can only be interpreted with the assumption that eddy-mixing processes are predominant over mean motions in the lower stratosphere. (Feely and Spar, 1960; Friend *et al.*, 1961). The crucial test should perhaps come from the meteorological wind observations. During the past year several of my colleagues working with Professor V. P. Starr have completed portions of a climatological study of the International Geophysical Year stratospheric data and have found equatorward motion in the 100–30 mb region over a wide zone in middle latitudes. The observations again reveal motions opposite to those expected prior to a detailed examination of the meteorological data. It should be noted that the data were processed so as to reveal not only the mean meridional motions directly but also to reveal the mean meridional motion necessary to balance the angular momentum budget of the region. The former work has been reported by Barnes (1962) and Oort (1962); the latter by Dickinson (1962). A similar mean meridional cell pattern exists for the southern hemisphere (Obasi, 1963).

The atmosphere in the 30–100 km region has also been supposed to possess a mean meridional cell structure. Kellogg and Schilling (1951) have invoked such a scheme, while Murgatroyd and Singleton (1961) and Haurwitz (1961) have actually calculated mean meridional motions using various assumptions. The validity of these predictions cannot be judged properly until substantial data on meteorological motions is available for these levels from a global network of stations. It is clear that such a desirable state of affairs will not prevail for many years. Nevertheless a small step in this direction has been taken by the establishment of the Meteorological Rocket Network (Webb *et al.*, 1961; Joint Scientific Advisory Group, 1961). Attempts are made to send rockets to heights of 60 km on a synoptic basis during four periods each year, in order to permit wind velocities and temperatures to be measured. Because of the limited coverage it is not possible to estimate the effects of mean motions or standing eddies. However, it is legitimate to examine the observations for possible systematic effects of *transient* eddies and to estimate roughly the magnitudes of any such effects discovered. This is one of the objects of this paper.

2. Procedure and results of wind study

The Meteorological Rocket Network began synoptic measurements over North America in the fall of 1959. Data were made available from the White Sands Missile Range where the observations for each station are collected and recorded (Inter-Range Instrumentation Group 1960, 1961 and 1962). Wind components and temperatures are presented in graphical form against altitude, in feet, with each individual determination plotted. Values of the wind components were selected from these graphs at 20,000-ft intervals. When the observed components oscillated from one point to the next a mean line was drawn through the curves on the assumption that such oscillations represented tracking errors. When several points defined an oscillation, however, it was retained. Observational summaries were prepared for each station and time cross-sections for each component were constructed, an example of which appears in Fig. 1. It appears that two general regimes of flow occur in the 100,000–200,000 ft region, the oscillating westerly regime of winter and the relatively steady easterly regime of summer. Velocities and the fluctuation amplitudes increase upwards and the fluctuation amplitudes are larger in winter than in summer. The replacement of the easterly regime by westerlies appears to take place first at high altitudes and high latitudes and then to progress downwards and southwards, there being a period of about two weeks between the reversal occurrence at the highest altitudes of the northernmost station and at the lowest altitude (100,000 ft) of the southernmost station.

It was desired to calculate angular momentum transports from the wind data, and this procedure required as large a data sample as possible. All the measurements available were therefore included in either the winter regime or summer regime. The dates of seasonal division were chosen, after inspection of all the time sections such as Fig. 1, as 15 September and 22 April. At some future date when the number of observations is sufficiently large, a division into four seasons will be attempted. For each station and at 20,000-ft intervals between 80,000 and 200,000 ft calculations were made of the mean zonal and meridional wind components ($\bar{u}$ and $\bar{v}$ where the bar denotes a time mean), their standard deviations $\sigma(u)$ and $\sigma(v)$ and the transient eddy angular momentum transport (proportional to $\overline{u'v'}$ where the primes denote departures from the means). The winter mean zonal and meridional components are shown in Table 1 together

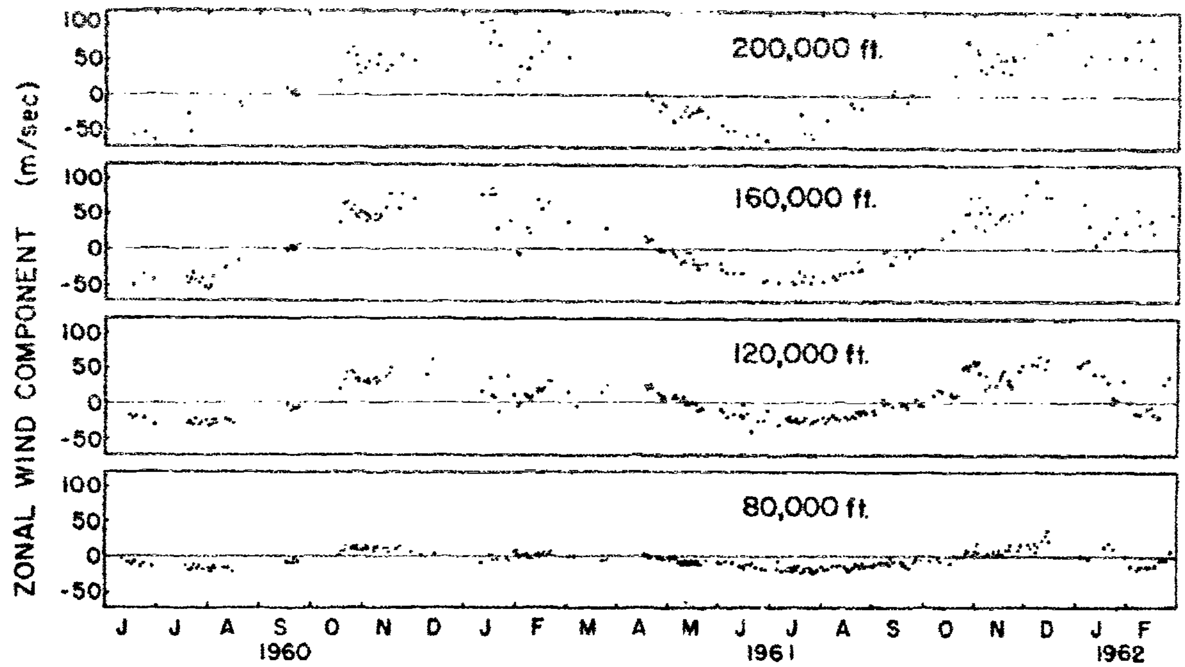

FIG. 1. The zonal wind component at White Sands, N. Mex., as measured during 1960-1962. Units: m sec^{-1}.

with the standard deviation of the meridional component. Zonal winds increase with height above 80,000 ft (25 km) as has been recognized for many years. These data alone do not delineate the altitude of the high level maximum which seems to be higher still. In a horizontal plane maxima appear to be in middle latitudes at these longitudes. The meridional components *cannot be interpreted as "mean meridional cells"* because of the limited longitude coverage. The large meridional values may be evidence of standing eddy components and indeed such standing eddies are in evidence on the charts drawn by various authors for the region. Standard deviations of the meridional component (computed only when ten or more observations were available) appear to increase steadily with height and, insofar as many of the wind profiles were obtained with chaff, it would seem that this increase is not due solely to instrumental problems but may well represent a real increase in eddy activity with height. The standard deviations are largest at the two northernmost stations as is the case also for the lower stratosphere (Murakami, 1962; Newell, 1963b). Table 2 contains corresponding information for the summer regime. Zonal winds, now easterly, and standard deviation values again increase with height but absolute values are smaller than in the winter.

The covariances between the zonal and meridional wind components, which are proportional to the horizontal transport of angular momentum, are shown for winter and summer in Tables 3 and 4, respectively. In the 100,000-200,000 ft layer the winter values show evidence of a systematic tendency for a northward transport of momentum at all levels with the smallest values at the bottom of the layer. This immediately suggests that the high level jet stream may be maintained by quasi-horizontal eddy processes in the same general manner as the tropospheric middle-latitude jet stream. Included in Table 3 is the result of combining all the stations in the 28-38 deg latitude belt. The covariances are larger than those reported by Starr and White (1954) for the tropospheric jet. The limitation in longitude must be borne in mind when making such comparisons. An earlier study by Starr (1951) was restricted to North American longitudes and referred to a single winter month but the covariance values found are exceeded by the present results. Such large northward transports would make a mean northward drift unnecessary to maintain the westerly jet; on the other hand if it turns out that an overabundance of zonal momentum is transported by the eddies, then it may be necessary to invoke a southward drift.

The summer data available so far indicate a very small northward eddy transport in the layer between 100,000 and 200,000 ft. Eddy activity appears to be considerably smaller than in the winter. But individual stations show large differences between levels and the general lack of pattern suggests that the sampling is not yet adequate for this season.

It is quite difficult to estimate the importance of these systematic eddy processes in the winter regime from the limited observational material available. The most

TABLE 1. Mean zonal and meridional components of Rocket Network winds.
Units: m sec^{-1}. Number of observations (N) in parentheses.

Winter (1959–1962)

Height		64.0N, 145.7W Ft. Greely, Alaska				58.8N, 94.3W Ft. Churchill				38.0N, 116.5W Tonopah Range, Nev.				37.8N, 75.5W Wallops Is., Va.				34.1N, 119.1W Pt. Mugu, Calif.			
km	kft	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$
61.0	200	+13	−8	(2)		+28	+17	(4)		+24	−2	(3)		+66	+7	(18)	12	+59	+10	(14)	19
54.9	180	+25	−1	(11)	16	+37	−2	(17)	15	+23	+6	(3)		+56	+10	(31)	11	+53	+11	(40)	14
48.8	160	+19	−6	(19)	19	+32	−4	(24)	17	+11	0	(2)		+63	+11	(50)	10	+49	+8	(54)	12
42.7	140	+19	−7	(31)	19	+21	−8	(35)	17	+38	+6	(13)	9	+48	+9	(64)	12	+34	+2	(64)	11
36.6	120	+17	−1	(40)	19	+17	−7	(49)	16	+12	−1	(27)	7	+36	+3	(69)	10	+22	+3	(75)	7
30.5	100	+12	0	(42)	15	+11	−4	(58)	14	+12	0	(28)	6	+20	+5	(65)	10	+9	+1	(77)	5
24.4	80	+10	+1	(41)	8	+11	−3	(62)	12	−1	−2	(23)	4	+8	+2	(51)	4	+2	0	(74)	4
18.3	60	+11	+3	(33)	6	+11	−2	(55)	9	+6	−3	(12)	4	+15	0	(43)	7	+8	−1	(56)	4

Height		32.9N, 106.1W Holloman, N. Mex.				32.4N, 106.5W White Sands, N. Mex.				30.5N, 86.5W Eglin Field, Fla.				28.2N, 80.6W Cape Canaveral, Fla.			
km	kft	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$
61.0	200	+58	+15	(7)		+55	+11	(69)	12	+43	0	(11)	8	+58	+3	(8)	
54.9	180	+31	+9	(8)		+52	+10	(88)	13	+51	+5	(12)	8	+45	+8	(34)	14
48.8	160	+32	0	(12)	13	+45	+11	(89)	12	+56	+8	(14)	10	+31	+9	(49)	12
42.7	140	+28	−4	(16)	15	+34	+5	(96)	10	+48	+4	(15)	12	+19	+4	(50)	9
36.6	120	+17	−1	(19)	8	+25	+2	(120)	8	+37	−3	(14)	5	+13	−1	(56)	8
30.5	100	+4	−2	(18)	4	+12	+2	(130)	7	+17	+4	(11)	3	+6	+3	(53)	6
24.4	80	+9	−3	(4)		+4	+1	(133)	4	+3	−4	(1)		+2	+1	(29)	3
18.3	60	+10	+2	(4)		+11	+1	(115)	7					+14	+1	(11)	2

TABLE 2. Mean zonal and meridional components of Rocket Network winds.
Units: m sec^{-1}. Number of observations (N) in parentheses.

Summer (1960–1961)

Height		64.0N, 145.7W Ft. Greely, Alaska				58.8N, 94.3W Ft. Churchill				38.0N, 116.5W Tonopah Range, Nev.				37.8N, 75.5W Wallops Is., Va.				34.1N, 119.1W Pt. Mugu, Calif.			
km	kft	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$
61.0	200	−5	+8	(1)		−3	+3	(1)						−29	+10	(5)		−48	−4	(9)	
54.9	180	−22	+13	(4)		−41	+3	(1)						−19	+3	(14)	7	−27	+4	(32)	6
48.8	160	−11	+9	(4)		−25	−4	(2)						−17	+3	(35)	7	−24	+6	(55)	5
42.7	140	−3	+3	(9)		−19	+1	(5)		−17	+3	(9)		−14	+3	(38)	5	−18	0	(64)	5
36.6	120	−3	+2	(11)	4	−14	0	(11)	7	−5	−2	(23)	4	−9	0	(40)	3	−10	0	(66)	3
30.5	100	−3	0	(12)	3	−11	−1	(20)	6	−4	0	(23)	3	−6	+1	(44)	3	−10	0	(69)	3
24.4	80	−3	+1	(15)	4	−4	0	(25)	6	−7	+1	(22)	2	−6	+1	(39)	2	−10	0	(58)	2
18.3	60	−2	−1	(8)		+1	−2	(24)	5	+1	+2	(8)		0	0	(36)	5	−1	+2	(45)	3

Height		32.9N, 106.1W Holloman, N. Mex.				32.4N, 106.5W White Sands, N. Mex.				30.5N, 86.5W Eglin Field, Fla.	28.2N, 80.6W Cape Canaveral, Fla.			
km	kft	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$	$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$		$\bar{u}$	$\bar{v}$	(N)	$\sigma(v)$
61.0	200	−13	+9	(1)		−30	+7	(31)	8	(No data received)	−20	+7	(11)	16
54.9	180	−26	+6	(4)		−33	+5	(55)	9		−31	+6	(32)	8
48.8	160	−13	+8	(7)		−27	+7	(72)	5		−34	+4	(43)	9
42.7	140	−13	+4	(9)		−20	+2	(82)	6		−27	+2	(46)	6
36.6	120	−10	+2	(10)	3	−16	+1	(104)	4		−19	+1	(45)	5
30.5	100	−7	+2	(10)	3	−14	+1	(113)	4		−18	+1	(44)	3
24.4	80					−13	0	(122)	2		−16	0	(17)	2
18.3	60					−3	0	(107)	3		−1	−2	(8)	

TABLE 3. Covariance of meridional and zonal wind components. Winter regime. Units: meters² sec⁻². Number of observations in parentheses.

Station	Height (kft) 200	180	160	140	120	100	80	60
Ft. Greely, Alaska 64°00'N 145°44'W		+116 (11)	+95 (19)	+79 (31)	+113 (40)	+12 (42)	+8 (41)	−2 (33)
Ft. Churchill, Canada 58°47'N 94°17'W		+74 (17)	+181 (24)	+238 (35)	+65 (49)	+38 (58)	−4 (62)	−20 (55)
Tonopah Range, Nev. 38°00'N 116°30'W				+60 (13)	+92 (27)	+43 (28)	+13 (23)	+11 (12)
Wallops Is., Va. 37°50'N 75°29'W	+137 (18)	+77 (31)	+81 (50)	+27 (64)	+108 (69)	+37 (65)	+8 (51)	−3 (43)
Point Mugu, Calif. 34°07'N 119°07'W	+53 (14)	+97 (40)	+139 (54)	+96 (64)	+66 (75)	+7 (77)	+17 (74)	+4 (56)
Holloman, N. Mex. 32°51'N 106°06'W			−107 (12)	+79 (16)	+108 (19)	+11 (18)		
White Sands, N. Mex. 32°23'N 106°29'W	+47 (69)	+49 (88)	+104 (89)	+48 (96)	+63 (120)	+45 (130)	+22 (133)	+37 (115)
Eglin Field, Fla. 30°30'N 86°30'W	+13 (11)	−48 (12)	−43 (14)	+94 (15)	+23 (14)	+35 (11)		
Cape Canaveral, Fla. 28°14'N 80°36'W		+221 (34)	+126 (49)	−13 (50)	+1 (56)	+13 (53)	+9 (29)	−8 (11)
Weighted average for 28°–38°	+59 (112)	+86 (205)	+94 (268)	+48 (318)	+66 (380)	+29 (382)	+17 (310)	+18 (237)

TABLE 4. Covariance of meridional and zonal wind components. Summer regime. Units: meters² sec⁻². Number of observations in parentheses.

Station	Height (kft) 200	180	160	140	120	100	80	60
Ft. Greely, Alaska 64°00'N 145°44'W					−16 (11)	−1 (12)	+3 (15)	
Ft. Churchill, Canada 58°47'N 94°17'W					+30 (11)	+27 (20)	+6 (25)	−5 (24)
Tonopah Range, Nev. 38°00'N 116°30'W					+14 (23)	−4 (23)	−2 (22)	
Wallops Is., Va. 37°50'N 75°29'W		+29 (14)	−34 (35)	−9 (38)	−3 (40)	+5 (44)	+2 (39)	−5 (36)
Point Mugu, Calif. 34°07'N 119°07'W		+14 (32)	+43 (55)	+16 (64)	+11 (66)	−1 (69)	+2 (58)	+2 (45)
Holloman, N. Mex. 32°51'N 106°06'W					0 (10)	−14 (10)		
White Sands, N. Mex. 32°23'N 106°29'W	−61 (31)	−7 (55)	+2 (72)	−13 (82)	+2 (104)	−10 (113)	0 (122)	+1 (107)
Eglin Field, Fla. 30°30'N 86°30'W								
Cape Canaveral, Fla. 28°14'N 80°36'W	−190 (11)	0 (32)	0 (43)	+18 (46)	+3 (45)	+2 (44)	−3 (17)	
Weighted average for 28°–38°	−95 (42)	+3 (133)	+6 (205)	+2 (230)	+5 (288)	−4 (303)	0 (258)	0 (188)

straightforward procedure is to apply, in a crude fashion, the techniques of balance reguirements extensively used in the troposphere to the polar cap north of 35N and above 30-km altitude. (Henceforth the discussion will proceed in terms of kilometers.) The total content of relative angular momentum of the region can be found from the observed distribution of zonal wind and density and the change in the content over a period, for example between summer and winter, can be evaluated and compared with the flux of relative angular momentum across the vertical wall formed by 35N latitude. Such a procedure has been followed; the integration used to estimate the content was carried out over 5 km height intervals commencing at 30 km and in 5-degree latitude increments. The wind cross-sections presented by Batten (1961) were used and density values were taken from a paper by Quiroz (1961). The winter polar cap content of relative angular momentum, counted positive for eastward motion, was $+8.3\times10^{30}$ gm cm^2 sec^{-1} and the summer content was -8.5×10^{30} gm cm^2 sec^{-1}. If the change from summer to winter is assumed to occur over a three month period, the flux involved is 2.1×10^{24} gm cm^2 sec^{-2}. The total eddy flux into the volume is

$$R \cos\phi \int_0^{2\pi} \int_{30\text{ km}}^{\infty} \rho\overline{u'v'}dzd\lambda,$$

where λ represents longitude, ϕ is latitude, R is the radius of the earth and ρ is density. The mean values in Table 3 were taken to represent the flux at 35N in which case the northward flux into the polar cap is 11.5×10^{24} gm cm^2 sec^{-2}. The eddy flux is therefore more than five times greater than the required flux deduced from change requirements. Indeed the eddy flux could produce the observed change within about 17 days. The lack of a representative sample of data with respect to longitude is a serious drawback to such estimates, and will not be remedied until a Rocket Network is operating on a global basis.

If the above estimates are borne out by more extensive observations, then a suitable sink must be sought for the momentum transported northward, as the polar vortex does not continuously increase its momentum content throughout the winter. In the troposphere there is a yardstick to gauge such a contingency not available in the present case, namely that the zonal momentum transported northwards into the middle-latitude jet stream must be balanced largely by that lost from the jet through vertical transport and friction at the earth's surface. The momentum budget of the winter high-level jet may be balanced either by a downward flux by one or more processes or by a mean southward drift within the region. The downward flux could be performed by a downward mean motion accompanying the vertical shear between 30 and 50 km, or by a systematic flux by large-scale eddies such as might be evidenced by negative values of the covariance $u'w'$ where w is the vertical velocity, or by so-called frictional processes accompanying the vertical shear which would correspond to motions on a scale too small to be observed by the network. There are presently no methods of estimating the first two possibilities.

The third possible process, internal friction, may be described by a law of the form $\tau=-\rho K\partial\bar{u}/\partial z$ where τ is the stress, K the vertical eddy diffusion coefficient and $\partial\bar{u}/\partial z$ the shear of the mean zonal wind. K is one of the enigmas of modern meteorology. An effective K for the polar lower stratosphere has been estimated from the vertical spread of radioactive tungsten by Feely and Spar (1960) as $\sim10^4$ cm^2 sec^{-1}; this would include all scales of motion. As reported elsewhere (Newell, 1963b) the climatological variances of the vertical velocity (appropriate to the large-scale eddies) decrease with altitude between the levels of 18 and 22 km; no observations are available for higher altitudes. The variances have been converted to effective K values (Newell, 1963b) by an empirical approach based upon the more extensive knowledge about the troposphere. The effective K for the polar lower stratosphere thus found was also 10^4 cm^2 sec^{-1} suggesting that the small-scale friction contributes very little to the total eddy friction in the region. Even allowing for an increase of both small-scale and large-scale K's with height between 25 and 35 km, in conjunction with the observed increase in horizontal exchange coefficients, it seems that the largest value of K that may be reasonably ascribed to the small-scale friction at 35 km is about 5×10^4 cm^2 sec^{-1}. With the mean zonal shear estimated from Table 1 the total downward flux with this value of K is about 2×10^{23} gm cm^2 sec^{-2} which is almost two orders of magnitude smaller than the northward eddy advection. If this estimate is borne out by future observations and if the first two processes mentioned also prove inadequate to provide a balanced momentum budget a mean southward drift may have to be invoked, in which case the additional factor of the earth's angular momentum will have to be considered. As in many other instances it is possible that the true resolution of the problem will only come when vertical velocities can be calculated from a larger and more complete Rocket Network so that such quantities as $[\overline{u'w'}]$ and $[\bar{u}][\bar{w}]$ can properly be calculated (square brackets here denote latitudinal averages).

3. Possible origin of the eddy fluxes

By analogy with the troposphere it could be argued that the transient eddies that apparently transport considerable zonal momentum northwards in winter in the 30–60 km region do so incidentally in the process of transporting heat energy northwards as the atmosphere in this region endeavours to balance its heat budget. The radiation budget calculations such as those presented by Murgatroyd and Goody (1958) or Murgatroyd and Singleton (1961) certainly indicate that the requirement for such a heat transport exists and that

without it the polar cap region would get very much colder than is actually observed in the winter. Murgatroyd and Singleton in fact calculated the hypothetical mean meridional motions which would be necessary to balance the heat budget, although they recognized that ultimately eddy effects would have to be taken into account.

A detailed specification of the atmosphere above 30 km is not yet available but it is perhaps not too early to make a crude estimate of some of the terms in the energy budget of the high-level jet. Differential cooling in the winter hemisphere, which is indicated by the radiational calculations, in conjunction with the observed temperature structure would be expected to generate zonal available potential energy. It is not presently known that such energy generated is transformed to eddy available potential energy and hence to eddy kinetic energy and ultimately zonal kinetic energy as in the troposphere but it is of interest to examine the directions and magnitudes of the transformation quantities. The appropriate equations for the zonal available potential energy, for its generation and for the zonal kinetic energy, have been presented by Lorenz (1955). Numerical values appropriate to the atmosphere have been estimated by Lorenz and evaluated on a daily basis for a two week period by Winston and Krueger (1961). The generation term has been evaluated twice daily for a one month period by Wiin-Nielsen and Brown (1962). The studies so far have been essentially confined to the troposphere. It was decided to evaluate the three quantities mentioned for the region above the troposphere divided into a series of layers.

The appropriate equations in the order above (after Lorenz) are:

$$\bar{A}_z=\frac{1}{2}\int_{p_2}^{p_1}\frac{1}{(\Gamma_D-\bar{\Gamma})}\frac{1}{\bar{T}}\overline{[T]'^2}dp, \qquad (1)$$

$$\bar{G}_z=\frac{1}{g}\int_{p_2}^{p_1}\frac{\Gamma_D}{(\Gamma_D-\bar{\Gamma})}\frac{1}{\bar{T}}\overline{[T]'[Q]'}dp, \qquad (2)$$

$$\bar{K}_z=\frac{1}{2g}\int_{p_2}^{p_1}\overline{[u]^2}dp, \qquad (3)$$

where the symbols are as follows:

$\bar{A}_z$ = zonal available potential energy
$\bar{G}_z$ = generation of zonal available potential energy
$\bar{K}_z$ = kinetic energy of zonal flow
g = acceleration of gravity
p = pressure, T = temperature
$\Gamma_D=g/c_p \quad \Gamma=-\partial T/\partial z$
c_p = specific heat of air at constant pressure
u = zonal wind component
Q = rate of diabatic heating

An overbar denotes a mean over an isobaric surface. Square brackets denote a mean value around a latitude circle.

Although Lorenz applied the equations to the entire atmosphere there appears to be no objection to their application to a series of layers. An assumption in their derivation is that the pressure variation over an isentropic surface is small compared with the average pressure. With the present observations the assumption appears to be satisfied at least as well as in the extreme example selected by Lorenz which applied to the 1000–300 mb region. When more detailed observations become available at the higher levels the point may well have to be reconsidered.

Cross-sections of zonal wind and temperature from the surface to 80 km were constructed for both winter and summer by Mrs. Dorothy Berry, the author's assistant. Such cross-sections are redrawn each year for presentation to students taking a course of lectures on the upper atmosphere. They are not meant to represent standard atmospheres suitable for engineering use in any sense. The cross-sections are changed as the data accumulate. Temperature data for the lowest 100 mb were taken from the work of Peixoto (1960) and zonal winds for this region from Crutcher (1961). For the 100–10 mb layer mean values were obtained from my colleagues on the Planetary Circulations Project who are presently engaged in a study of the stratospheric circulation. These data all refer to average values over the entire northern hemisphere. Data for the 30–80 km region were taken from the rocket grenade results (Stroud, Nordberg and Walsh, 1956; Stroud, Nordberg, Bandeen, Bartman and Titus, 1960; Nordberg and Stroud, 1961) and the rocket network results referred to earlier. Several sets of high level radiosondes (Arnold and Lowenthal, 1959) and rocket results prior to the establishment of a network (Smith, 1960) were also included. The point of difference with previous cross-sections, for example that of Murgatroyd (1957), is that no results from anomalous sound propagation measurements were included. For completeness the zonal wind and temperature cross-sections which resulted for winter are shown in Figs. 2 and 3. Obviously they are not representative of the global averages, as the majority of the observations were collected over North America and over the Pacific.

These crude cross-sections were used to evaluate the expressions for $\bar{G}_z$, $\bar{A}_z$ and $\bar{K}_z$ with 5-km height increments between 15 and 30 km and 10-km increments above. Height, rather than pressure, was used as a vertical coordinate and values at the mid-point of each selected layer were taken as representative of the entire layer. The calculations covered the region from 10N to 65N except for the summer where for the layers from 60 to 80 km, the region was 30N to 60N. Diabatic heating values were taken from the work of Murgatroyd and Singleton. A previous set of values for the higher layers

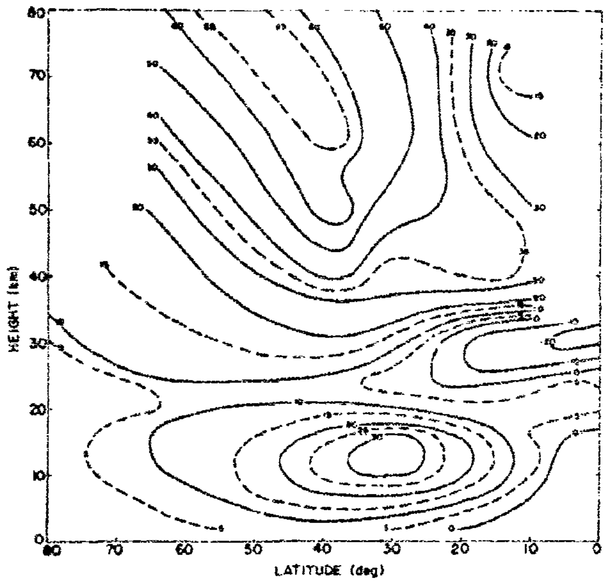

FIG. 2. Meridional cross-section of zonal wind component. Winter regime. Units: m sec^{-1}.

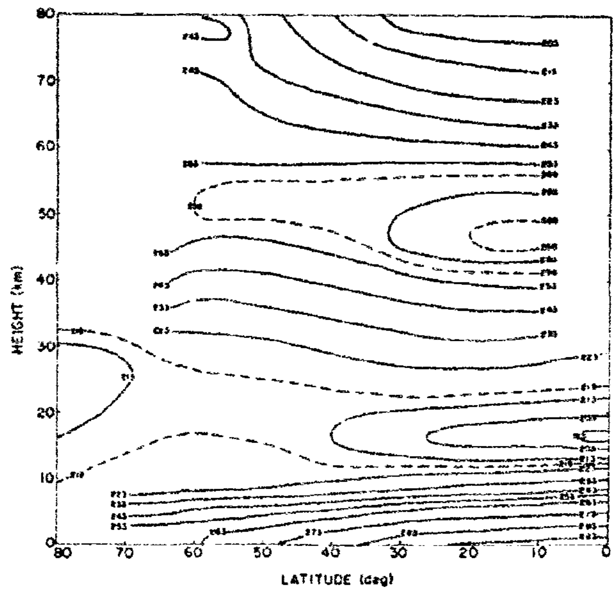

FIG. 3. Meridional cross-section of temperature. Winter regime. Units: degrees Kelvin.

as presented by Murgatroyd and Goody was also used for comparison. It is appreciated that Murgatroyd and Singleton's values had been adjusted to fit in with other radiation calculations, notably those by Ohring (1958), and to fit in with their meridional motion ideas but the differences are comparatively minor. The results are shown in Table 5; all $\bar{G}_z$ values were based on Murgatroyd and Singleton except those in parenthesis which are based on Murgatroyd and Goody. Table 6, to be considered in conjunction with Table 5, shows the time period necessary to produce the computed $\bar{A}_z$ and $\bar{K}_z$ from zero, or to reduce $\bar{A}_z$ to zero, on the basis of the computed values of $\bar{G}_z$. Winter results will be discussed first commencing with the lowest layer.

In the 15–20 km layer zonal available potential energy is being destroyed by the differential heating at a rapid rate. At the calculated destruction rate it would be reduced to zero in about six days hence there must be advection of energy into the region, presumably from below. There are several items of information which suggest that kinetic energy is converted to available potential energy, although not necessarily to zonal available potential energy, within the region. The author (Newell, 1961) has pointed out that lines of equal ozone concentration slope downwards with increasing latitude in the 10–55 deg region more steeply than the isentropic surfaces. Radioactivity measurements also support this picture. If the slopes of the isolines of trace substance concentration represent the motion of air particles then a little consideration shows that such motions would act to convert kinetic to potential energy in the region (the relationship between air particle paths and the slope of isentropic surfaces has been discussed by Eady, 1949, 1950; Kuo, 1956a; and Green, 1960) and would also give rise to an apparent northward flux of heat. If attention is fixed on a particular point on an isentropic surface then air parcels reaching the point from the south will be descending and warming adiabatically and therefore be warmer than the local mean temperature. These concepts are consistent with the observations of White (1954), Peixoto (1960) and Murakami (1962) which showed a countergradient heat flux in the lower stratosphere, of White and Nolan (1960) and Jensen (1960) which showed the conversion of kinetic to potential energy in the lower stratosphere and of Molla and Loisel (1962) which showed that vertical and meridional components of the motions in the region are negatively correlated. The sequence of energy transport and transformation in the 15–20 km layer would thus seem to be as follows: energy is advected into the region from below (whether this advection is in the form of a systematic transport of kinetic energy or of energy in the form of sensible heat has not yet been established from observations), kinetic energy is converted to available potential energy, and zonal available potential energy is destroyed by differential heating.

An additional point of interest is the near-equality of the available potential energy and the kinetic energy. Lorenz estimated that the available potential energy is about ten times larger than the kinetic energy; his estimate refers mainly to the troposphere. Murakami (1962) pointed out that in the lower stratosphere the two quantities are of the same order of magnitude.

The layer 20–25 km also requires energy from a source

TABLE 5. Distribution of $\bar{G}_s$, $\bar{A}_s$, and $\bar{K}_s$ with height for zone from 10N to 65N.

Layer km	Winter $\bar{G}_s$ ergs cm^{-2} sec^{-1}	Winter $\bar{A}_s$ ergs cm^{-2}	Winter $\bar{K}_s$ ergs cm^{-2}	Summer $\bar{G}_s$ ergs cm^{-2} sec^{-1}	Summer $\bar{A}_s$ ergs cm^{-2}	Summer $\bar{K}_s$ ergs cm^{-2}
15–20	−214.1	104.0×10^{6}	86.3×10^{6}	−227.4	148.3×10^{6}	18.9×10^{6}
20–25	−0.4	0.8×10^{6}	6.2×10^{6}	−21.6	9.2×10^{6}	14.4×10^{6}
25–30	+7.5	1.2×10^{6}	9.2×10^{6}	−1.2	0.6×10^{6}	12.5×10^{6}
30–40	+10.7 (+5.4)	4.0×10^{6}	17.5×10^{6}	+0.3	0.3×10^{6}	10.0×10^{6}
40–50	+4.7 (+5.5)	1.5×10^{6}	12.8×10^{6}	+0.5	4.1×10^{5}	52.5×10^{5}
50–60	+0.8 (+0.5)	0.3×10^{5}	56.0×10^{5}	+0.8	2.3×10^{5}	30.9×10^{5}
60–70	−1.5 (−2.1)	1.7×10^{5}	19.0×10^{5}	0.0	0.5×10^{5}	9.6×10^{5}
70–80	−0.4 (−0.3)	1.9×10^{5}	5.5×10^{5}	+0.2	0.9×10^{5}	1.9×10^{5}

TABLE 6. Time in days necessary to establish or destroy $\bar{A}_s$ and $\bar{K}_s$.

Layer km	Winter $\bar{A}_s$	Winter $\bar{K}_s$	Summer $\bar{A}_s$	Summer $\bar{K}_s$
15–20	5.6	—	7.5	—
20–25	21.6	—	4.9	—
25–30	1.9	14.4	6.0	—
30–40	4.3	18.9	10.3	413
40–50	3.8	31.9	9.0	115
50–60	0.4	77.2	3.5	47.1
60–70	1.3	—	—	—
70–80	5.7	—	4.7	9.6

other than differential heating and it would seem that this again may be supplied from below. The layers between 25 and 60 km all exhibit internal generation of potential energy and, as the lowest two apparently could produce their own observed kinetic energy in 14 and 19 days, there is no demonstrated requirement for a further supply. The use of Murgatroyd and Goody's values does not change $\bar{G}_s$ by more than a factor of two. While the zone from 25 to 40 km may be self sustaining the two layers above, that is 40–60 km, appear to need considerably longer periods in order that the kinetic energy can be generated *in situ*. Certainly it would be reasonable to claim that the 50–60 km region receives some of its kinetic energy from below. Without consideration of further energy sources the two layers from 60–70 and 70–80 also appear to need advection of energy, presumably from below, to supply their kinetic energy. Notice that the available potential energy is smaller than the kinetic energy for all layers above 20 km so that a rapid response of the kinetic energy to changes in the differential heating would be expected. There is no large reservoir of potential energy that can support the motions, such as exists in the troposphere.

The numbers presented here, while admittedly somewhat crude, may well be correct as to sign and order of magnitude. Accepting them as they stand for the present, it would then appear that there are two essentially independent vortices at different levels in winter with the upper vortex producing sufficient energy *in situ* to run itself without communication of energy from below. Theoretical considerations by Charney and Drazin (1961) also pointed to the fact that the upper vortex is mechanically independent of the troposphere. There is a region of minimum activity in between the vortices as is evident from an inspection of zonal wind cross-sections.

Above about 55 km in winter there is found a situation similar to that above the tropospheric jet, namely that energy apparently has to be supplied to balance radiational destruction and that the temperature increases northwards. If these conditions are a general accompaniment of the upper portion, that is the energy consuming portion, of these large-scale eddy structures then the high temperatures over the pole may simply be a reflection of the presence of quasi-horizontal eddies in which kinetic energy is being converted to potential energy. From another point of view the air parcel motions in the upper portions of the eddies (above about 55 km) are tending to force the isentropic surfaces in the region to slope downwards from equator to pole whereas the radiational effects are working in the opposite direction. The mean isentropic slope actually represents an equilibrium between the two effects. There are not yet sufficient observations in the region to test whether a countergradient heat flux is present as would be predicted by the above scheme. Kellogg (1961) has suggested that the high temperatures in the winter polar mesosphere are produced by recombination of atomic oxygen in a subsiding current and Young and Epstein (1962) have presented some further calculations

which support this contention. If the situation outlined here is an approximation to events then atomic oxygen recombination need not be invoked.

In order to illustrate that the pattern of potential energy destruction in winter in the lower part of the region is not simply a consequence of the particular cross-sections chosen, the calculations of $\bar{A}_z$, $\bar{G}_z$ and $\bar{K}_z$ were performed for mean monthly cross-sections along 75W for October and December 1957 and February 1958 (U. S. Weather Bureau, 1961). The results, shown in Table 7, exhibit a general pattern of values that is similar to those found for the winter regime. There is destruction in the 15–20 km layer throughout the season with largest values in February after the change in temperature structure that had accompanied a so-called sudden warming in January. Generation of energy is largest in the 20–30 km region in December. The magnitudes are somewhat larger than those for the winter regime; this is no doubt due to the selection of 75W which is a longitude along which there is a larger horizontal temperature contrast than is the case for the global average. The zonal potential and kinetic energies are of the same order of magnitude in this region except in the month of February when the potential energy increases in the 15–20 km layer and the kinetic energy decreases in the 20–25 km layer, with the result that in both layers the potential energy becomes an order of magnitude larger than the kinetic energy.

There has been considerable discussion about the nature of the disturbances which are responsible for the energy conversions and momentum transports observed in the 15–30 km region in the winter season. Zubyan (1959) has discussed the inter-latitudinal exchange of warm and cold air which results from stratospheric differential heating. Boville, Wilson and Hare (1961) have described some of the disturbances in the 15–30 km region in detail and have evaluated energy conversions from potential to kinetic energy as a function of wave number at 25 mb (~25 km). Their general conclusion is that the disturbances in the polar-night vortex have many features in common with the baroclinic waves of the troposphere. Barnes (1962) has also calculated energy conversions for this region. For the higher regions the existence of large-scale disturbances has been stressed in the analyses of Teweles (1961, 1962) and Keegan (1961, 1962), and the synoptic charts presented by these authors show evidence of northward momentum transports in the appearance of tilted troughs. Until this relatively large number of observations from the Rocket Network became available, however, it was not possible to estimate quantitatively the momentum transports brought about by the disturbances.

In the case of the summer season (Table 5) available potential energy is destroyed from 15 to 30 km. There must therefore be considerable advection of energy, presumably from below, into the region. In the 30–60 km region there is production of available potential energy by the differential heating but apparently it

TABLE 7. Distribution of $\bar{G}_z$, $\bar{A}_z$ and $\bar{K}_z$ for three cross-sections along 75W.

Layer km	$\bar{G}_z$ ergs cm^{-2} sec^{-1}	$\bar{A}_z$ ergs cm^{-2}	$\bar{K}_z$ ergs cm^{-2}
		October 1957	
15–20	−199	87 $\times 10^6$	23 $\times 10^6$
20–25	+3	2.9$\times 10^6$	8.3$\times 10^6$
25–30	+22	14.5$\times 10^6$	14.8$\times 10^6$
		December 1957	
15–20	−127	92 $\times 10^6$	99 $\times 10^6$
20–25	+48	41 $\times 10^6$	42 $\times 10^6$
25–30	+48	106 $\times 10^6$	34 $\times 10^6$
		February 1958	
15–20	−358	520 $\times 10^6$	65 $\times 10^6$
20–25	−36	27 $\times 10^6$	2.3$\times 10^6$
25–30	+6	3.2$\times 10^6$	4.0$\times 10^6$

would take a considerable length of time (see Table 6) for the observed kinetic energy to be produced if generated *in situ*. There are several possible energy sources. The energy may come from below or from the southern hemisphere where the southern winter polar vortex may be in operation. Such a possibility can only be examined when this latter vortex has also been extensively observed.

It has been argued above that in the 30–60 km region in winter differential cooling produces zonal available potential energy which is realized in large-scale eddies that transport momentum, and possibly heat energy, northwards. It would be desirable to calculate the heat flux from observations and compare it with the requirements of the radiation budget, but there are not yet sufficient observations of temperature in the region. If events are similar to those in the troposphere the further argument could be made that the zonal available potential energy is converted to eddy available potential energy which in turn is converted to eddy kinetic energy and ultimately into the kinetic energy of the mean motion. The last item in the chain can be roughly estimated from the wind observations presented above. Its value has been calculated for the troposphere by Starr (1953) and is given by the integral

$$2\pi R^2 \int\int \rho\left[\overline{u'v'}\right] \cos^2\phi \frac{\partial}{\partial\phi} \frac{[\bar{u}]}{R\cos\phi} d\phi dz.$$

Division by the area of interest gives the rate of working of the Reynold's stresses per unit area which may be compared with the generation of available potential energy. For the present estimates the observed values of $\bar{u}$ for the 20–60N region were used in conjunction with the weighted average values of $\overline{u'v'}$ as quoted in Table 1. Values for the winter regime for the layers 30–40, 40–50 and 50–60 km are 14.9, 4.1 and 3.8 ergs cm^{-2} sec^{-1},

respectively. These are very similar to the generation values and this similarity gives further weight to the argument that the sequence of energy transformations in high level jet is the same as in the tropospheric jet except for ground friction effects. If the similarity is verified when more comprehensive observations become available then the previous suggestions concerning the smallness of the internal friction will be upheld.

4. Concluding remarks

Meteorological Rocket Network measurements have been used to demonstrate that the layer of the atmosphere between 30 and 60 km can run on its own energy supply in the winter season without the benefit of energy advected from below. The disturbances, that apparently arise to transfer heat in a direction required by the differential cooling, transport relative angular momentum northwards and are thereby responsible for the formation of the high level jet stream. The radiation calculations coupled with mean winter cross-sections of temperature and wind indicate that available potential energy is generated *in situ* and the winter wind observations permit calculations which demonstrate that eddy kinetic energy is being converted to mean zonal kinetic energy at about the same rate as the generation. The implication from the available data is thus that the quasi-horizontal large-scale eddy motions are of prime importance in the heat and momentum budgets of the region in winter as opposed to the mean meridional motions previously suggested. The finding is then essentially the same as the earlier findings for the troposphere and lower stratosphere.

While the large static stability of the 30–60 km region may well increase the horizontal scale size of any baroclinic disturbances present it does not appear to prohibit such disturbances.

There are some rather obvious drawbacks in the computations in Sections 2 and 3 due to the small data samples presently available. Some of these which can be removed in the near future, as examination of the region 30–60 km proceeds, are outlined below.

a) In order to improve the estimates of $[\overline{u'v'}]$ and to weigh the contributions of standing eddies and ultimately mean meridional motions, the Rocket Network needs to be extended around the globe. Careful attention to the selection of firing times in such an extended network would enable the tidal components to be properly examined and their interaction with the non-tidal components assessed. All the data should be published frequently in an easily accessible form such as was done for Point Mugu, California, by Masterson, Hubert and Carr (1961) and for the North American stations by the Inter-Range Instrumentation Group.

b) An increased number of reliable temperature observations are required from the region above 30 km. A new economical method of measuring temperature above 45 km will have to be developed, as is widely recognized. Heat transport estimates will then be forthcoming.

c) The distribution of trace substances needs to be measured above 30 km. Water vapor is particularly important as it has been assumed to have a very low concentration in the radiation calculations made so far for the region, whereas much of the current experimental work indicates relatively high concentrations at 30 km. If the experimental work is verified, the radiation calculations will have to be repeated. It would also be of considerable interest to have available the proper distributions of ozone and carbon dioxide for the region as well as the various radioactive trace materials. Distributions of the latter may enable the paths of air motions relative to isentropes to be traced and will afford much information on mass transfer complementary to that discussed on momentum and heat transfer.

d) When the variations of ozone concentration and water vapor in the 30–60 km region are properly known as functions of latitude and longitude the radiational calculations can be extended to include variations of heating and cooling rates along latitude circles and estimates can be made of the possible generation of available potential energy thereby produced.

The radiation calculations apply to the northern hemisphere for summer and winter. If it can be assumed as a rough approximation that they represent global conditions at a given time then a qualitative picture of the heat budget of the entire 30–60 km region can be obtained. In the case where it is winter in the northern hemisphere there is a requirement for a quasi-horizontal heat flux northwards across the equator if the region is energetically independent of the lower atmosphere. Simultaneously relative angular momentum must be transported northwards if the same assumptions of symmetry are made about the wind structure. Available potential energy will be generated by the differential heating and cooling. A calculation for the 30–40 km region from 65N to 65S based upon interpolated temperatures in the equatorial region gave a zonal potential energy production rate similar to that previously obtained for the winter season. It does not seem profitable to pursue such matters further until global observations are available; then it will be of interest to apply the present approach to observations of winds and temperatures in the equatorial region to seek evidence of the trans-equatorial transports which are apparently necessary. Finally it should be noted that consideration has not yet been given specifically to the theoretical problem of large-scale atmospheric disturbances which span the equator.

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Poleward Flux of Atmospheric Angular Momentum in the Southern Hemisphere

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ABSTRACT

Tabulations of daily winds from IGY data were made at eight pressure levels (850, 700, 500, 400, 300, 200, 100 and 50 mb) for 121 Southern Hemisphere plus 22 Northern Hemisphere equatorial stations during the calendar year 1958, in order to study by seasons the poleward flux and convergence of relative angular momentum.

The overwhelming importance in both seasons of the transports of relative angular momentum by the transient eddies is demonstrated. These transports are enough to balance the drain of atmospheric angular momentum in the belt of surface westerlies. The mean meridional motions and the standing eddies are ineffective in the fulfilment of the angular momentum balance requirements of this hemisphere. On the other hand the standing eddies are the main agents in the interhemispheric exchanges of relative angular momentum. There are insignificant seasonal variations in the vertically integrated horizontal momentum transport across latitude belts equatorward of 50S, although there is evidence for marked seasonal vertical fluctuations of the zones of maximum flux of relative angular momentum.

The sign of the mean surface zonal stress computed from the angular momentum convergence is in good agreement with the distribution of the surface winds; the magnitude is comparable to that computed by Priestley (1951), except in the zones of strong surface westerlies.

1. Introduction

In recent years much attention has been given to the angular momentum balance of the Northern Hemisphere (see, e.g., Widger, 1949; Starr, 1951, 1952; Starr and White, 1951; Buch, 1954; Mintz, 1951; and Tucker, 1960). The upper wind observations collected in the Southern Hemisphere during the International Geophysical Year (IGY) were good enough to justify for the first time a study of the poleward flux of relative angular momentum. The data also enabled us to study the mechanisms responsible for these poleward fluxes and to estimate the probable interaction between the two hemispheres.

The first attempt to employ the concept of angular momentum to the general circulation of the Southern Hemisphere was made through use of surface climatological data by Priestley (1951), who computed the frictional stresses over all the southern oceans. Other preliminary studies include those of Palmén (1955), Berson and Troup (1961), and Gruza (1962). The study of Berson and Troup was restricted to the eastern part of the Southern Hemisphere, while that of Gruza employed geostrophic winds and was limited to only the 700-mb level and to the eastern half of the hemisphere (10W to 170W).

A second objective of this paper is to compute the mean frictional stress from angular momentum convergence. The stress thus indirectly obtained is compared with that of Priestley (1951), which was derived using surface climatological wind observations. Since the Southern Hemisphere is mainly oceanic, the result is expected to be of value in the understanding of the frictional coupling between the southern oceans and the atmosphere.

The following notation will be employed:

$$\frac{1}{t_1-t_0}\int_{t_0}^{t_1} x\,dt=\bar{x}=\text{time average of } x.$$

$$\frac{1}{2\pi}\int_0^{2\pi} x\,d\lambda=[x]=\text{zonal average of } x.$$

$x'=x-\bar{x}$ = departure from time average,
$x^*=x-[x]$ = departure from zonal average,
a = radius of the earth,
g = acceleration due to gavity,
λ = longitude,
φ = latitude,
p = pressure,
p_0 = pressure at the ground,
t = time,

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$u = a \cos\varphi(d\lambda/dt)$ = eastward component of the wind,
$v = a(d\varphi/dt)$ = northward component of the wind,
$\omega = dp/dt$ = individual pressure change,
Ω = angular velocity of the earth,
T_λ = eastward component of viscous force per unit mass.

2. Data and their representativeness

The data consist of all wind observations taken during the IGY for the calendar year 1958. The primary data employed were 0000Z wind observations except in cases where the observations were missing or insufficient. On such occasions 1200Z observations were substituted. In addition a few African stations reporting at 0600Z and some South American stations reporting at 1800Z were used. We feel justified in using observations made at irregular hours for the following reasons: first, there is little diurnal variation of tropospheric winds, except close to the ground, and secondly, the statistical methods employed in this study require that the data be chosen at random. Data based on observations at regular time intervals are not exactly random samples, but for a large number of observations they may be considered as such, unless the period of observation coincides with a period in the fluctuations of the wind.

All the winds observed were tabulated for the eight pressure levels: 850, 700, 500, 400, 300, 200, 100 and 50 mb. The winter has been defined to include the months April–September while summer includes the months October to December and January to March.

Fig. 1 contains the 143 stations which made wind observations. Those stations which reported rawins are designated by R while those stations using theodolite (pilot balloon) methods are designated by W. Of these stations, 22 are actually not southern hemispheric stations but are situated slightly north of the equator. They were included in order to obtain a better analysis in the equatorial regions. The present data made our analysis more complete than any other similar hemispheric study, and we are thus able to say something about the seasonal interchange of relative angular momentum between the two hemispheres. If we assume that a station's observations are representative over an area of a circle of radius 300 n mi, then throughout the Southern Hemisphere all the continents are well represented except for the interior of Brazil.

We received data from only two South Atlantic Ocean stations, viz., station 68–906 (Gough Island), and station 82–400 (Fernando Noronha). Unfortunately data from stations 83–650 (Ilha da Trinidad) and 61–901 (St. Helena Island) were not available during the analysis. On the other hand, the Indian Ocean had good coverage. Both Malagasy and nearby Vacoas provided good coverage. However data from 96–996 (Cocos Island) were not available during the analysis. Berson and Troup (1961) studied the angular momentum balance in the equatorial trough zone of the eastern hemisphere and they tabulated the values of $\bar{u}$, $\bar{v}$, and $\overline{u'v'}$ from all the

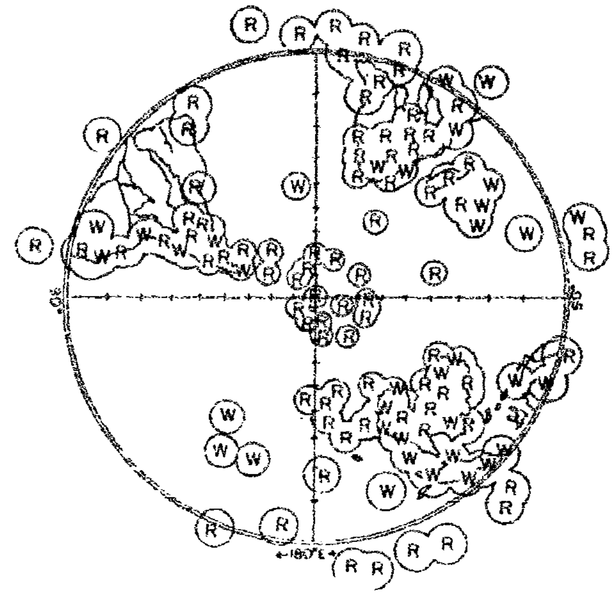


FIG. 1. The geographical location of the stations and the representativeness of the data. *R* and *W* indicate rawin and pilot balloon (theodolite) stations, respectively.

data available from this island during the period 1956–59. Their computations were made only for the months December through February. During the analysis of our summer data, the values of Berson and Troup were used as a guide only.

The region of greatest uncertainty in the present analysis is the Pacific Ocean. Not a single observation was available in the sector bounded roughly by the equator and latitude 60S, and by longitudes 90W and 140W. During the summer months, the pilot balloon observations obtained on Cruise VII of the ship *Carnegie* were used as a guide in the analysis of $\bar{u}$ and $\bar{v}$ for the regions bounded by latitudes 10S and 20S and longitudes 100W and 172W. During the summer months, whaling ships report surface synoptic observations from oceanic regions of the Southern Hemisphere. Techniques have been devised to construct the 500-mb height patterns from the surface circulation pattern plus the 1000-mb heights (see, for example, Schmitt 1953; Taljaard and Van Loon, 1960), However it is our intention in this study to avoid the use of geostrophic winds, and little advantage was gained from use of surface observations.

Aside from the scatter of geographical location of the observing stations, there is the question of the representativeness of the observed winds. In general, pilot balloon observations are less representative than radio winds. Pilot balloon observations made under conditions of cloudiness and/or strong winds are usually terminated at low levels. The longer runs are made when the winds are light and the sky relatively clear. Therefore the data from the pilot balloon observing stations are usually biased in favor of light winds and fair weather.

In addition, pilot balloon observations usually do not extend above the 300-mb level. Fortunately, in regions where the circulations are intense, radio wind equipment was used. Most of the rawin stations have fairly complete sets of data at least up to 100 mb where in winter 50 stations and in summer 66 stations reported more than one half of the possible 183 or 182 observations. During the winter the percentage of reports drops off markedly at 50 mb, especially over the Antarctic continent. This is attributed in part to the large percentage of balloon bursts due to the extremely cold stratosphere. The temperature of the Antarctic stratosphere occasionally drops to −90C at night (Alt, Astanpenko and Ropar, 1959).

3. Equations

The steady state absolute angular momentum balance for a polar cap of the spherical earth can be written in the form

$$\Omega a^3g^{-1}\cos^3\varphi\int_0^{p_G}\int_0^{2\pi}\bar{v}d\lambda dp+a^2g^{-1}\cos^2\varphi\int_0^{p_G}\int_0^{2\pi}\overline{uv}d\lambda dp$$
$$=\int_a^{a+H}\int_\varphi^{\pi/2}\int_0^{2\pi}r^2\frac{\overline{\partial p}}{\partial\lambda}\cos\varphi d\lambda d\varphi dr+a^3g^{-1}\int_0^{p_G}\int_\varphi^{\pi/2}\int_0^{2\pi}T_\lambda\cos^2\varphi d\lambda d\varphi dp. \quad (1)$$

By taking averages over longitude the equation becomes

$$2\pi\Omega a^3g^{-1}\cos^3\varphi\int_0^{p_G}[\bar{v}]dp+2\pi a^2g^{-1}\cos^2\varphi\int_0^{p_G}[\overline{uv}]dp$$
$$=2\pi\int_a^{a+H}\int_\varphi^{\pi/2}\left[r^2\frac{\overline{\partial p}}{\partial\lambda}\right]\cos\varphi d\varphi dr+2\pi a^3g^{-1}\int_0^{p_G}\int_\varphi^{\pi/2}[\bar{T}_\lambda]\cos^2\varphi d\varphi dp. \quad (2)$$

Since in the long run, there is no net mass shift, we can write

$$\int_0^{p_G}[\bar{v}]dp\approx 0. \quad (3)$$

Over the six month's period used, this term is not zero, due to the difficulty in the measurement of $[\bar{v}]$, and so the constraint of equation (3) is used by defining at each level the quantity $[\bar{v}]''$:

$$[\bar{v}]-\{[\bar{v}]\}=[\bar{v}]'', \quad (4)$$

where

$$\frac{1}{p_G}\int_0^{p_G}[\bar{v}]dp=\{[\bar{v}]\}, \quad (5)$$

and

$$\int_0^{p_G}[\bar{v}]''dp\equiv 0. \quad (6)$$

This correction then allows us to use the values of $[\bar{v}]''$ at each level as a corrected estimate of $[\bar{v}]$.

The torque equation (2) then states that for the polar cap, the relative angular momentum flux through the latitudinal wall is balanced by pressure differentials across mountain barriers and frictional torques exerted at the ground surface of the polar cap.

We shall not be able to measure the separate effects of the pressure differential and frictional torques independently from wind observations but their combined effects will be determined when we consider the angular momentum convergence in Section 7 of this paper.

4. Flux of relative angular momentum

The term

$$2\pi a^2g^{-1}\cos^2\phi\int_0^{p_G}[\overline{uv}]dp$$

can be written in the form

$$2\pi a^2g^{-1}\cos^2\varphi\int_0^{p_G}([\overline{u'v'}]+[\bar{u}^*\bar{v}^*]+[\bar{u}][\bar{v}]'')dp.$$

The term $[\overline{u'v'}]$ represents a contribution to the momentum flux which depends upon the presence of a time correlation between the instantaneous values of u and v at individual points along a latitude circle. This is associated with the temporal variations of the wind at a given station, and will be referred to as flux due to *transient eddies*. The term $[\bar{u}^*\bar{v}^*]$ represents a contribution to the momentum flux which is due to the presence of a spatial correlation between the time means $\bar{u}$ and $\bar{v}$

along a latitude circle. It is associated with the asymmetry of the seasonal mean streamline pattern, and is referred to as flux due to *standing eddies*. The term $[\bar{u}][\bar{v}]''$ represents a contribution to the momentum flux due to a net meridional mass flux $[\bar{v}]''$ during the entire period at a given level. This component depends on the existence of *mean meridional* circulations.

A) WINTER

1. Transient eddies. The zonally averaged values of the relative momentum transport by the transient eddies are shown in Table 1. A negative value of the covariance represents a poleward (south pole) transport of relative angular momentum, while positive values indicate equatorward transport of relative angular momentum. The table indicates that negative values predominate in the region between 5S and 50S. The maximum values are found between 30S and 35S at 300 mb. Between 55S and 60S equatorward fluxes are predominant. The maximum flux is found in the layer 300–400 mb between 60S and 70S.

When the vertical integral is evaluated and the units expressed in gm cm² sec⁻², it is found, as shown by the full curve of Fig. 2, that the maximum poleward flux of relative angular momentum by the transient eddies is at 32S and the maximum equatorward transport is at 64S. The corresponding magnitudes are respectively

TABLE 1. Zonally averaged values of the transient eddy transport of momentum $[\overline{u'v'}]$ in winter 1958. Units are in m² sec⁻².

Lat. (°S)	Pressure level in millibars 50	100	200	300	400	500	700	850	*Vertical integral	Relative angular momentum flux (10^{25} gm cm² sec⁻²)
80	−0.39	−0.03	12.61	18.72	9.75	8.58	2.33	—	+56.47	+0.44
75	0.05	0.72	15.92	30.44	24.31	13.92	8.83	—	+100.97	+1.76
70	2.31	5.08	17.22	34.36	28.08	14.64	14.08	8.83	+121.24	+3.70
65	1.17	7.39	14.14	25.11	26.14	12.17	11.58	7.36	+126.89	+5.89
60	0.64	6.36	6.94	11.42	14.69	5.69	5.89	4.56	+67.40	+4.38
55	0.97	3.17	−3.78	−1.44	2.50	1.78	1.56	2.14	+10.60	+0.90
50	1.45	−0.44	−16.06	−14.00	−10.08	−2.72	−2.92	−0.31	−49.27	−5.29
45	1.06	−4.22	−22.75	−23.94	−19.83	−7.47	−6.72	−2.83	−98.22	−12.76
40	0.72	−7.44	−29.92	−34.61	−25.86	−10.92	−7.72	−5.36	−137.38	−20.94
35	0.25	−7.61	−34.36	−44.03	−30.44	−13.17	−7.08	−4.25	−156.06	−27.23
30	−0.81	−7.08	−31.58	−39.92	−27.00	−13.25	−6.78	−4.33	−145.89	−28.42
25	−0.94	−5.00	−25.31	−29.33	−22.39	−11.14	−6.39	−3.97	−118.31	−25.24
20	−0.36	−1.78	−15.78	−20.05	−13.61	−8.42	−5.39	−4.47	−83.16	−19.08
15	−1.28	0.53	−7.31	−11.75	−8.44	−5.22	−4.22	−4.42	−53.22	−12.90
10	−0.89	2.17	−1.28	−5.11	−5.81	−2.14	−1.92	−4.17	−27.19	−6.85
5	−1.14	2.42	3.42	−0.22	−3.08	−0.50	−0.47	−2.81	−6.31	−1.63
0	−1.78	2.00	6.67	2.89	−0.78	0.81	−0.08	−1.19	+7.36	+1.91

* The entries in the column "vertical integral" in Tables 1–6 are equal to the weighted sum of the columns to the left. The weights were 50 mb, 0.75; 100 mb, 0.75; 200 mb, 1.0; 300 mb, 1.0; 400 mb, 1.0; 500 mb, 1.5; 700 mb, 1.75; and 850 mb, 2.25. In latitudes 65S–70S, where $p_0 \sim 850$ mb, the 850-mb weight was 0.75. In latitudes 75S and 80S, where $p_0 \sim 700$ mb, the 850-mb weight was zero and the 700-mb weight was 1.0.

TABLE 2. Zonally averaged value of the standing eddy transport of momentum $[\bar{u}^*\bar{v}^*]$ in winter 1958. Units are in m² sec².

Lat. (°S)	Pressure level in millibars 50	100	200	300	400	500	700	850	*Vertical integral	Relative angular momentum flux (10^{24} gm cm² sec⁻²)
80	2.48	0.97	−0.43	1.51	−1.29	−2.80	−5.24	—	−7.06	−0.55
75	3.98	3.40	−0.16	−0.94	−2.80	−4.38	−6.50	—	−11.44	−2.00
70	5.64	3.82	1.13	−0.71	−2.73	−3.48	−4.66	−5.41	−13.99	−4.27
65	4.65	2.80	2.18	−0.24	−1.77	−1.29	−1.70	−1.97	−3.66	−1.70
60	1.56	1.83	1.41	−0.74	−1.09	0.23	−0.83	0.79	2.81	1.82
55	−0.39	−0.39	−1.19	−0.13	−0.47	0.19	−0.95	0.97	−1.56	−1.35
50	1.48	0.16	0.71	1.22	1.08	1.30	0.03	1.13	8.78	9.44
45	3.04	0.45	0.81	0.26	2.27	1.12	0.25	0.83	9.93	12.84
40	2.47	0.26	−1.61	−0.48	0.24	0.81	−0.16	−0.05	1.03	1.57
35	0.70	0.31	−2.55	−0.92	−1.36	0.04	−0.04	−0.11	−4.80	−8.40
30	−0.04	0.86	−0.11	−0.76	−1.22	0.06	1.38	0.37	1.86	3.64
25	−0.23	1.70	1.35	0.03	−0.18	−0.11	0.82	0.32	4.30	9.20
20	−0.23	1.81	−0.51	0.10	−0.02	−0.58	−0.73	−0.50	−2.51	−5.78
15	0.02	1.52	−0.40	0	−0.22	−0.47	−0.89	−0.57	−3.01	−7.33
10	−0.12	0.95	1.82	0.10	0.09	0.12	0.39	−2.43	−1.98	−5.01
5	0.03	0.46	1.35	0.04	0.09	0.39	1.15	0.39	5.31	13.75
0	−0.09	1.08	1.23	0.03	−0.07	0.18	0.34	0.02	2.83	7.38

* See Table 1 for details.

28.8×10^{25} gm cm^2 sec^{-2} and 6×10^{25} gm cm^2 sec^{-2}. It is noted that there is a small flow of relative angular momentum across the equator into the Northern Hemisphere.

2. Standing eddies. The transport of relative angular momentum by these eddies was obtained through the use of the uncorrected values of $[\bar{v}]$ in equation (4) such that

$$\bar{v}^* = \bar{v} - [\bar{v}]$$

and

$$\bar{u}^* = \bar{u} - [\bar{u}].$$

The zonally averaged values of the standing eddy flux $[\bar{u}^*\bar{v}^*]$ is shown in Table 2. Unlike the transient eddies, there is no systematic arrangement of the signs of the standing eddy transport of relative angular momentum. Since this table, except the last column, shows linear momentum transports rather than angular momentum transport, one has to multiply each value by $\cos^2\phi$ in order to compare the relative angular momentum transport across each latitude circle. If one takes this factor into consideration, then the largest equatorward fluxes are found in the upper troposphere of the equatorial and tropical regions. Other large equatorward fluxes are observed at the 50-mb level in the latitude belt 60S to 70S. The largest poleward fluxes are observed at the 200-mb level in the belt 35S–40S. The standing eddies in the Antarctic troposphere transport relative angular momentum poleward, while in the lower stratosphere they transport the relative angular momentum equatorward. The last column of the table shows that the total relative angular momentum fluxes across the higher latitudes are poleward. This is opposite to the direction of the flux due to the transient eddies. Between latitudes 10S and 20S, the transport by these eddies is poleward, as is the flux due to the transient eddies.

The dotted curve of Fig. 2 shows these total fluxes by the standing eddies as a function of latitude. The maximum flux of 1.40×10^{25} gm cm^2 sec^{-2} is muchs maller than the maximum due to the transient eddies (28.8×10^{25} gm cm^2 sec^{-2}). This important result is to be compared with the results obtained by Starr and White (1954) and the yearly results of Buch (1954) for the Northern Hemisphere. Their results indicate that in middle latitudes, the standing eddies transport (poleward) as much as 20 per cent of the maximum flux of relative angular momentum transported by the transient eddies.

3. Mean meridional motion. The exact contribution of this term in the relative angular momentum transport is quite uncertain. This is because of the difficulty in the measurement of the mean meridional motion. However, the values of $[\bar{v}]''$ in accordance with equation (4) have been used in the computation of the mean meridional flux of zonal momentum across various latitude circles as shown in Table 3. Because of the uncertainties inherent in the tabulated values, no detailed discussion of this table will be made. The last column of the table shows the vertically integrated total flux across various latitude circles. The fluxes are poleward in the regions 50S to 80S and from the equator to 19S. Equatorward fluxes are observed between 20S and 45S. These fluxes are in an opposite sense to the fluxes due to the transient eddies. The maximum poleward fluxes are observed between 55S and 60S while the maximum equatorward fluxes are observed between 25S and 30S.

Again the dashed curve of Fig. 2 shows the flux due to the mean meridional circulation. This curve shows a maximum equatorward flux of 5.3×10^{25} gm cm^2 sec^{-2} at about 27S.

B) SUMMER

1. Transient eddies. The zonally averaged values of the relative momentum transport by the transient eddies is shown in Table 4. The table shows that poleward fluxes occur almost at all levels between the equator and

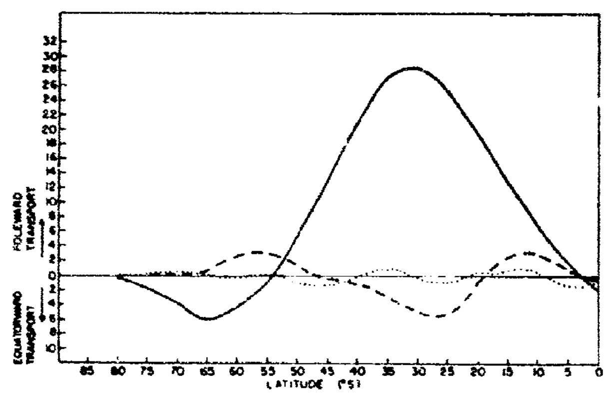


FIG. 2. Vertically integrated mean relative angular momentum flux across various latitudes in winter 1958. The full curve is flux due to the transient eddies, the dashed curve due to mean meridional motion and the dotted curve is due to the standing eddies. Units are in 10^{24} gm cm^2 sec^{-2}.

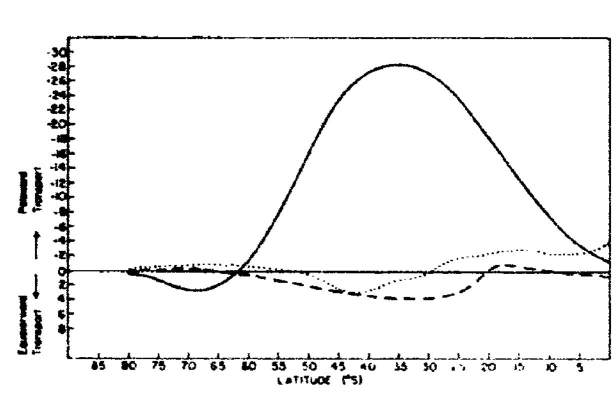


FIG. 3. Vertically integrated mean relative angular momentum flux across various latitudes in summer 1958. The full curve is flux due to the transient eddies, the dashed curve is due to mean meridional motion and the dotted curve is due to the standing eddies. Units are in 10^{24} gm cm^2 sec^{-2}.

TABLE 3. Zonally averaged values of the mean meridional transport of momentum $[\bar{u}][\bar{v}]''$ in winter 1958. Units are in $m^2\ sec^{-2}$.

Lat. (°S)	Pressure level in millibars 50	100	200	300	400	500	700	850	*Vertical integral	Relative angular momentum flux (10^{24} gm cm² sec⁻²)
80	2.90	−0.78	1.14	−0.85	−1.51	−0.48	0.03	—	−0.32	−0.03
75	0.79	−1.91	3.44	−1.48	−0.05	−0.84	0.17	—	−0.02	0
70	−7.06	−2.48	2.56	0.11	−0.57	−2.73	0.63	−1.68	−9.31	−2.83
65	−7.90	−7.90	−1.17	2.81	−1.14	−4.97	1.70	0.24	−15.65	−7.28
60	−12.85	−18.40	−7.51	−2.06	−6.33	−5.67	2.79	2.75	−36.77	−23.90
55	−10.79	−19.08	−9.78	−2.36	−7.88	−1.47	3.86	3.68	[illegible]	−25.33
50	−7.48	−10.85	−6.31	−2.89	−5.56	3.49	2.06	1.96	−15.26	−16.40
45	−4.12	−4.16	−1.98	5.07	−1.45	4.98	0.46	0.86	5.64	7.34
40	−0.37	−1.54	1.95	7.27	3.69	4.13	−1.97	−2.67	8.22	12.54
35	1.64	−0.23	6.77	12.08	2.93	2.44	−2.90	−1.92	17.10	29.86
30	−0.75	−1.51	13.81	14.55	2.09	1.86	−2.45	−0.48	26.18	51.05
25	−0.45	−2.61	12.46	11.56	0.82	2.06	−1.37	−0.02	23.19	49.53
20	0.40	−1.91	0.64	2.83	−0.24	0.91	−0.11	−0.04	3.18	7.30
15	0.83	−2.32	−3.73	−0.90	−0.26	−0.47	−0.15	−1.68	−10.76	−26.11
10	0.75	−1.05	−2.26	−0.79	−0.03	0.77	−0.62	−3.72	−11.60	−29.26
5	−0.54	0	0.96	1.79	−0.53	1.47	−1.42	−2.59	−4.29	−11.07
0	−1.58	0.47	3.77	3.28	−2.00	0.85	−1.27	−1.36	0.21	0.55

TABLE 4. Zonally averaged values of the transient eddy transport of momentum $[\overline{u'v'}]$ in summer 1958. Units are in $m^2\ sec^{-2}$.

Lat. (°S)	Pressure level in millibars 50	100	200	300	400	500	700	850	*Vertical integral	Relative angular momentum flux (10^{25} gm cm² sec⁻²)
80	−0.22	0.08	8.33	12.92	1.78	4.81	7.64	—	43.51	0.34
75	−1.25	2.22	11.81	20.17	11.81	9.75	10.39	—	77.32	1.35
70	−2.69	3.64	8.94	16.25	15.08	11.25	9.08	7.00	89.50	2.73
65	−3.19	1.61	0.58	3.28	9.31	5.36	4.91	6.08	42.30	1.97
60	−3.89	−2.72	−10.01	−9.39	0.27	−2.28	−0.97	3.11	−22.21	−1.45
55	−4.97	−7.47	−21.50	−20.03	−8.67	−8.56	−6.11	−2.44	−88.55	−7.59
50	−4.97	−10.58	−33.80	−30.19	−16.27	−11.94	−10.13	−9.39	−148.69	−16.01
45	−3.56	−11.38	−42.08	−38.14	−21.69	−12.42	−11.61	−12.56	−180.32	−23.51
40	−2.91	−9.81	−45.50	−40.72	−22.38	−11.83	−10.36	−10.42	−177.46	−27.15
35	−2.33	−10.86	−46.56	−36.19	−20.06	−12.00	−10.03	−6.14	−162.06	−28.36
30	−0.72	−13.83	−39.83	−27.17	−17.97	−11.81	−9.44	−3.75	−138.36	−27.05
25	−0.11	−14.92	−28.06	−18.31	−14.61	−11.61	−7.72	−3.39	−110.80	−23.73
20	0.17	−12.36	−15.69	−10.28	−8.53	−10.94	−5.89	−4.25	−79.92	−18.40
15	0.14	−7.86	−6.22	−3.47	−4.89	−9.00	−4.31	−4.94	−52.52	−12.78
10	−0.47	−4.69	−0.83	0.97	−2.22	−5.36	−2.31	−4.94	−29.14	−7.37
5	−2.92	−3.86	−1.69	1.86	−0.81	−2.28	−0.56	−2.11	−14.87	−3.85
0	−2.17	−3.00	−3.58	1.67	0.56	0	0.47	0	−4.41	−1.15

TABLE 5. Zonally averaged values of the standing eddy transport of momentum $[\bar{u}^*\bar{v}^*]$ in summer 1958. Units are in $m^2\ sec^{-2}$.

Lat. (°S)	Pressure level in millibars 50	100	200	300	400	500	700	850	*Vertical integral	Relative angular momentum flux (10^{24} gm cm² sec⁻²)
80	2.61	1.02	−2.45	−7.30	−5.87	−6.42	−3.77	—	−26.29	−2.08
75	0.80	0.18	−3.39	−5.59	−6.76	−6.22	−4.46	—	−28.79	−5.01
70	2.05	0.62	−2.46	−3.06	−5.51	−6.64	−4.14	0.02	−25.84	−7.93
65	4.17	1.00	−0.24	−0.23	−3.13	−6.62	−3.79	−1.01	−17.04	−7.93
60	2.09	0.01	0.21	−0.51	−0.43	−6.45	−1.15	0.31	−10.14	−6.59
55	−1.00	0.38	0.35	0.40	1.26	−2.87	−0.18	0.03	−3.01	−2.60
50	−2.47	1.02	1.25	2.76	2.71	0.43	0.16	−0.41	5.65	6.07
45	−1.46	1.75	4.31	6.02	2.80	3.43	0.86	−0.64	18.56	24.17
40	−0.90	1.79	5.05	5.42	1.64	2.93	1.11	−0.35	18.34	28.03
35	−0.87	0.84	1.57	2.27	0.25	1.92	1.12	−0.43	7.94	13.90
30	−0.40	1.17	−1.35	−2.03	−1.13	0.72	1.63	0.06	1.40	2.73
25	0.19	0.94	−4.46	−3.43	−2.56	−0.49	0.46	0.17	−9.09	−17.88
20	−0.17	−0.72	−2.63	−2.80	−2.79	−0.69	−0.39	0.19	−10.17	−23.42
15	−0.82	−1.21	−2.18	−2.59	−2.25	−0.71	−1.15	−0.06	−11.74	−28.56
10	−1.06	−0.73	−3.10	−1.61	−0.55	−0.17	−0.74	−0.46	−9.17	−23.19
5	−0.37	0.44	−6.18	−1.88	0.85	−0.13	−0.12	−1.09	−10.01	−25.91
0	0.30	−1.22	−8.45	−1.93	1.39	0.12	−0.20	−2.39	−15.23	−39.72

* See Table 1 for details.

60S. Between 65S and 80S equatorward fluxes are observed at all levels except at 50 mb. The maximum relative angular momentum fluxes due to these eddies are found between 30S and 35S at 200 mb. The maximum equatorward flux of relative angular momentum is found at 300 mb at latitude 75S.

The full curve of Fig. 3 shows the profile of the vertically integrated total flux of relative angular momentum due to the transient eddies in the summer. The maximum poleward flux (28.4×10^{25} gm cm^2 sec^{-2}) is centered at 35S. The maximum equatorward flux (3×10^{25} gm cm^2 sec^{-2}) is observed at 68S. At the equator, there is an influx of 1.2×10^{25} gm cm^2 sec^{-2} from the Northern Hemisphere.

2. *Standing eddies*. Table 5 shows the zonally averaged values of relative zonal momentum flux due to the standing eddies in the summer. Poleward fluxes predominate between the equator and 30S. The maximum poleward flux of relative angular momentum due to these eddies is observed at the equator at the 200-mb level.

The vertically integrated total flux of relative angular momentum due to the standing eddies is shown by the dotted curve of Fig. 3. The curve shows that the maximum poleward flux (4×10^{25} gm cm^2 sec^{-2}) is observed at the equator. Between the equator and 30S and between 55S and 80S the fluxes are poleward, while between 30S and 55S the fluxes are equatorward. The maximum equatorward flux is less than 3×10^{25} gm cm^2 sec^{-2}.

3. *Mean meridional motion*. Table 6 shows the zonally averaged values of the relative zonal momentum flux due to mean meridional motion. Poleward fluxes predominate in the regions where direct cells are observed. Equatorward fluxes are predominant in regions occupied by the indirect cell of the middle latitudes. The fluxes

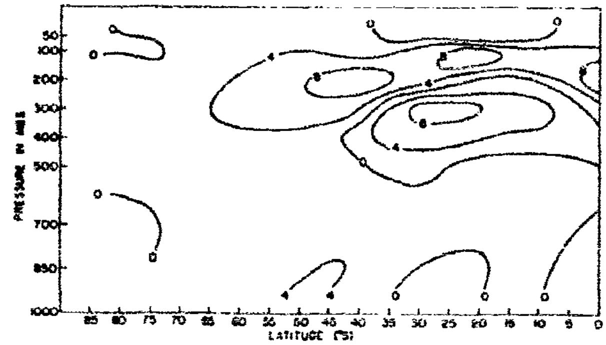


FIG. 4. Seasonal variation of relative angular momentum transport (poleward) by the transient eddies. The curves indicate winter transport minus summer transport. The unit are in 2.6×10^{21} gm cm^3 sec^{-2} mb^{-1}.

at all levels in the tropical regions are very small (less than 1 m^2 sec^{-2}).

The dashed curve of Fig. 3 shows the profile of the vertically integrated flux of relative angular momentum due to the mean meridional motion. The curve shows a maximum poleward flux (0.9×10^{25} gm cm^2 sec^{-2}) at 17.5S. The maximum equatorward flux (4×10^{25} gm cm^2 sec^{-2}) is observed at 32.5S. This latitude corresponds approximately to the latitude where the maximum transient eddy poleward flux of relative angular momentum is observed.

5. Seasonal variation of relative angular momentum transport

Fig. 4 shows the seasonal variation of relative angular momentum transport due to the transient eddies. The curves were obtained by subtracting the summer transports at each grid point of the $p-\varphi$ plane from the

TABLE 6. Zonally averaged values of the mean meridional transport of momentum $[\bar{u}][\bar{v}]''$ in summer 1958. Units are in m^2 sec^{-2}.

Lat. (°S)	Pressure level in millibars 50	100	200	300	400	500	700	850	*Vertical integral	Relative angular momentum flux (10^{24} gm cm^2 sec^{-2})
80	−3.22	−2.57	−0.79	−0.01	−0.04	0.08	−0.11	—	−5.17	−0.41
75	−3.96	−2.95	−1.55	−0.30	0.54	0.62	−0.24	—	−5.80	−1.01
70	−2.95	−0.94	−1.74	−0.08	0.40	−0.03	0.01	−2.10	−5.94	−1.81
65	0.41	4.99	−1.44	1.41	−0.16	−0.96	−0.38	−0.42	1.44	0.67
60	2.16	9.10	2.20	4.17	−0.12	−4.10	−0.16	−0.30	7.59	4.93
55	2.87	10.79	9.42	9.37	1.97	−7.85	2.38	−3.28	16.02	13.71
50	2.23	11.86	16.13	14.54	3.21	−10.35	2.81	−6.84	18.45	19.83
45	0.99	10.40	17.81	15.52	4.80	−8.87	0.28	−6.42	19.41	25.25
40	0.35	11.47	15.88	13.07	3.67	−2.21	−2.79	−5.02	21.99	33.56
35	−0.11	12.89	8.66	13.56	3.11	−2.65	−2.72	−1.91	21.88	38.20
30	−0.74	8.81	5.00	12.53	1.52	−1.28	−2.15	−0.44	18.43	35.94
25	−2.17	4.82	4.52	6.82	0.94	0.25	−0.77	0.24	13.84	29.56
20	−2.65	1.65	−1.14	1.20	−0.43	−0.07	−0.09	0.03	−1.32	−3.03
15	−1.07	0.55	−1.10	−0.33	−0.81	−0.13	−0.13	0.06	−2.92	−7.09
10	0.40	0.18	0	−0.08	−0.20	0	−0.28	0	−0.34	−0.86
5	1.10	−0.07	0.66	−0.09	0	0.29	−0.40	−0.18	0.67	1.73
0	0.07	−0.10	0.92	−0.01	−0.10	0.42	−0.07	0.21	1.77	4.60

* See Table 1 for details.

winter transports. The regions of maximum variability are observed in the upper troposphere and lower stratosphere of the latitude belt 15S to 60S. In both seasons, as shown by Tables 1 and 4, the transient eddy relative angular momentum fluxes are poleward (except very near latitude 60S). We conclude that in this region of maximum variability, positive values of Fig. 4 represent regions where the summer poleward fluxes exceed the winter poleward fluxes (and vice versa for negative values).

The largest seasonal variations are observed in the troposphere and lower stratosphere in the region 450 mb to 50 mb from latitude 60S to the equator. The maximum negative values are observed at 300 mb between latitudes 30S and 20S. The maximum positive values of about the same magnitude is observed at 200 mb between latitudes 47.5S and 35S. Hence a marked vertical fluctuation (200 mb to 300 mb) of the region of maximum flux of relative angular momentum from season to season is indicated. A latitudinal shift is also observed. The latitude and levels where the maximum fluxes occur are higher in the summer than in the winter. A second zone of maximum fluxes of relative angular momentum is observed in the lower stratosphere in the latitude belt 27.5S to 17.5S. Fig. 10 also shows the seasonal variations as a function of pressure and latitude. The figure shows that the seasonal variation at 30S is more pronounced in the vertical shifts of zones of maximum fluxes while at 55S the seasonal variation is more pronounced, with little vertical displacements.

From the solid curves of Figs. 2 and 3, the seasonal variations of the vertically integrated total fluxes of relative angular momentum due to the transient eddies can be inferred. The peaks of the poleward fluxes during the winter and summer are at 32S and 35S, respectively. There is a difference of 0.4×10^{25} gm cm^2 sec^{-2} in the amplitudes. The peaks of the equatorward fluxes in the same seasons are at 64S and 68S, respectively. The difference in the amplitudes is 3.0×10^{25} gm cm^2 sec^{-2}. The summer curve is much broader than the winter curve. The two curves show that the major seasonal variations occur between 35S and 70S. The largest variation of about 10.8×10^{25} gm cm^2 sec^{-2} is observed at 47S. A variation of 3.0×10^{25} gm cm^2 sec^{-2} is observed at the equator.

The dotted curves of the figures show the vertically integrated standing eddy fluxes. Unlike the transient eddies, there is no well organized distribution of the standing eddy flux of relative angular momentum in the winter. The largest variation of 4.7×10^{25} gm cm^2 sec^{-2} is observed at the equator. The seasonal variations are greatest between the equator and 5S. In other latitudes the variations are less than 2×10^{25} gm cm^2 sec^{-2}.

The dashed curves of the figures show the vertically integrated transport of relative angular momentum by the mean meridional motion. The largest seasonal variation (less than 3×10^{25} gm cm^2 sec^{-2}) is observed at 10S. Other regions of notable seasonal variations are found between 45S and 65S.

6. Hemispheric variation of relative angular momentum flux

The yearly mean values of the momentum flux are still being processed, but it is assumed that the winter and summer averages of the fluxes will give a representative idea of the yearly flux. It is unfortunate that no published seasonal studies are available for the entire Northern Hemisphere. For this reason our comparison will be based on yearly fluxes, using the 1950 data presented by Buch (1954) for the Northern Hemisphere.

Fig. 5 shows the annual poleward flux of relative angular momentum by the transient eddies in the Southern Hemisphere. Large poleward fluxes are observed in a deep layer of the troposphere at the latitude belts 20S to 50S. The largest fluxes are observed between the 200- and 300-mb layers in the latitude belt 30S to 35S. South of 60S the fluxes are everywhere equatorward.

Fig. 6 shows the fluxes due to the transient eddies in the Northern Hemisphere. Large poleward fluxes are observed in a shallower and narrower layer than in the Southern Hemisphere. The largest fluxes are observed

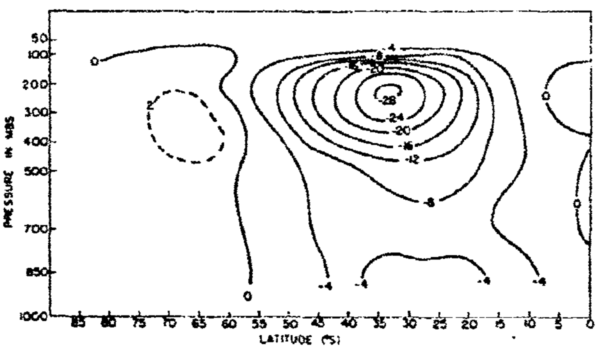


FIG. 5. Poleward flux of relative angular momentum due to the transient eddies for the year 1958. The curves are obtained by taking the winter and summer mean. The units are in 2.6×10^{21} gm cm^2 sec^{-2} mb^{-1}.

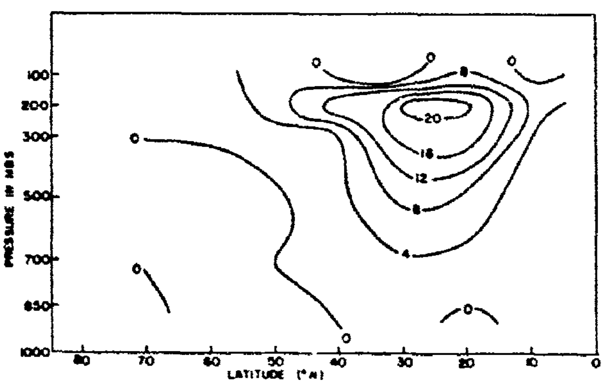


FIG. 6. Poleward flux of relative angular momentum due to the transient eddies for the year 1950. The curves are obtained by using Buch's data. The units are in 2.6×10^{21} gm cm^2 sec^{-2} mb^{-1}.

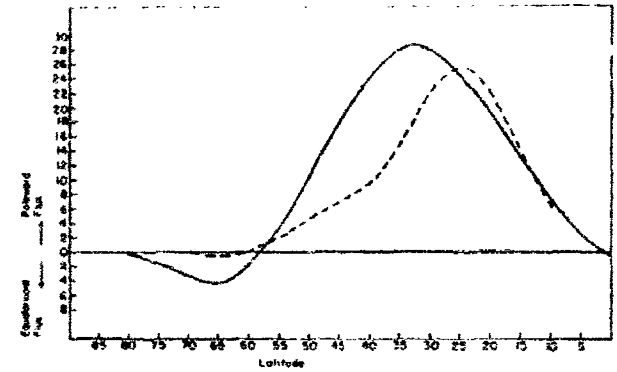


FIG. 7. Vertically integrated mean relative angular momentum flux across various latitudes by the transient eddies. The full curve is the summer and winter mean for the Southern Hemisphere in 1958. The dashed curve is the result obtained by Buch for the Northern Hemisphere 1950. Units are in 10^{25} gm cm^2 sec^{-2}.

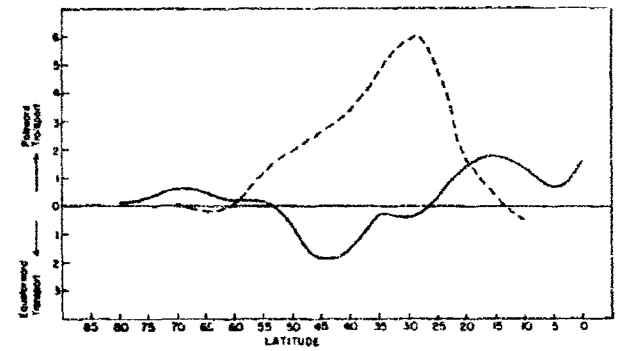


FIG. 8. Vertically integrated mean relative angular momentum flux across various latitudes by the standing eddies. The full curve is the summer and winter mean for the Southern Hemisphere in 1958. The dashed curve is the result obtained by Buch for the Northern Hemisphere 1950. Units are in 10^{25} gm cm^2 sec^{-2}.

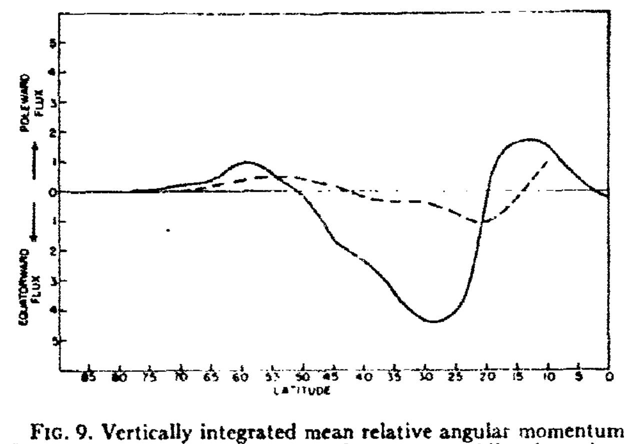


FIG. 9. Vertically integrated mean relative angular momentum flux across various latitudes by the mean meridional motion. The full curve is the summer and winter mean for the Southern Hemisphere in 1958. The dashed curve is the result obtained by Buch for the Northern Hemisphere 1950. Units are in 10^{24} gm cm^2 sec^{-2}.

at the 200-mb level in the latitude belt 20N to 30N. The maximum flux at this level is about 70 per cent of the maximum flux in the Southern Hemisphere. Poleward of 60 deg the equatorward fluxes appear to be larger in the Southern Hemisphere than in the Northern Hemisphere.

Fig. 7 shows the vertically integrated fluxes due to the transient eddies in the two hemispheres. The curves show that the maximum poleward transient eddy flux is at 24N (25.4×10^{25} gm cm^2 sec^{-2}) in the Northern Hemisphere, while in the Southern Hemisphere, this flux s 28.5×10^{25} gm cm^2 sec^{-2} at 32S. The maximum[5] equatiorward flux in the Southern Hemisphere (4.5×10^2 gm cm^2 sec^{-2}) is at 66S, while in the Northern Hemisphere, this flux is 0.5×10^{25} gm cm^2 sec^{-2} at 65N. The curves show that the largest differences are observed between latitudes 30 and 50 deg, and between 60 and 80 deg.

Fig. 8 shows the standing eddy fluxes. (Note the expanded vertical scale.) The solid curve is for the Southern Hemisphere and the dashed curve for the Northern Hemisphere. The maximum poleward flux in the Southern Hemisphere is 1.8×10^{25} gm cm^2 sec^{-2} at 15S, while in the Northern Hemisphere a poleward flux of 6×10^{25} gm cm^2 sec^{-2} is observed at 29N. In the Southern Hemisphere the standing eddies transport relative angular momentum equatorward over a broad region (between 53S and 28S), while in the Northern Hemisphere, there are no regions of significant equatorward transports.

Fig. 9 shows the flux of relative angular momentum due to mean meridional motion. The maximum poleward flux of 1.8×10^{25} gm cm^2 sec^{-2} is at 14S. A smaller maximum of 0.9×10^{25} gm cm^2 sec^{-2} is at 60S. In the Northern Hemisphere a maximum poleward flux of 1.0×10^{25} gm cm^2 sec^{-2} is suggested at 10N, a smaller one of 0.4×10^{25} gm cm^2 sec^{-2} is at about 60N. The maximum equatorward flux in the Southern Hemisphere is 4.4×10^{25} gm cm^2 sec^{-2} at 29S. In the Northern Hemisphere this maximum is about 1.0×10^{25} at 20N. Owing to the uncertainties in the measurement of the mean meridional motions, it is safer to assume that the curves of Fig. 7 which represent the transient eddy fluxes are more representative for their respective hemispheres.

Another distinction between the two hemispheres is indicated by the curves of Fig. 10. The curves show the vertical profiles of the transient eddy transport of momentum on a linear pressure scale showing the seasonal variations. At 30S, the maximum poleward flux (41 m^2 sec^{-2}) during the winter is at 270 mb. During the summer this maximum decreased by 1 m^2 sec^{-2} and is displaced vertically to the 220-mb level. The variation below the 500-mb level remains small. The maximum variation (12 m^2 sec^{-2}) is observed at the 300-mb level. This result is compared with the profiles presented by Starr and White (1952) for the Northern Hemisphere at

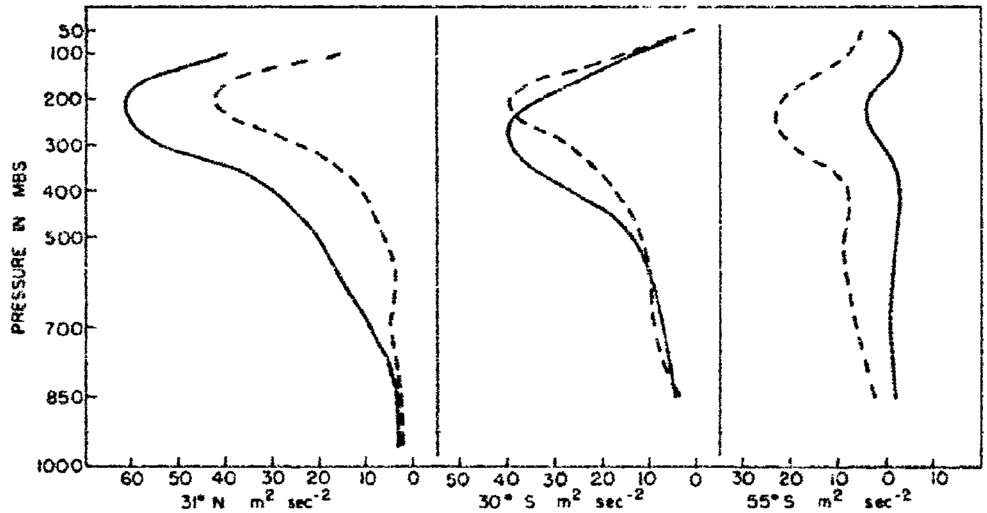


FIG. 10. Vertical profiles of the eddy transport of momentum on a linear pressure scale showing the seasonal variations. The full curves are for winter and the dashed curves for summer 1958. The curves for 31 N were taken from Starr and White (1952).

latitude 31N. Their summer includes the months of May to October 1949 and their winter includes the six months February–April 1949, November–December 1949 and January 1950. Their profiles show that during the winter there is a maximum flux of 63 m^2 sec^{-2} at 230 mb. The maximum flux in the summer is 43 m^2 sec^{-2} found at 200 mb. The largest variation is about 23 m^2 sec^{-2} at 230 mb. Large variations exist down to 700 mb. Since the largest variations in any latitude of the Southern Hemisphere are found at about 55S, the profile for this latitude is included in Fig. 10. It is observed that the maximum flux in the winter is 4 m^2 sec^{-2} at 230 mb. In the summer this flux is 25 m^2 sec^{-2} at 250 mb. In general, the fluxes are larger in the summer than in the winter, the reverse of what is observed at 31N. The maximum variation at 55S is 21 m^2 sec^{-2} at 250 mb. Though this value appears to be the largest zonal momentum variation in the whole hemisphere, when one thinks in terms of relative angular momentum, the value is actually small.

7. Computation of the mean surface stress from the relative angular momentum flux

The zonal equation of motion can be written in the form

$$\frac{\partial \bar{u}}{\partial t}+\frac{\partial \bar{u}^2}{a\cos\varphi\partial\lambda}+\frac{\partial \overline{uv}\cos^2\varphi}{a\cos^2\varphi\partial\varphi}+\frac{\partial\overline{u\omega}}{\partial p}-f\bar{v}=-\frac{\partial\Phi}{a\cos\varphi\partial\lambda}+\bar{T}_\lambda.$$

If we take zonal averages, we obtain

$$\frac{\partial[\bar{u}]}{\partial t}+\frac{\partial[\overline{uv}]\cos^2\varphi}{a\cos^2\varphi\partial\varphi}+\frac{\partial[\overline{u\omega}]}{\partial p}-f[\bar{v}]=[\bar{T}_\lambda].$$

If one now introduces the approximate relation

$$[\bar{T}_\lambda]=-g\frac{\partial}{\partial p}[\bar{\tau}_x]$$

one obtains, after integration with respect to pressure, and use of the boundary conditions $\omega=0$ and $\bar{\tau}_x=0$ at $p=0$, the equation

$$\int_{p_G}^{0}\frac{\partial}{\partial t}[\bar{u}]dp+\int_{p_G}^{0}\frac{\partial[\overline{uv}]\cos^2\varphi}{a\cos^2\varphi\partial\varphi}dp-[\overline{u\omega}]_{p=p_G}=g[\bar{\tau}_x]_{p=p_G}.$$

Since we are dealing with mean conditions for a season, where $[\bar{u}]$ is the zonally averaged time mean of u, it follows that $\partial[\bar{u}]/\partial t=0$, approximately. If we further assume that the zonally averaged correlation between u and ω at the ground (p_G) is very small then we can neglect the term $[\overline{u\omega}]_{p=p_G}$. Near mountain ranges this assumption may not be valid strictly, but probably no large error is involved. We are thus left with the convergence equation

$$(ag\cos^2\varphi)^{-1}\frac{\partial}{\partial\varphi}\int_{p_G}^{0}([\overline{u'v'}]+[\bar{u}^*\bar{v}^*]+[\bar{u}][\bar{v}]'')\cos^2\varphi dp=[\bar{\tau}_x]_{p=p_G}.$$

In the evaluation of the integrand, we shall neglect the effect of the mean meridional circulations. It is felt that their omission will not lead to too serious errors.

The computed mean surface stress in the winter is shown in Fig. 11. The solid curve is the stress computed from the convergence of the relative angular momentum due to both transient and standing eddies. The dashed-

dot curve is the stress computed from the convergence due to the transient eddies. The dashed curve is the value obtained by Priestley (1951) for July using surface climatological wind data over the Southern Hemisphere oceans.

The solid curve shows that between 32.5S and 64S, $[\tau_s]$ is negative. (Attention must be given here to the sign convention implied by the equations written above.) There is a numerical maximum of 1.2 dynes cm^{-2} at about 44S. The belt of the 850-mb level westerlies in winter (Obasi, 1963) extends from about 24S to 66S. The maximum value of the surface westerlies is suggested to be at 45S. From considerations of the 850-mb level westerlies in winter, it appears that the belt of surface westerlies is in good agreement with the frictional stress exerted by the atmosphere on earth. Between the equator and 32.5S, the values of $[\tau_s]$ are positive. A maximum of 0.54 dyne cm^{-2} is observed at 9S. The 850-mb level winds indicate that easterlies exist in these regions and that the maximum is at 10S. Again a good agreement is thus obtained. Beyond 64S positive values of $[\tau_s]$ are observed. At the 850-mb level, easterlies are again observed from about 66S to the coast of the Antarctic.

The dashed curve from Priestley (1951) is the average of the surface stress over all the oceans in the Southern Hemisphere in July. Aside from the uncertainties in the drag coefficients, his values did not include the stress over the land surfaces and the torque due to pressure differentials over the mountains of the Southern Hemisphere. For these reasons care must be exercised in the comparison of his curve with those obtained from momentum convergence studies. Making allowance for these effects, his mean surface stress appears to be numerically too small in the region 45S to 60S. From studies conducted by Wilson (1960), using all the reported values of the drag coefficient from 47 different authorities, it is found that the value of this coefficient for light winds (less than 20 mph) is 1.49 $\pm 0.83 \times 10^{-3}$, while for strong winds (greater than 20 mph) the value is $2.37 \pm 0.56 \times 10^{-3}$. Priestley employed the value 1.30×10^{-3} for his computations throughout the hemisphere. The present data show that at the 850-mb level the maximum westerlies (24 mph) can be grouped into the category of strong winds. It is probable that Priestley has used a too small value of the drag coefficient in the region between 45S and 60S where there is most disagreement. His curve agrees quite well in the latitude belts north of 45S except between 10S and 15S.

Fig. 12 shows the frictional stress as a function of latitude in the summer. The solid curve is for all eddies, the dashed-dot curve is for transient eddies. The dashed curve is the value obtained by Priestley for all Southern Oceans in January using surface climatological data.

The solid curve shows that between 32S and 69S $[\tau_s]$ is negative. A numerical maximum of 1.5 dynes cm^{-2} is found at about 54S. Obasi (1963) has shown that at 850-mb level, westerlies are observed between 26S and 64S. The maximum westerlies are found at about 47S. These distributions are in good agreement with the computed stress. Between the equator and 32S and poleward of 69S the values of $[\tau_s]$ are positive. A maximum of 0.48 dyne cm^{-2} is observed at 16S. The 850-mb level winds indicate that easterlies exist in these regions and that the maximum is at 15S. In the polar regions surface easterlies are observed poleward of 64S. Priestley's curve is in good agreement with the solid and dashed-dot curves equatorward of 50S. Poleward of this latitude his values appear to be too small.

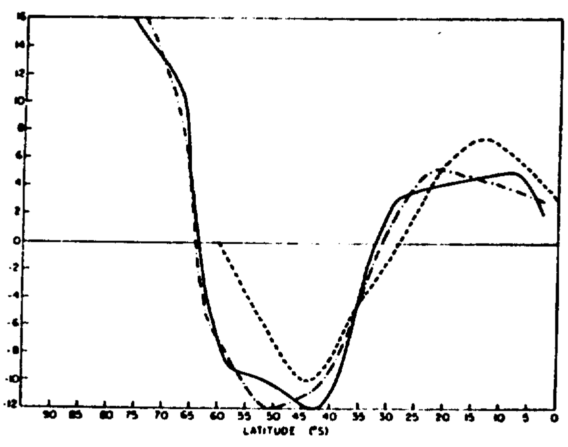
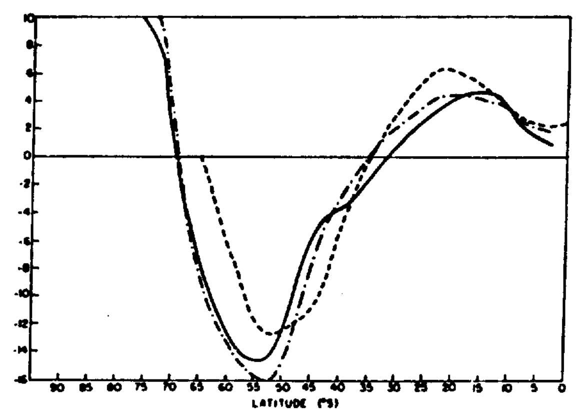


FIG. 11. The mean zonal stress of the atmosphere on the earth. The solid and dashed-dotted curves are computed from the convergence of the total eddy and transient eddy flux of relative angular momentum, respectively, for the winter 1958. The dashed curve is the mean stress estimated directly from the surface winds over all southern oceans in July by Priestley. Units are in 10^{-1} dyne cm^{-2}.



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8. Summary

The studies reported in this paper have demonstrated the overwhelming importance of the transport of relative angular momentum by the transient eddies in both seasons of the Southern Hemisphere. The results show that they transport enough relative angular momentum from the belt of surface easterlies to the belt of surface westerlies to balance the drain of atmospheric angular momentum in the latter belt. In the winter the maximum transient-eddy poleward transport of relative angular momentum is observed at 32S, while the maximum equatorward transport is observed at 64S. In the summer these maxima are observed, respectively, at 35S and 68S. In both seasons the maximum poleward transport is observed to be less than 30×10^{25} gm cm^2 sec^{-2}.

The fluxes of relative angular momentum due to the mean meridional motions are observed to be opposite to those of the transient eddies in the belt of maximum poleward flux (30S–35S). The maximum flux due to the mean meridional motions in both seasons is less than 6×10^{25} gm cm^2 sec^{-2}.

The standing eddy transport of relative angular momentum is insignificant in winter in all latitude belts, but it may be important in the momentum balance of the stratosphere.

In the summer the standing eddies play the important role in the interhemispheric exchanges of relative angular momentum. However their transports are opposite to those of the transient eddies poleward of 30S. The maximum flux of relative angular momentum by the standing eddies is observed in the summer at the equator and is of magnitude 4×10^{25} gm cm^2 sec^{-2}.

There are insignificant seasonal variations of the integrated relative angular momentum flux equatorward of 50S. However, there is evidence for a marked vertical displacement of the zones of maximum transports from 300 mb in winter to 200 mb in summer. The Northern Hemisphere study of Starr and White (1952) shows a marked seasonal variation in the momentum flux across 31N but there is, on the other hand, less vertical fluctuation of the maximum from season to season.

The vertically integrated summer and winter means show a maximum of transient eddy flux of relative angular momentum of 28.5×10^{25} gm cm^2 sec^{-2} at 32S. Buch's data for the Northern Hemisphere show a maximum of 25×10^{25} gm cm^2 sec^{-2} at 24N during 1950.

During the winter there is a flux of relative angular momentum into the Northern Hemisphere. In the summer larger fluxes into the Southern Hemisphere were observed. This summer result is in agreement with a speculation of Widger (1949).

The wind stresses derived from the angular momentum convergence are in agreement with those of Priestley (1951) except in the belt of strong surface westerlies.

North of 70S, the zonally averaged mean frictional stress is everywhere less than 2 dynes cm^{-2}. This result has an important application to the dynamics of the Antarctic Circumpolar Current. In their model of this current, Munk and Palmén (1951) used a mean zonally averaged frictional stress of 2 dynes cm^{-2} between latitudes 55S and 65S (Drake Passage). The present study indicates that a mean of 1 dyne cm^{-2} is more appropriate. It is probable that this lower value of frictional stress will affect their conclusions with respect to the insufficiency of the lateral frictional stress in the balancing of the wind stress.

The zonally averaged mean stress distribution is expected to be of value in studying the mean baric slopes and other oceanographic parameters for all the southern oceans. Since this hemisphere is mainly oceanic, it is of interest to examine more closely the mechanisms for the upward transport of momentum from the tropical oceans to the atmosphere and the corresponding downward transport in the midlatitude oceans through which the atmospheric momentum may be, at least in part, passed to the solid earth.

To appreciate more fully the importance of the transient eddies in the maintenance of the normally observed circulation of the Southern Hemisphere, the following additional computations were made. The integrated summer and winter mean relative angular momentum of the atmosphere over the polar cap of the Southern Hemisphere south of latitude 35S is 6.36×10^{32} gm cm^2 sec^{-1}. The flux of relative angular momentum through the latitudinal wall of 35S is about 2.9×10^{26} gm cm^2 sec^{-2}. It follows that if no relative angular momentum were transported across this wall by the transient eddies, and if the values of the frictional and mountain torques were to remain constant in spite of decreasing zonal momentum, then the atmosphere over the polar cap south of latitude 35S will lose all its relative angular momentum in about 25 days.

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On the Vertical Transport of Angular Momentum in the Atmosphere

By PETER A. GILMAN[1])

Summary – The possible modes of vertical transport of angular momentum in the atmosphere are considered. Momentum balance calculations for both hemispheres show the possibility of countergradient transport by vertical eddies in the region of the mid-latitude jet. As a consequence, it is pointed out that the transport of momentum downward from the region of maximum westerlies would have to be accomplished by the mean meridional motions, through the action of 'Coriolis torques'. The same mechanism may account for a large part of the upward transport in the tropics. The very approximate nature of the calculations must, however, be borne clearly in mind.

1. *Introduction*

It is a generally accepted fact that the momentum of the westerlies is maintained against ground friction by the large scale horizontal eddy transports from the latitudes of surface easterlies (cf. STARK [4][2])). However, how this momentum is transported vertically from the surface in low and very high latitudes, and from the level of maximum westerlies to the ground in middle latitudes is not quantitatively understood. There can be but two agents: one, the mean meridional circulations; the other, the vertical eddies. Molecular friction, above the earth-air interface, is of negligible importance.

The vertical eddies include motions of widely varying time and space scales, from a few seconds to several days, and from a few centimeters to a few thousand kilometers. There is no reason to assume that all these turbulent motions behave in the same way as regards momentum transport. Those small enough and of short enough duration presumably act to transport momentum down the momentum gradient (the inertial sub range). The large scale vertical eddies, that is, rising and sinking motions associated with wave cyclones and anticyclones, and even larger scale waves, may very well transport momentum *up* the gradient. As an example, upward motions to the east of troughs in the 500 mb map are located on the average in the presence of stronger westerlies then are the accompanying sinking motions

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[2]) Numbers in brackets refer to References, page 166.

west of the trough, and therefore would in the mean transport momentum upward toward the level of maximum westerlies. It is not clear, then, what the net effect of all scales of vertical eddies would be.

The mean meridional motion can transport relative momentum vertically in two ways. First, through direct correlation of the vertical component with the mean zonal wind; and second, through the action of Coriolis forces on the horizontal meridional motion. In the latter case, the vertical transport is obtained by the addition, due to the Coriolis forces, of westerly angular momentum at one level, and easterly angular momentum at another, as continuity requires a return flow. In non-rotating coordinates, the mechanisms above represent in sum the vertical advection of absolute angular momentum.

If all but one of the elements in the momentum balance are known, the remaining one may be inferred for balance to be maintained. In practice, only the horizontal eddy transports and the mean zonal wind itself can be measured with reasonable accuracy. The horizontal meridional motions are known only to within a factor of 2 or 3 from direct measurements. The mean vertical motions can not, of course, be measured at all directly, and if they are inferred via continuity from the horizontal motions, they inherit their uncertainty. Recently, an attempt has been made by STARR and DICKINSON [5] to evaluate the vertical eddy transport for two periods of one month, using 'adiabatic' vertical motions (cf. JENSEN [2]). The results were inconclusive, however, and the monthly averages were probably too short for significance.

Despite all of these difficulties, it may be possible to get some indication of the direction in which the vertical eddies as a whole are transporting momentum, by using the best obtainable estimates for the other quantities. In this regard, calculations were made for both the northern and southern hemispheres, using data of BUCH [1] for the former, and recent data of OBASI [3] for the latter. Some of OBASI's values were corrected by him and it is these that are used here. It seemed appropriate to calculate the vertical transports due to all scales of eddies in terms of an *empirical* eddy viscosity. Then positive values of the viscosity could indicate a transport in the direction of the gradient, while negative values would indicate countergradient transport.

2. *Mathematical formulation and results*

We may somewhat crudely represent the mean wind stress on a horizontal surface in the zonal direction by the expression

$$[\bar{\tau}] = - g \, \varrho^2 \, \nu \, \frac{\partial [\bar{u}]}{\partial p} , \tag{1}$$

where ϱ is density, g gravity, ν the kinematic eddy viscosity, u the zonal wind, p the pressure and τ the wind stress. The bar, $(^{-})$, indicates a time average, and the brackets, $[(\,)]$, a zonal average. Now the frictional stress in the pressure coordinate system is related to the friction force per unit mass, $[\bar{\chi}]$, by the expression

$$[\bar{\chi}] = - g \, \frac{\partial [\bar{\tau}]}{\partial p} \tag{2}$$

which, using (1), becomes

$$[\bar{\chi}] = \nu g^2 \varrho^2 \left(\frac{\partial^2[\bar{u}]}{\partial p^2} + \frac{2}{\varrho}\frac{\partial \varrho}{\partial p}\frac{\partial[\bar{u}]}{\partial p}\right). \tag{3}$$

Here ν has been assumed not a function of pressure. Now

$$\frac{\partial \varrho}{\partial p} = \frac{\varrho}{p}\left(1 + \frac{R}{g}\frac{\partial T}{\partial h}\right),$$

where R is the gas constant for air, T is the temperature, and h is the vertical space coordinate. Letting

$$B = 1 + \frac{R}{g}\frac{\partial T}{\partial h},$$

$$[\bar{\chi}] = \nu g^2 \varrho^2 \left(\frac{\partial^2[\bar{u}]}{\partial p^2} + \frac{2B}{p}\frac{\partial[\bar{u}]}{\partial p}\right). \tag{4}$$

If we take $\partial T/\partial h = -6.5°$ K km^{-1} in the troposphere (taken here to be below 200 mb) and $\partial T/\partial h = 0$ in the stratosphere, B then has the values 0.83 and 1.0, respectively.

Now the time and zonally averaged steady state zonal equation of motion may be written as

$$Z[\bar{v}] - [\bar{\omega}]\frac{\partial[\bar{u}]}{\partial p} - G_H + [\bar{\chi}] = 0, \tag{5}$$

where Z is the absolute vorticity

$$\left(Z = f - (a\cos\phi)^{-1}\frac{\partial}{\partial\phi}[\bar{u}]\cos\phi\right),$$

v the meridional wind, $\omega = dp/dt$, and G_H the horizontal eddy convergence of momentum. That is

$$G_H = -(a\cos^2\phi)^{-1}\frac{\partial}{\partial\phi}([\overline{u'v'}] + [\bar{u}^*\bar{v}^*])\cos^2\phi,$$

where a is the radius of the earth, ϕ the latitude, and where a prime, ('), indicates a deviation from a time average and a star, (*), a deviation from a zonal average. Then, writing (4) as $[\bar{\chi}] = -\nu R_F$ and solving (4) for ν, we obtain, finally,

$$\nu = \frac{Z[\bar{v}] - [\bar{\omega}]\dfrac{\partial[\bar{u}]}{\partial p} - G_H}{R_F} \tag{6}$$

The numerical values obtained from (6) were not considered to be significant due to the errors mentioned earlier, so that only the signs are presented Figures 1 and 2. Figure 1 gives results for the southern hemisphere, using data of OBASI [3]. Figure 2 is for the northern hemisphere, using data of BUCH [1]. In both cases, $[\bar{\omega}]$ was found from the directly measured $[\bar{v}]$ via continuity.

The striking feature of Figure 1 and 2 is that each contains large areas of all negative eddy viscosity. Furthermore, the mid-latitude jet stream is in all cases well within the negative regions, indicating that near the jet the net effect of vertical eddies of all scales is to transport momentum into the jet. The signs are generally positive at lower levels (particularly 775 mb), seen most clearly for the southern hemisphere summer, and the northern hemisphere year.

Now the most reliable signs obtained from (5) come when both numerator and denominator have their largest values. This occurs primarily in the regions of the negative values. It is possible that with better data, there will be fewer negative signs, but there is no reason to expect that all the signs will become positive. Therefore, to the degree that these crude calculations may be trusted, the usual assumption in numerical models of a positive eddy viscosity, if it is used to represent

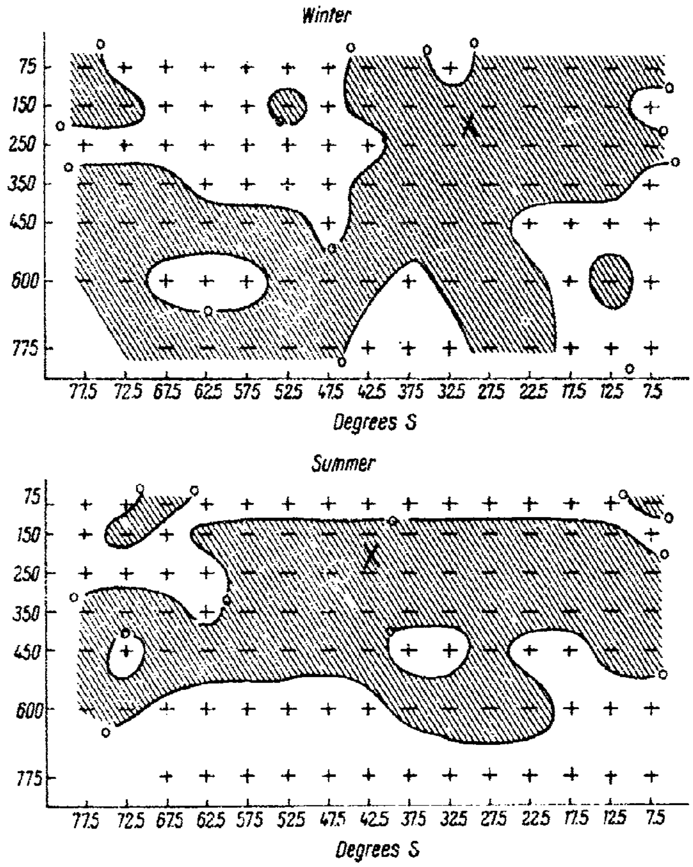

Figure 1
Signs of the vertical eddy viscosity for the southern hemisphere, winter and summer seasons, obtained from data of OBASI [3]. X denotes the approximate position at the mid-latitude jet stream

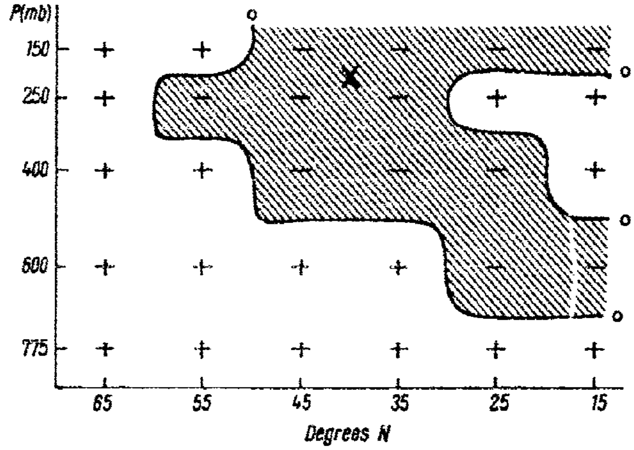

Figure 2
Signs of the vertical eddy viscosity for the northern hemisphere year 1950, from data of BUCH [1]. X denotes the approximate position of the mid-latitude jet stream

eddies including scales large enough to give some counter gradient transport contributions, may be open to question.

Finally, since the net effects of all the possible mechanisms for vertical transport of momentum must be to carry momentum downward from the level of maximum westerlies to the ground, this transport must be effected in the regions of negative eddy viscosity in some manner by the mean meridional circulation. In the region between the equatorial direct cell and the mid-latitude indirect cell, the vertical component of the circulation, $[\bar{\omega}]$, through the term $[\bar{\omega}]\,\partial[\bar{u}]/\partial p$, will give transport in the correct sense (see e.g. (5)). This term is, however, generally smaller than the term $Z[\bar{v}]$, which, in the region of the indirect mid-latitude cell, represents effectively the extraction at high levels, and the injection at low levels, of easterly angular momentum due to the 'Coriolis torques'. Since the surface stress changes sign in the vicinity of 30° N or S, which is also the approximate dividing line between the equatorial Hadley cell and the mid-latitude Ferrel cell, this Coriolis torque mechanism may be the major one effecting the downward transport. Below a certain level, the vertical eddies would also work in the same direction, and probably would predominate in the surface layer. In the tropics, this same Coriolis torque mechanism may also effect a large part of the upward transport of angular momentum, though its role is less clear from the results presented.

3. *Concluding Remarks*

It should be stressed again that the above calculations are to be considered very approximate in nature. What are needed are time average values of the mean motion and the eddies for, say, 5 years, rather than one year or season. Such an extensive study is underway at M.I.T. in conjunction with the Travelers Research Center. When the statistics are compiled, the relative roles in the vertical transport

of momentum of the mean meridional circulation and the vertical eddies may become much clearer.

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I. THE MOMENTUM BUDGET

B. Local Rotation Systems

CONTRIBUTIONS TO THE STUDY OF PLANETARY ATMOSPHERIC CIRCULATIONS

SOME ASPECTS OF THE DYNAMICS OF CYCLONES

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ABSTRACT

The angular momentum equation for a cyclonic disturbance is examined. It is found that the increase of this momentum in a given region during the onset of an extratropical cyclone is due mainly to a flux of such momentum into the region from the environment. A synoptic example is included.

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1. INTRODUCTION

One of the most primitive attributes of a cyclonic disturbance in the atmosphere is the fact that air particles involved in it possess angular momentum relative to the earth about a vertical axis located in the vicinity of (but not necessarily exactly at) the center. At a given instant, one may consider the volume integral of this angular momentum within a circular cylinder whose axis is thus specified, extending from the earth's surface to some appropriately high elevation above which only a negligible fraction of mass is present. Although serious questions may arise in the case of shallow and weak cyclones, there can be no doubt that stronger systems are sufficiently intense to impart a mean rotation to the entire atmosphere, and hence this integral would, in a sense, constitute a measure of the disturbance, if the radius of the cylinder is chosen properly. Having selected the region, we may consider its walls to be fixed in space relative to the earth during the prior onset and subsequent disappearance of the cyclone, assuming that the latter is a transient phenomenon. A fundamental problem of meteorology may be formulated by asking how the rise and fall in the total relative angular momentum within this region of space is to be accounted for.

Although a truly complete solution is far beyond our present state of knowledge, there are several specific questions of a less comprehensive nature to which theory and modern observational material might yield some answers:

(a) Is the relative angular momentum which appears during the onset generated within the region or is it communicated to the region by a flux across the boundary from the environment?

(b) Is the later disappearance likewise due to a net destruction within the region or is it due to a net outward flux across the boundary?

(c) Is it feasible to measure the flux across the boundary or the generation, whatever the algebraic signs of these effects may be?

In short, the questions posed above are whether the relative angular momentum, as it has been defined, obeys a source-free equation of continuity, and whether it is possible to demonstrate the fulfillment of this equation in the atmosphere by actual measurement. Notwithstanding the fact that these considerations alone are not sufficient for a complete solution, it is difficult for the present writer to see how basic understanding may be achieved and recognized as such unless this subject receives close scrutiny.

2. THEORETICAL CONSIDERATIONS

In order to arrive as simply as possible at the crux of the matter it suffices to consider the surface of the earth to be a plane and to use a cylindrical polar coordinate system (r, θ, z) in which r is outward radial distance, θ is azimuth counted positive counterclockwise from East and z is elevation. With sufficient accuracy the equation of motion for the tangential direction may be written in the form,

$$\rho \frac{d}{dt}(rc_T) - \rho\lambda r c_r = -\frac{\partial p}{\partial \theta}, \tag{1}$$

where C_T and C_r are the tangential and radial (inward) components of wind velocity, ρ is density, p pressure, λ the Coriolis parameter and t time. The continuity equation for mass,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{c} = 0, \tag{2}$$

where c is the (three-dimensional) vector wind, may be used to rewrite Eq. (1) in the form

$$\frac{\partial}{\partial t}(\rho r c_T) = -\nabla \cdot \rho r c_T \mathbf{c} + \rho \lambda r c_r - \frac{\partial p}{\partial \theta}. \tag{3}$$

A volume integral may now be taken over the cylindrical region assumed, and written

$$\frac{d}{dt}\int_v \rho r c_T\, dv = \int_{s_0} \rho r c_T\, w ds_0 + R\int_s \rho c_T c_r\, ds + \int_v \rho \lambda r c_r\, dv. \tag{4}$$

Here v is the volume of the cylinder whose radius is R, s_0 is the area of the (plane) bottom, w is the vertical upward component of velocity and s is the area of the lateral wall.

By definition, the left-hand member of Eq. (4) is the time rate of change of the total relative angular momentum within the cylinder. The first two integrals on the right measure the flux of such momentum across the horizontal bottom and the vertical wall, respectively. The last integral may be looked upon as a source term, since it cannot be reduced to a transport across the walls. In a sense Eq. (4) is an equation of continuity for the type of angular momentum here considered. It is apparent that it is not source free, since a contribution may be obtained from the last term in addition to that from the transfer terms.

If it is supposed that the plane bottom s_0 is located a few feet from the earth's surface, the flux across it may be identified with the effect of the surface wind stress. During the onset of cyclonic conditions it would not, of course, be possible to ascribe the increase in angular momentum within the cylinder to this effect, since it represents a drain.

On the assumption that the Coriolis parameter λ is uniform in space, the last term may be rewritten as follows:

$$\int_v \rho \lambda r c_r\, dv = \lambda \int_0^R r^2 \int_0^{2\pi} \int_0^\infty \rho c_r\, dz\, d\theta\, dr. \tag{5}$$

Since $g\rho = -\partial p/\partial z$ on hydrostatic principles, it follows from Eq. (2) that, for all values of r,

$$\int_0^{2\pi}\int_0^\infty \rho c_r\, r dz\, d\theta = \frac{1}{g}\int_0^r \int_0^{2\pi} r \frac{\partial p_0}{\partial t}\, d\theta\, dr, \tag{6}$$

where g is the (spatially uniform) acceleration of gravity and p_0 is surface pressure. Upon substitution, Eq. (5) now gives a multiple integral of the form

$$\frac{\lambda}{g}\int_0^R r_1 \int_0^{r_1}\int_0^{2\pi} r \frac{\partial p_0}{\partial t}\, d\theta\, dr\, dr_1, \tag{7}$$

r_1 being a running variable. In view of the fact that during the onset of a cyclone the surface pressure normally decreases, an expression of the form of Eq. (7) would be negative. Therefore, the effect represented by it cannot be used for explaining an increase of total angular momentum within the cylinder.

On the other hand, one may suppose that the Coriolis parameter is not spatially uniform, but instead increases linearly with latitude so that $\lambda = \lambda_0 + \beta y$, λ_0 being the value at the center and y being northward distance, i.e., $r \sin \theta$. The effect of the variable part of λ, namely, βy, may be examined as follows. Again from the last term of Eq. (4),

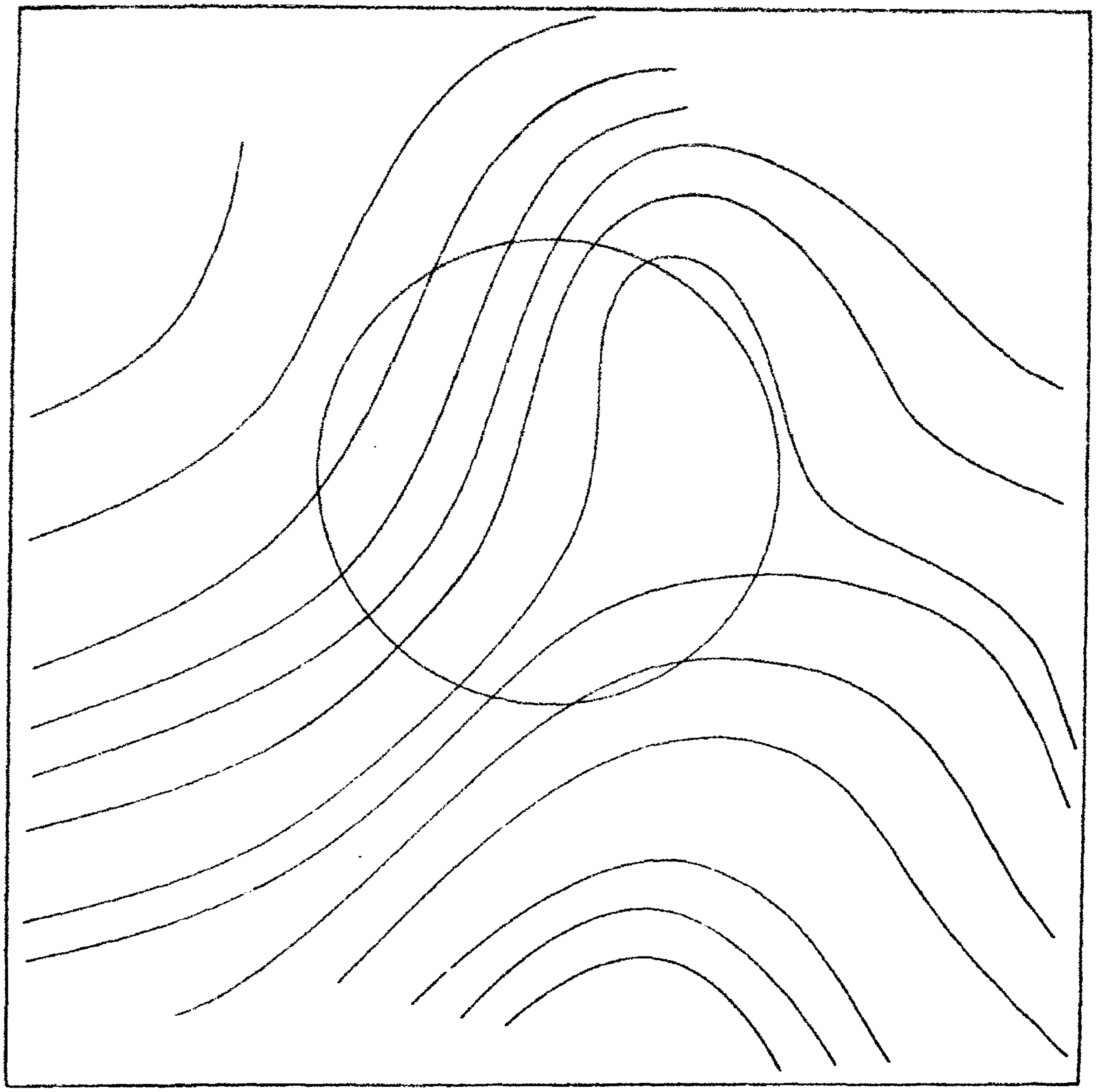

Fig. 1. Isobaric contour map for 500 mb for 18 January 1949 at 1600 GMT showing the cylinder used. The contour interval is 200 ft. The map projection is Lambert conformal conic with standard parallels at 30° and 60° N.

$$\beta \int_{v} \rho y r c_r \, dv = \beta \int_{0}^{R} r^2 \int_{0}^{2\pi} \sin\theta \left[\int_{0}^{\infty} \rho c_r r \, dz \, d\theta \right] dr. \tag{8}$$

The quantity set off by brackets in Eq. (8) is the radial mass inflow across an elementary sector at a given radius. In order to achieve a finite positive value for the entire integral, this inflow should be positively

correlated with $\sin \theta$. Normally, however, during the onset of cylonic conditions (in the northern hemisphere) southerly winds prevail in the cylinder, implying an inflow from the south over the locations where $\sin \theta < 0$ and an outflow where $\sin \theta > 0$. It is thus suggested that, even though the Coriolis parameter is taken as variable, the source term of Eq. (4) is not suitable in order to account for an increase in the total relative angular momentum under the assumed circumstances. This leaves the second term on the right of Eq. (4) as the only remaining possibility.

The second term on the right of Eq. (4) has the nature of a Reynolds stress effect, since the net flow of mass across the lateral wall is not large. It is similar to the eddy stress term which appears in the equation for the hemispheric balance of angular momentum discussed by various authors. (See Starr and White (1951) and the references given there.) It may be resolved into several physically distinct components as in the case of the general circulation problem just cited, although some simplification of the scheme seems to be desirable. Thus, an averaging process with respect to the circumference of the cylinder may be introduced and denoted by square brackets so that

$$\rho c_r = [\rho c_r] + (\rho c_r)', \tag{9}$$

where the prime denotes a departure from the corresponding average. An equation similar to Eq. (9) may be written for c_T. The lateral stress term may then be subdivided into two parts, namely,

$$2\pi R^2 \int_0^\infty [c_T]\,[\rho c_r]\,dz + R^2 \int_0^\infty \int_0^{2\pi} c'_T(\rho c_r)'\,d\theta\,dz. \tag{10}$$

In view of the fact that $[\rho c_r]$ measures the mass convergence, the first part measures the effect of this factor. The second part is dependent upon the correlation between c_T and ρc_r along the circumference of the cylinder at individual levels in the atmosphere. Positive values of this quantity are therefore associated with certain characteristic horizontal streamline patterns which lead to the necessary correlation. One such pattern of flow is illustrated in Fig. 1.

3. OBSERVATIONAL EXAMPLE

Any adequate test of Eq. (4) involving the use of meteorological data is a rather formidable undertaking, and in certain respects a serious question exists as to whether sufficiently extensive and accurate direct wind measurements are available for this purpose. One may nevertheless expect that some very general evidence of the validity of the conclusions drawn from the equation should not be completely absent. Accordingly, an example has been calculated, although its shortcomings are all too evident.

The issue involves to a large extent a means of measuring the second term on the right of Eq. (4), as already stated. Similar tasks in the case of the angular momentum balance of the general circulation have been treated, not without success, through the use of geostrophic winds notably by Widger (1949) and by Mintz (1951). Encouraged by these efforts, as an experiment the writer has used similar methods for the purpose at hand, mindful, however, that appreciable differences may exist between the two cases.

In the first place, it is to be noted that the use of geostrophic winds reduces the last term in Eq. (4) to zero, directly as it stands. Apparently there is then no simple way to measure its effect from such winds.

As far as the second term on the right is concerned, it may be seen from Eq. (8) that only its second part can probably be determined, since the first part involves $[\rho c_r]$ which measures convergence — a quantity not suited for evaluation geostrophically.

A synoptic situation was selected which was characterized by the subsequent formation of an extremely intense cyclone, namely, that of 18 January 1949 over the central United States. A cylinder 6.5° latitude in radius centered at 37.5° N, 95° W was used. The wind components C_T and C_r were determined at 16 points along the circumference, taking the latitude variations into account in the geostrophic formula. This was done at the 850-mb, 700-mb, 500-mb, 300-mb, and 100-mb levels for which analyzed synoptic charts drawn previously for other purposes by my colleague R. Reed were available. From these components, the second term in Eq. (4) was computed for the range from 850 to 100 mb. The value obtained was about 6×10^{25} gm cm^2 sec^{-2} in the cyclonic sense. The maximum effect per unit isobaric layer was obtained from the 300-mb level, with much smaller contributions from the levels above and below. The value of $[c_T c_r]$ at 500-mb approximated the vertical average with respect to pressure quite well. A portion of the synoptic chart for 500 mb is shown in Fig. 1. At the time there was a somewhat weak but rapidly developing closed circulation at the ground, centered more or less below the cylinder.

In order to secure an easily grasped appraisal of the efficacy of the torque measured, one might ask how intense a solid rotation could it produce in one hour if acting alone on the cylinder considered. The answer may be stated in terms of the change in linear peripheral velocity of the solid rotation, namely, about 5 m sec^{-1}.

4. DISCUSSION

A number of points concerning the material presented are worthy of note. Among these are the following:

(a) The basic physical principle involved in Eq. (4) is a mechanical one depending upon the inertial properties of matter and the conservation of mass. As such, the integral requirement stated by it should be fulfilled, regardless of such factors as the presence or absence of nonadiabatic processes or of internal vertical motions, etc. This does not mean that factors such as these do not influence air motions. Rather, we have deliberately selected an equation in which they do not appear *explicitly*, and in doing so we have gained certain obvious advantages and lost others.

(b) In order to gain an appreciation of magnitudes in actual cases, it is necessary to secure relative orders of magnitude of the terms of Eqs. (7) and (8) as compared with the second term on the right of Eq. (4). Plausible estimates of them for the case studied suggest that they are at least one order of magnitude smaller.

(c) Likewise plausible estimates suggest that, for the situation studied, the surface frictional torque on the cylinder was probably between one and two orders of magnitude smaller than the eddy torque obtained.

(d) On the supposition that the estimates made are roughly correct, it is to be inferred that at the time the observations were made the eddy torque was acting virtually unopposed. It is difficult to imagine

that this situation could endure for any appreciable length of time, because the resulting circulations would soon become too intense to be reasonable. A check at 500 mb 12 hours later showed that the eddy torque for the same cylinder had decreased to a negligible value. Meanwhile, the cyclone at the ground underwent a development of almost explosive violence, and its center moved rapidly northward to a position a little beyond the circumference of the cylinder. The extremely rapid changes in the flow pattern aloft as well as the surface, although interesting in themselves, detract somewhat from the utility of the situation as a model case, since the tendency of the angular momentum cannot be extrapolated for a 12-hour period. Apparently there was a positive change in this quantity for the fixed cylinder, but only a relatively moderate one.

(e) Equation (4) does not in itself distinguish between the effect of mere translation of existing systems and the effect of true development. Also, it should, with proper interpretations, be equally applicable to anticyclonic systems as well as to tropical cyclones, but these uses may involve additional difficulties for various reasons.

(f) Going beyond the specific information contained in Eq. (4), it is natural to speculate further concerning the physical processes that transpire during the actual development of an extratropical cyclone. Such speculation must perforce be rather vague and general, but by such means some grains of truth may be captured which through further insight can later be rendered more precise and specific. It may be observed that the particular equation dealt with for the cylinder does not stipulate anything concerning the vertical redistribution of angular momentum, once it has appeared in this space. If it is assumed that the increase is indeed mainly due to the horizontal eddy processes which are most intense at upper levels and quite negligible near the surface (as was the case in the example), then a question arises as to the manner in which a strong closed circulation develops near the surface.

On synoptic grounds, it would be difficult to escape from the concept that intense cyclonic circulations at low levels are associated with low-level convergence which implies upper-level divergence. This is essentially a classical view. On the basis of the present arguments, however, the primary action is the tendency of the eddy torques to establish cyclonic circulations aloft. Such a tendency cannot be immediately successful because, to put the matter crudely, the new mid-air cyclone would tend to fill up from below (and perhaps from above also). In other words, the palpable result of the tendency of the eddy torques to establish an upper circulation would be upward vertical motion from below. Since the latter must be arrested at the surface, the vertical stretching involved would then lead to low-level convergence and development of a surface cyclone. Hydrodynamic models could perhaps be calculated to illustrate a portion of this chain of events.

Two points concerning the picture envisaged above remain to be considered. First, the requisite convergence of angular momentum aloft due to eddy processes implies a positive correlation between c_T and ρc_r, i.e., certain favorable kinematic properties of the flow pattern aloft are required. It is probable that this initial condition necessitates the existence of suitable kinds of trough and ridge formations aloft at the very outset. Second, these favorable formations are perhaps the consequence of the solenoidal modifications of the upper flow throughout a large portion of the troposphere.

(g) No prognostic uses of the material discussed are contemplated here. It is felt that the mere exploration of what does happen in cyclonic areas from the present viewpoint is a sufficiently difficult task for the immediate future.

5. CRITICAL REMARKS

A number of shortcomings and approximations exist in the material as outlined, which, although not crucial, deserve specific mention. The more important of these are the following.

(a) Strictly speaking, the earth's surface should not be approximated by a plane. A better approximation is, of course, a spherical surface, in which case it is more appropriate to deal with a conical region in the place of the cylinder.

(b) The equation of motion could be written more exactly, especially so as to include molecular viscosity, or a separation of the scales of eddies could be introduced so as to provide a small-scale eddy-viscosity effect.

(c) It has been tacitly assumed that the ground surface is free of orographic irregularities. Actually if a mountainous terrain is present the bottom of the cylinder need not be level, so that the pressure term in Eq. (3) does not drop out upon taking the volume integral. In that case a mountain-pressure torque term appears in Eq. (4). Such a mountain torque term is probably of much significance in the study of lee pressure troughs and the like, as was suggested by the writer's colleague, E. N. Lorenz.

(d) Some readers may regard the consideration of angular momentum about a fixed vertical as being somewhat awkward for analytical purposes, and would prefer vorticity concepts. Such an alternative approach is indeed possible, although certain interpretations involved are then altered. In terms of Cartesian coordinates x, y and z, in which the corresponding eastward, northward and upward velocity components are u, v and w, respectively, one would then deal with the vorticity about the vertical of the *momentum*, as has been used by Lorenz (1950) and by Kuo (1951). From the two equations of motion for the horizontal under the same assumptions as before, with the aid of the (general) mass continuity equation, it is simple to derive the following equation in which the pressure is again eliminated

$$(\overline{(\rho v)_x - (\rho u)_y})_t = (\overline{\rho uu} - \overline{\rho vv})_{xy} - (\overline{\rho uv})_{xx} + (\overline{\rho uv})_{yy} - (\lambda\overline{\rho u} + \rho_0 v_0 w_0)_x - (\lambda\overline{\rho v} - \rho_0 u_0 w_0)_y. \quad (11)$$

Here the subscripts x, y and t denote partial differentiation and the bars indicate vertical integrals with respect to z throughout the atmosphere. Subscript zeros signify surface values. The equation gives the rate of change of the vertically integrated momentum vorticity for a column in terms of the several terms on the right-hand side. Like Eq. (4) it expresses a purely mechanical principle and is independent of various thermodynamic and other factors.

The reader will no doubt observe that, if a closed system of equations capable of solution in the mathematical sense is desired, additional equations must be used with Eq. (11). Thus, although by use of this single equation it is theoretically possible to obtain an instantaneous tendency of the integrated vorticity by evaluating the right-hand side from observations, additional information would be needed in order to iterate the process for successive time steps. In substance at least, such an iterative procedure has been devised by the writer's colleague, P. D. Thompson (1952). The advantage of his treatment is that the problem is reduced to a two-dimensional one in space, without sacrifice of solenoidal effects or of the effects of vertical motion.

(e) Not much has been said relative to the disappearance of cyclonic angular momentum from a given region after the passage of a disturbance. Qualitatively, the eddy effect seems then to operate in the reverse sense. Due to the fact that at this stage surface friction may be substantial, the decay need not be brought about entirely by the other terms.

(f) A little more should be said concerning the first term in Eq. (10) which is not given by geostrophic winds. Actually a positive contribution of a given level to this integral consists of a transfer of mass *already possessing* a mean relative circulation into the cylinder by convergence. Moreover, since there exists a *small* net mass outflow rather than inflow, different levels tend to produce opposing effects. On the whole, it is not likely that the net contribution is of much importance.

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DISPLACEMENT AND INTENSIFICATION ASSOCIATED WITH VARIATIONS OF LOCAL ANGULAR MOMENTUM

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ABSTRACT

The rate of change of the total angular momentum within a cylinder about an arbitrary vertical axis results largely from the horizontal transport of angular momentum across the vertical boundary of the cylinder. This transport may be resolved into the transport due to the mean wind, i.e., the wind averaged vertically with respect to pressure, and the departure of the total transport from the transport due to the mean wind. The case is presented for regarding such a resolution as a resolution into displacement and intensification.

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Even a casual inspection of a typical sequence of weather maps reveals the presence of certain outstanding features of the weather pattern, such as cyclones and anticyclones, whose identities are usually preserved from one map to the next. A closer study shows that even though certain features of one map may at times appear virtually unaltered on the following map, aside from a change in geographical location, at other times these features may undergo marked variations in intensity. It is not surprising, therefore, that variations of the state of the atmosphere have often been regarded as consisting partly of displacement of the prominent features, and partly of intensification of these features.

In order to study quantitatively the variations of the state of the atmosphere, and their resolutions into displacement and intensification, one must first choose some quantity or some set of quantities as a measure of the existing state of the atmosphere. The way in which particular variations will be resolved will depend upon what quantity is chosen. By far the most commonly used quantity would seem to be pressure, in view of the almost universal use of sea-level pressure maps in synoptic forecasting. In numerous studies, however, vorticity rather than pressure has been used as the basic quantity.

Recently, Starr (1953) (see pp. 9-25) has used the relative angular momentum about an arbitrary vertical axis, integrated throughout a circular cylinder about this axis, as a measure of the state of the atmosphere in the region of the cylinder. Such angular momentum may be called local angular momentum, to distinguish it from the more widely studied angular momentum about the earth's axis. It is the purpose of this paper to examine a method of resolving the theoretical rate of change of local angular momentum into displacement and intensification, and to compare this resolution with the resolutions which occur when more familiar quantities are used as measures of the state of the atmosphere.

A natural choice for a basic quantity is pressure, since many meteorologists are accustomed to think in terms of pressure, identifying particular weather phenomena with the pressure patterns which accompany them. Particularly at sea level, pressure can be measured with a high degree of accuracy. It is therefore possible to resolve observed pressure changes into displacement and intensification, using some method such as the one used by Austin and Shapiro (1951), where the pressure change which would have occurred at a point, if a nearby pressure system had moved without changing its shape, is assumed to consist of displacement, and the remainder of the actual pressure change is assumed to consist of intensification. An alternative method of resolution, suggested by Austin (1952), is based upon the behavior of isallobaric centers rather than isobaric centers. It is evident that these methods do not yield identical resolutions, since changes in the form of isobaric centers do not imply similar changes in the form of isallobaric centers.

Pressure possesses the disadvantage, however, that the value of the theoretical expression for its rate of change cannot readily be computed from observational data. To resolve the theoretical rate of change into a computable displacement and a computable intensification is clearly out of the question.

Recently vorticity has been widely used as a measure of the state of the atmosphere. The mean vorticity, i.e., the vorticity of the mean wind, i.e., of the wind vector averaged vertically with respect to pressure, is especially suitable. As a first approximation, its theoretical rate of change is given by the mean horizontal advection of vorticity, i.e., by the advection computed at individual levels and then averaged vertically. The mean vorticity and the mean advection of vorticity may both be determined fairly accurately from observational data, if the geostrophic approximation is used for purposes of computing. A knowledge of these quantities appears to give considerable information concerning the accompanying weather phenomena. Thus it is that the mean vorticity has served as a basic quantity in many of the recent methods of numerical weather prediction (see Charney (1949) and Thompson (1952)).

Since the mean advection of vorticity is a quadratic function of the wind field, it is not determined by the mean wind. As an approximation, however, one may replace the mean advection of vorticity by the advection of the mean vorticity by the mean wind. This procedure yields a simplified form of the barotropic vorticity equation. This equation has the well-known property that it cannot lead to the appearance of new values of mean vorticity, but can merely redistribute the existing values, and so, from the point of view of vorticity, cannot lead to intensification (see Charney, Fjörtoft and von Neumann (1950)). There is available, therefore, a natural method of resolving the theoretical rate of change of vorticity into displacement and intensification: the advection of mean vorticity by the mean wind represents displacement, and the excess of the mean advection of vorticity over this quantity represents intensification.

Although this method of resolution may seem to be the most natural, other methods are possible. A more general form of the barotropic vorticity equation occurs when the wind speed is assumed to vary with elevation, while the wind direction remains fixed. This equation again leads to an advection of mean vorticity, but by a wind somewhat stronger than the mean wind (Charney (1949)), and therefore cannot lead to intensification in the sense of introducing new values of mean vorticity. It might, therefore, be more logical to let the intensification be represented by the departure of the mean advection of vorticity from the advection of vorticity by a wind which equals the mean wind, multiplied by a suitable function of elevation.

It should be noted that the alternative methods of resolution just described differ not only from each other, but also from the methods which arise when pressure rather than vorticity is used as the basic quantity. It can hardly be expected that advection of vorticity will preserve the strengths of maxima and minima in the pressure field. Numerous other methods of resolution could presumably be justified also. There is probably no one "best" method; at most, there may be best methods of resolution for particular problems.

In the previously mentioned paper, Starr (1953) showed that at the onset of an extratropical cyclone, the increase of local angular momentum within a cylinder must result from the horizontal flow of already-existing local angular momentum across the vertical boundary of the cylinder. In the present paper, this flow will be taken as a first approximation to the change of total local angular momentum. Both the local angular momentum and the flow of local angular momentum can be determined fairly accurately from observational data if the geostrophic approximation is used for purposes of computing.

Like the mean vorticity, the total local angular momentum is determined by the mean wind. Like the mean advection of vorticity, the total transport of local angular momentum is a quadratic function of the wind field, and so is not determined by the mean wind. It may, however, be resolved into the transport due to the mean wind, and the departure of the total transport from the transport due to the mean wind. In this paper the case will be presented for regarding such a resolution as a resolution into displacement and intensification.

In the following paragraphs, it will be assumed that the portion of the earth's surface under consideration may be approximated by a plane. In this plane, polar coordinates (r, θ) may be introduced. If variations of the surface pressure p_0 are neglected, the total local angular momentum M within a cylinder of radius R whose vertical axis passes through the origin is given by

$$M = \frac{p_0}{g} \int_0^R \int_0^{2\pi} r^2 \bar{c}_T \, d\theta \, dr \, ; \tag{1}$$

and the horizontal flow τ of local angular momentum across the vertical boundary of the cylinder is given by

$$\tau = \frac{p_0}{g} \int_0^{2\pi} R^2 \overline{c_R c_T} \, d\theta. \tag{2}$$

In these expressions g is the acceleration of gravity, c_R and c_T are the radial (inward) and tangential (counterclockwise) components of the wind velocity $\mathbf{c}$, and a bar denotes a vertical average throughout the atmosphere with respect to pressure. The first approximation to the rate of change of M is

$$\frac{\partial M}{\partial t} \sim \tau. \tag{3}$$

The well-known rule that the average value of a product equals the product of the average values plus the average value of the product of the departures from average may now be applied. The approximation (3) then becomes

$$\frac{\partial M}{\partial t} \sim \frac{p_0}{g} \int_0^{2\pi} R^2 \bar{c}_R \bar{c}_T \, d\theta + \frac{p_0}{g} \int_0^{2\pi} R^2 \, \overline{c_R' c_T'} \, d\theta \tag{4}$$

where a prime denotes a departure from the kind of average denoted by a bar. It is the first and second terms on the right side of approximation (4) which are claimed to represent displacement and intensification, respectively.

The justification for this claim depends on the relation between local angular momentum and vorticity. The vorticity ζ is given by

$$\zeta = \frac{1}{r}\left(\frac{\partial c_R}{\partial \theta} + \frac{\partial r c_T}{\partial r}\right). \tag{5}$$

If the mean wind is assumed to be nondivergent, an assumption which is equivalent to neglecting variations of p_0, and if variations of the Coriolis parameter are neglected, the approximate relation

$$\frac{\partial \bar{\zeta}}{\partial t} \sim -\overline{\mathbf{c} \cdot \nabla \zeta} \tag{6}$$

may be obtained, expressing the rate of change of mean vorticity $\bar{\zeta}$ as the mean advection of vorticity. This expression may be rewritten

$$\frac{\partial \bar{\zeta}}{\partial t} \sim -\bar{\mathbf{c}} \cdot \nabla \bar{\zeta} - \overline{\mathbf{c}' \cdot \nabla \zeta'}. \tag{7}$$

According to the previous discussion, the first and second terms on the right of approximation (7) may be regarded as displacement and intensification, respectively.

The relation between M and $\bar{\zeta}$ will now be established. If

$$C(r_1) = \int_0^{r_1} \int_0^{2\pi} \bar{\zeta} r \, d\theta \, dr, \tag{8}$$

it follows from Eq. (5) that

$$C(r_1) = \int_0^{2\pi} \bar{c}_T(r_1, \theta) r_1 \, d\theta. \tag{9}$$

Equations (8) and (9) merely express the familiar relation between circulation and vorticity. Comparison of Eq. (9) with Eq. (1) shows that

$$M = \frac{p_0}{g} \int_0^R r_1 C(r_1) \, dr_1, \tag{10}$$

whence from Eq. (8),

$$M = \frac{p_0}{g} \int_0^R r_1 \int_0^{r_1} \int_0^{2\pi} \bar{\zeta} r \, d\theta \, dr \, dr_1. \tag{11}$$

A change in the order of integration reduces Eq. (11) to the simpler expression

$$M = \frac{p_0}{g}\int_0^R \int_0^{2\pi} \frac{1}{2}(R^2 - r^2)\bar{\zeta} r \, d\theta \, dr. \tag{12}$$

Thus M is proportional to a weighted average value of $\bar{\zeta}$ over the circle of radius R, the weighting factor being greatest at the center, and falling off to zero at the boundary.

It follows that exact changes of M are determined by exact changes of the distribution of $\bar{\zeta}$. The approximate rate of change of M consistent with the approximate rate of change of $\bar{\zeta}$ associated with displacement, as given by the first term on the right of (7), will now be determined.

Since (7) is based upon the assumption that the mean wind $\mathbf{c}$ is nondivergent, this same assumption may be used to introduce a stream function ψ, such that $\bar{c}_T = \psi_r$ and $\bar{c}_r = r^{-1}\psi_\theta$, where the subscripts r and θ denote partial differentiation. Then

$$\bar{\zeta} = \nabla^2\psi = r^{-1}(r\psi_r)_r + r^{-2}\psi_{\theta\theta}. \tag{13}$$

The approximate value of $\partial\bar{\zeta}/\partial t$ associated with displacement may be written

$$\frac{\partial\bar{\zeta}}{\partial t} \sim r^{-1}(-\psi_r\bar{\zeta}_\theta + \psi_\theta\bar{\zeta}_r). \tag{14}$$

The value of $\partial C/\partial t$ consistent with Eq. (8) and approximation (14) is

$$\frac{\partial C(r_1)}{\partial t} \sim \int_0^{2\pi} \psi_\theta\bar{\zeta}(r_1, \theta) \, d\theta, \tag{15}$$

and the value of $\partial M/\partial t$ consistent with Eq. (10) and approximation (15) is

$$\frac{\partial M}{\partial t} \sim \frac{p_0}{g}\int_0^{2\pi} R\psi_r\psi_\theta \, d\theta. \tag{16}$$

Evidently the right side of (16) is identical with the first term on the right of (4). It follows that the approximate value of $\partial M/\partial t$, as defined by (16) or by the first term on the right of (4), is equal to the value which would result if the value of $\partial\bar{\zeta}/\partial t$ at every point within the circle were equal to the value associated with displacement, as defined by (14) or by the first term on the right of (7). If this definition of displacement is accepted, the first term on the right of (4) must also represent displacement. The remainder of the approximation (4), i.e., the second term on the right, then represents intensification.

Thus, there is available a natural method for resolving the rate of change of local angular momentum into displacement and intensification. Needless to say, it is not the only possible method.

It is generally accepted that atmospheric motion tends to conserve absolute vorticity rather than relative vorticity. Hence a more accurate approximation than (6) or (7) is

$$\frac{\partial\bar{\zeta}}{\partial t} \sim -\overline{\mathbf{c}\cdot\nabla\zeta} - \overline{\mathbf{c}\cdot\nabla\lambda}, \tag{17}$$

where λ is the Coriolis parameter. If (17) is accepted in place of (6), the corresponding approximation (3) or (4) must also be modified by terms involving λ. These terms are discussed by Starr (1953). It is significant that these terms, and also the last term in (17), are linear functions of the mean wind field, since λ does not vary with elevation. Hence they may be combined with the terms previously regarded as representing

displacement, to represent displacement in the sense of leading to no new values of absolute vorticity. The terms in (4) and (7) representing intensification are therefore unaltered by the addition of terms involving λ to (4) and (7).

The term in (4) representing intensification, and involving $\overline{c_a' c_T'}$, takes on a simple form if it is assumed that the wind, but not the wind shear, varies with elevation. In this case, if the geostrophic approximation is used for purposes of computing, c_a' and c_T' may be regarded as components of the thermal wind. The term in question may then be regarded as the transport of the angular momentum of the thermal wind by the thermal wind. Its value is related to the configuration of the isotherms in the same way that the value of the term representing displacement is related to the mean streamlines. The reasoning leading to this conclusion is analogous to the reasoning which leads to the conclusion that the term in (7) representing intensification may be regarded as the advection of the vorticity of the thermal wind by the thermal wind (see Charney, Fjörtoft and von Neumann (1950) and Fjörtoft (1951)).

In discussing local angular momentum, Starr (1953) suggested choosing the axis of the cylinder near the center of a cyclone. It is possible to extend this procedure, and consider the total local angular momentum in each of many cylinders. Then M becomes a function of the coordinates of the center of the cylinder.

To express this function analytically, it is most convenient to introduce rectangular coordinates (x, y), and to let $\bar{u}(x, y)$ and $\bar{v}(x, y)$ be the components of $\bar{c}$ in the directions of the x- and y- axes. Then,

$$M(x, y) = \frac{p_0}{g} \iint\limits_{A(x,y)} [(x' - x)\bar{v}(x_1' y') - (y' - y)\bar{u}(x', y)]dx'\, dy', \tag{18}$$

where $A(x, y)$ represents the area of the circle of radius R centered at (x, y). An interesting question which now arises is whether maxima and minima of $M(x, y)$ are preserved under changes associated with displacement, as defined by the first term on the right of Eq. (4). Evidently this question must be answered in the negative. According to Eq. (12), the field of local angular momentum may be regarded as a smoothed field of vorticity. Even if no new values of $\bar{\zeta}$ occur, more extreme smoothed-values may occur if high or low values of $\bar{\zeta}$ become more closely packed together. Such a situation might arise if the amplitudes of long wavelengths in the field of $\bar{\zeta}$ increase at the expense of the amplitudes of short wavelengths. Nevertheless, it would seem that very pronounced changes in the maxima and minima in the field of M, and hence in the smoothed field of $\bar{\zeta}$, could result only from changes in the maxima and minima in the unsmoothed field of $\bar{\zeta}$, and would hence be associated with intensification.

Finally, one may ask whether $M(x, y)$ is really a good measure of the state of the atmosphere, and, in particular, whether large values of M imply strong cyclonic activity. An indication that this is so comes from Eq. (12), which expresses M as a smoothed vorticity, and which may be written, symbolically,

$$M(x, y) = \frac{p_0}{g} \iint\limits_{A(x,y)} \frac{1}{2} (R^2 - r^2)\bar{\zeta}\, dA, \tag{19}$$

where r represents distance from (x, y). An additional indication comes from the expression for M as a stream function deficit, i.e., a deficit of the average stream function over an area below the average stream function over the boundary of the area, namely,

$$M(x, y) = 2A \frac{p_0}{g} \left[\frac{1}{S} \int\limits_{S(x,y)} \psi\, dS - \frac{1}{A} \iint\limits_{A(x,y)} \psi\, dA\right] \tag{20}$$

where $S(x, y)$ is the circumference of the circle whose area is $A(x, y)$. Equation (20) follows from Eq. (19) when $\tilde{\zeta}$ is expressed in terms of ψ. Another equation relating M and ψ, namely,

$$M(x, y) = \frac{p_0}{g}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\iint_{A(x,y)} \frac{1}{2}(R^2 - r^2)\,\psi\, dA \tag{21}$$

also follows from Eq. (19).

The alternative Eqs. (19), (20) and (21) for M in terms of $\tilde{\zeta}$ and ψ suggest that conversely the fields of $\tilde{\zeta}$ and ψ may be determined by the field of M, together with suitable boundary conditions. If this is so, it might even be possible to set up a system of numerical forecasting with M rather than $\tilde{\zeta}$ as a basic quantity. In any case, it is strongly suggested that both the field of M and individual values of M are indicative measures of the state of the atmosphere.

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ANGULAR MOMENTUM AS A PARAMETER IN THE INVESTIGATION OF CYCLONE-SCALE CIRCULATIONS

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ABSTRACT

Angular momentum relative to a fixed vertical axis is used as a measure of rotational flow on the scale of cyclones and anticyclones. An exact equation derived by Starr for the time rate of change of this momentum is expressed in an approximate form, and a long-period observational study is made in two regions to determine the validity and prognostic implications of the resulting relationship.

It is found that the fluctuations of angular momentum can be accounted for primarily by a single effect — the horizontal transport of momentum across the boundaries of the regions. Moreover, a sufficient lag relationship is found to exist so that the transport serves as a predictor of the 12-hour change of momentum.

1. Introduction

Although an exact solution of the general equations governing the "meteorological behavior" of the atmosphere remains beyond the scope of even the most powerful techniques of mathematical analysis, useful information about the atmosphere can still be obtained by formulating and solving less general, though non-trivial, problems. In this connection, one may cite the successful application of the principle of conservation of angular momentum, on a global basis, in studies of the dynamics of the general circulation (see, for example, Lorenz [2], Mintz and Kao [4], and Starr and White [8]).

Recently, a relationship has been derived by Starr [7] which extends the application of the angular-momentum principle to large-scale rotational systems whose axes of rotation do not necessarily coincide with the axis of the earth (e.g., cyclones and anticyclones). Starr uses angular momentum relative to a fixed vertical axis as a measure of cyclonic activity within a circular cylinder about this axis. On the basis of theoretical considerations and an observational case study of a cyclone, he suggests that the increase of *local*[2] momentum within the cylindrical volume is attributable primarily to the horizontal flux of this momentum from the environment.

In the present study, local angular momentum is taken as a measure of the state of rotation in two large cylindrical volumes — one located over the central United States and the other over the North Atlantic Ocean. As shown in fig. 1, the dimensions of these cylinders are such that they capture the effects of the large-scale horizontal eddies in the atmosphere. The objectives of the study are (1) to determine, by means of an extensive day-to-day empirical test, the degree to which the observed fluctuations of local angular momentum in the two regions can be accounted for by the horizontal transport of local momentum across the surrounding walls, and (2) to explore the possibility that a sufficient lag relationship exists so that this transport may be used as a predictor of the local momentum in the regions.

2. The local angular momentum tendency equation, exact and approximate

Referred to a cylindrical polar coordinate system (r, θ, z), the equation derived by Starr for the time rate of change of local angular momentum within a cylindrical volume, V, extending through the depth of the atmosphere, may be written in the form

$$\frac{\partial}{\partial t}\int_V \rho r c_T \, dV = R\int_S \rho c_T c_R \, dS - \int_V \frac{\partial p}{\partial \theta} dV + \int_V \rho f r c_R \, dV + \int_V \rho r D \, dV, \quad (1)$$

where S is the vertical surface and R the radius of the volume, c_T and c_R are the tangential (positive counterclockwise) and radial (positive inward) components of the wind velocity, respectively, ρ is density, p pressure, f the Coriolis parameter, D the tangential component of the frictional force per unit mass, and t is time.

This equation states that the total local angular momentum within the cylinder may vary as a result of the horizontal transport of such momentum across the walls of the cylinder and as a result of torques due to pressure gradient, Coriolis and frictional forces.

As noted by Starr [7], the pressure-torque term,

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[2] After Lorenz [3], angular momentum relative to axes other than that of the earth will hereafter be called *local* angular momentum.

$\int_V (\partial p/\partial \theta)\, dV$, vanishes identically unless topographical irregularities are present within the cylinder. In this study we are concerned with cylindrical volumes whose underlying surfaces (the North Atlantic Ocean and the central plains of the United States) are relatively flat, thereby excluding any consideration of "mountain torque" effects.

While friction is known to be of considerable importance in the general circulation, its influence on vertically-integrated cyclonic-scale circulations, at least for short periods of time, is probably not very great.

Furthermore, as noted by Starr [7], the Coriolis effect represented by the third term on the right is probably not of primary importance in producing changes of local momentum.

On the basis of these considerations, one might venture the guess that the most important single contribution to the growth and decay of rotational motion in a region free of mountain barriers comes from the horizontal transport of local angular momentum, as represented by the first term on the right-hand side of (1). Accordingly, as a first approximation, (1) may be written in the form

$$\partial \boldsymbol{M}/\partial t = \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{M} = \int_V \rho r c_T\, dV$, and $\boldsymbol{\tau} = R \int_S \rho c_T c_R\, dS$.

The remainder of this article is devoted to a study of the validity and prognostic implications of this approximate momentum-tendency equation by means of correlation and spectral analysis of observational data.

3. Data and computational procedure

Estimates of $\boldsymbol{M}$ and $\boldsymbol{\tau}$ for the North Atlantic cylinder were computed geostrophically with use of data at the surface and at the 700-, 500-, 300- and 100-mb levels, at 24-hr intervals, for January and February 1949 (a total of 59 cases). These estimates will be denoted by M^* and τ^*.

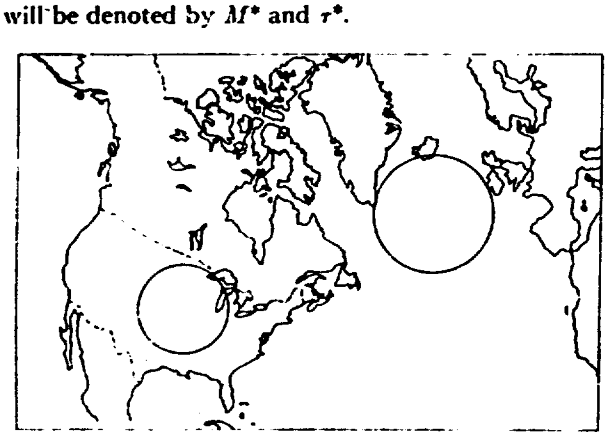

FIG. 1. Top view of cylindrical volumes selected for observational study.

It was considered desirable to study a longer series of data with smaller time intervals between successive values. Although additional data at high levels were lacking, sufficiently long series of 500-mb data were available at 12-hr intervals. Accordingly, time series of momentum and momentum transport, estimated by the use of 500-mb data alone (henceforth denoted by M and τ), were computed in addition to the aforementioned series, for both the United States and North Atlantic cylinders. It should be noted that, *to the extent that the wind at the 500-mb level approximates the vertically-averaged wind*, the use of M and τ in place of $\boldsymbol{M}$ and $\boldsymbol{\tau}$ reflects the difference between an assumed barotropic model and the true baroclinic atmosphere. This follows from the fact that the departure of the *transport of mean momentum by the mean wind* (proportional to τ) from the *mean transport of momentum* (proportional to $\boldsymbol{\tau}$) is directly attributable to the baroclinicity of the atmosphere. In general, however, the 500-mb wind is not exactly equal to the vertical mean wind. Hence, the use of M and τ in place of $\boldsymbol{M}$ and $\boldsymbol{\tau}$ in (2) contains, in addition to the assumption of barotropy, the implicit assumption that the effects of the Coriolis term and the vertical convergence of momentum are negligible *at the 500-mb level*. The ultimate justification for the use of 500-mb data alone is the extent to which the relationship $\partial M/\partial t = \tau$ is observed to apply in the atmosphere.

For the United States cylinder, M and τ were computed at 12-hr intervals from 500-mb streamline and isotach charts for March and April 1952 (122 cases). Observed winds, supplemented by gradient winds in regions of sparse data, served as the basis for the analyses of these charts. The observed winds were derived from radiosonde, rawin and pilot-balloon reports, while the gradient winds were computed from 500-mb charts extracted from the Massachusetts Institute of Technology official map series.

For the North Atlantic cylinder, M and τ were computed geostrophically at 12-hr intervals for January to April 1949 (239 cases). The data used in obtaining the estimates of the momentum and momentum transport for this region were derived from the following sources:

1. 24-hr sea-level and 500-mb maps from the northern-hemisphere historical map series, January to April 1949;
2. 1500 GCT 500-mb maps of the La Guardia Field, International Aviation Forecast Unit, January to April 1949;
3. 12-hr 700-mb maps of the Extended Forecasting Section, U. S. Weather Bureau, January to April 1949; and
4. 24-hr 300- and 100-mb maps prepared by the University of California General Circulation Project, January to February 1949.

Computational procedure.—M^* and τ^* were computed by numerical integration, with use of the values of the local momentum and local momentum transport

at each of the five levels. As defined above, M and τ were computed from 500-mb data only. A pilot study indicated that a 12-point grid network at each level along the circumference of the cylinder provided adequate resolving power for these measurements.

The formula for the computation of the momentum transport per unit pressure difference at a given pressure level, p, may be written in terms of the product, $c_T c_R$, at each of the grid points, in the form

$$\tau(p) = \frac{\pi R^2}{6\,g}\left\{\sum_{n=1}^{12} (c_T c_R)_n\right\}_p, \tag{3}$$

where g is the acceleration of gravity.

The formula for the momentum per unit pressure difference at level p may be written in the form

$$M(p) = (2\pi/g)\int_0^R r^2[c_T]_p\,dr, \tag{4}$$

where $[(\ )] = \frac{1}{2}\pi^{-1}\int_0^{2\pi} (\ )\,d\theta$. Strictly speaking, it is necessary to know the value of $[c_T]$ along each of several concentric rings to evaluate this integral. With sufficient accuracy, however, it can be assumed that $[c_T]$ is uniform in the radial direction. This assumption is especially valid if the peripheral value of $[c_T]$ is chosen as the representative radially-constant value, since this quantity is weighted most heavily in the integral (4). Accordingly, the formula for the computation of $M(p)$ becomes

$$M(p) = \frac{\pi R^3}{18\,g}\left\{\sum_{n=1}^{12} (c_T)_n\right\}_p. \tag{5}$$

4. Correlation analysis

Integration of the simple differential equation (2) over the time interval t_i to t_f leads to an expression of the form

$$(\Delta M)_{i-f} = \bar{\tau},$$

where $(\Delta M)_{i-f} \equiv M_f - M_i$, and the bar denotes the time integral. Correlations between $(\Delta M)_{0-24}$ and $\bar{\tau}$ (where the change of momentum and the integration of the momentum transport are taken over the same 24-hr period) are presented in table 1. Here $\bar{\tau}_{0,12,24}$ represents an estimate of $\bar{\tau}$ obtained by use of three successive values of τ, the middle (12-hr) value being weighted by a factor of two in accordance with the trapezoidal rule.

Although not perfect, these correlations demonstrate the high degree to which the simple relationship (2) is fulfilled in the atmosphere, even when data at the 500-mb level only are used to estimate M and τ. It should be noted that, even if (2) governed the variables *exactly*, a perfect correlation would be impossible by virtue of random errors in the raw wind and pressure data, in the analyses of these data, and in the finite-difference computations from the resulting charts. The higher correlation obtained for the United States cylinder may, in fact, be a reflection of the greater accuracy of observed wind information based on a rather extensive network of reporting stations.

Table 1. Correlations of time-integrated value of τ with simultaneous change of M over 24-hr interval.

	U. S.	N. Atl.
$\bar{\tau}_{0,12,24}$ *vs.* $(\Delta M)_{0-24}$:	+0.78	+0.70
Number of pairs:	121	238

As noted by Panofsky [5], there is no fully acceptable way of demonstrating the "significance" of correlations of the type presented here. The fact that the correlations in table 1 are based on many pairs of data and are founded on a physical basis is good reason for believing that they are not merely accidental.

One might expect even higher correlations when values of momentum and momentum transport based on data at several levels of the atmosphere are used. As noted earlier, such "vertically integrated" data were available for the North Atlantic region for 59 cases at 24-hr intervals only. In the absence of the middle (12-hr) information, the reliability of the estimates of the time integral $\overline{\tau^*}$ is reduced considerably. Consequently, the correlation between $\overline{\tau^*}_{0,24}$ and $(\Delta M^*)_{0-24}$ turns out to be rather low (+0.54); but, when this correlation is compared with that obtained by extracting the corresponding 59 pairs of data from the previously discussed series of 238 pairs based on 500-mb information alone, the vertically-integrated model shows an improvement as expected (see table 2)[3]. Although the "significance" of this improvement is open to question, one may speculate, on the basis of the result, that if 12-hr values of M^* and τ^* were available the simple derivative relationship (2) might be verified to an even higher degree than is indicated by the correlations presented in table 1.

Prediction.—Correlations between the instantaneous value of τ and the *subsequent change* of M over 12 hr are presented in the first row of table 3. These correlations give evidence that τ_0 is of value as a predictor of the 12-hr change of local momentum, accounting for 30 to 40 per cent of the variance of $(\Delta M)_{0-12}$.

The prediction question may be looked upon from a slightly different viewpoint. The correlations presented

[3] The importance of the additional 12-hr information is clearly demonstrated by the following consideration: when $\bar{\tau}_{0,24}$ is replaced by $\bar{\tau}_{0,12,24}$ in the first line of table 2, the correlation with $(\Delta M)_{0-24}$ for the *same* 58 pairs is raised to +0.70.

Table 2. Correlations using 500-mb data, compared with those using corresponding vertically-integrated data.

$\bar{\tau}_{0,24}$ *vs.* $(\Delta M)_{0-24}$:	+0.29
$\overline{\tau^*}_{0,24}$ *vs.* $(\Delta M^*)_{0-24}$:	+0.54
Number of pairs:	58

in the second row of table 3 show that approximately 60 per cent of the variance of M_{12} can be accounted for by M_0, that is, by persistence. The usefulness of momentum transport as a predictor of future values of momentum may be measured by the extent to which τ_0 can account for the remaining variance of M_{12}. In this connection, correlations between M_{12} and the best possible linear combination of M_0 and τ_0 were computed; these are presented in the third row of table 3. A comparison of these correlations with the persistence correlations shows that the momentum-transport information accounts for 40 per cent of the remaining variance, indicating that τ_0 is indeed a useful parameter in the prediction of M_{12}.

While predictions of the type presented here might not be superior to those prepared by forecasters using empirical procedures, there is some intrinsic value in being able to attribute future changes in atmospheric flow to recognizable dynamical processes in the atmosphere. This has, in fact, already been a major achievement of the barotropic numerical-forecasting experiments, in which vorticity rather than local angular momentum has served as the primary physical parameter. The connection between the vorticity and momentum approaches has been discussed in detail by Lorenz [3].

5. Spectral analysis

The time variations of the momentum and momentum transport may be considered to consist of a superposition of harmonic fluctuations of many different scales or frequencies. It is of interest to determine the extent to which these harmonic components individually satisfy (2). For example, it is conceivable that a physical effect, such as friction, influences one scale of motion more than another. This would lead to systematic shortcomings in the verification of (2) for the affected frequency bands. The technique of spectral analysis, which has been developed in comparatively recent years, is well suited for a study of phenomena of this type. This technique is based largely on a theorem by Weiner [12], which demonstrates that a *variance spectrum* of a "random" time series can be defined and computed. The variance spectrum is a graph showing the contribution of each small band of harmonics to the total variance (or power) of a given series, and, as such, often provides the clearest characterization of the series.

In addition to the spectrum of a single variable, a complex cross-spectrum for two variables such as $M(t)$ and $\tau(t)$ can be defined. This cross-spectrum measures the power common to both series.

TABLE 3. 12-hr prediction correlations.

	U.S.	N. Atl.
τ_0 *vs.* $(\Delta M)_{0-12}$:	+0.66	+0.54
M_0 *vs.* M_{12}:	+0.78	+0.79
$(A\tau_0 + BM_0)$ *vs.* M_{12}:	+0.88	+0.87
Number of pairs:	121	238

By comparing the spectra computed from observed data with those which would be expected in theory if (2) were valid over the entire frequency range, one can obtain a measure of the extent to which the components of momentum and momentum transport in different portions of the frequency scale satisfy (2). This application of spectral analysis is somewhat different from previous applications in meteorology, in which the method has been used to determine a representative picture of the scales of eddies which are important in turbulence (see, for example, Taylor [9], and Panofsky and McCormick [6]). In the present study, the spectra are not obtained primarily for the purpose of discovering the dominant fluctuations in the data, but rather as a means for testing (2) in the frequency domain.

Theory.[4]—Weiner [12] has shown that the variance spectrum $\Phi_{ff}(\omega)$, of a given function $f(t)$, may be expressed as the Fourier transform of the autocorrelation function in the form

$$\Phi_{ff}(\omega) = \tfrac{1}{2}\pi^{-1}\int_{-\infty}^{\infty}\phi_{ff}(\epsilon)\cos\omega\epsilon\, d\epsilon, \tag{6}$$

where the autocorrelation function, $\phi_{ff}(\epsilon)$, is given by

$$\phi_{ff}(\epsilon) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f(t)\, f(t+\epsilon)\, dt. \tag{7}$$

In these equations, f may represent any variable, for example, M or τ (where the symbols f, M and τ now refer to deviations from mean values); ϵ represents time lag, and ω is frequency.

Similarly, the cross-spectrum for M and τ is defined by the relationship

$$\Phi_{\tau M}(\omega) = \tfrac{1}{2}\pi^{-1}\int_{-\infty}^{\infty}\phi_{\tau M}(\epsilon)\, e^{-i\omega\epsilon}\, d\epsilon, \tag{8}$$

[4] For auxiliary material pertaining to the theory of spectral analysis, the reader is referred to Wadsworth *et al* [11] or Lee [1].

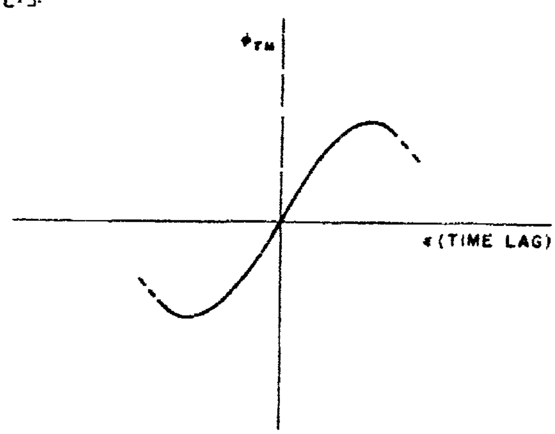

FIG. 2. Idealized cross-correlation function for two variables, M and τ, which satisfy $\partial M/\partial t = \tau$.

where the cross-correlation function, $\phi_{\tau M}(\epsilon)$, is given by

$$\phi_{\tau M}(\epsilon) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\tau(t)\,M(t+\epsilon)\,dt. \qquad (9)$$

The cross-spectrum is composed of both a real and imaginary component (sometimes called the cospectrum and quadrature spectrum, respectively). These components will hereafter be denoted by $\mathrm{Re}\{\Phi_{\tau M}\}$ and $\mathrm{Im}\{\Phi_{\tau M}\}$, respectively.

By introduction of (2) into the definitions of the auto- and cross-correlation functions [(7) and (9)], the following relations are obtained:

$$\phi_{\tau\tau}(\epsilon) = \partial[\phi_{\tau M}(\epsilon)]/\partial\epsilon,$$

and

$$\phi_{\tau M}(\epsilon) = -\,\partial[\phi_{MM}(\epsilon)]/\partial\epsilon. \qquad (10\text{ a,b})$$

Since ϕ_{MM} is by definition an even function, $\phi_{\tau M}$ should, in consequence of (10b), be an odd function as pictured in fig. 2.

Further, by introducing (10) into the definitions of the power spectra, (6) and (8), one obtains

$$\mathrm{Im}\{\Phi_{\tau M}(\omega)\} = \omega\,\Phi_{MM}(\omega),$$

$$\Phi_{\tau\tau}(\omega) = \omega\,\mathrm{Im}\{\Phi_{\tau M}(\omega)\},$$

and

$$\mathrm{Re}\{\Phi_{\tau M}(\omega)\} = 0. \qquad (11\text{ a,b,c})$$

These relationships must be satisfied by the data if (2) is truly valid. The observed departures from this behavior, if such exist, may serve as a measure of the extent to which the hypothesized relationship (2) is not fulfilled by components of M and τ in different frequency bands.

Data and computational procedure.—The data for the United States cylinder were chosen for the application of the spectral analysis. The standard formula for the correlation coefficient, computed at discrete lags from zero to ±40 half-days, was used to approximate the auto- and cross-correlation functions. The evaluation of the integrals defining the spectral functions was carried out numerically by the trapezoidal rule; the formulae for this numerical integration are

$$\Phi_{ff}(\omega_n) = \frac{\Delta\epsilon}{2\pi}\Big[\phi_{ff}(0) + 2\sum_{\epsilon=1}^{39}\phi_{ff}(\epsilon)\cos\omega_n\epsilon + \phi_{ff}(40)\cos 40\,\omega_n\Big], \qquad (12)$$

$$\mathrm{Re}\{\Phi_{\tau M}(\omega_n)\} = \frac{\Delta\epsilon}{4\pi}\Big[\phi_{\tau M}(-40)\cos(-40\,\omega_n) + 2\sum_{\epsilon=-39}^{39}\phi_{\tau M}(\epsilon)\cos\omega_n\epsilon + \phi_{\tau M}(40)\cos 40\,\omega_n\Big], \qquad (13)$$

and

$$\mathrm{Im}\{\Phi_{\tau M}(\omega_n)\} = \frac{\Delta\epsilon}{4\pi}\Big[\phi_{\tau M}(-40)\sin(-40\,\omega_n) + 2\sum_{\epsilon=-39}^{39}\phi_{\tau M}(\epsilon)\sin\omega_n\epsilon + \phi_{\tau M}(40)\sin 40\,\omega_n\Big], \qquad (14)$$

where discrete values of frequency [$\omega_n = n\pi/(40)$; $n = 0, 1, 2, \ldots, 40$] are used, and $\Delta\epsilon$ is the unit time interval of one half-day.

To reduce errors due to the truncation of the correlation functions at a given lag, Tukey and Hamming [10] recommend a smoothing process, known as "hanning." The smoothed spectra, $\Phi'(\omega_n)$, obtained from the values $\Phi(\omega_n)$ by the formula

$$\Phi'(\omega_n) = \tfrac{1}{4}[\Phi(\omega_{n-1}) + 2\Phi(\omega_n) + \Phi(\omega_{n+1})], \qquad (15)$$

were computed in the present study. It is recognized that, while this smoothing process reduces the "smearing" effect resulting from the use of a finite amount of data, it does so at the expense of resolution in all frequency bands.

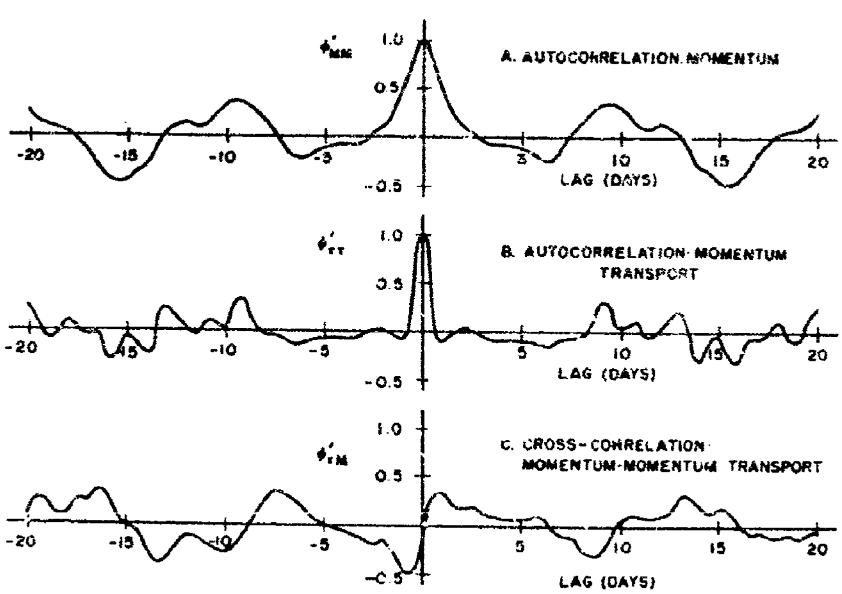

FIG. 3. Correlation functions for United States data. (ϕ' denotes *normalized* correlation functions.)

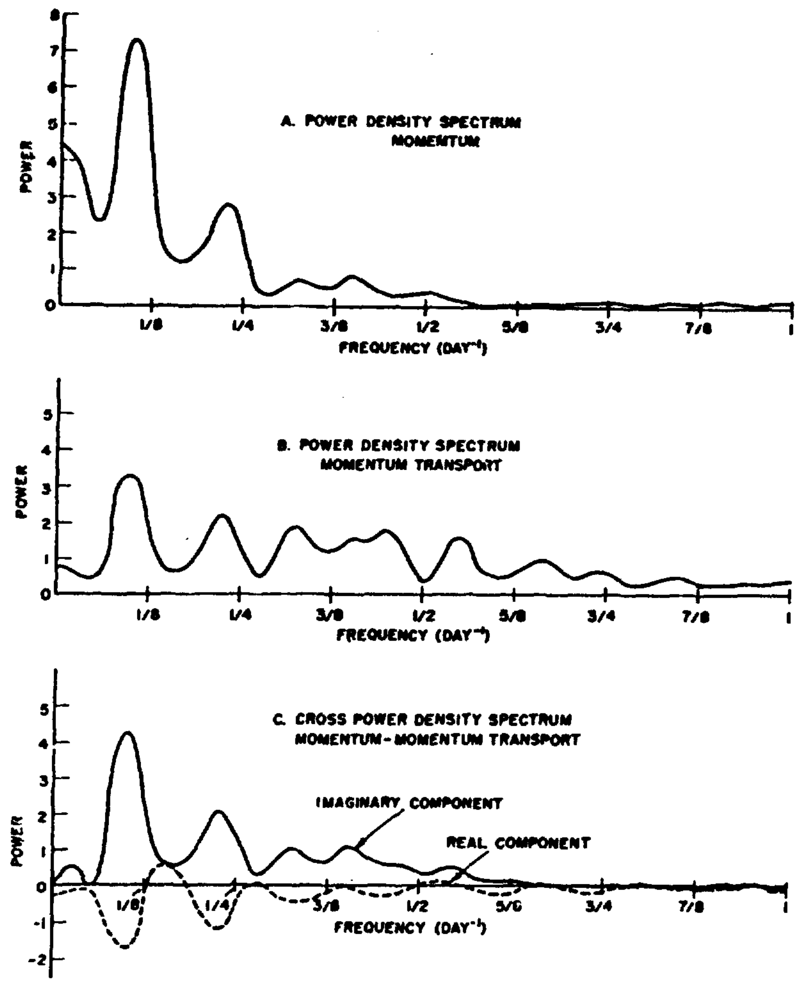

Fig. 4. Power-density spectra for United States data.

The spectra.—The correlation functions and spectra are pictured in figs. 3 and 4, respectively. The auto- and cross-correlation curves appear to be consistent with (10). It may be noticed, for example, that $\phi_{\tau M}$ is essentially an odd function, in good agreement with fig. 2.

The spectrum of momentum, shown in fig. 4a, indicates that there is considerable variance in frequencies ranging from 1/4 to 1/5 day^{-1}, and from 1/8 to 1/10 day^{-1}. These frequencies characterize the scales of transient disturbances which were dominant in the region during the particular time interval studied. It is worthy of note that *all* the spectra have maxima in these same frequency bands. This is in accordance with the fact that there is a significant relationship between $M(t)$ and $\tau(t)$.

Let us now examine the spectra in connection with (11). The imaginary component of the cross-spectrum, computed from the spectrum of M by means of (11a), is compared in fig. 5 with that computed directly from the equation which defines $\mathrm{Im}\{\Phi_{\tau M}\}$. In addition, the "theoretical" spectrum of τ, computed from the

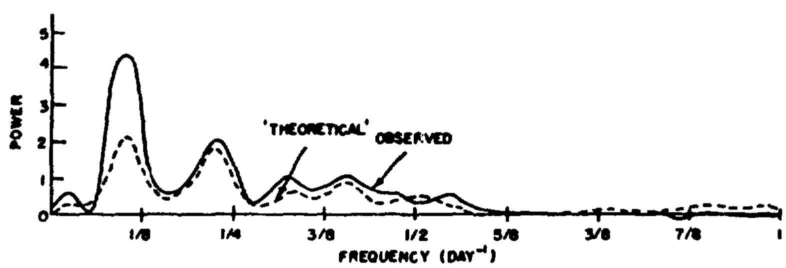

Fig. 5. "Theoretical" and observed imaginary cross-power density spectrum.

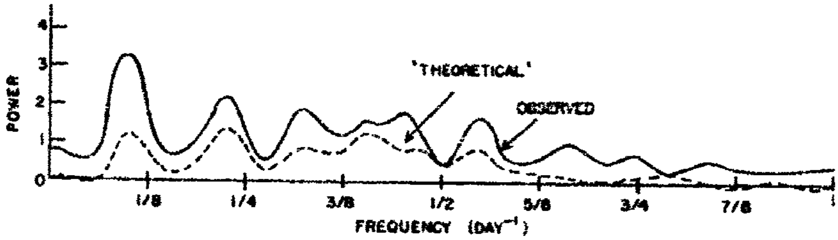

FIG. 6. "Theoretical" and observed power-density spectrum for momentum transport.

imaginary component of the cross-spectrum by means of (11b), is pictured in fig. 6 along with that computed directly from (6). The real part of the cross-spectrum, which according to (11c) should be zero everywhere, is shown in fig. 4c.

Although there are differences in amplitude, there is good agreement in the distribution of power as a function of frequency between the "theoretical" and "observed" spectra shown in figs. 5 and 6. Fig. 4c points to a deviation of the data from (2), however, since the real part of the cross-spectrum contains noticeable power in certain frequency bands. The implications of this departure will be discussed below.

Phase considerations.—It is of interest to examine the phase differences between the harmonic components in M and those of the same frequency in τ. These phase differences, $\theta(\omega)$, may be obtained as the argument of the cross-spectrum by the formula

$$\theta(\omega) = \arctan \frac{\text{Im } \{\Phi_{\tau M}(\omega)\}}{\text{Re } \{\Phi_{\tau M}(\omega)\}} \tag{16}$$

(see Wadsworth *et al* [11]). For the frequency bands in which (2) is satisfied, θ should take on the value $\pi/2$.

The observed phase differences are plotted in fig. 7. The values at the high and low ends of the spectrum are extremely sensitive to error, since both components of the cross-power are very nearly zero in these regions (fig. 4c). For this reason, they are not entered on the graph.

The phase differences are seen to lie in the general vicinity of $\pi/2$, although they tend toward higher values in the frequency bands in which the cross-power is large. There is no clear indication, however, that the deviations from $\pi/2$ at one end of the frequency scale are selectively greater than those at the other end. This suggests that the dynamics governing the entire frequency range pictured are substantially the same.

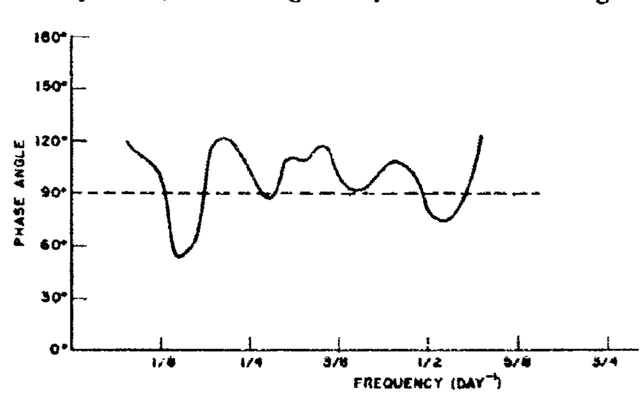

FIG. 7. Phase-angle difference between individual harmonics in $M(t)$ and those of same frequency in $\tau(t)$.

The tendency for the values of the phase lag to be somewhat greater than $\pi/2$ is directly related to the fact that the real part of the cross-spectrum is not everywhere zero. If, for example, there were no power in $\text{Re}\{\Phi_{\tau M}\}$ (which would be the case if $\phi_{\tau M}$ were *perfectly* odd), it follows from (16) that the phase angles would be *exactly* $\pi/2$. The nature of this departure suggests that a more appropriate governing equation might be

$$dM/dt - kM = \tau \tag{17}$$

(where k is a small positive constant); for, if this equation is satisfied, the phase differences between harmonics in M and those of the same frequency in τ must be greater than $\pi/2$ by an amount which depends on k. It should be noted that the departures which lead to the term $-kM$ are small and may be peculiar to the particular series tested. Furthermore, there is no obvious physical connection between this term and any of the factors which were neglected in the study. For these reasons, caution should be exercised in generalizing this result.

While the validity of the term $-kM$ may be questionable as a general result, the gross characteristics of figs. 5, 6 and 7 demonstrate that the simple expression (2) is the *dominant* relationship governing the data over the entire frequency range studied.

6. Some remarks on the role of friction

Studies of the angular-momentum balance in connection with the general circulation have clearly demonstrated the important influence of surface friction on global-scale motions. In section 2, we have speculated that friction plays a considerably less important role in cyclonic-scale circulations of the type considered in this article. To obtain more concrete evidence concerning the efficacy of friction, an effort was made to test whether the incorporation of the friction term in (1) helps to account for any of the

variance of the momentum change not otherwise accounted for by the momentum transport. The simplest assumption regarding surface friction was made, namely, that it is proportional to the surface wind. With use of geostrophic wind data for the United States region, it was found that the correlation between $(\Delta M)_{0-24}$ and the best possible linear combination of $\bar{\tau}_{0,12,24}$ with surface friction shows practically no improvement over the correlations between $(\Delta M)_{0-24}$ and $\bar{\tau}_{0,12,24}$ alone (table 1). This is evidence that, in a statistical sense at least, surface friction has little influence on the short-period changes of the vertically-integrated, cyclonic-scale motions.

It might be noted that in studies of the prediction of the zonal index with use of angular momentum about the earth's axis as a parameter (Lorenz [2]; Mintz and Kao [4]), improvement in forecasting was obtained by incorporating friction even though this friction was taken proportional to the upper-level (500-mb) wind rather than the surface wind. The basic equation for these studies is of the form

$$dM'/dt + kM' = b\tau',$$

where the primes indicate that the angular momentum is measured relative to the axis of the earth, and k and b are positive constants. In the present study, a friction term of the form $+kM$ would certainly not appear to be valid, since an effect which is opposite in sign manifests itself in the data as shown by (17).

7. Conclusions

The empirical study has demonstrated the high degree to which the *observed* motions of the atmosphere are consistent with the principle of conservation of local angular momentum, and to which the approximate equation (2) expresses this principle, even when information at the 500-mb level only is used. Although the spectral analysis has pointed to possible deficiencies of the approximations, one may safely conclude that variations of the local angular momentum in the regions studied can be accounted for primarily by the horizontal transport of this momentum across the boundaries of the regions. Furthermore, no selective effects are found which influence one scale of fluctuations more than another. The results indicate also that the instantaneous transport of local angular momentum is of value as a predictor of the 12-hr change of local momentum in the regions.

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NOTES

Note on Simple Assumptions Regarding the Baroclinic Structure of the Atmosphere

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(Manuscript received November 11, 1954)

1. Background

In a series of recent articles (STARR 1953, LORENZ 1953, PFEFFER and SALTZMAN 1954) 'local' angular momentum has been used as a basic physical parameter in the study of atmospheric flow. In the last of these references the following approximate dynamical equation was shown to be valid for regions of the atmosphere free of mountain barriers;

$$\frac{\partial M}{\partial t} = \tau, \tag{1}$$

where t is time, M is the total angular momentum within an arbitrarily-located, vertically-oriented, fixed cylindrical volume extending through the depth of the atmosphere, measured relative to the axis of the volume, and τ is the horizontal transport of this 'local' momentum across the fictitious walls of the volume.

LORENZ (1953) has shown that local momentum, M, represents a space-weighted average of the vorticity in a given region of the atmosphere. He has shown, also, that it is possible to make a distinction between displacement and intensification processes by resolving the local momentum transport, τ, into barotropic and baroclinic components, similar to the conventional resolution of the vorticity advection. In more concrete terms we may write,

$$\tau = \frac{p_0}{g} R^2 \int_0^{2\pi} \overline{C_T C_R}\, d\theta \tag{2}$$

or,

$$\tau = \frac{p_0}{g} R^2 \int_0^{2\pi} \overline{C_T} \cdot \overline{C_R}\, d\theta + \frac{p_0}{g} R^2 \int_0^{2\pi} \overline{C_T' C_R'}\, d\theta, \tag{3}$$

where p_0 is the surface pressure (assumed uniform), g is the acceleration of gravity, R is the radius of the cylindrical volume, C_T and C_R are the tangential (positive counter-clockwise) and radial (positive inward) components of the wind velocity respectively, θ is the polar coordinate, the bar denotes a vertical average throughout the atmosphere with respect to pressure, and the primes denote a departure from this average. In (3) the first integral

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represents the barotropic component of τ, while the second integral gives the baroclinic contribution resulting from the vertical variation of the horizontal wind field.

It is the purpose of this note to report some observed short-comings of the evaluation of the baroclinic term of (3) by means of an assumption regarding the vertical variation of the wind, as compared with a "three-dimensional" evaluation of the baroclinic contribution by means of data at five levels of the atmosphere. These results may have some bearing on numerical prediction attempts which aim to capture the effects of baroclinicity by similar assumptions.

2. Procedure

In the previous study (PFEFFER and SALTZMAN, 1954) the integral (2) was evaluated directly on a daily basis, using geostrophic winds, for the two-month period January—February, 1949, for a cylindrical volume located over the North Atlantic Ocean. Data at the surface and the 700, 500, 300, and 100 mb levels were employed. The 300, and 100 mb charts were kindly loaned by the U.C.L.A. General Circulation Project and are believed to be one of the best series of maps available at these levels. In addition, the barotropic part of (3) was evaluated for the same region and period, with 500 mb taken as the level of the mean wind.

As an extension of this study it was decided to test whether sufficiently accurate estimates of τ could be obtained by using an assumed wind profile to measure the baroclinic term of (3). Specifically, it was assumed that the tangential and radial components of the wind vary linearly with pressure, the rate of variation (or thermal wind per unit pressure difference) in the troposphere being determined by the 700—300 mb thickness in accordance with the geostrophic principle. Although it does not affect the results appreciably, for greater realism the wind was assumed to decrease above the 200 mb level in a simple linear fashion. These are similar to the assumptions used in the so-called $2^1/_2$-dimensional numerical prediction models.

By subtracting the barotropic component from the "three-dimensional" momentum transport it is possible to obtain a measure of the daily baroclinic contributions. It would be hoped that the baroclinic term computed on the basis of the assumed vertical variation of the wind compares favorably with these values. As it turns out, however, the correlation coefficient between these two quantities for the two-month period is disappointingly small (+.20) and the root mean square deviation is of precisely the same magnitude as the baroclinic term itself. It is recognized that these results may, in part, be attributed to inaccuracies in the estimation of the barotropic term using 500 mb data. It is most likely, however, that the results are mainly a consequence of the crudeness of the representation of the vertical variation of the momentum transport through the use of the assumed wind profile.

One may further inquire as to whether there is an improvement in the verification of the dynamical relationship (1) as a result of using the values of τ based on data at five levels of the atmosphere rather than the values based on the assumed wind profile. Verification of (1) may be measured by the correlation coefficient between the change of local momentum over 24 hours and the time-integrated momentum transport over the same period. As noted in reference 2, estimates of this time-integrated momentum transport over 24 hours, obtained by summing the values of τ at the beginning and end of the period, are far less satisfactory than those obtained when the intermediate (12-hourly) information is incorporated. Unfortunately, in the present case only 24-hourly data were available at upper levels for the "three-dimensional" measurement of the momentum transport and, for comparison purposes, it was necessary to use 24-hourly data in the case of the barotropic and "$2^1/_2$-dimensional" quantities also. Consequently, the correlation coefficients presented in Table 1 are all rather low and can be taken only as a very crude indication of the true relative differences in the verification of the three models[1]. Subject to this reservation, these coefficients suggest that the improvement obtained over the barotropic model by the

[1] When 12-hourly information is included in estimates of $\int_0^{24} \tau\, dt$ the +.29 correlation coefficient for the barotropic case becomes +.70 (see reference 2).

"2 $^1/_2$-dimensional" model is small compared with the improvement obtained when baroclinic effects are taken into account in a more elaborate manner by means of multi-level computations.

Table 1. Correlation coefficients between the time-integrated transport of local momentum and the simultaneous change of local momentum over a 24-hour period, using data at 24-hour intervals. Number of pairs: 58.

barotropic	+.29
"2 ½-dimensional" baroclinic	+.32
"3-dimensional" baroclinic	+.54

As would be expected, it is observed that the greatest improvement over the barotropic verification occurs on specific days when the baroclinic component of the local momentum transport is large. On these same days it is often found that the baroclinic term computed by means of the assumption fails to give an adequate measure of this baroclinicity.

These findings are, of course, of a limited nature and cannot be regarded as a conclusive test of the "2 $^1/_2$-dimensional" assumption. Strictly speaking, the results apply to a model which has angular momentum rather than vorticity as the primary physical parameter. The results may be of some general interest, however, in that they suggest possible difficulties in obtaining significant improvement over barotropic numerical forecasting procedures by incorporating simple assumptions regarding the baroclinic structure of the atmosphere.

Acknowledgements

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CONCERNING THE MECHANICS OF HURRICANES

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ABSTRACT

The balance of angular momentum in the mature hurricane is examined quantitatively. It is found that the cyclonic angular momentum which ultimately appears in the hurricane is first introduced into the atmosphere within the circulation around each of the neighboring high-pressure cells. From here it is transferred into the cyclonic core of the hurricane through the agency of "horizontal-eddy" processes, which are characterized by mass exchanges in *horizontal* planes. Such mass exchanges are identified with the so-called "Rossby regime" of convection. Within the cyclonic core of the hurricane, the angular momentum is transported inward through the agency of "vertical-eddy" processes, which are characterized by mass exchanges in *vertical* planes passing through the axis of rotation of the hurricane. Such mass exchanges are identified with the "Hadley regime". The hurricane thus possesses characteristics of two distinctly different regimes of convection.

1. Introduction

When a fluid contained in a rotating cylindrical vessel is heated differentially in an axially symmetrical fashion, and allowed to reach a state in which the statistical properties of the motion are quasi-steady, it is observed that one portion of the fluid generally rotates faster, and another portion slower, than the vessel. In the slower-moving portion the vessel tends to drag the fluid along, and hence to transfer angular momentum to it. In the faster moving portion the vessel tends to retard the fluid, and hence to remove angular momentum from it. In the quasi-steady state there must, therefore, be a transport of angular momentum from the slower to the faster-moving portion in order to maintain this state of affairs. It has been found, largely as a result of the theoretical work of Kuo (1953; 1954; 1955; 1956a; 1956b) and Lorenz (1954), and the experimental results of Fultz (1953) and Hide (1953), that for different rates of rotation and intensities of heating, different physical processes accomplish the required transport of angular momentum. That this result has important implications in regard to the dynamics of rotating systems in the earth's atmosphere has already been demonstrated in the case of the global wind circulation. Investigations of the balance of angular momentum about the earth's polar axis and the balance of energy on a hemispheric scale (see for example Starr and White, 1954, and references therein) have shown that the general circulation of the atmosphere resembles the convective regime which develops in the model experiments under conditions of high rotation and weak heating.

Although the model analogue may not be as good in the case of other rotating systems in the atmosphere, such as the hurricane and the extratropical cyclone, it may still be possible on the basis of similar studies to identify these systems with specific convective regimes of the rotating model experiments, the dynamics which can be studied under controlled conditions. It is the purpose of the present article to make a beginning in this direction in the case of the hurricane through an examination of the way in which the motions of the atmosphere are organized to transfer angular momentum between the mature hurricane and its surroundings.

2. Wind structure of the mature hurricane and angular momentum considerations

For the purposes of the present discussion we shall take as a frame of reference a system of spherical coordinates $(r, \Delta\phi, \theta)$ with origin at the center of the earth, where r is linear distance from the origin measured along a vertical axis which coincides approximately with the axis of rotation of a hurricane, $\Delta\phi$ is angular distance from this axis and θ is azimuth measured positive in the counterclockwise sense. As shown in fig. 1, a constant value of $\Delta\phi$ defines a conical wall. In the fig., a is the radius of the earth, $R \equiv r \sin \Delta\phi$ is linear distance from the axis,[3] S is area measured along the conical wall and ϕ_0 is the latitude of the axis. The following definitions will also be of use in the present study: $[Q] \equiv (1/2\pi)\oint Q d\theta$, $\bar{Q} \equiv (1/P_s)\int_0^{P_s} Q dP$; $Q' \equiv Q - [Q]$ and $Q'' \equiv Q - \bar{Q}$.

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[2] This material was presented at the Cambridge Seminar of the American Meteorological Society on 7 November 1955 (see Pfeffer, 1955), and as Scientific Report No. 4 of the M.I.T. General Circulation Project, May 1956.

[3] Since the atmosphere is extremely shallow, R is sensibly constant with altitude within the atmosphere and may be taken equal to $a \sin \Delta\phi$ for practical purposes.

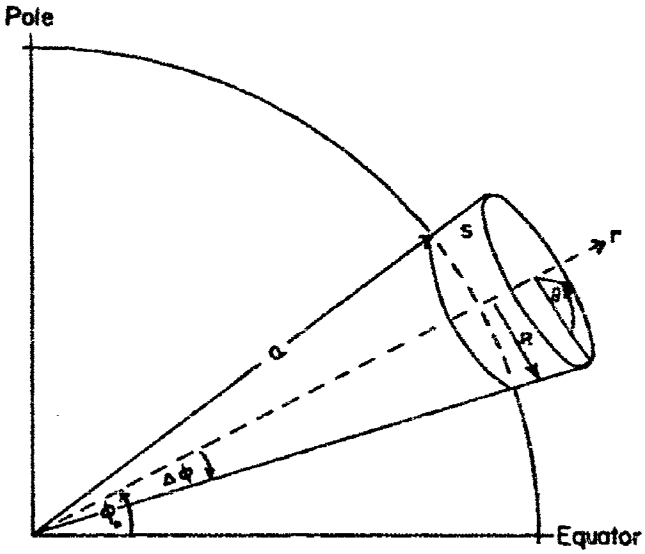

FIG. 1. Schematic picture of coordinate system and conical wall $\Delta\phi$ = constant.

where P is pressure (employed in place of r as the vertical coordinate), p_s is surface pressure and Q may be any quantity. The square brackets and bar thus denote averages with respect to θ and P along the conical wall, and the single and double primes denote deviations from these averages, respectively. C_T and C_N will be taken as the horizontal components of the wind velocity tangent (positive counterclockwise) and normal (positive inward) to the conical walls $\Delta\phi$ = constant.

Figs. 2 and 3, based on composite wind charts prepared by Jordan (1952) and Hughes (1952), give the spacial distribution of $[C_N]$ and $[C_T]$, respectively, in the mature hurricane. Fig. 2 shows that the azimuthally-averaged normal velocity is directed inward throughout most of the troposphere, and outward at higher levels. Between the primary maximum of inflow near sea level and the secondary maximum around 18,000 ft there is a layer of minimum inflow. According to Simpson (1954), the moisture distribution in

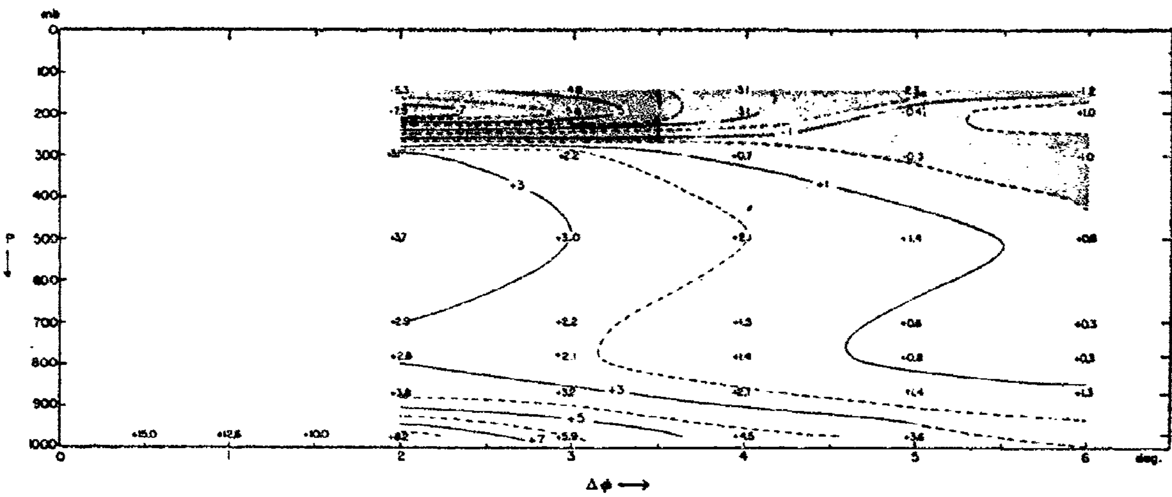

FIG. 2. Distribution of $[C_N]$ in the mature hurricane, based on the composite wind charts of Jordan and Hughes. 1 unit = 1 m sec^{-1}. Shading denotes negative values (*i.e.*, outflow).

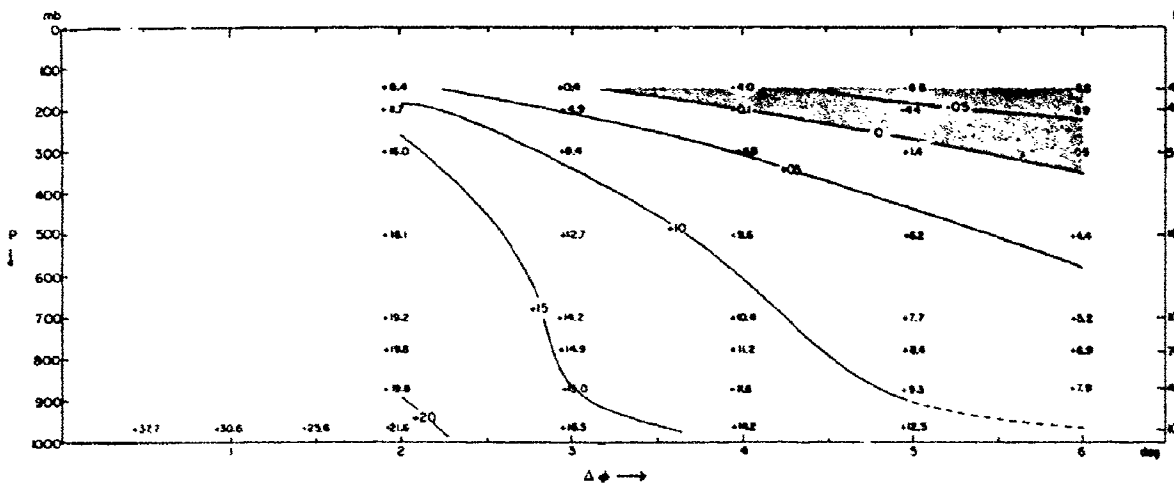

FIG. 3. Distribution of $[C_T]$ in the mature hurricane, based on the composite wind charts of Jordan and Hughes. 1 unit = 1 m sec^{-1}. Shading denotes negative values (*i.e.*, anticyclonic winds).

the hurricane displays a similar character, in that it consists of alternating layers of cloud and dry air.

Fig. 3 shows that an extensive body of air undergoes cyclonic rotation about a vertical axis located in the vicinity of the eye of the hurricane. Both the intensity and the radial extent of this rotation are greatest near sea level and diminish with height.[4] With this type of wind structure a comparatively large surface area of the cyclonic wind core of the hurricane is exposed to frictional effects at the lower surface. As a result, the earth must exert a large clockwise torque on the atmosphere, and there must be a fairly intense flux of angular momentum from the atmosphere into the earth within the limits of the sea-level cyclonic winds. An estimate of this drain based on Hughes' data for 1000 ft indicates that it is sufficient to reduce the cyclonic angular momentum of the hurricane to zero within 48 hr.

It is probably safe to say that there is no progressive transfer of angular momentum from the atmosphere into the earth. Since there is a frictional loss of angular momentum within the cyclonic core, there must, therefore, be a supply at some other place, time, or both. This can take place only where the surface winds possess an anticyclonic component *about the hurricane axis*. Synoptic evidence suggests that the required anticyclonic component is to be found in those portions of the neighboring high pressure cells which lie farthest from the hurricane, as shown schematically in fig. 4. At such great distances the magnitude of the surface wind velocity does not have to be very great in order to bring about the necessary transfer of angular momentum into the atmosphere.

To complete the cycle, angular momentum must be carried from the region in which it is introduced into the atmosphere to the cyclonic core of the hurricane, where it is removed through surface friction. For the average mature hurricane the mean angular momentum flux into the cyclonic core should be roughly equal to the mean rate of removal of angular momentum by surface friction.

It should be remarked that not all of the angular momentum which is introduced into the atmosphere within the anticyclonic circulations must be carried into the hurricane. Undoubtedly, a portion is transported into the surrounding atmosphere. The fact that such exchanges with the environment do take place makes it invalid to treat the hurricane as a closed system.

[4] Another feature of the flow pattern which, however, is not revealed in fig. 3, since it represents a departure of the wind field from axial symmetry, is the fact that the tangential component of the wind is systematically greater to the right of the path of motion of the hurricane (looking downstream) than to the left. It is shown elsewhere (Pfeffer, 1957) that this feature plays an important role in the motion of the hurricane.

3. Transport of angular momentum

We next investigate the way in which the motions of the atmosphere are organized to transport angular momentum into the hurricane. The flux of angular momentum across a fixed conical wall such as that shown in fig. 1 may be expressed in the form,

$$\int_S \rho M C_N dS = \frac{2\pi R P_s}{g} \times \{\overline{[M][C_N]} + \overline{[M]''[C_N]''} + \overline{[M'C_N']}\}, \quad (1)$$

where M is the absolute angular momentum per unit mass about the axis of the cone, ρ is density, and g is the acceleration of gravity. In (1) it is tacitly assumed that the density variation along the wall at each level is negligible.

The first term on the right represents the contribution due to a net mass flux across the wall, while the last two terms represent contributions associated with mass *exchanges* between the volume and the surroundings. Except in those rare instances when the total *absolute* angular momentum of the atmosphere within the conical volume is negative, the first term is always of the wrong sign to account for the observed behavior of the atmosphere. For example, it gives a positive contribution when mass accumulates within the volume. Observations show, however, that "filling" is accompanied by a *decrease* in the intensity of cyclonic rotation. Furthermore, plausible estimates, based on the magnitude of surface pressure changes, indicate that this term is at least one order of magnitude smaller than the others.[5] Thus, significant net transports of angular momentum are accomplished only through the agency of processes which involve mass exchanges with the surroundings.

The second term depends on the covariance of $[M]$ and $[C_N]$ in the *vertical* direction. It will give a positive contribution when there is a net mass inflow at those levels at which the azimuthal-mean angular momentum is large, and outflow at those levels at which it is small. This process will hereafter be called the "vertical-eddy" transport. The term "eddy," as used here, is not meant to imply the size of the circulation. Any circulation, no matter how large, which affects an *exchange* of mass will be regarded as an eddy.

The last term depends on the covariance of M and C_N in the *horizontal* direction along the conical wall. It will give a positive contribution when, *at each level*, the angular momentum is greater at inflow points than it is at outflow points. This term will be called the "horizontal-eddy" transport.

[5] The form of this term is such, however, that when it is evaluated from wind data it may assume large spurious values due to the inadequacies of the data.

The absolute angular momentum per unit mass may be resolved into the sum of two components, one associated with the motion of the atmosphere *relative* to the surface of the earth, $M_R \equiv RC_T$, and the other associated with the rotation of the atmosphere with the earth, $M_\omega \equiv R^2\omega \sin \phi_0$, where ω is the angular velocity of the earth. Large transports of M_ω may be effected at individual levels by vertical-eddy processes, and at individual points at a given elevation by horizontal-eddy processes. But, these must be compensated by oppositely directed transports at other levels and points, since M_ω is sensibly constant along the conical wall. Consequently, no significant *net* transports of M_ω take place through mass exchange processes. Accordingly, (1) may be rewritten in the form,

$$\int_S \rho M C_N dS \approx \frac{2\pi R^2 P_s}{g} \{\overline{[C_T]''[C_N]''} + \overline{[C_T'C_N']}\}. \quad (2)$$

As noted by Starr (1953), the last term in this expression is related to certain asymmetries in the horizontal streamline pattern.

4. Data and computational procedure

If a dense network of simultaneous wind observations in individual hurricanes were available, it would be a simple matter to evaluate the integrals in (2) and thereby to determine what role each of the eddy processes plays in the hurricane mechanism. Present day data coverage at upper levels in hurricanes is, however, inadequate for such purposes. As a result, we shall have to rely on the composite wind charts of Jordan and Hughes as the source of data for the present quantitative study. Since these charts are in the nature of ensemble averages of the velocity field taken over a large number of hurricanes they cannot, however, be expected to contain all of the information that might be present on synoptic charts which are based on simultaneous wind observations. In particular, features which may be common to all hurricanes, but which do not have a preferred location within the hurricane, tend to become obscured due to the averaging. This limitation in the data should be kept in mind when interpreting the results of the present study.

The composite wind charts were used to evaluate the horizontal and vertical-eddy transports of angular momentum across the vertical walls $\Delta\phi = 2°, 3°, 4°, 5°$ and $6°$. No attempt was made to adjust the data for mass continuity since it was felt desirable to make the observational study as objective as possible. In this connection, it should be noted that the measured value of each of the eddy terms is not sensitive to violations in mass continuity. The horizontal averages were computed as linear means of the values at 12 equally-spaced grid points along each of the walls at 1000 ft, and at 8 equally-spaced points along each of the walls at the higher levels. In the evaluation of the vertical averages, the data at 1000, 4000, 7000, 10,000, 18,000, 30,000, 40,000 and 45,000 ft were considered to apply over the pressure intervals 1000 to 920, 920 to 825, 825 to 740, 740 to 600, 600 to 400, 400 to 250, 250 to 175 and 175 to 125 mb, respectively. Since the 1000-ft data do not extend to a distance of 6° lat from the hurricane center, the contribution at this level to the vertical-eddy transport of angular momentum across $\Delta\phi = 6°$ was estimated from extrapolated values of $[C_T]$ and $[C_N]$ taken from figs. 2 and 3. Furthermore, since the term $[C'_T C'_N]$ cannot be estimated by such extrapolations, the contribution of the lowest layer was omitted in the computation of the horizontal-eddy transport across $\Delta\phi = 6°$. It is felt that this omission is not serious.

In order to determine the extent to which the estimated angular momentum transport satisfies the frictional requirements, measures of the rate at which angular momentum is removed by friction were computed from the formula,

$$\int_V \rho R D dV \approx -2\pi\rho_0 K \int_0^{\Delta\phi} R^2[|C|C_T]_0 a\, d(\Delta\phi), \quad (3)$$

where D is the tangential component of the viscous force per unit mass, $|C|$ is the wind speed, K is the skin friction coefficient and the subscript zero refers to the values measured at 1000 ft. The value $K = 0.0015$, chosen by Hughes as a representative value for the friction coefficient at 1000 ft, was used in the present study; ρ_0 was taken as 1.1903×10^{-3} gm cm^{-3} and the integral $\int_0^{\Delta\phi} R^2[|C|C_T]_0 a\, d(\Delta\phi)$ was evaluated by means of the trapezoidal rule using values of $[|C|C_T]_0$ measured at $\Delta\phi = 0.5°, 1°, 1.5°, 2°, 3°, 4°$ and $5°$; at $\Delta\phi = 6°$ the extrapolated value of $[C_T]_0^2$ was used in place of $[|C|C_T]_0$.

5. Results

The results of the present computations are presented in table 1. Comparison of the values in the last two columns shows that, within the limits of

TABLE 1. Estimates of the angular momentum transport and frictional drain based on the composite wind charts.
1 unit = 10^{22} gm cm^2 sec^{-2}.

$\Delta\phi$	Horizontal-eddy transport	Vertical-eddy transport	Total eddy transport	Frictional drain
2°	+0.2	+ 3.5	+ 3.7	+ 3.1
3°	+0.8	+ 6.9	+ 7.7	+ 6.8
4°	+2.9	+10.1	+13.1	+11.6
5°	+3.7	+10.3	+14.1	+17.6
6°	+6.9	+ 9.0	+15.8	+23.9

accuracy of the measurements, the present estimates of the angular momentum transport satisfy the estimated frictional requirements. Owing to the uncertainties involved in the measurement of friction, this agreement does not, however, rule out the possibility that features which do not show up on the composite wind charts also transport significant amounts of angular momentum. Such transports may, in fact, account for the larger discrepancies at $\Delta\phi = 5°$ and $\Delta\phi = 6°$.

Turning to the first two columns we find that at small values of $\Delta\phi$ the total transport of angular momentum is accomplished almost entirely by vertical eddies. This is in agreement with generally accepted ideas concerning the hurricane mechanism. With increasing distance from the center of the hurricane, however, the horizontal-eddy transport takes on greater and greater relative importance. This process continues to increase in magnitude out to the limit of the data, whereas the vertical-eddy transport appears to reach a peak somewhere in the range $4° < \Delta\phi < 5°$ and then decrease. At $\Delta\phi = 6°$ the difference in magnitude between the two eddy processes is no longer very great. Thus, horizontal eddies, which transport only small amounts of angular momentum into the hurricane at small values of $\Delta\phi$, seem to play a significant role in the transport process at greater distances from the hurricane center.

6. Hurricane Connie, 1955

The writer has examined data from several recent hurricanes in an effort to find more direct evidence relating to the relative importance of the two eddy processes at various distances from the center of the hurricane. Although wind observations at upper levels in individual hurricanes are too few to permit their use in quantitative studies, sea-level data coverage is sometimes adequate for such purposes. This was the case in hurricane Connie on 9 August, 1955, at 1830 GCT. It was decided, therefore, to utilize these data to investigate further into the nature of the angular momentum transport in the hurricane.

The transport of angular momentum per unit mass at a single level, say sea level, may be expressed in the form,

$$\oint_s C_T C_N R^2 d\theta = 2\pi R^2 [C_T]_s [C_N]_s + 2\pi R^2 [C_T' C_N']_s = \boldsymbol{\tau} + \boldsymbol{\tau}', \quad (4)$$

where the subscript, s, signifies that the quantity is measured at sea level. Here $\boldsymbol{\tau} \equiv 2\pi R^2 [C_T]_s [C_N]_s$ represents that fraction of the sea-level transport which is associated with a net mass flux at that level, and $\boldsymbol{\tau}' \equiv 2\pi R^2 [C_T' C_N']_s$ represents the sea-level contribution to the horizontal-eddy transport. The integral, $\boldsymbol{\tau}$ can be resolved further into the sum of three components, namely

$$\tau_1 \equiv 2\pi R^2 [C_T]_s'' [C_N]_s'', \quad \tau_2 \equiv 2\pi R^2 \overline{[C_T]} [C_N]_s''$$

and

$$\tau_3 \equiv 2\pi R^2 (\overline{[C_T]} + [C_T]_s'') \overline{[C_N]}.$$

τ_1 measures the sea-level contribution to the vertical-eddy transport; τ_2 measures that portion of the sea-level transport which is compensated at other levels due to an oppositely directed mass flux at those levels; and τ_3 measures the contribution to the sea-level transport due to an accumulation of mass in the volume.

In order to determine the relative importance of vertical and horizontal-eddy processes at sea level we should compare the magnitude of τ_1 with $\boldsymbol{\tau}'$. With the use of data at a single level, however, only $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$ can be evaluated, whereas τ_1, τ_2 and τ_3 cannot. In practice, therefore, we must find an alternate method of obtaining the desired information. One such method consists in reducing the relationship between τ_1 and $\boldsymbol{\tau}$ to a more direct one, and then comparing the latter with $\boldsymbol{\tau}'$. This may be accomplished with the aid of the following considerations:

In the first place, it may be noted that the integral τ_3 is negligible in comparison with τ_1 and τ_2, since it depends on a net mass flux into the volume. This component could, however, give an erroneous contribution to $\boldsymbol{\tau}$ if incomplete or inaccurate data are used to evaluate $\boldsymbol{\tau}$. Since data coverage in hurricane Connie was quite good, however, we shall assume that the errors are small, and regard the measured value of $\boldsymbol{\tau}$ as being given by the sum of τ_1 and τ_2 only.

According to fig. 3, the distribution of $[C_T]$ in the mature hurricane is such that $[C_T]_s''$ and $\overline{[C_T]}$ (and therefore τ_1 and τ_2) are of the same sign within a substantial distance from the center of the hurricane. Thus, $\boldsymbol{\tau}$ must be of the same sign as τ_1 and greater in magnitude than it. This relationship will enable us to gain information about the comparison between τ_1 and $\boldsymbol{\tau}'$ by comparing, instead, $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$. The latter comparison is made in table 2. On the basis of the

Table 2. Comparison of $\boldsymbol{\tau}$ and $\boldsymbol{\tau}'$ in hurricane Connie. 1 unit = 10^{19} cm^4 sec^{-2}.

$\Delta\phi$	2°	3°	4°	5°	6°	7°	8°	9°	10°
$\boldsymbol{\tau}'$	− 1.5	− 1.8	− 2.2	+ 6.3	+ 6.6	+10.0	+15.9	+20.1	+47.7
$\boldsymbol{\tau}$	+22.6	+38.1	+40.2	+31.6	+32.3	+29.9	+27.4	+28.0	+20.3

results the following statements may be made: 1. *Within a few hundred kilometers from the center of the hurricane, at sea level, vertical-eddy processes are of the correct sign to account for the inward transport of angular momentum, whereas horizontal-eddy processes are of the wrong sign. 2. Beyond a certain point, the horizontal-eddy processes increase consistently with distance from the hurricane center, and finally become the primary agency for the inward flux of angular momentum.*

The distances over which the above statements apply are evidently greater in the case of the sea-level quantities in table 2 than they are in the case of the vertically integrated quantities in table 1. This is consistent with the fact that cyclonic winds at sea-level extend over a greater distance than does the vertically integrated cyclonic core of the hurricane.

Fig. 5, which shows the surface streamline chart for hurricane Connie, reveals the synoptic features which are responsible for the observed horizontal and vertical-eddy transports of angular momentum. In this figure the circle represents the intersection of the vertical wall $\Delta\phi = 12°$ with the ground. Within about 8° lat from the center the hurricane is characterized by an extensive cyclonic circulation with a net radial inward component of motion near the surface. This inward spiraling air constitutes the lower branch of the vertical-eddy circulation. In the region surrounding this circulation, the character of the flow is very different, however. Here the exchange of mass between the hurricane and its surroundings takes place predominantly in the horizontal plane. The main synoptic features which effect this exchange in the present case are two large anticyclones—one centered in the vicinity of Portland, Maine and the other in the mid-Atlantic Ocean—and a minor anticyclonic cell located near Chattanooga, Tennessee. The significant characteristic of these anticyclonic circulations is their orientation with respect to concentric circles about the center of the hurricane. As a result of the "tilt" of these systems, mass which enters the hurricane at sea level carries with it a greater quantity of absolute angular momentum than mass which leaves the hurricane *at the same level.* It can be seen, for example, that air entering the circle from the east and southeast possesses a cyclonic component about the hurricane axis, while air leaving in the west possesses an anticyclonic component about the axis. In this way cyclonic angular momentum is transported into the circle.

7. Discussion

The results of the present investigation serve to indicate the nature of some of the important links in the angular momentum cycle. In order to see how these fit together let us trace the course of the angular momentum transfer in the mature hurricane with the aid of the various figures and tables presented in the previous sections. In the first place we may note that the frictional transfer of cyclonic angular momentum from the earth into the atmosphere takes place in the outer branches of the circulation around each of the large-scale high-pressure cells which border the hurricane, where the surface winds possess an anticyclonic component about the axis of the hurricane (fig. 4). A certain portion of the angular momentum which is received by the atmosphere in these regions must be transported into the hurricane in order to maintain the rotation against frictional dissipation. The rest must

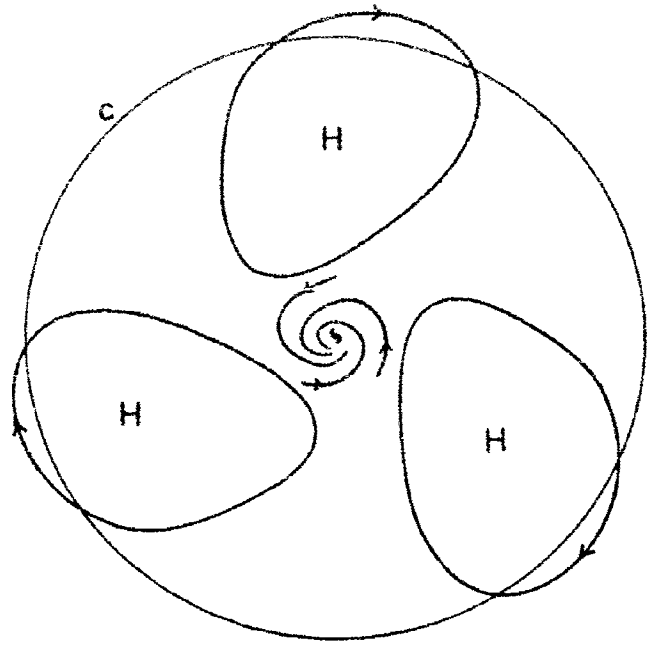

FIG. 4. Idealized picture showing the hurricane vortex and associated high-pressure cells. The tangential component of the motion around circuit, C, is in the anticyclonic sense.

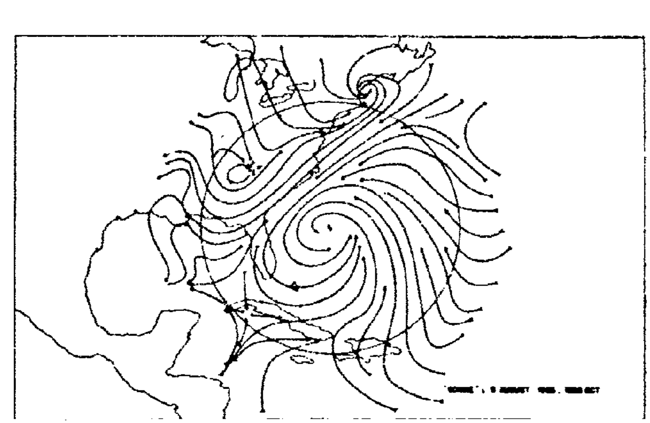

FIG. 5. Sea-level streamlines: hurricane "Connie," 1830 GCT 9 August 1955.

be carried away into the surrounding atmosphere. Focusing attention on that portion which serves to maintain the hurricane circulation, we find that it is transported from the source region into the cyclonic core primarily through the agency of horizontal-eddy processes (tables 1 and 2). This transport is associated with certain asymmetries in the flow around each of the neighboring high-pressure cells (fig. 5). The supply of angular momentum which is accumulated in this way is then transported closer to the center of the hurricane by a vertical-eddy circulation which operates within the cyclonic core. This circulation maintains the high wind speeds immediately surrounding the eye of the hurricane.

As an additional consideration it should be remarked that, in order to satisfy the frictional requirements, the angular momentum transport must increase with distance from the center of the hurricane within the cyclonic core, and decrease with distance from the center within the anticyclonic belt. The maximum transport is, therefore, found somewhere between these two regions.

The significance of the present results can best be appreciated when viewed in the light of the recent theoretical and experimental work dealing with rotating fluids (see for example Kuo, 1956b, and Fultz, 1953). The reader will recall that the type of process which accomplishes the required transports of angular momentum and energy in the rotating model experiments depends on the rate of rotation of the vessel and on the intensity of the radial heating gradient. Specifically, it is found that, when the heating is intense and the rotation rate small, free vertical-eddy convection develops and dominates the flow. This is characteristic of the so-called "Hadley regime," in which vertical-eddy processes serve as the primary agency for the transport of angular momentum and energy. On the other hand, when the heating is weak and the rotation rate large, the required transports are accomplished principally through the agency of horizontal-eddy processes. This is characteristic of the "Rossby regime." In the hurricane, we find that a combination of vertical and horizontal eddies is required to accomplish the necessary transports of angular momentum. The hurricane thus possesses characteristics of *both* the Hadley and the Rossby regimes.

A legitimate question might be raised concerning the validity of the analogy between the convective regimes of the hurricane and those of the model experiments, since the manner in which the heating is controlled is quite different in the former from what it is in the latter. In the way of justification for this approach, it may be remarked that the type of regime which develops at a given rate of rotation seems to depend on the *magnitude* of the heating gradient rather than on the way in which the heating is controlled.

8. Concluding remarks

From time to time theoretical models will be proposed which attempt to explain the mechanism of the hurricane in terms of cause and effect. It will be necessary to find some criterion by which to judge the validity of the conclusions reached on the basis of such models. Regardless of any other considerations it may be stated that if the model satisfies the principles of conservation of mass, momentum and energy *in the same way that the atmosphere does* then it constitutes a valid explanation of the behavior of the atmosphere. Thus, the burden of proof rests on studies in which an effort is made to determine, quantitatively, the physical processes which take place in the atmosphere. The present investigation represents an attempt in this direction. Owing to the limitations of the data it must, however, be regarded as merely a first look at the hurricane problem. It is hoped that more accurate and complete data will soon become available so that additional investigations of the angular momentum and energy budgets can be undertaken, for it is only after large quantitites of data have been processed, and the results interpreted properly, that firm conclusions concerning the hurricane mechanism can be reached.

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I. THE MOMENTUM BUDGET

C. Model Experiments

THE FLUX OF ANGULAR MOMENTUM IN ROTATING MODEL EXPERIMENTS

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ABSTRACT

The flux of angular momentum in rotating model experiments, in which relative motions (in water) are generated by differential heating, is investigated observationally. For appreciable rates of rotation a close similarity is obtained with the atmosphere at the jet-stream level, not only in regard to the appearance of the flow patterns, but also in the mode of poleward transfer of angular momentum as measured quantitatively.

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1. INTRODUCTION

The aim of this discussion is to describe a study of the flux of angular momentum in an experimental model of fluid flow, involving a rotating system in which the relative motions are generated by differential heating. The experiments were performed by R. R. Long at the hydrodynamics laboratory of the University of Chicago. Inasmuch as a complete account of the technical aspects of the experiments has been presented elsewhere by Long (1951a) and Fultz (1951), only a brief general statement is made here concerning the form of the model used. In addition to the descriptive results outlined in the reference already given and partly repeated below, it seemed desirable to investigate the radial transport of angular momentum in the flows obtained by the same methods as were used by Starr and White (1951) for the analogous purpose in the atmosphere, in order to observe such similarities or differences as might be revealed in the two instances.

2. THE MODEL

The apparatus consisted of a circular vessel (actually a dishpan) mounted on a rotating table so that a given angular velocity could be maintained during the course of each experiment. The vessel was heated by means of an electrical resistance coil located underneath it and forming a flat annular ring, whose outside diameter was equal to that of the vessel and whose inside diameter was somewhat smaller. The vessel was filled with water to a given depth and rotated until solid rotation was established before the heating was begun. The motion at the free surface was followed by means of a tracer consisting of aluminum powder sprinkled upon the water.

In order to observe the motions of the fluid relative to the rotating pan directly, the experiments were viewed, and also photographed, with the aid of a rotoscope, Long (1951b). When properly adjusted, this piece of equipment enables the observer to eliminate the basic rotation from the image seen. Thus the pan itself appears in the image with zero rotation, and the tracer indicates only the relative motions.

Streak photographs of the relative motion were obtained at the rate of approximately one per revolution by using one-half second time exposures. A strong flash of illumination at the end of each time exposure was used to indicate the direction of the motion given by the streaks.

The pan was 15 cm in radius and was filled to a depth of 2 cm in all of the experiments. The temperature difference between the rim and the center was recorded continuously during the course of each experiment. In all, 108 usable photographs were obtained. In all cases the sense of rotation was counterclockwise when viewed from above, thus being analogous to that of the atmosphere in the northern hemisphere. For reasons of simplicity, meteorological and geographical terms will be used in the descriptions which follow.

3. QUALITATIVE RESULTS

It has been pointed out by Long (1951), that two characteristic circulation regimes are observed which depend on the rate of rotation of the dishpan. The streamline patterns obtained in the cases of high rotation exhibit a striking resemblance to the corresponding atmospheric flow patterns at the jet-stream level, as shown by appropriate northern hemisphere isobaric charts (see Fig. 18). The most notable characteristic

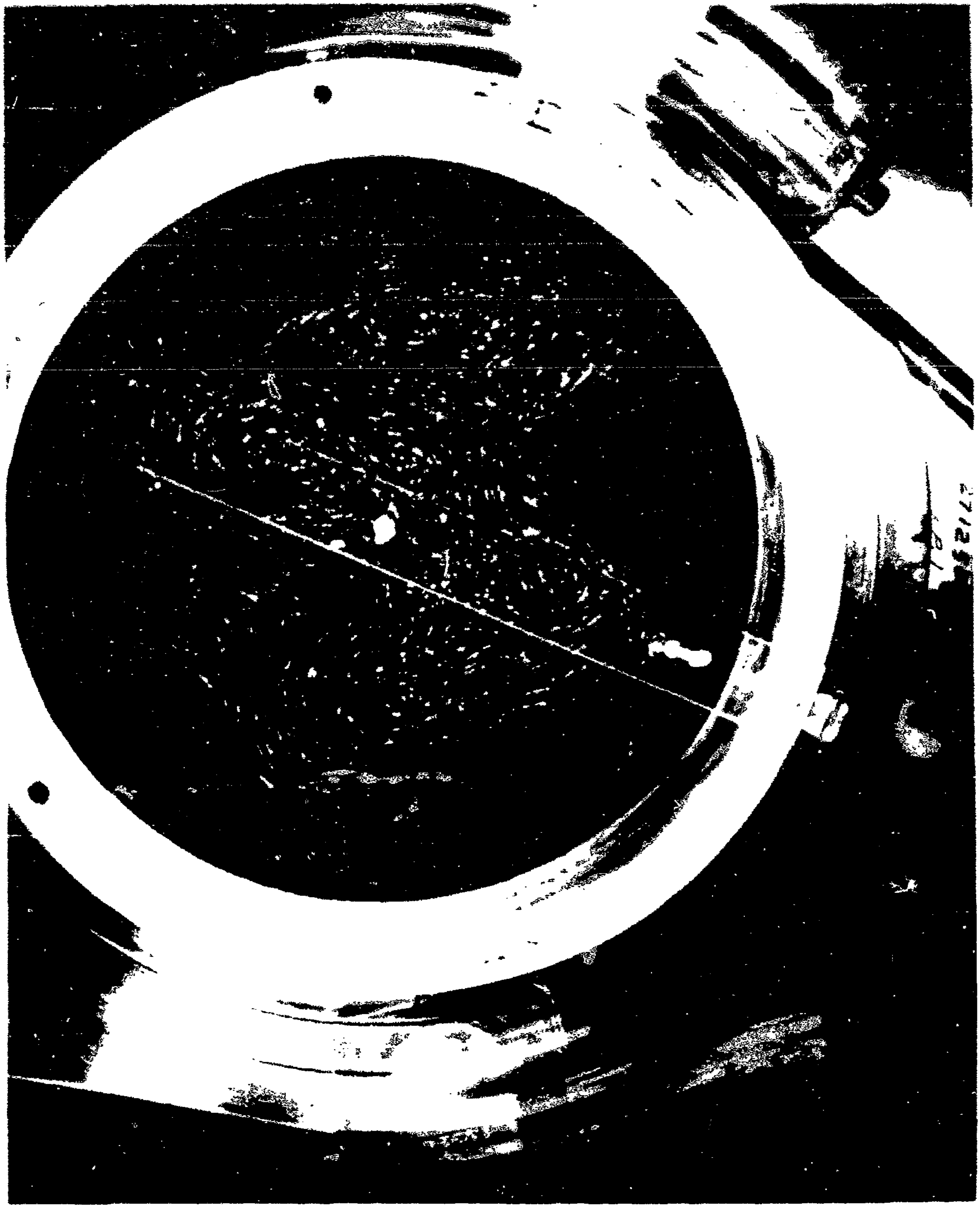

Fig. 18. A high-rotation (4.81 rpm) streak-flash photograph of fluid motions induced in a rotating cylinder heated at the perimeter. The flow patterns bear resemblance to those found in free atmosphere.

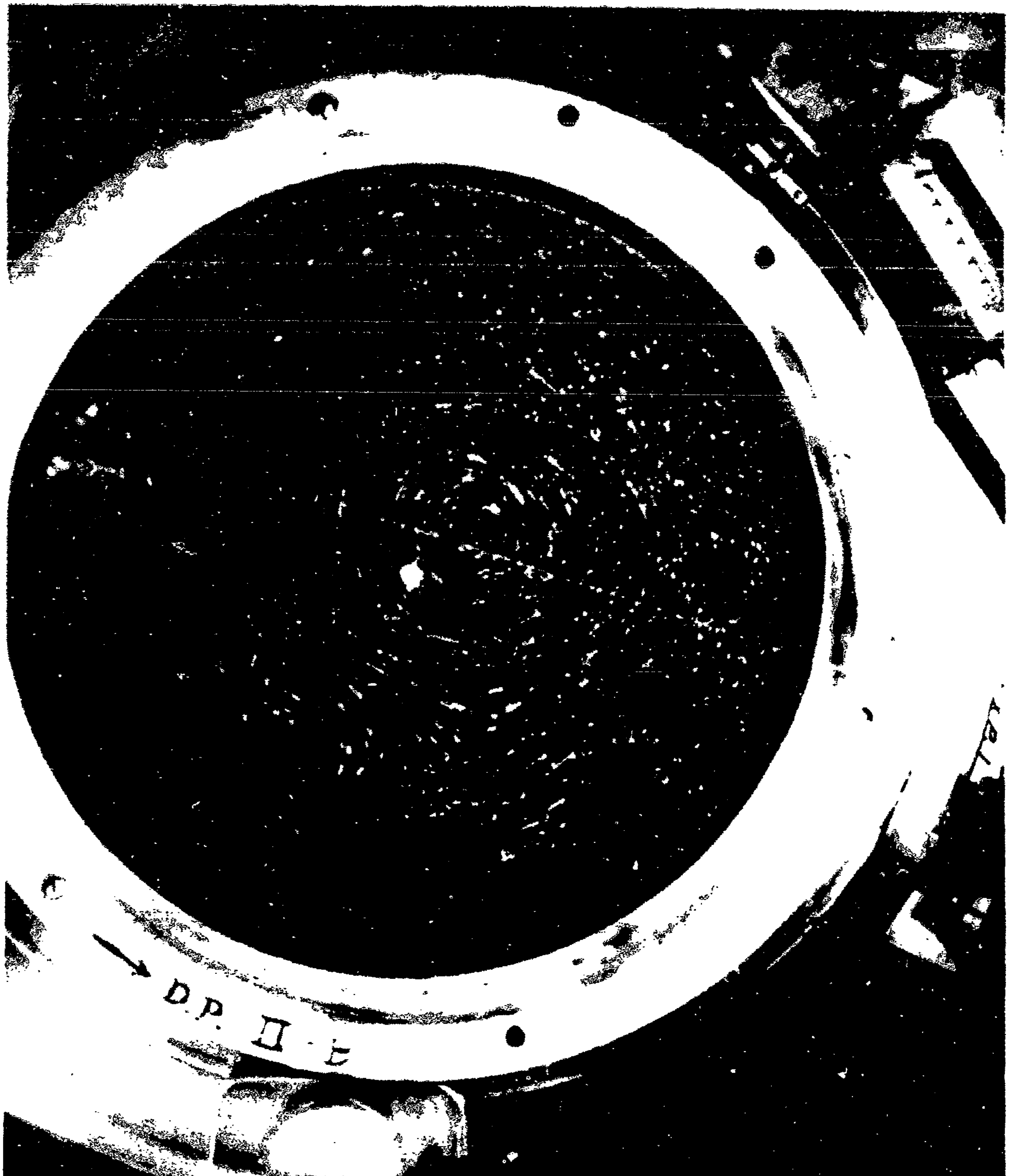

Fig. 19. A low-rotation (2.92 rpm) streak-flash photograph of fluid motions induced in a rotating cylinder heated at the perimeter. The flow pattern is more symmetrical than in Fig. 18.

is the presence of a meandering jet stream of westerlies, with several large wave-like undulations progressing slowly eastward in general. The wave number and ratio of amplitude to wavelength were about the same as in the atmospheric case. Occasionally the loops in the jet tend to pinch off and form closed cyclonic and anticyclonic centers which persist for some time. The zonal motion is from west to east, on the average, from the pole to the equator (rim), although strongest in the middle latitudes.

The circulation in the high rotation case is strongly influenced by varying the rotation and the intens of heating. The particular choice of about 4 rpm for the given amount of heating was not accidental. effort was made to choose a condition similar to that at high levels in the middle and high latitudes of t northern hemisphere. Two criteria were used to obtain this similarity. The first involved an equality model and prototype of the nondimensional number $R = U/fL$, where U is a representative relative veloci L is a representative length, and f is the Coriolis parameter. This number is the ratio of inertia force Coriolis force, and its equality is a necessary but not a sufficient condition that two rotating fluid systems dynamically similar. In this application U was taken to be a representative velocity in the jet stream and was the radius of the pan (distance from pole to equator on the earth).

There was some difficulty in deciding the best value of rotation for the experiment. A prelimina choice indicated a rotation of 5 rpm for the cylinder. The second criterion was then employed, namel that the westerly jet have a mean position at about 10 cm from the center and that the general appearan of the motion be similar to that in the high troposphere. If it is permissible to associate this circle with tl 30° latitude circle on the earth, the jet would then occupy a similar position in model and prototype. / 5 rpm it was found that the jet remained too close to the edge of the pan. The rotation was then decrease to about 4 rpm, and from qualitative visual observation it appeared that the jet was at the proper positio After the numerical computations were completed, however, it was found that the latitude of the mean j was still farther south than anticipated. Further experimentation with a rotation rate below 4 rpm woul therefore be desirable.

It is of considerable interest to investigate the value of R at this rotation. The velocity in the jet wa then close to 0.75 cm sec^{-1}, f was 0.8 sec^{-1}, and L was 15 cm. The ratio R was therefore 0.06 for the mode If the corresponding atmospheric quantities are taken as $U = 6 \times 10^3$ cm sec^{-1}, $f = 10^{-4}$, and $L = \frac{1}{2}$ $\times 6.4 \times 10^8$, the two values of R become nearly equal. The two uncertain quantities in the atmospher calculation, U and f, are nevertheless both very reasonable. The implication is that the two criteria a not independent and that kinematic similarity in the flow pattern is a consequence of similarity in th characteristic ratio R.

In the cases of low rotation the flow patterns again show the presence of mean westerlies from the pol to the equator (see Fig. 19). However, the jet stream is now replaced by a broad maximum at somewh less than one half the distance from the pole to the rim, and there is little evidence of the large meandering to be seen, although some lack of radial symmetry about the pole is usually present. At the time of th writing, only a single sequence of 6 pictures was available for study. Any conclusions based upon such small sample would certainly be questionable. Therefore, the remainder of the discussion will be confine to the high rotation case.

4. QUANTITATIVE DATA

For purposes of securing quantitative data for use in calculating the transfers of angular momentum, grid of six latitude circles and sixteen longitude lines was fixed relative to the pan and reproduced on all th

photographs. The latitude circles were taken at the six radii 2½, 5, 7½, 10, 12½ and 13¾ cm, thus corresponding in a sense to 75°, 60°, 45°, 30°, 15° and 7.5° latitude on the earth. The intersections of this grid then provided 88 systematically located "stations" at which the "wind" measurements were made (only 8 longitudes were used at the 2½-cm circle). The wind direction was measured from the streaks in the vicinity of each station and recorded in terms of 36 compass points. Likewise the speed was measured from the length of the streaks and the duration (automatically recorded) of the time-exposures. The speeds were recorded to the nearest hundredth of a centimeter per second. Because of the uneven distribution of the tracer, it was at times impossible to make measurements, but the data were almost perfect in this regard as compared with similar atmospheric actual wind observations. The material was tabulated by "days" and grouped according to the six latitudes for each experiment.

5. COMPUTATION OF TRANSPORTS

The computation of the transports of angular momentum follow along the plan used by Starr and White (1951, 1952). Since these methods and the considerations on which they are based have already been expounded on previous occasions, it is assumed that readers who desire a detailed exposition will refer to the papers in question. It may nevertheless not be out of place to state that in the scheme u and v denote the eastward and northward components of wind velocity, the square brackets denote instantaneous space averages along the length of a complete latitude circle, bars indicate time averages and primes are used for departures from the averages. Curly brackets signify arithmetic means over the total number N of observations at a given latitude during the course of an experiment or group of experiments. The coefficient of linear correlation between N pairs of u and v is denoted by r, and the number of photographs from an experiment or group of experiments is given by n.

The total transport of linear eastward relative momentum, per unit length of the latitude circle and per unit mass in the vertical $\overline{[uv]}$, is expressible as the sum of the three terms $\overline{[u]}\overline{[v]}$, $\overline{[u]'[v]'}$ and $\overline{[u'v']}$ called the transports of the first, second and third species, respectively. These three quantities have distinct physical meanings useful for the present purpose. They are discussed further below and also in the references already given. The total transport may also be calculated differently, as a check, in the form $\{u\,v\}$. In this form it may be resolved into the two terms $\{u\}\{v\}$ and $\{u\,v\} - \{u\}\{v\}$. The first is analogous to the transport of the first species and the second is analogous to the sum of the transports of the remaining two species. This equivalence becomes necessarily exact when no individual wind reports are missing.

6. NUMERICAL RESULTS

In view of the fact that it is desired to compare the results with the corresponding atmospheric findings, the form of Table 4 is basically similar to Table 3 given by Starr and White (1951) and also to Tables 2, 3 and 4 given by the same authors (1952). The main differences are in the velocity units, the inclusion of five latitudes in the place of one and the separation by individual experiments. The laboratory results are also necessarily for the free surface only.

For the sake of added information, the rates of rotation and the average temperature differences between the rim and the center are given for all the experiments. Since a more or less steady regime of motion was

Table 4. Numerical analysis of high rotation data. The radius is given in centimeters; all velocities are in centimeters per second.

1	2	3		4		5		6	7	8		9	10	11	12	13	14	15
Radius	$\overline{\{u\}}$	$\overline{\{v\}}$		$\overline{\{u\}\{v\}}$		$\overline{\{uv\}}$		$\overline{\{u\}}\,\overline{\{v\}}$	$\overline{\{u\}'\{v\}'}$	$\overline{\{u'v'\}}$		n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
Experiment 1. Rotation 3.872 rpm. Thermal Gradient 11.4° C. Starting Time 197 sec.																		
2.50	+0.0570	−0.0029		+0.0001		−0.0003		−0.0002	+0.0003	−0.0004		36	+0.0570	−0.0030	−0.0003	−0.0001	−0.01	288
5.00	+0.1296	−0.0081		−0.0008		−0.0020		−0.0010	+0.0002	−0.0013		36	+0.1279	−0.0085	−0.0020	−0.0009	−0.03	568
7.50	+0.2495	−0.0093		−0.0023		+0.0032		−0.0023	0.0000	+0.0055		36	+0.2500	−0.0093	+0.0032	+0.0055	+0.12	571
10.00	+0.4460	−0.0093		−0.0047		+0.0183		−0.0041	−0.0006	+0.0230		36	+0.4460	−0.0093	+0.0183	+0.0224	+0.31	576
12.50	+0.6561	−0.0003		+0.0011		+0.0209		−0.0002	+0.0013	+0.0198		36	+0.6561	−0.0004	+0.0208	+0.0211	+0.38	571
13.75	+0.5616	−0.0056		−0.0035		+0.0070		−0.0031	−0.0004	+0.0105		36	+0.5616	−0.0056	+0.0070	+0.0101	+0.33	540
Experiment 2. Rotation 3.992 rpm. Thermal Gradient 8.1° C. Starting Time 383 sec.																		
2.50	+0.0737	−0.0075		−0.0004		+0.0034		−0.0006	+0.0002	+0.0038		16	+0.0727	−0.0076	+0.0032	+0.0038	+0.09	127
5.00	+0.0879	−0.0209		−0.0012		−0.0015		−0.0018	+0.0006	−0.0003		16	+0.0877	−0.0202	−0.0017	+0.0001	0.00	250
7.50	+0.2235	−0.0098		−0.0017		+0.0080		−0.0022	+0.0005	+0.0097		16	+0.2228	−0.0099	+0.0080	+0.0103	+0.21	252
10.00	+0.4799	−0.0040		−0.0004		+0.0215		−0.0019	+0.0015	+0.0219		16	+0.4798	−0.0039	+0.0215	+0.0234	+0.39	255
12.50	+0.7216	−0.0003		+0.0002		+0.0120		−0.0002	+0.0004	+0.0118		16	+0.7220	−0.0001	+0.0120	+0.0121	+0.21	254
13.75	+0.5670	−0.0050		−0.0026		−0.0022		−0.0028	+0.0002	+0.0004		16	+0.5670	−0.0050	−0.0022	+0.0006	+0.03	240
Experiment 3. Rotation 3.829 rpm. Thermal Gradient 9.2° C. Starting Time 332 sec.																		
2.50	+0.0850	+0.0013		+0.0005		+0.0066		+0.0001	+0.0004	+0.0061		21	+0.0849	+0.0013	+0.0066	+0.0065	+0.17	168
5.00	+0.1976	+0.0053		+0.0017		+0.0068		+0.0010	+0.0007	+0.0051		21	+0.1976	+0.0051	+0.0068	+0.0057	+0.07	336
7.50	+0.2847	+0.0023		+0.0007		+0.0144		+0.0007	0.0000	+0.0137		21	+0.2845	+0.0019	+0.0140	+0.0135	+0.24	332
10.00	+0.4317	−0.0226		−0.0101		+0.0168		−0.0098	−0.0003	+0.0269		21	+0.4316	−0.0226	+0.0169	+0.0267	+0.26	333
12.50	+0.7242	−0.0009		−0.0014		+0.0232		−0.0007	−0.0007	+0.0246		21	+0.7243	−0.0012	+0.0229	+0.0238	+0.28	331
13.75	+0.6830	−0.0027		−0.0018		+0.0094		−0.0018	0.0000	+0.0113		21	+0.6830	−0.0027	+0.0093	+0.0111	+0.25	313
Experiment 4. Rotation 3.858 rpm. Thermal Gradient 7.2° C. Starting Time 390 sec.																		
2.50	+0.0743	−0.0034		−0.0011		+0.0006		−0.0003	−0.0008	+0.0017		35	+0.0745	−0.0036	+0.0006	+0.0009	+0.02	276
5.00	+0.1441	+0.0015		+0.0001		+0.0068		+0.0002	−0.0001	+0.0067		35	+0.1442	+0.0014	+0.0069	+0.0067	+0.13	552
7.50	+0.2262	−0.0031		−0.0006		+0.0160		−0.0007	+0.0001	+0.0166		35	+0.2262	−0.0031	+0.0160	+0.0167	+0.32	549
10.00	+0.3316	−0.0055		−0.0025		+0.0242		−0.0018	−0.0007	+0.0267		34	+0.3300	−0.0055	+0.0241	+0.0259	+0.31	537
12.50	+0.8103	+0.0024		+0.0026		+0.0333		+0.0019	+0.0007	+0.0306		35	+0.8107	+0.0024	+0.0332	+0.0313	+0.37	551
13.75	+0.7206	−0.0093		−0.0080		+0.0055		−0.0067	−0.0013	+0.0135		35	+0.7206	−0.0093	+0.0055	+0.0122	+0.27	525
Combined Total																		
2.50	+0.0705	−0.0029	±0.0051	−0.0003	±0.0006	+0.0019	±0.0025	−0.0002	−0.0001	+0.0021	±0.0024	108	+0.0704	−0.0030	+0.0019	+0.0021	+0.05	859
5.00	+0.1413	−0.0043	±0.0069	−0.0001	±0.0011	+0.0026	±0.0031	−0.0006	+0.0005	+0.0027	±0.0029	108	+0.1410	−0.0043	+0.0026	+0.0032	+0.06	1706
7.50	+0.2449	−0.0051	±0.0051	−0.0011	±0.0013	+0.0102	±0.0041	−0.0012	+0.0001	+0.0113	±0.0038	108	+0.2450	−0.0052	+0.0102	+0.0115	+0.23	1704
10.00	+0.4119	−0.0099	±0.0067	−0.0044	±0.0030	+0.0203	±0.0048	−0.0041	−0.0003	+0.0248	±0.0037	107	+0.4117	−0.0099	+0.0203	+0.0244	+0.30	1701
12.50	+0.7290	+0.0004	±0.0062	+0.0010	±0.0046	+0.0240	±0.0060	+0.0003	+0.0007	+0.0231	±0.0037	108	+0.7291	+0.0004	+0.0239	+0.0236	+0.33	1707
13.75	+0.6375	−0.0061	±0.0043	−0.0045	±0.0031	+0.0056	±0.0043	−0.0039	−0.0006	+0.0101	±0.0032	108	+0.6374	−0.0061	+0.0056	+0.0095	+0.25	1618

not established until the lapse of some time after heating was begun, the actual taking of photographs was therefore likewise delayed for several minutes. The time interval between the instant when heating was begun and when the first photograph was taken is given as the "starting time" for each experiment.

After protracted periods of time the surface of the fluid becomes contaminated and the increase in surface tension interferes with the free motion of the tracer particles. This happens quite suddenly and is easily recognized in the photographs. In order to obtain a long period average, the individual experiments were combined at the bottom of the tables. This introduces some minor questions in regard to the transports of the second species which involve a temporal correlation between $[u]$ and $[v]$, but since the experiments show a good degree of reproducibility this procedure probably is otherwise free from serious objection. Confidence limits of the time means of the various quantities defined as twice the standard error are given for these combined totals.

7. DISCUSSION OF NUMERICAL RESULTS

Many features of the results are apparent immediately from an inspection of Table 4. The more important of these are the following.

(a) There is a tendency for the quantities $[v]$ and $\{v\}$ to be negative implying, from continuity considerations, a slight net northward return flow at lower levels.

(b) There is, except for a few instances in individual experiments, a net flow of relative linear momentum $\overline{[u\,v]}$ toward the north at all latitudes. This is due mainly to the transport of the third species, i.e., $\overline{[u'\,v']}$. Since the transport of the second species is almost zero, this contrast is reflected also in the values of $\{u\,v\} - \{u\}\,\{v\}$.

(c) Aside from considerations of absolute magnitude of the velocities, comparison with the numerical results for the atmosphere at latitude 30° N, as obtained in the references quoted, reveals striking similarities. Thus the data at the radius of 10 cm show: (1) a relatively strong positive correlation between u and v, (2) a strong northward total flux of momentum and (3) a predominance of the transport of the third species in this poleward momentum flux. All four of these characteristics are shown also by the atmospheric data at the jet-stream level.

(d) The individual experiments give results that are essentially the same, showing that a necessary condition for the experiments has been achieved, namely, reproducibility.

8. INTERPRETATION OF RESULTS

Assuming that the numerical findings reflect the actual physical behavior of the upper strata of the fluid, the following suggestions appear to be conveyed by them.

(a) When the rotation is high, the convection does not follow a simple scheme, but instead a circulation regime ensues characterized by quasi-horizontal exchanges represented by the large-scale meanderings of the

jet stream. This reflects an important characteristic of rotating fluids. If the relative velocities are small compared to the basic rotations, i.e., if R is small, the motion tends to be two-dimensional When R becomes large, however, as in the low rotation case, vertical motions may increase. These tendencies have been investigated theoretically and experimentally by Proudman (1916) and Taylor (1923).

(b) In both high and low rotation cases, westerlies develop at the top. With low rotation these seem to be dependent upon the Coriolis forces associated with the net poleward mass flux. In the high rotation case they are probably due to Reynolds stresses associated with the quasi-horizontal exchanges, and not so simply related to the Coriolis forces.

(c) Since it would be impossible for mean westerlies to prevail throughout the fluid in a quasi-steady regime, it therefore follows that there should be easterlies at some latitudes next to the bottom or at the rim at lower levels.

(d) The first two experiments with high rotation show transports of angular momentum southward across the 5-cm circle, although the combined average does not show this feature. It is interesting on this account to speculate whether polar easterlies may at times be present at the bottom near the pole, but no definite conclusions can be reached from the data in this regard.

9. CRITICAL REMARKS

It is a matter of interest to examine the computational procedures used with the view of determining their validity in certain respects, and to note various comparisons which may be made with the corresponding applications of them to the atmospheric case. One may note the following items.

(a) The general smallness of $[v]$ and $\{v\}$ indicate that probably conditions to the rear and in front of troughs (or ridges) were sampled in rather random fashion, as in the atmospheric calculations.

(b) Although the irregular spacing and location of stations is a critical problem in the atmosphere, no such difficulty arises in the experimental case. One may use a much more detailed grid, the main limitation being the labor involved.

(c) Whereas wind soundings are selective in favor of light wind conditions in the atmosphere, this factor is absent in the experiments. It is true, nevertheless, that some "reports" were missing in the laboratory experiments as already explained, but it is most unlikely that this situation is associated with selective effects (at some latitudes the data were actually complete for certain individual experiments).

(d) The upper surface of the water is a special level at which there is a large density discontinuity, and at which surface tension phenomena are present. It may be that these factors are of some importance in influencing the motions. More experimentation involving such modifications as the measurement of velocities slightly below the surface are needed in order to throw light upon these questions.

(e) Because of the fact that the data suffer less from missing reports than for the atmospheric case, various quantities calculated may exhibit serial correlation in time so that the confidence limits computed are too small. The quantities $[v]$, $[\bar{u}\, v]$, $[u][v]$, $[u'\, v']$ give correlations at one lag for the 10-cm circle of -0.11, $+0.10$, -0.09 and $+0.23$, respectively, showing that there may be some tendency for persistence, but not enough to warrant more elaborate computation of the limits.

10. GENERAL COMMENTS

Several broad topics arise naturally from the subject dealt with in this paper. Two of these are especially worthy of note. For years the question as to whether or not laboratory experiments could be devised which would duplicate in some measure the large-scale processes of the atmosphere has been discussed by meteorologists. Many have evinced varying degrees of despair concerning such possibilities because of the numerous difficulties in reproducing natural conditions. Others have nevertheless proceeded in the hope that similarity in certain features could be achieved, and consequently there is a growing literature on the subject. The studies here described form a continuation of these developments.

The reader can easily supply almost any number of items of dissimilarity between our experiments and the atmospheric processes. This, however, does not rule out the possibility that certain important elements of the basic dynamic mechanism of the general circulation have been captured in the experiments. The general appearance and behavior of the flow patterns, together with the numerical results concerning the flow of angular momentum for the two systems, constitute prima facie evidence of similarity in these respects, although much more must be learned from further experimentation, principally as to the field of motion in the strata of the fluid beneath the upper surface.* Furthermore, one can, at the present moment, merely raise the question as to whether more experimentation might not reveal similarities with respect to smaller synoptic features, such as occluding cyclones with axes tilted westward in the vertical and anticyclones of the cold and warm type, etc.† Moreover, such additional exploration is not attended by difficulties that are beyond the scope of a well-equipped physical laboratory, so that additional information is to be expected without undue delay. It will be of considerable interest, therefore, to see to what added extent the large-scale atmospheric phenomena can be studied, so to speak, in vitro and to note whether our understanding of the natural processes is thereby enhanced.

As mentioned previously, the results for high rotation suggest that in this regime the mean relative zonal motions are maintained by eddy processes. In this role the eddy processes must then act in a sense contrary to that of genuine viscosity, for genuine viscosity can only tend to produce rotation of the fluid as a solid. It is therefore quite inappropriate to use the concept of a virtual viscosity in order to describe these eddy processes, unless one is prepared to admit the existence of a negative virtual viscosity in certain portions of the system, Kuo (1951). These considerations point to a source of danger in many customary methods of dealing with the large-scale eddy processes in the atmosphere. This could lead to much needless confusion, if the basic assumption involved is not first subjected to sufficiently close scrutiny.

* As in the atmosphere one may define a system of equipotential surfaces within the fluid used in the experiments. In the direction normal to these surfaces (very nearly in the vertical) the counterpart of the thermal wind relationship should be present. A preliminary check as to orders of magnitude suggests that the temperature difference from the rim to the center is accompanied by the proper increase of the mean zonal westerlies from zero at the bottom to the observed values at the free surface in the jet stream.

† Since writing this paper, Dr. Fultz has been able to reproduce such systems.

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II. THE ENERGY BUDGET & RELATED SUBJECTS

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ON THE PRODUCTION OF KINETIC ENERGY IN THE ATMOSPHERE

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(Manuscript received 25 March 1948)

ABSTRACT

In the present paper the production of kinetic energy in the atmosphere is examined from a hydrodynamical point of view. The results indicate that the intensity of the primary source of horizontal kinetic energy at any point in the atmosphere is equal to the pressure multiplied by the horizontal divergence of the velocity. Regions of horizontal velocity convergence appear as hydrodynamical sinks for kinetic energy, in addition to frictional effects. It is found that kinetic energy may be transferred through advection and through work done by pressure forces. It appears that diverging anticyclones are of primary importance in providing kinetic energy for the general circulation.

1. Introduction

One of the basic problems in the science of meteorology relates to the manner in which thermal energy received by the atmosphere through short-wave solar radiation becomes in part transformed into kinetic energy of motion relative to the rotating earth. Plausible estimates show that the fraction of the total energy so transformed is very small, but must nevertheless be sufficient to account for all air motions, in the absence of any other significant energy sources. Since the kinetic energy of organized motions is continually degraded and ultimately dissipated by turbulence and viscosity, the process of kinetic energy production must be a continuous one with, probably, certain fluctuations about a mean rate when the whole atmosphere is considered. The purpose of this paper is to examine this production process from a hydrodynamical point of view.

Changes in the kinetic energy of a particle or system of particles can result only from the action of mechanical forces, and hence the rate of kinetic-energy production can be discussed in terms of the joint action of such forces and the kinematics of existing motions. In this light it is not essential to inquire how systems of such forces and such motions in the atmosphere are related to the thermodynamical processes which are ultimately responsible for their existence. In order to demonstrate the particular point in question as simply as possible we shall first consider an example of fluid motion under somewhat artificial circumstances, but still having theoretical interest. In view of the fact that the kinetic energy of vertical motions in the atmosphere is very small compared with the kinetic energy of the large-scale horizontal motions, we shall consider only the latter.

The approach used is one suggested by the beautiful classic paper of Osborne Reynolds (1895) entitled "On the dynamical theory of incompressible viscous fluids and the determination of the criterion." Whereas Reynolds was concerned only with the dissipation of kinetic energy, his treatment must be modified in order to envisage also the process which creates kinetic energy. For this reason his assumption of incompressibility will be abandoned. Also, our restriction to the study of the kinetic energy of horizontal motions introduces certain changes, although these changes are not actually in the nature of approximations.

2. Study of a simple system

Let it be supposed that a mass of gas is confined in a chamber with a plane bottom and vertical walls, under the action of gravity which we assume to be acting vertically downward. If the chamber is of sufficiently great height, it is not necessary that it have a top. Likewise, the gas need not be an ideal one, since no use will be made of an equation of state. Coriolis forces will, for the present, be omitted. Let it be supposed further that the gas is in some state of motion induced by differential heating and cooling.

If we take x, y, z to be a cartesian coordinate system with the positive z-axis vertical, we may write the equations of motion for the horizontal directions in the form

$$\begin{aligned} \frac{du}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + F_x, \\ \frac{dv}{dt} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_y. \end{aligned} \tag{1}$$

Here u, v are the velocity components in the directions x, y; ρ is the density; p the pressure; t time; and F_x, F_y are the components of the viscous forces in the x, y directions. Generally speaking, the motions in the chamber might be turbulent. If we wish to regard the

dependent variables in equations (1) as representing mean values free of the turbulence components, we shall assume that the only change necessary is to include eddy-stress effects in the quantities F_x, F_y after the manner of Reynolds. More will be said concerning this point later.

The kinetic-energy equation corresponding to the system (1) is

$$\rho\frac{\partial}{\partial t}\frac{V_h^2}{2}+\rho u\frac{\partial}{\partial x}\frac{V_h^2}{2}+\rho v\frac{\partial}{\partial y}\frac{V_h^2}{2}+\rho w\frac{\partial}{\partial z}\frac{V_h^2}{2}=-\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right)-d. \quad (2)$$

We use the symbol d to represent the rate at which the turbulence and viscosity are decreasing the kinetic energy per unit volume, and $V_h^2 \equiv u^2+v^2$. It is possible to rewrite (2) in the following form:

$$\frac{\partial E}{\partial t}+\frac{\partial Eu}{\partial x}+\frac{\partial Ev}{\partial y}+\frac{\partial Ew}{\partial z}=-\left(\frac{\partial pu}{\partial x}+\frac{\partial pv}{\partial y}\right)+p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-d, \quad (3)$$

where use has been made of the continuity equation

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}+\frac{\partial \rho w}{\partial z}=0, \quad (4)$$

which in any case must be true, and where $E \equiv \frac{1}{2}\rho V_h^2$ is the horizontal kinetic energy per unit volume. The quantity represented by the last three terms on the left-hand side of (3) is the divergence of the (three-dimensional) kinetic energy transport vector $E\mathbf{V}$. The quantity in the first parentheses on the right is the divergence of the horizontal vector $p\mathbf{V}_h$. If equation (3) is integrated over an arbitrary volume, both of these quantities may be represented as surface integrals with the aid of the divergence theorem. Thus, if the limits are fixed, we may write

$$\frac{\partial}{\partial t}\iiint E\,dx\,dy\,dz=\int EV_n\,dS-\iint p(v\,dx-u\,dy)dz+\iiint p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dx\,dy\,dz-\iiint d\,dx\,dy\,dz, \quad (5)$$

where V_n is taken to be the inward component of velocity at the boundary, and dS is a surface element.

Equation (5) may now be given the following interpretation. The total horizontal kinetic energy (T) in a fixed region may be changing in consequence of:

a. An advection of new fluid having kinetic energy across the boundary. This is represented by the term

$$A=\int EV_n\,dS.$$

This is then one mode of ***redistribution*** of kinetic energy.

b. The performance of work by pressure forces at the boundary in virtue of the displacements due to the horizontal velocity components. This is represented by the term

$$W=-\iint p(v\,dx-u\,dy)dz.$$

This is a second mode of ***redistribution*** of kinetic energy.

c. A production of kinetic energy within the volume itself. This is represented by the term

$$S=\iiint p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dx\,dy\,dz,$$

which contains the ***primary source*** of kinetic energy.

d. The action of frictional forces. This effect would ordinarily consist of a dissipation and is represented by the term

$$D=\iiint d\,dx\,dy\,dz.$$

If the limits of integration include all of the fluid in the fixed chamber, it is clear that the surface integrals must vanish, so that in a *mechanically closed* system (5) reduces to

$$\frac{\partial T}{\partial t}=S-D. \quad (6)$$

Since for such a system the frictional effect would ordinarily lead to dissipation, it follows that S must be positive if the kinetic energy T is to remain constant or increase. If a more or less constant amount of kinetic energy is to be present, the dissipation must be balanced by a corresponding positive average rate of production.

The production S may be looked upon as the integral of the contributions from the various horizontal layers of fluid present and written as

$$S=\int\left[\iint p\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dx\,dy\right]dz. \quad (7)$$

In view of the fact that the surface integral

$$\iint\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dx\,dy$$

must vanish if the horizontal velocity is zero across the fixed walls, it follows that a given horizontal stratum of fluid cannot give a positive contribution to S unless larger values of the pressure p are associated with

areas of horizontal divergence than are associated with areas of convergence. Thus areas of ***horizontal*** divergence represent primary kinetic energy sources while areas of convergence represent sinks for kinetic energy. Furthermore, in a mechanically closed system of the kind here considered it is impossible to have source regions for kinetic energy without at the same time having sinks of a hydrodynamic nature, entirely independent of frictional effects.

3. Equations for the atmosphere

Before embarking upon a discussion of the meteorological implications of the material presented above, it is desirable to develop the concepts involved in more general terms, so as to render it possible to perform integrations over the entire mass of the atmosphere.

To a sufficiently close degree of approximation the shape of the geopotential surfaces may be considered as spherical so that we may make use of spherical polar coordinates in which r is the radius, ϕ is latitude, and λ is longitude. By analogy with the cartesian case we may then write the equations of motion for the horizontal directions (see Brunt, 1939) in the form

$$\left.\begin{aligned} \frac{du}{dt} - \frac{uv}{r}\tan\phi + \frac{uw}{r} + 2\Omega(w\cos\phi - v\sin\phi) \\ = -\frac{1}{\rho}\frac{\partial p}{\partial x} + F_x, \\ \frac{dv}{dt} + \frac{u^2}{r}\tan\phi + \frac{vw}{r} + 2\Omega u\sin\phi \\ = -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_y, \end{aligned}\right\} \quad (8)$$

where u, v, w are the linear velocity components in the northward, eastward, and upward directions, respectively, and x, y are measures of linear distance eastward and northward, respectively; Ω is the angular velocity of the earth. The analogous energy equation in this case may be written as

$$\begin{aligned} \rho\frac{d}{dt}\frac{V_h^2}{2} + \rho\frac{V_h^2}{r}w + 2\rho\Omega uw\cos\phi \\ = -\left(\frac{\partial pu}{\partial x} + \frac{\partial pv}{\partial y} - \frac{pv}{r}\tan\phi\right) \\ + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \frac{v}{r}\tan\phi\right) - d. \quad (9) \end{aligned}$$

Making use of the observational fact that the last two terms on the left-hand side of (9) are of a very small order of magnitude, these terms will be dropped.[1]

[1] In reality these terms represent a conversion of kinetic energy of horizontal motions into kinetic energy of vertical motions, and as such do not involve a production of kinetic energy. Indeed, by methods similar to those used in this paper one can investigate separately the kinetic energy of motions in each of the three directions, namely, zonal, meridional and vertical. In that case other conversion terms of a similar nature arise.

The manipulation of the remaining term on the left side may now be carried out with the aid of the continuity equation much as before, since this operation is independent of the specific coordinate system used, so that we may write

$$\frac{\partial E}{\partial t} + \mathrm{div}_3 EV = -\mathrm{div}_2 pV_h + p\,\mathrm{div}_2 V_h - d. \quad (10)$$

A volume integral of (10) may now be taken and written in the form

$$\begin{aligned} \frac{\partial}{\partial t}\int E\,d\tau = \int EV_n\,dS - \iint p(v\,dx - u\,dy)dr \\ + \int p\,\mathrm{div}_2 V_h\,d\tau - \int d\,d\tau, \quad (11) \end{aligned}$$

where $d\tau$ is a volume and dS a surface element. Equation (11) is physically identical with (5) and has, therefore, the same interpretation. In symbolic form we may write

$$\frac{\partial T}{\partial t} = A + W + S - D, \quad (12)$$

which states that the rate of increase of horizontal kinetic energy for a fixed volume is equal to the net rate of advection of such kinetic energy into the region, plus the rate at which work is being done by the surroundings on the fluid in the region through horizontal motions, plus the production of kinetic energy in the volume, minus the frictional dissipation. For a system which is mechanically closed A and W again vanish. This is therefore true when the entire atmosphere is considered. In this case the surface integral of the horizontal divergence over each closed geopotential surface must vanish as in the case of the chamber previously considered.

4. Conclusions

Although it is possible to form other energy integrals for fluid motion, as pointed out in standard texts on hydrodynamics,[2] the particular merit of the procedure followed above is that the expression for production of kinetic energy assumes a form which is of interest in meteorological problems. The implications of equation (12) may be stated in brief as follows:

a. The intensity of the primary source of horizontal kinetic energy at a given point in the atmosphere is given by the product of the pressure into the divergence of the horizontal velocity.

b. Positive primary sources must always occur in combination with negative sources or sinks independent of frictional effects, when the entire atmosphere is considered.

[2] See, for example, Bjerknes *et al.* (1933).

c. In addition to the action of the sources and frictional effects, the horizontal kinetic energy in a fixed region not embracing the entire atmosphere may change due to advection of kinetic energy across the boundary and due to the redistribution of kinetic energy through the boundary by work done by pressure forces and horizontal velocity components at the boundary.

From the standpoint of the general circulation it would appear that the sources of kinetic energy are to be found in the regions of horizontal divergence. The net contribution from a given level results from the fact that areas of divergence generally occur at a different pressure than do the areas of convergence. Thus at lower levels it is common for horizontal divergence to be present in anticyclonic areas while convergence takes place in cyclonic areas, the net result being positive. We as yet do not have sufficient observational material concerning the distribution of divergence at higher levels, but the fact that the pressure decreases with elevation would seem to indicate that the importance of the higher levels rapidly diminishes. Generally speaking, it would thus appear that the energy sources for the general circulation are to be found principally in the subtropical high-pressure cells, the migratory polar anticyclones and the subsiding cap of cold air over the polar regions. From these primary centers the kinetic energy is continually transferred to the cyclonic areas with convergence which act as sinks in addition to the action of friction.

One might ask why it is that if the diverging anticyclones act as primary sources of kinetic energy, they are not the scenes of major activity. Actually, however, the generation process cannot be present in such systems without the simultaneous operation of the transfer processes. If divergence exists in an anticyclone, the peripheral outward motion results in a rapid outward flow of kinetic energy through work done by pressure forces and through advection.

We have made the tacit assumption in the development given above that the "frictional" term D leads to a dissipation of kinetic energy. If only molecular viscosity and small-scale turbulent viscosity are included in this term the assumption is undoubtedly valid. However, if relatively large-scale eddies and other large features of the atmospheric motions are included in the form of a gross turbulence as distinguished from the remaining mean motion, it is apparent that the quantity D may then embrace energy-producing systems and it is possible that it may change sign. Thus, for example, if only the average zonal circulation of the atmosphere be considered as the true mean motion so that the cyclones, anticyclones and other nonzonal motions appear as turbulence, there is no clear ***a priori*** reason for assuming that the term D represents a dissipation.

Finally, it is interesting to compare the results obtained here with those of Margules (1905) in his classic paper, "On the energy of storms." Very broadly speaking the two approaches deal with essentially the same process. We have simply enlarged the "chamber" containing the gas used by Margules so as to include the whole atmosphere. Furthermore, whereas Margules considered a discrete process, we have replaced it by a continuous one and restricted our attention to the production, redistribution, and dissipation of kinetic energy of horizontal motions only. Also, we have recognized that under these circumstances the pressure multiplied by the ***horizontal*** divergence is the measure of the rate at which other forms of energy such as potential and internal energy are being converted into kinetic energy.[3] When the divergence is negative the sense of this conversion process is reversed. It should be noted that this result is independent of the physical nature of the "working substance," which might indeed be partly liquid (or even solid), with the gaseous and liquid components undergoing changes of phase. The result therefore automatically embraces the consequence of all condensation phenomena insofar as they contribute to the horizontal kinetic energy.

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[3] It is worthy of note that the present treatment gives no information as to whether the bulk of the kinetic energy generated in the atmosphere represents a conversion from geopotential energy or whether it represents a conversion directly from internal heat energy.

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Transport of kinetic energy in the atmosphere

By VICTOR P. STARR
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30 January 1949

In the issue of the JOURNAL for October 1948 there appeared an article entitled "On the production of kinetic energy in the atmosphere," written under my authorship. Since that time it has occurred to me that certain principles set forth in the paper are capable of further elaboration leading to a better insight concerning the nature of the general circulation and its relation to the so-called secondary circulations.

It was pointed out in the article mentioned that a transfer of kinetic energy of horizontal motions across the boundary of a region which is not mechanically closed may be brought about by advection of existing kinetic energy and through the work done by pressure forces in virtue of the components of horizontal velocity across the boundary. Thus, if one considers a symmetrical polar cap extending from the north pole to some middle latitude ϕ and embracing the entire vertical extent of the atmosphere, the expression for the transfer of kinetic energy across the vertical southern boundary at the latitude ϕ is

$$\int (\tfrac{1}{2}\rho V_h^2 v + pv)ds \approx \int pv\, ds = \frac{R}{m}\int \rho T v\, ds,$$

where ds is an element of area, R/m is the gas constant for air, and T is the absolute temperature, while the other symbols have the same significance as previously. The approximate equality of the first two integrals is based upon the fact that the advection of kinetic energy in the atmosphere is of a smaller order of magnitude than the contribution of the term pv except possibly at very high levels. The final form depends also upon the feasibility of applying the ideal equation of state to the atmosphere. If these simplifications are accepted, the following observations may be made.

The last integral is proportional to the advection of internal heat energy northward, and hence is in all probability positive. This would indicate that there is normally a poleward flow of kinetic energy across middle latitudes from the tropics and subtropics which apparently serve as important source regions for such energy. Since this flow must cease as the polar regions are approached, it follows that the cyclone belts in middle and polar latitudes serve as dissipative mechanisms for this kinetic energy through friction and through the horizontal convergence present in them.

It is a matter of common synoptic experience that an extratropical cyclone is more apt to intensify if there is a relatively large contrast in the heat advection on its eastern and western sides. According to the present discussion it is not essential that the increase of kinetic energy in such cases be produced *in situ* through conversion from other forms of energy. The intensification may be brought about through the increased local poleward transport of kinetic energy from the general source region in lower latitudes, as measured by the large net local heat transport poleward.

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APPLICATIONS OF ENERGY PRINCIPLES TO THE GENERAL CIRCULATION

By VICTOR P. STARR

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INTRODUCTION

Theoretical hydrodynamics and thermodynamics furnish the basic equations of energy which in the end must describe the energy transformations which take place in the atmosphere. These equations in themselves are not capable of furnishing a sufficient rational explanation of the causes of atmospheric processes, but nevertheless provide a guide to systematic exploration for purposes of finding empirically important facts concerning the behavior of the atmosphere. Thus their utility is much enhanced if consideration is given to observational data.

The most important problem which confronts us in such an effort is therefore not one of merely stating the several pertinent equations in a formally complete manner, but rather one of discussing atmospheric processes as given by observations in terms of these relationships. To this end the principles involved must be moulded and recast in such a form as to permit the desired applications to be made. In this procedure the failure to give proper cognizance to the special circumstances characteristic of the atmospheric processes dealt with can only lead to endless complications and needless confusion.

Much of what has been written concerning this subject has been deficient in two respects. In the first place, investigators have been prone to lump together various diverse forms of energy, thereby losing the advantages to be gained from the fact that each form of energy is produced from and converted to other forms in its own characteristic fashion, permitting individual study. Likewise for each form there exist specific modes of transfer and redistribution. Unless these specific characteristics are subject to scrutiny in detail, only very broad generalizations can be reached. Even here, however, the implications of the balance of total energy for the globe have not yet been studied in sufficient detail as will be discussed later.

In the second place, the modes of energy transfer within the atmosphere are so effective that no feature such as a cyclone can be treated independently without due allowance for exchanges of energy between it and the remaining atmosphere. It is therefore inappropriate to treat such a feature as in any sense a closed system. Modern trends are beginning to give proper cognizance to this circumstance, although much of the too restricted point of view permeates meteorological thought.

In the present discourse only certain phases of the subject are discussed by way of illustrating a general approach. Thus the discussion which follows treats only the global balance of kinetic energy as an example of the study of one individual form of energy. In the last section the total energy balance is re-examined. It is of course true, as has already been stated, that it is also possible to study the global balance of other individual forms of energy such as geopotential and internal heat energy. A beginning in this direction has been made by Van Mieghem [10].

GLOBAL BALANCE OF KINETIC ENERGY

General Considerations. One of the basic problems in the science of meteorology relates to the manner in which thermal energy received by the atmosphere through short-wave solar radiation becomes in part transformed into kinetic energy of motion relative to the rotating earth. Plausible estimates show that the fraction of the total energy so transformed is very small, but must nevertheless be sufficient to account for all air motions, in the absence of any other significant energy sources. Since the kinetic energy of organized motions is continually degraded and ultimately dissipated by turbulence and viscosity, the process of kinetic energy production must be a continuous one with, probably, certain fluctuations about a mean rate when the whole atmosphere is considered. The purpose of this discussion is to examine this production process from a hydrodynamical point of view.

Changes in the kinetic energy of a particle or system of particles can result only from the action of mechanical forces, and hence the rate of kinetic-energy production can be discussed in terms of the joint action of such forces and the kinematics of existing motions. In this light it is not essential to inquire how systems of such forces and such motions in the atmosphere are related to the thermodynamical processes which are ultimately responsible for their existence. In order to demonstrate the particular point in question as simply as possible we shall first consider an example of fluid motion under circumstances which are somewhat artificial, but which still have theoretical interest. In view of the fact that the kinetic energy of vertical motions in the atmosphere is very small compared with the kinetic energy of the large-scale horizontal motions we shall consider only the latter.

The approach used is one suggested by the beautiful classic paper of Osborne Reynolds [6] entitled "On the Dynamical Theory of Incompressible Viscous Fluids and the Determination of the Criterion." Since Reynolds was concerned only with the dissipation of kinetic energy, his treatment must be modified in order to envisage also the process which creates kinetic energy. For this reason his assumption of incompressibility will be abandoned. Also, our restriction to the study of the

kinetic energy of horizontal motions introduces certain changes, although these changes are not actually in the nature of approximations.

Study of a Simple System. Let it be supposed that a mass of gas is confined in a chamber with a plane bottom and vertical walls, under the action of gravity which we assume to be acting vertically downward. If the chamber is of sufficiently great height, it is not necessary that it have a top. Likewise, the gas need not be an ideal one, since for the time being no use will be made of an equation of state. Coriolis forces will, for the present, be omitted. Let it be supposed further that the gas is in some state of motion induced by differential heating and cooling.

If we take x, y, and z to be a Cartesian coordinate system with the positive z-axis vertical, we may write the equations of motion for the horizontal directions in the form

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Here u and v are the velocity components in the directions x and y; ρ is the density; p the pressure; t time; and F_x and F_y are the components of the viscous forces in the x and y directions. Generally speaking, the motions in the chamber might be turbulent. If we wish to regard the dependent variables in equations (1) as representing mean values free of the turbulence components, we shall assume that the only change necessary is to include eddy-stress effects in the quantities F_x, F_y after the manner of Reynolds. More will be said concerning this point later.

The kinetic-energy equation corresponding to the system (1) is

$$\rho\frac{\partial}{\partial t}\frac{V_h^2}{2} + \rho u\frac{\partial}{\partial x}\frac{V_h^2}{2} + \rho v\frac{\partial}{\partial y}\frac{V_h^2}{2} + \rho w\frac{\partial}{\partial z}\frac{V_h^2}{2} = -\left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) - d. \tag{2}$$

We use the symbol d to represent the rate at which the turbulence and viscosity are decreasing the kinetic energy per unit volume, and $V_h^2 = u^2 + v^2$. It is possible to rewrite (2) in the following form:

$$\frac{\partial E}{\partial t} + \frac{\partial Eu}{\partial x} + \frac{\partial Ev}{\partial y} + \frac{\partial Ew}{\partial z} = -\left(\frac{\partial pu}{\partial x} + \frac{\partial pv}{\partial y}\right) + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - d, \tag{3}$$

where use has been made of the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0, \tag{4}$$

which in any case must be true, and where $E = \frac{1}{2}V_h^2$ is the horizontal kinetic energy per unit volume. The quantity represented by the last three terms on the left-hand side of (3) is the divergence of the (three-dimensional) kinetic energy transport vector $E\mathbf{V}$. The quantity in the first parenthesis on the right is the divergence of the horizontal vector $p\mathbf{V}_h$. If equation (3) is integrated over an arbitrary volume, both of these quantities may be represented as surface integrals with the aid of the divergence theorem. Thus, if the limits are fixed, we may write

$$\frac{\partial}{\partial t}\iiint E\,dx\,dy\,dz = \int EV_n\,dS - \iint p(v\,dx - u\,dy)\,dz + \iiint p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dx\,dy\,dz - \iiint d\,dx\,dy\,dz, \tag{5}$$

where V_n is taken to be the inward component of velocity at the boundary, and dS is a surface element.

Equation (5) may now be given the following interpretation. The total horizontal kinetic energy (T) in a fixed region may be changing in consequence of:

1. An advection of new fluid having kinetic energy across the boundary. This is represented by the term $A = \int EV_n\,dS$. This is then one mode of *redistribution* of kinetic energy.
2. The performance of work by pressure forces at the boundary in virtue of the displacements due to the horizontal velocity components. This is represented by the term $W = -\iint p(v\,dx - u\,dy)\,dz$. This is a second mode of *redistribution* of kinetic energy.
3. A production of kinetic energy within the volume itself. This is represented by the term
$$S = \iiint p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dx\,dy\,dz,$$
which contains the *primary source* of kinetic energy.
4. The action of frictional forces. This effect would ordinarily consist of a dissipation and is represented by the term $D = \iiint d\,dx\,dy\,dz$.

If the limits of integration include all of the fluid in the fixed chamber, it is clear that the surface integrals must vanish, so that in a *mechanically closed* system (5) reduces to

$$\frac{\partial K}{\partial t} = S - D. \tag{6}$$

Since for such a system the frictional effect would ordinarily lead to dissipation, it follows that S must be positive if the total horizontal kinetic energy K is to remain constant or increase. If a more or less constant amount of kinetic energy is to be present, the dissipation must be balanced by a corresponding positive average rate of production.

The production S may be looked upon as the integral of the contributions from the various horizontal layers of fluid present and written as

$$S = \int \left[\iint p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dx\, dy \right] dz. \tag{7}$$

In view of the fact that the surface integral

$$\iint \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dx\, dy$$

must vanish if the horizontal velocity is zero across the fixed walls, it follows that a given horizontal stratum of fluid cannot give a positive contribution to S unless larger values of the pressure p are associated with areas of horizontal divergence than are associated with areas of convergence. Thus areas of *horizontal* divergence represent primary kinetic energy sources, while areas of convergence represent sinks for kinetic energy. Furthermore, in a mechanically closed system of the kind here considered it is impossible to have source regions for kinetic energy without at the same time having sinks of a hydrodynamic nature, entirely independent of frictional effects.

Equations for the Atmosphere. Before embarking upon a discussion of the meteorological implications of the material presented above, it is desirable to develop the concepts involved in more general terms, so as to render it possible to perform integrations over the entire mass of the atmosphere.

To a sufficiently close degree of approximation the shape of the geopotential surfaces may be considered as spherical so that we may make use of spherical polar coordinates in which r is the radius, ϕ is latitude, and λ is longitude. By analogy with the Cartesian case we may then write the equations of motion for the horizontal directions (see Brunt [2]) in the form

$$\left.\begin{aligned} &\frac{du}{dt} - \frac{uv}{r}\tan\phi + \frac{uw}{r} + 2\Omega(w\cos\phi - v\sin\phi) \\ &\qquad = -\frac{1}{\rho}\frac{\partial p}{\partial x} + F_x, \\ &\frac{dv}{dt} + \frac{u^2}{r}\tan\phi + \frac{vw}{r} + 2\Omega u\sin\phi \\ &\qquad = -\frac{1}{\rho}\frac{\partial p}{\partial y} + F_y, \end{aligned}\right\} \tag{8}$$

where u, v, and w are the linear velocity components in the eastward, northward, and upward directions, respectively; x and y are measures of linear distance eastward and northward, respectively; and Ω is the angular velocity of the earth. The analogous energy equation in this case may be written as

$$\begin{aligned} &\rho\frac{d}{dt}\frac{V_h^2}{2} + \rho\frac{V_h^2}{r}w + 2\rho\Omega uw\cos\phi \\ &\qquad = -\left(\frac{\partial pu}{\partial x} + \frac{\partial pv}{\partial y} - \frac{pv}{r}\tan\phi\right) \\ &\qquad\quad + p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \frac{v}{r}\tan\phi\right) - d. \end{aligned} \tag{9}$$

If we make use of the observational fact that the last two terms on the left-hand side of (9) are of a very small order of magnitude, these terms will be dropped.[1] The manipulation of the remaining term on the left side may now be carried out with the aid of the continuity equation much as before, since this operation is independent of the specific coordinate system used, so that we may write

$$\frac{\partial E}{\partial t} + \mathrm{div}_3\, E\mathbf{V} = -\mathrm{div}_2\, p\mathbf{V}_h + p\,\mathrm{div}_2\,\mathbf{V}_h - d. \tag{10}$$

A volume integral of (10) may now be taken and written in the form

$$\begin{aligned} \frac{\partial}{\partial t}\int E\, d\tau = \int EV_n\, ds - \iint p(v\, dx - u\, dy)\, dr \\ + \int p\,\mathrm{div}_2\,\mathbf{V}_h\, d\tau - \int d\, d\tau, \end{aligned} \tag{11}$$

where $d\tau$ is a volume and ds a surface element. Equation (11) is physically identical with (5) and has, therefore, the same interpretation. In symbolic form we may write

$$\frac{\partial K}{\partial t} = A + W + S - D, \tag{12}$$

which states that the rate of increase of horizontal kinetic energy for a fixed volume is equal to the net rate of advection of such kinetic energy into the region, plus the rate at which work is being done by the surroundings on the fluid in the region through horizontal motions, plus the production of kinetic energy in the volume, minus the frictional dissipation. For a system which is mechanically closed, A and W again vanish. This is therefore true when the entire atmosphere is considered. In this case the surface integral of the horizontal divergence over each closed geopotential surface must vanish as in the case of the chamber previously considered.

Although it is possible to form other energy integrals for fluid motion, as pointed out in standard texts on hydrodynamics (*e.g.*, [1]), the particular merit of the procedure followed above is that the expression for production of kinetic energy assumes a form which is of interest in meteorological problems. The implications of equation (12) may be stated in brief as follows:

1. The intensity of the primary source of horizontal kinetic energy at a given point in the atmosphere is given by the product of the pressure and the divergence of the horizontal velocity.

2. Positive primary sources must always occur in combination with negative sources or sinks independ-

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1. In reality these terms represent a conversion of kinetic energy of horizontal motions into kinetic energy of vertical motions, and as such do not involve a production of kinetic energy. Indeed, by methods similar to those used in this paper one can investigate separately the kinetic energy of motions in each of the three directions, namely, zonal, meridional, and vertical. In that case other conversion terms of a similar nature arise.

ent of frictional effects, when the entire atmosphere is considered.

3. In addition to the action of the sources and frictional effects, the horizontal kinetic energy in a fixed region not embracing the entire atmosphere may change due to advection of kinetic energy across the boundary and due to the redistribution of kinetic energy through the boundary by work done by pressure forces and horizontal velocity components at the boundary.

From the standpoint of the general circulation it would appear that the sources of kinetic energy are to be found in the regions of horizontal divergence. The net contribution from a given level results from the fact that areas of divergence generally occur at a different pressure than do the areas of convergence. Thus at lower levels it is common for horizontal divergence to be present in anticyclonic areas while convergence takes place in cyclonic areas, the net result being positive. We do not as yet have sufficient observational material concerning the distribution of divergence at higher levels, but the fact that the pressure decreases with elevation would seem to indicate that the importance of the higher levels rapidly diminishes. Generally speaking, it would thus appear that the energy sources for the general circulation are to be found principally in the subtropical high-pressure cells, the migratory polar anticyclones, and the subsiding cap of cold air over the polar regions. From these primary centers the kinetic energy is continually transferred to the cyclonic areas with convergence which act as sinks in addition to the action of friction.

One might ask why it is that if the diverging anticyclones act as primary sources of kinetic energy, they are not the scenes of major activity. Actually, however, the generation process cannot be present in such systems without the simultaneous operation of the transfer processes. If divergence exists in an anticyclone, the peripheral outward motion results in a rapid outward flow of kinetic energy through work done by pressure forces and through advection.

It was pointed out that a transfer of kinetic energy of horizontal motions across the boundary of a region which is not mechanically closed may be brought about by advection of existing kinetic energy and through the work done by pressure forces in virtue of the components of horizontal velocity across the boundary. Thus, if one considers a symmetrical polar cap extending from the north pole to some middle latitude ϕ and embracing the entire vertical extent of the atmosphere, the expression for the transfer of kinetic energy across the vertical southern boundary at the latitude ϕ is

$$\int (\tfrac{1}{2}\rho V_h^2 v + pv)\, ds \approx \int pv\, ds = \frac{R}{m}\int \rho T v\, ds, \quad (13)$$

where ds is an element of area, R/m is the gas constant for air, and T is the absolute temperature, while the other symbols have the same significance as previously. The approximate equality of the first two integrals is based upon the fact that the advection of kinetic energy in the atmosphere is of a smaller order of magnitude than the contribution of the term pv except possibly at very high levels. The final form depends also upon the feasibility of applying the ideal equation of state to the atmosphere. If these simplifications are accepted, the following observations may be made.

The last integral is proportional to the advection of internal heat energy northward, and hence is in all probability positive. This would indicate that there is normally a poleward flow of kinetic energy across middle latitudes from the tropics and subtropics which apparently serve as important source regions for such energy. Since this flow must cease as the polar regions are approached, it follows that the cyclone belts in middle and polar latitudes serve as dissipative mechanisms for this kinetic energy through friction and through the horizontal convergence present in them.

It is a matter of common synoptic experience that an extratropical cyclone is more apt to intensify if there is a relatively large contrast in the heat advection on its eastern and western sides. According to the present discussion, it is not essential that the increase of kinetic energy in such cases be produced *in situ* through conversion from other forms of energy. The intensification may be brought about through the increased local poleward transport of kinetic energy from the general source region in lower latitudes, as measured by the large net local heat transport poleward.

We have made the tacit assumption in the development given above that the "frictional" term D leads to a dissipation of kinetic energy. If only molecular viscosity and small-scale turbulent viscosity are included in this term, the assumption is undoubtedly valid. However, if relatively large-scale eddies and other large features of the atmospheric motions are included in the form of a gross turbulence as distinguished from the remaining mean motion, it is apparent that the quantity D may then embrace energy-producing systems and it is possible that it may change sign. Thus, for example, if only the average zonal circulation of the atmosphere be considered as the true mean motion so that the cyclones, anticyclones, and other nonzonal motions appear as turbulence, there is no clear a priori reason for assuming that the term D represents a dissipation.

Finally, it is interesting to compare the results obtained here with those of Margules [5] in his classic paper, "On the Energy of Storms." Very broadly speaking, the two approaches deal with essentially the same process. We have simply enlarged the "chamber" containing the gas used by Margules so as to include the whole atmosphere. Furthermore, whereas Margules considered a discrete process, we have replaced it by a continuous one and restricted our attention to the production, redistribution, and dissipation of kinetic energy of horizontal motions only. Also, we have recognized that under these circumstances the pressure multiplied by the *horizontal* divergence is the measure of the rate at which other forms of energy such as potential and internal energy are being converted into kinetic

energy.[2] When the divergence is negative the sense of this conversion process is reversed. It should be noted that this particular result is independent of the physical nature of the "working substance," which might indeed be partly liquid (or even solid), with the gaseous and liquid components undergoing changes of phase. The result therefore automatically embraces the consequence of all condensation phenomena insofar as they contribute to the horizontal kinetic energy.

GLOBAL BALANCE OF TOTAL ENERGY

Basic Equations. Thus far we have found it convenient to deal with the kinetic energy problem alone, since this quantity can be changed only by mechanical forces, and hence may be studied separately in terms of the systems of such forces considered as given by observational data. However the problems connected with the total global energy balance must in the end be of significance in the further understanding of atmospheric and oceanic circulations. For this reason we shall now attempt to formulate certain relationships involved in this more general subject.

Proceeding along more classical lines, let us consider the statement of the general physical energy equation written in the form

$$\rho \frac{dq}{dt} + \psi = \rho \frac{dU}{dt} + p\rho \frac{d\alpha}{dt} = \rho \frac{dU}{dt} - \frac{p}{\rho}\frac{d\rho}{dt}. \quad (14)$$

Here $\rho\, dq/dt$ is the rate of external heat addition per unit volume, U is the total internal energy per unit mass, $\alpha \equiv 1/\rho$ is the specific volume, ψ is the rate of generation of heat by friction per unit volume, while the other symbols have already been defined. With the aid of the continuity equation

$$\frac{d\rho}{dt} + \rho\nabla\cdot\mathbf{c} = 0, \quad (15)$$

where $\mathbf{c}$ is the total vector particle velocity, we may write that

$$\rho \frac{dq}{dt} + \psi = \rho \frac{dU}{dt} + p\nabla\cdot\mathbf{c}. \quad (16)$$

We next proceed to evaluate the last term in (16) from the dynamical equation of motion written in vectorial form as follows:

$$\rho \frac{d\mathbf{c}}{dt} = -\nabla p - \rho\nabla\Phi - 2\rho\mathbf{\Omega}\times\mathbf{c} - \mathbf{F}, \quad (17)$$

where Φ is geopotential energy per unit mass, $\mathbf{\Omega}$ is the constant angular velocity of the earth's rotation, and $\mathbf{F}$ is the vectorial retarding force per unit volume due to friction.

The scalar product of (17) with the particle velocity yields the corresponding equation of energy which may be written after slight rearrangement as

$$\rho \frac{d}{dt}\frac{c^2}{2} = p\nabla\cdot\mathbf{c} - \nabla\cdot p\mathbf{c} - \nabla\cdot\rho\Phi\mathbf{c} + \Phi\nabla\cdot\rho\mathbf{c} - d. \quad (18)$$

In (18), c is the magnitude of $\mathbf{c}$, and $d \equiv \mathbf{c}\cdot\mathbf{F}$ is the rate at which work is done by the fluid against frictional forces per unit volume. Assuming that the geopotential Φ is constant with time at a fixed point with respect to the earth (this is true except for such things as the small tide-producing disturbances), we may write, with the aid of the continuity equation in the form

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{c} = 0, \quad (19)$$

that

$$\Phi\nabla\cdot\rho\mathbf{c} = -\frac{\partial}{\partial t}(\rho\Phi). \quad (20)$$

Using (18) and (20), we can now rewrite (16) in the form

$$\rho\frac{dq}{dt} + \psi - d = \rho\frac{d}{dt}\left(U + \frac{c^2}{2}\right) + \frac{\partial}{\partial t}(\rho\Phi) + \nabla\cdot(p + \rho\Phi)\mathbf{c}. \quad (21)$$

Since with the aid of (19) it follows that

$$\rho\frac{d}{dt}(\quad) = \frac{\partial\rho(\quad)}{\partial t} + \nabla\cdot\rho(\quad)\mathbf{c},$$

we finally have the equation

$$\rho\frac{dq}{dt} + \psi - d = \frac{\partial}{\partial t}\left(\rho U + \rho\frac{c^2}{2} + \rho\Phi\right) + \nabla\cdot\left(\rho U + \rho\frac{c^2}{2} + \rho\Phi + p\right)\mathbf{c}. \quad (22)$$

The various considerations which have entered into the formulation of equation (22) are true for any fluid medium without significant approximation. We may therefore apply the equation to the entire fluid envelope of the earth or portion of it, making no distinction between the atmosphere and the hydrosphere. We can thus integrate it over an equatorial belt between latitudes $-\phi$ and $+\phi$ and include all bodies of water such as the oceans, rivers, lakes, etc. Considering again the average conditions so that local time variations disappear we have, with the aid of the divergence theorem, that

$$H \equiv \int\left(\rho\frac{dq}{dt} + \psi - d\right)d\tau = \int\left(\rho U + \rho\frac{c^2}{2} + \rho\Phi + p\right)v_n\, ds, \quad (23)$$

where $d\tau$ is a volume element. Equation (23) has of course a very simple interpretation and could have indeed been written directly from general considerations. If we include under friction only the effects of

2. It is worthy of note that the present treatment gives no information as to whether the bulk of the kinetic energy generated in the atmosphere represents a conversion from geopotential energy or whether it represents a conversion directly from internal heat energy.

molecular viscosity or of small-scale disturbances which can produce no significant tangential stresses at the boundaries $-\phi$ and $+\phi$, the contribution of the term d in the integral on the left-hand side may be assumed to represent the mean rate of dissipation of kinetic energy into heat within the equatorial belt and therefore cancels the contribution of the term ψ. It follows therefore that H is the total net rate of heating of the air in the belt. Equation (23) simply states that this net incoming energy is transferred meridionally in the form of (1) internal energy ρU per unit volume, (2) kinetic energy of existing motions $\rho c^2/2$, and (3) potential energy $\rho\Phi$, as well as (4) through work done by pressure forces p. We shall refer to these four items as advective modes of energy transfer.

It is here assumed that there is no advection of energy through the surface of the lithosphere. This is essentially correct except for processes such as volcanism and seismological phenomena, but these are deemed to be too unimportant for the present considerations. Also it is assumed that there is no advection of energy through the top of the atmosphere, which therefore neglects the effect of interchange of molecules with astronomical space and of the mass accretions of meteoric origin. The quantity H, representing the net heat gain within the equatorial belt by processes other than advection, may be very closely identified with the net heat received through exchange of radiation with the extraterrestrial environment. This identification neglects such processes as conduction of heat from the interior of the earth, which is of appreciable importance only locally in connection with volcanism, and it also neglects heat liberated (or consumed) by net progressive chemical changes such as oxidation or photosynthesis processes. Net heat gain through exchange of radiation with other portions of the earth is likewise neglected. All the various corrections mentioned are however in all probability insignificant.

If we take H to be the net gain of heat through exchange of radiation with space, various necessarily crude estimates of this quantity have been prepared. A convenient arrangement of one set of such estimates has been presented by Bjerknes [1]. As is well known, the estimates give positive values of H for all choices of $\pm\phi$ between the equator and the poles with a maximum for about $\phi = \pm 45°$ latitude. It therefore follows that there must be an advective transport of energy poleward by the combination of terms indicated in (23), with a maximum at $\phi = \pm 45°$ latitude in the mean.

It is apparent that the important problem posed by the global energy balance concerns itself with the partition of the poleward energy transport among the several terms in the integrand of the right-hand member of equation (23).

Discussion. Unfortunately our observational information concerning the problem posed by the global energy balance is very sketchy and incomplete. We shall nevertheless endeavor to discuss such aspects of it as are possible with existing knowledge. In the first place, the contribution of the hydrosphere to the transfer integral is probably small (see Sverdrup [9]), but directed toward the poles. A reasonable estimate of this contribution would appear to be about ten per cent of H. Denoting this fraction by h, let us next turn our attention to the state of affairs within the atmosphere.

It has been previously pointed out that the advection of existing kinetic energy $\rho c^2/2$ meridionally is very small, relatively speaking. We are therefore again justified in omitting it. The internal energy U may be considered as being the sum of the internal heat energy and the latent heat of water vapor. Since there is assumed to be practically no net meridional mass transport in the atmosphere, it will suffice to assume that the internal heat energy is given by $c_v T$, c_v being the mean specific heat at constant volume. It thus follows that $U \approx c_v T + \epsilon L$, where ϵ is the specific humidity and L is the latent heat of condensation, assumed to be constant.[3] The term involving the work done by pressure forces may again be transformed according to the ideal equation of state, and finally combined with the internal heat-energy term using the relation between the specific heats of a gas. In the end (23) may be written in the form

$$H = h + \int (c_p T + \epsilon L + \Phi)\rho v_n \, ds, \tag{24}$$

where c_p, the specific heat at constant pressure, is assumed to have a constant mean value.

The contribution of the term involving the latent heat may be estimated from the mean excess of evaporation over precipitation in the equatorial belt. Using data of this kind given by Conrad [3], the writer has estimated that the magnitude of this effect is about one-half of H for an equatorial belt extending to $\pm 40°$ latitude. Let us denote this quantity by l.

The remaining terms may be examined as follows. If we write

$$\rho v_n = \overline{\rho v_n} + \{\rho v_n\}, \tag{25}$$

where $\overline{\rho v_n}$ is the average of ρv_n along the entire length of a closed latitude circle and $\{\rho v_n\}$ is the deviation from this average, it is clear that the identical vanishing of $\overline{\rho v_n}$ implies absence of closed mean meridional circulations, while its presence is required for the existence of such circulations. Here we neglect all topographic inequalities of the earth's surface. In view of the fact that Φ is constant along a latitude circle at any given elevation and that $\{\overline{\rho v_n}\}$ is zero, it follows that (24) may be rewritten in the form

$$H = h + l + \int c_p T\{\rho v_n\}\, ds + \int (c_p T + \Phi)\overline{\rho v_n}\, ds, \tag{26}$$

3. The latent heat as ordinarily discussed is the sum of the change in specific internal energy plus the work done in the expansion during evaporation. Strictly speaking, we are here concerned only with the first quantity, although the difference is not great enough to be of much significance.

where the last integral now represents the contribution of mean meridional circulations of the Hadley type to the poleward energy flux.

It should be remarked that, in a stable atmosphere, a meridional cell of the so-called direct type produces a poleward flow of energy, while one of the indirect type produces a flow in the opposite sense. This can easily be shown from the form of the last integral in (26).

Very preliminary estimates by the writer of the value of the third term on the right-hand side, made from geostrophic wind data for individual Northern Hemisphere maps for various levels seem to suggest that this contribution is somewhere in the vicinity of one-half of H at 40°N latitude. All in all it would thus appear that the contribution of the mean meridional circulations is small or even negative in middle latitudes, although very little reliance can be placed on the figures given or on the value of H obtained from the data given by Bjerknes. Suffice it to say that in all cases reasonable orders of magnitude are obtained, which in itself is somewhat encouraging.

Concluding Remarks. Most classical models for the general circulation of the atmosphere have followed along the lines originally proposed by Hadley [4] in that they assume the existence of large convectively driven closed circulations in meridional planes, at least in the average conditions. The development of the mean zonal motions is then ascribed to the effect of the earth's rotation on these primary circulations. In such a scheme the meridional circulations are a necessary mechanism in the production of kinetic energy. Also, according to this model the necessary meridional transport of angular momentum could be achieved if the poleward branches of the circulations carry more angular momentum than the returning ones at other levels.

For a number of reasons modern meteorologists have come to view models of the Hadley type with skepticism. A discussion of the basis for this current skepticism has been recently given by Rossby [7]. The writer rather inclines to the view that, although some mean meridional circulations in all probability do exist, their role in the energy balance and in the horizontal transport of angular momentum, at least in middle latitudes, may be overshadowed by the characteristics of other types of motion. Thus, following an original suggestion by Jeffreys, the writer has pointed out elsewhere [8] that the transport of angular momentum could be achieved through the observed properties of horizontal motions. This contention has since received a certain amount of corroboration by the observational studies of Widger [11].

The general views expressed in the present paper indicate that atmospheric meridional circulations likewise may not be essential for the global energy balance. Much more could be said if a more satisfactory appraisal were available for the magnitudes of the terms appearing in equation (26), since the values given are useful only for purposes of orientation. Further work in this direction is currently in progress at the Massachusetts Institute of Technology.

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ON THE ENERGY BALANCE OF THE ATMOSPHERE

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Abstract--The net radiative energy surplus or deficit of the atmosphere in middle and high latitudes is inferred from a consideration of energy transport processes in the atmosphere. It is shown that the results are consistent with estimates made through direct consideration of the radiative processes, thus providing an independent check upon them.

Introduction--Information concerning the radiation balance of the atmosphere may be obtained indirectly through procedures which do not involve measurement of the radiative processes themselves. Such determinations provide an independent estimate of the net radiative energy surplus or deficit within given regions of the atmosphere. The orders of magnitude of the radiative surplus or deficit derived in this manner for the mid-latitude and polar regions are shown to agree favorably with other measurements of this quantity which are available from direct consideration of the radiative processes.

The formulation of the problem--The problem may be described in the following manner. In the Earth's atmosphere, a poleward flux of total energy in all forms is necessitated by the unequal net heating of equatorial and polar regions in the presence of a mean hemispherical temperature distribution which remains relatively constant over long periods of time. Let us consider the entire volume of the atmosphere which is contained between two latitudes forming the equatorial belt between $+\phi$ and $-\phi$. Over long time periods the sole source of energy for this region of the atmosphere is the solar energy which it receives from space by radiation. It is generally agreed that conduction of heat energy through the Earth's crust is unimportant for the present problem. A given amount of energy is also lost from this volume of the atmosphere by long-wave radiative processes. The difference between the amount of short-wave radiative energy which is received and the outgoing long-wave radiation results in a surplus for the equatorial belt of the atmosphere extending from approximately +45°N to -45°S. Poleward of these latitudes it is generally considered that there exists a net radiative energy deficit.

It is probable that the energy storage within the atmosphere, oceans, or solid earth, over time periods of the length which are considered here is quite small since there appears to be no progressive change of the temperature of the atmosphere, oceans, or solid earth during such periods. Some energy storage within the earth-atmosphere system may be possible during periods of increase or decrease of the polar ice caps. However, this effect should be quite small since the time periods being considered are much shorter than the time between marked changes in the extent of the ice cap. It is necessary, therefore, that the radiative energy surplus be transported out of equatorial regions into the regions of radiative energy deficit.

STARR [1951] has studied this problem recently and has formulated it in a convenient manner as follows

$$H = h + \int_s (C_p T + Lw + \Phi + c^2/2)\rho V ds \quad \ldots\ldots (1)$$

where H is the net rate of external heat addition to the volume of air plus hydrosphere which in this case may be identified with the net effective radiation, h is the energy flux within the hydrosphere, C_p is the specific heat of air at constant pressure, T is the temperature, w is the specific humidity, L is the latent heat of water vapor which may be considered constant in this instance, Φ is the potential energy per unit mass of air, ρ is the density, c is the total vector particle velocity, V is the meridional component of velocity considered positive when the wind is from the south, and ds is an element of the conical surface erected at latitudes $\pm\phi$ and the integration is to be performed over the entire surface. This equation can be inferred directly from general considerations. However, the reader is referred to STARR [1951] for a better development of this equation from the general physical energy equation and the equations of motion. The interpretation of this equation is quite simple. It states that the total flux of energy which must be directed from equator to pole may be considered to be comprised of a flux of (a) sensible heat ($\int_s C_p T \rho V ds$), (b) latent energy of

water vapor ($\int_s Lw\rho V ds$), (c) kinetic energy ($\int_s (\rho c^2/2) V ds$), (d) potential energy ($\int_s \Phi \rho V ds$), and (e) a contribution due to the poleward transport of energy by the oceans (h). Thus it becomes theoretically possible to arrive at estimates of the net radiative energy surplus within given latitude belts in the atmosphere by measuring the terms on the right-hand side of (1). This method of approach has not been feasible until very recent years in view of our inability to measure the terms on a hemispherical basis with sufficient accuracy. However, with an expanded network of observing stations it has now become possible to make a preliminary attempt at measuring them.

A very real question arises when attempts are made to measure the meridional flux of energy in its various forms. This question stems from the nature of one's approach to the mode of operation of the general circulation. If it is supposed that the energy exchanges of the atmosphere are accomplished by the large-scale horizontal eddy motions, such as those associated with eddies the size of cyclones and anticyclones, then it becomes feasible to measure the magnitudes of the terms in (1) from hemispherical maps of standard meteorological quantities. If, on the other hand, it is supposed that the principal mode of energy exchange is accomplished through a process which in the main depends upon the existence of significant meridional cells operating in meridional planes so that the energy exchange process is essentially a vertical mass exchange process, then the problem becomes one of infinite complexity and difficult to compute from currently available data because of our inability to measure the magnitude of such cells. In the case of the energy-exchange process it is felt that the large-scale horizontal eddy exchange processes are the dominant ones. In what follows this assumption will be made.

The energy transport within the oceans--A certain amount of the total energy transport is accomplished by the currents of the oceans. In meteorology it is generally assumed that the transport of energy by ocean currents as internal energy of the water is quite small. SVERDRUP [1942] points out that this question has never been thoroughly examined principally because of lack of data. However, he agrees that when dealing with averages for the entire Earth this assumption is essentially correct. In localized areas, however, Sverdrup points out that the contribution of the ocean currents to the total energy transport may be as large as ten per cent of the total transport. These regions are thought to occur principally in middle latitudes where the principal north-south branches of the ocean currents are found. He also points out that in the southern oceans the north-south circulations and the corresponding temperature contrasts between currents flowing meridionally are very small. In view of this it seems that no great error will be introduced if the transport of energy by ocean currents is neglected in what follows, since great accuracy in the overall picture cannot in any case be expected.

The transport of existing kinetic energy--It is generally accepted [STARR, 1949] and it can be shown observationally, as has been done by the author in an unpublished doctoral thesis, that the transfer of existing kinetic energy is about two orders of magnitude smaller than the meridional transfer of energy as sensible heat and may be safely neglected in comparison with it.

The potential-energy transport--Since we have under consideration only horizontal eddy exchange processes it is obvious that there can be no transport of energy in this form since the geopotential is constant at any given elevation.

We thus arrive at the result that probably the energy-exchange processes in the atmosphere may be considered to be largely comprised of a transport of sensible heat and a transport of latent energy. This is fortunate since it is possible to arrive at estimates of the hemispherical transport of these quantities.

The latent-energy transport--The latent-energy transport may be evaluated in several ways. One approach is to use the observed moisture and wind data which are available from radiosonde and radio wind observations. The horizontal eddy transport of latent energy may then be obtained from such data. PRIESTLEY [1949] has suggested a method for utilizing such data and computed the latent energy transport for one station, Larkhill, England, for a period of two years. Using a method similar to Priestley's, WHITE [1951a] has obtained a measure of the latent-energy transport for one summer month and one winter month for a large number of stations extending from the tropics to the poles over the North American sector of the hemisphere. Although it is possible to use these rough estimates of the latent-energy transport obtained in this fashion, they suffer the serious drawback that they have been obtained from very restricted regions of the hemisphere for short time periods.

A second approach and one which is favored in this case is the computation of the long-term mean transport of latent energy from hemispherical evaporation and precipitation data. Over long

periods of time it may be assumed that there is no net change in the moisture of the atmosphere. If, therefore, there exists an excess of evaporation over precipitation within a given latitude belt in the atmosphere, there must exist over long time periods a transport of moisture and hence of latent energy out of the region. On the other hand, when the precipitation exceeds the evaporation, there must exist a transport of moisture into this latitude belt. Use of this indirect method of determining the flux of latent energy into given regions of the atmosphere does not assume that the transport of latent energy is accomplished solely by horizontal processes.

The knowledge of the net rate of evaporation from the land surfaces is extremely uncertain. However, it is possible to arrive at some estimates of the difference between precipitation and evaporation over large areas by a consideration of the amount of water discharged by the rivers into the oceans. Over a long period of time the difference between the precipitation and evaporation must equal the amount of water which is discharged into the oceans by the rivers. Such a treatment is valid only when long time periods are considered because a knowledge of the storage capacity of the ground is necessary and must be taken into account over short time periods. Such river-discharge data are available over wide areas of the land surface of the hemisphere. WUST [1922] has made use of them in computing the difference between the precipitation and evaporation over land surfaces.

Wust has also made estimates of the difference between the evaporation and precipitation over the ocean areas of the hemisphere. He used widely scattered pan observations of evaporation over the oceans and the precipitation data available from island stations. Since the evaporation data from the oceans are more reliable than over the land areas it is felt that such estimates represent a fairly good first approximation to actual conditions.

CONRAD [1936] has extended Wust's data to include the entire surface of the Earth and has estimated the latitudinal distribution of the net evaporation. Over long periods of time we may consider that there is no progressive change in the moisture content of the atmosphere although it is realized that over geological epochs there is some evidence of such changes.

From continuity considerations, by treating first the regions around the pole, it is possible to obtain a measure of the transport of moisture which must exist into the various latitude belts of the atmosphere. The values of these transports are given in Table 1. It is the opinion of Wust that his estimates are correct to within 15 pct.

Table 1--The observed meridional transport of latent energy and sensible heat

Latitude north	Latent energy	Sensible heat	Latent energy + sensible heat
°	(cal/sec) $\times 10^{12}$	(cal/sec) $\times 10^{12}$	(cal/sec) $\times 10^{12}$
75	+30	+14	+43
65	+99	+329	+428
55	+224	+834	+1058
45	+397	+793	+1189
35	+425	+355	+780

Note: Internal consistency subject to rounding off errors. Positive signs indicate a poleward transport.

The transport of sensible heat--Estimates of the horizontal eddy flux of sensible heat can be made (with considerable effort) directly from northern hemisphere weather charts of pressure and pressure-height contours. The transport of sensible heat across a complete latitude circle at a given elevation can be written as

$$\oint C_p T\rho V dx \quad \ldots \ldots \ldots \quad (2)$$

where the integration is to be performed completely around the latitude circle and dx is an element of longitudinal distance. Because of the lack of direct wind observations over most of the hemisphere it becomes necessary to use a geostrophic measure of the wind given by the relation

$$V_g = (1/\rho f)\, \partial p/\partial x \quad \ldots \ldots \ldots \quad (3)$$

where f is the coriolis parameter and p is the pressure. A reliable measure of the temperature may be obtained if it is realized that the mean virtual temperature of any column of air is directly proportional to the height difference between two constant pressure surfaces. The product of the

mean geostrophic wind for a given layer and the mean virtual temperature may be formed at a large number of evenly distributed points around a given latitude and the integral (2) may be evaluated by finite difference methods. Such computations were carried out by WHITE [1951b] for the four winter months, November 1945 - February 1946, on a daily basis for every ten degrees of latitude extending from 35°N to 75°N. The geostrophic transport of sensible heat was computed up to the 500-mb surface in two layers: Surface to 700 mb, and 700 mb to 500 mb. For details of these computations the reader is referred to the investigation mentioned previously.

By virtue of the fact that a geostrophic measure of the wind was used, transports of sensible heat which are accomplished by mean meridional cells are automatically eliminated since

$$\oint \rho V_g \, dx = \oint (1/f) \, (\partial p/\partial x) \, dx = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Use of the values of the geostrophic transport of sensible heat computed by White in conjunction with estimates of the normal latent-energy transport requires a certain amount of justification. In the first place, the estimates of the latent-energy transport evaluated from evaporation and precipitation data have been made on a yearly normal basis. The geostrophic transport of sensible heat was computed for four months of one winter season. In addition, the question may legitimately be raised as to the nature of the difference between the geostrophic and the actual eddy transports.

Studies of the eddy sensible heat transport over North America [WHITE, 1951b] have shown that there is good agreement (within 20 pct in the mean) between the eddy sensible heat flux computed geostrophically and that obtained from observed winds. The other question which must be answered is of a more serious nature. How closely does the eddy sensible heat transport computed up to 500 mb for the winter season represent the mean yearly transport through the entire atmosphere?

It appears that the transport computed up to 500 mb is smaller than the total eddy transport of sensible heat through the entire atmosphere since the transport of sensible heat above 500 mb has been neglected. On the other hand, there is a compensating factor in that the transport of sensible heat during the winter is greater than it is during the summer and hence use of the transports based upon winter data alone as a typical yearly value will be considerably in overestimate.

Studies of the extent to which these two factors actually cancel [WHITE, 1951a], based upon investigations of the eddy sensible heat transport over regions of North America in which observed wind and temperature data were used, show that the transports of sensible heat up to 500 mb represent approximately 70 pct of the total across a given latitude circle up to 100 mb. On the other hand, the winter-time rate of transport computed up to 500 mb is approximately 45 pct greater than the mean rate of transport.

In view of these results it seems permissible to use the geostrophic eddy sensible heat transport computed up to 500 mb for one winter season as a first approximation to a typical yearly mean transport. It must be remembered that the determination of a reliable normal for the magnitude of this term will require the analysis of many years of complete northern hemisphere data up to very high levels. Since the purpose at hand is merely to demonstrate the feasibility of this method of attack upon the radiation problem and to arrive at orders of magnitude, the use of these values is sufficient.

The mean winter geostrophic eddy sensible heat transport below 500 mb based upon the four winter months of data is presented in Table 1 together with the sum of the transport of energy as latent energy and sensible heat.

With the data of Table 1 it is possible to determine the net radiative energy surplus or deficit within given latitude belts in the atmosphere and to compare them with similar estimates which have been made by considering the radiative processes. The procedure is quite simple. We find that 43 units (cal/sec × 10^{12}) are transported across latitude 75°N. The region of the atmosphere bounded on the south by this latitude must therefore radiate an amount of energy equal to this amount back to space in excess of that which it receives from space, otherwise there would be a progressive increase in the temperature of the atmosphere in this region. We may proceed in a similar fashion to the latitude belt 65°N - 75°N. Here we find 428 units transported in from the south, 43 units transported out to the north, and hence a surplus of 385 units of energy transported into this region. This amount must therefore represent the excess long-wave radiation by the atmosphere over the short-wave radiation which it receives from space. We may thus arrive

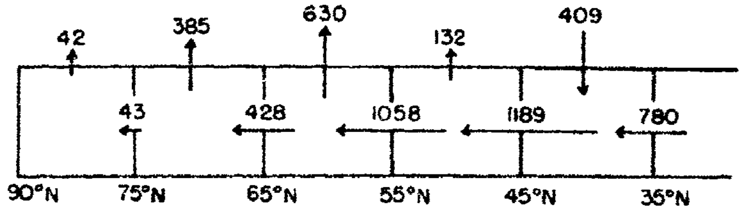

Fig. 1--Schematic diagram of the radiation balance deduced from observed energy transports; units in $10^{12} \times$ cal/sec; direction of arrows indicates direction of flux; internal consistency subject to rounding-off of errors

Table 2--Radiative energy surplus or deficit within various latitude belts as determined from radiative processes by various investigators and from observed atmospheric energy transports in this study

Latitude north From	Latitude north To	Albrecht	Houghton	Gabites	White
°	°	(cal/sec) × 10^{12}	(cal/sec) × 10^{12}	(cal/sec) × 10^{12}	(cal/sec) × 10^{12}
75	90	-126	-319	-222	- 43
65	75	-173	-392	-298	-385
55	65	-182	-405	-284	-630
45	55	-130	-322	-202	-132
35	45	- 3	-129	+ 29	+409

Note: Positive signs indicate a net surplus; negative signs a net deficit.

at an energy continuity diagram such as depicted in Figure 1. Here the amount of net radiative unbalance for each latitude belt is represented by the vertical arrows at the top of the diagram and the horizontal transport within the atmosphere by the horizontal arrows.

Comparison of results--It is of interest to compare the results of the computations described above with those of other investigators who have attacked this problem through a consideration of the radiative processes. Since the energy-transport data were available only for middle and polar latitudes, the discussion will be confined to these regions. Of the many investigations into this problem three have been selected for comparison, that of Albrecht and BJERKNES [1933, p. 665] and two of a more recent nature, that of Houghton (talk given at Cambridge Seminar of the American Meteorological Society, 1946) and that of Gabites (doctoral thesis, Massachusetts Institute of Technology, 1950). The results of Houghton's investigation should be considered as only preliminary in nature, as he is currently preparing a revised estimate. The values of the net radiative energy surplus or deficit reduced to the same units for the same regions which were used in this study are presented in Table 2, along with the results of the present investigation.

It is seen that the orders of magnitude of all these investigations are comparable, the more recent investigations agreeing more closely with the results of the present study. It is evident that the results of the present study give consistently higher values both for the net radiative deficit and the net radiative surplus except within the region between 75°N and the pole. This indicates that the intensity of the energy transport processes and the convergence of the energy transports are found to be more intense in the current study than would be indicated by the measurements based upon the radiative processes. It is also suggested that the region of net radiative heating in the atmosphere extends farther poleward than would be indicated by a consideration of the radiative processes. Nevertheless, it is quite gratifying, considering the crudity of the data which have been used to find that the estimates of the radiation balance deduced from radiative processes are fairly consistent with observations of the energy transport processes within the atmosphere. It appears that further studies of the energy transport processes might yield more reliable information which would provide an independent check upon the radiation balance as deduced from radiative processes. The desirability of further investigation is clearly indicated.

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The meridional eddy flux of energy

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Summary

An investigation of a large number of upper-air soundings over regions of North America shows that the eddy transport of sensible heat is poleward in the mid-latitude troposphere, reversing and becoming equatorward in the stratosphere. The eddy transport of sensible heat reaches its maximum in middle latitudes, decreasing both poleward and equatorward. It is shown wherein these results differ from those recently presented by Priestley. The transport of sensible heat has a high seasonal variation, being much stronger in winter than in summer. There are indications that at very low latitudes the eddy sensible-heat transport is directed equatorward.

The eddy transport of latent energy is described by means of the same soundings and is found to be poleward throughout middle and high latitudes with a maximum at the ground and decreasing with altitude. The principal seasonal variation in the transport of latent energy appears to be the northward shift of the mid-latitude maximum from winter to summer. This maximum is not very pronounced, the transport in both seasons being relatively constant through middle latitudes.

1. Introduction

A description of the character of the meridional flux of energy in the atmosphere is recognized to be of fundamental importance for a better understanding of the mode of operation of the general circulation. That a description of so basic a process in the atmosphere has not been clearly established can be attributed directly to the lack of observational material on a hemispherical basis, and to the tremendous task of reducing whatever data are available to comprehensible form. A method for the evaluation of certain components of the meridional energy flux has recently been reported by Priestley (1949). A study of the meridional flux of energy using a method similar to Priestley's but considerably more extensive in space, is described below.

2. Scope

To place the scope of this investigation in perspective it is necessary to consider the general continuity requirements of the global energy balance. Because there exists a difference between the net heating of the atmosphere in equatorial and polar regions, there must exist in the atmosphere a transport of energy directed meridionally from low to high latitudes. Starr (1949) has shown and indeed it may be inferred from general considerations that the total poleward energy flux may be closely approximated by an expression of the form

$$T^* \simeq \int_{z=0}^{z=\infty} \oint (c_p T + ql + \Phi)\rho v dx dz \qquad (1)$$

* This investigation was made possible through support extended by the Geophysical Research Directorate of the U.S.A.F. Cambridge Research Laboratories.

where c_p is the specific heat of air at constant pressure, T is the temperature, q is the specific humidity, l is the latent heat of water vapour, Φ is the potential energy per unit mass, ρ is the density, v is the meridional component of velocity considered positive when the wind is from the south, dx is an element of longitudinal distance and dz an element of vertical distance. The integration is to be performed over the entire vertical extent of the atmosphere and completely around the latitude circle. Eq. (1) has the simple interpretation that the poleward flux of energy is accomplished mainly as a flux of (a) sensible heat, (b) latent energy of water vapour, and (c) potential energy.

One way in which the terms in Eq. (1) may be further examined is as follows. We may write that

$$\rho \mathrm{v} = \left[\rho \mathrm{v}\right] + (\rho \mathrm{v})'$$

where $\left[\rho \mathrm{v}\right]$ is the average of $\rho \mathrm{v}$ along the entire length of a closed latitude circle and $(\rho \mathrm{v})'$ is the deviation from this average. It is clear that the identical vanishing of $\left[\rho \mathrm{v}\right]$ implies the absence of meridional circulations, while its failure to vanish implies the existence of such circulations. Here we neglect all topographic inequalities of the earth's surface. Since Φ is constant along a latitude circle at any given elevation and $\left[(\rho \mathrm{v})'\right]$ is zero, Eq. (1) may be rewritten as

$$T^* \simeq \int_{z=0}^{z=\infty} \oint (c_p T + ql)(\rho \mathrm{v})' dx dz + \int_{z=0}^{z=\infty} \oint (c_p T + ql + \Phi) \left[\rho \mathrm{v}\right] dx dz \qquad (2)$$

where the last integral now represents the contribution of mean meridional circulations of the Hadley type to the poleward energy-flux. The first integral in the right hand member of Eq. (2) represents the contribution of horizontal eddy-processes to the poleward energy-flux. It is the magnitude of these processes which can be estimated with available radio observations of wind, temperature and moisture content.

3. Procedure

Following Priestley we may write that the total meridional flux of any quantity across a complete circle of latitude is given very closely by,

$$\oint dx \int_{z=0}^{z=\infty} \rho F \mathrm{v} dz \qquad (3)$$

where F is the amount of the quantity per unit mass of the atmosphere. The integration is to be performed through the entire vertical extent of the atmosphere, and around a complete latitude circle. For practical purposes, it is more convenient to write this integral as

$$\frac{1}{g} \oint dx \int_0^{p_0} F \mathrm{v} dp \qquad (4)$$

where use has been made of the hydrostatic equation, and where p_0 is the sea level

pressure. Because observations around a complete circle of latitude are not available, it is impossible to compute this transport observationally. However, we may obtain highly useful information by considering the flux over a limited portion of the circle, which is given for a unit distance along a latitude circle by

$$\frac{1}{g}\int_0^{p_0} Fvdp \quad . \quad . \quad . \quad . \quad (5)$$

In expression (5) the sea level pressure p_0 is not constant. However, p_0 varies by only a small fraction of its absolute magnitude. In addition, in the computation of such a term over a large sample, p_0 may be considered as a constant if there is little correlation between Fv and p_0. We shall assume that the quantities used in the following analysis, the temperature and the mixing ratio multiplied by the meridional component of the velocity, are uncorrelated with the sea level pressure. Therefore, we shall treat p_0 as a constant in what follows.

In the following analysis it will be necessary to make use of both space and time means and deviations from these means. For convenience in notation, a bar is used to denote a time mean, a bracket a space mean. Primes denote deviations from the means and the subscripts x and t are used to denote the nature of the deviation. Thus we may write $F = \bar{F} + F_t'$ or $F = \left[F\right] + F_x'$ where F is the actual value of the quantity, F_t' is the deviation from the time mean and F_x' is the deviation from the space mean. In a similar fashion we may define a mean of F in both space and time as $\left[\bar{F}\right]$.

The mean meridional flux of F over both space and time, $\left[\overline{\mathrm{v}F}\right]$ is given by

$$\left[\overline{\mathrm{v}F}\right] = \left[\bar{\mathrm{v}}\right]\left[\bar{F}\right] + \left[\overline{\mathrm{v}'_{tx}F'_{tx}}\right] \quad . \quad . \quad . \quad (6)$$

which is derived by taking the mean of the product $\mathrm{v} = \left[\bar{\mathrm{v}}\right] + \mathrm{v}'_{tx}$ and $F = \left[\bar{F}\right] + F'_{tx}$. The mean flux of F is then composed of two terms of distinct physical significance. Following Priestley's terminology, let us call $\left[\bar{\mathrm{v}}\right]\left[\bar{F}\right]$ the advective flux. This is the flux of the quantity F at any given level in the atmosphere which is associated with a non-zero mean rate of flow $\left[\bar{\mathrm{v}}\right]$ at the level in question. The second term $\left[\overline{\mathrm{v}'_{tx}F'_{tx}}\right]$ is the mean eddy flux and depends upon the existence of a correlation between v and F at any given level.

It can be shown that the mean eddy flux may be further analysed into two terms of different physical significance. This is easily done by adding to and subtracting from the left hand side of the Eq. (6) the term $\left[\bar{\mathrm{v}}\bar{F}\right]$. After slight rearrangement it can be seen that

$$\left[\mathrm{v}'_{tx}F'_{tx}\right] = \left[\overline{\mathrm{v}'_tF'_t}\right] + \left[\bar{\mathrm{v}}'_x\bar{F}'_x\right] \quad . \quad . \quad . \quad (7)$$

in which the terms on the right hand side have the following meanings :

(a) $\left[\overline{\mathrm{v}'_tF'_t}\right]$ represents the space mean of the local eddy flux. This is the average over space of the eddy flux at individual stations due to the correlation in time of v and F.

(b) $\left[\bar{\mathrm{v}}'_x \bar{F}'_x\right]$ represents the standing eddy flux. This is the flux due to the correlation in space between the time means of F and v over a given period of time. In other words, this is the portion of the flux due to the semi-permanent features of the atmosphere.

Ideally, if we wished to compute the total northward eddy flux for a given period from actual wind-data, we should desire stations located around a complete circle of latitude. The more closely we could approximate a continuous distribution of stations, the more accurately we could obtain the eddy transport across that latitude circle. We could then compute the total eddy flux once or twice daily, and obtain the mean flux for any given period. In the absence of such a distribution of observing stations the existing correlation between v and F may be approximated by the use of a large number of stations distributed over a limited portion of the latitude circle. Because of the nature of the available data, all of the computations which follow have been made with reference to constant pressure surfaces.

In the case of the transport of sensible heat, $F = c_p T$. The mean eddy flux of sensible heat is then given by

$$\frac{c_p}{g}\int_0^{p_0}\left(\left[\overline{\mathrm{v}T}\right] - \left[\bar{\mathrm{v}}\right]\left[\bar{T}\right]\right)dp \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The mean eddy flux of latent heat may be written similarly as

$$\frac{l}{g}\int_0^{p_0}\left(\left[\overline{\mathrm{v}q}\right] - \left[\bar{\mathrm{v}}\right]\left[\bar{q}\right]\right)dp \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where l is the latent heat of water vapour per unit mass taken as 600 cal gm^{-1}, and q is the specific humidity.

In his interesting paper, "Heat transport and zonal stress between latitudes," Priestley (1949) selected a single station, Larkhill, England, and performed such computations over a period of two years. However, since he used only a single station, he was able to evaluate only the local eddy flux term.

He found that "the eddy flow of sensible heat at some levels is from regions of low towards regions of higher mean temperature and that the eddy fluxes of heat in sensible and latent forms are approximately equal." These conclusions must be modified however when the standing eddy flux of these quantities is also considered.

In order to include a measure of the standing eddy flux it is necessary to take into account some of the semi-permanent features of the general circulation. The investigation was extended to include subtropical and polar regions so as to be able to determine the variation of such eddy transports with latitude. The region of North America and the adjacent regions of the Atlantic Ocean were chosen for study. In order to eliminate any bias toward cloudy or clear weather only stations which take radiosonde and radio wind observations were selected*.

The months of February and August 1949 were selected for study. The upper-air data were taken from the *Daily upper air bulletin* published by the U.S. Weather

* The observing stations were distributed as follows : South of 30°N. — 6 ; 30° to 40°N. — 16 ; 40° to 50°N. — 14 ; 50° to 60°N. — 6 ; 60° to 70°N. — 6 ; 70° to 80°N. — 4.

Bureau. The data were broken down into three groups · (a) Feb. 0300 GMT, 1949, (b) Feb. 1500 GMT, 1949, (c) Aug. 1500 GMT, 1949. Only those soundings which reached a height of 300 mb were used in the computations. It was felt that such a restriction would enable more accurate comparison of low-level and high-level transports*.

The data were recorded at each station at each of the standard pressure levels : 1000, 850, 700, 500, 300, 200, and 100 mb. The products vT and vq were formed for each level and each time category for each latitude belt.

Then $\left[\overline{vT}\right]$, $\left[\overline{vq}\right]$, $\left[\bar{v}\right]$, $\left[\bar{T}\right]$, $\left[\bar{q}\right]$ were computed by a simple averaging process†. The subtractions $\left[\overline{vT}\right] - \left[\bar{v}\right]\left[\bar{T}\right]$ and $\left[\overline{vq}\right] - \left[\bar{v}\right]\left[\bar{q}\right]$ were performed for each level, time category, and latitude belt. These were then converted into the mean flux in cal sec^{-1} per mb of pressure difference in the vertical per cm of longitude. Because of the unrepresentativeness of observations taken near the surface (due to radiation and local topographic effects) the values of the transports of energy obtained for the 1000 mb surface were not used. Instead, in the integrations through height which were made in order to obtain the mean total eddy transport throughout the atmosphere, it was assumed that the transports at 850, 700, 500, 300, 200, and 100 mb were representative of the transports between the surface and 800 mb, 800 and 600 mb, 600 and 400 mb, 400 and 250 mb, 250 and 150 mb and 150 and 50 mb, respectively. It was assumed that the transports above 50 mb could be neglected.

The results of this investigation are presented in the form of a series of diagrams and tables. A short discussion of the nature of these data is necessary for proper interpretation. The data may be considered to be of two types, (a) the eddy transports of either sensible heat or latent energy or a combination of both at given levels and latitudes, and (b) the total eddy transports of sensible heat or latent energy or a combination of both, by which is meant the value of the transport after integration through height. It must be remembered that the results presented are valid only for regions of North America and only for the specific times for which they were computed.

Tables I and II present the total eddy transports of the two energy forms, by which is meant the values of the eddy transports after integration through height, across various latitude circles. This was done under the assumption that these transports are representative of the transports around a complete circle of latitude. The values of these total transports are given in units of 10^{11} cal sec^{-1} across a complete circle of latitude.

* Such a restriction undoubtedly has some effect upon the magnitudes of the energy transports because of the selectivity of radio wind observations. Dr. E. N. Lorenz (1950) has found in an investigation into this selectivity, a pronounced tendency for radio wind observations to be biased according to the wind speed. In periods of strong winds there is a decided tendency for balloons to be blown out of range or below the angle specified in observing manuals below which observations are considered unreliable and hence not reported. The effect of such a bias is to minimize the magnitude of the energy transports. Hence the magnitudes of the energy transports reported herein may lean in the direction of under-estimation rather than over-estimation.

† Such a simple averaging process would be entirely legitimate were the selected stations evenly spaced along latitude circles. However, the stations are irregularly spaced, and hence such an averaging process introduced a bias because each station is weighted equally. The results therefore may be biased in favour of the character of the eddy transport in those regions where the stations are more closely spaced. In order to correct this bias, proper weighting factors would have to be introduced. However, since the stations are irregularly spaced in both latitude and longitude, it is not clear how a proper weighting procedure could be devised.

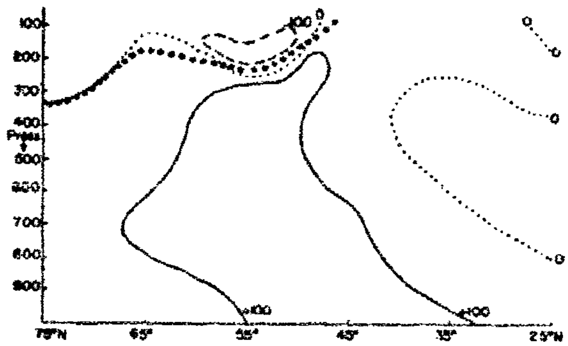

Figure 1. Transport of sensible heat computed from soundings for August 1949, 1500 GMT.

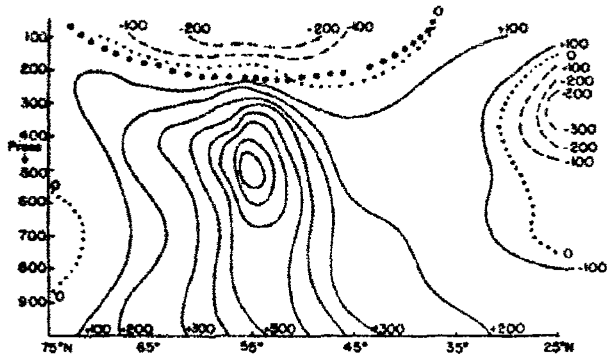

Figure 2. Transport of sensible heat computed from soundings for February 1949, 0300 GMT + 1500 GMT.

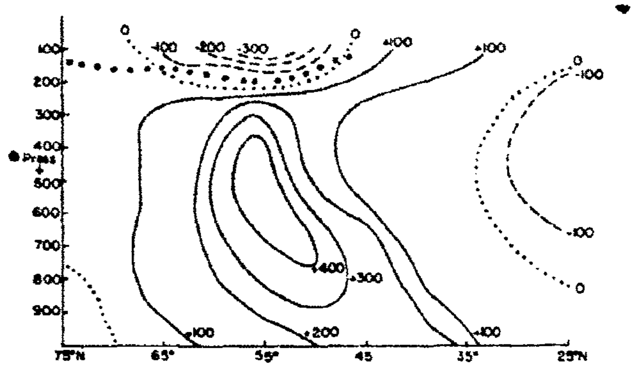

Figure 3. Transport of sensible heat computed from soundings for February 1949, 1500 GMT + August 1949, 1500 GMT.

← Latitude

Figs. 1-3. The mean eddy transport of sensible heat in the atmosphere over regions of North America. Isolines represent the eddy transport in units of cal/sec x 10' per degree of longitude per mb pressure difference in the vertical. Solid isolines indicate a northward transport ; dashed isolines a southward transport. The isoline of zero transport is represented by dots. The height of the mean tropopause for the cases examined is indicated by open dots.

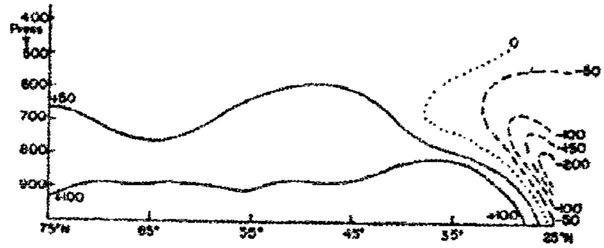

Figure 4. Latent-energy transport computed up to 500 mb from soundings for August 1949, 1500 GMT.

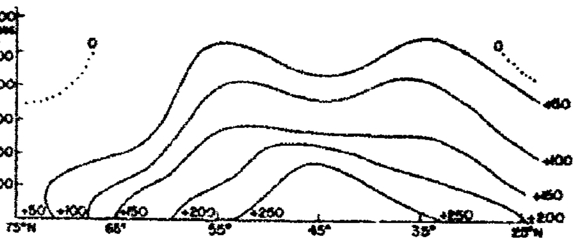

Figure 5. Latent-energy transport computed up to 500 mb from soundings for February 1949, 0300 GMT + 1500 GMT.

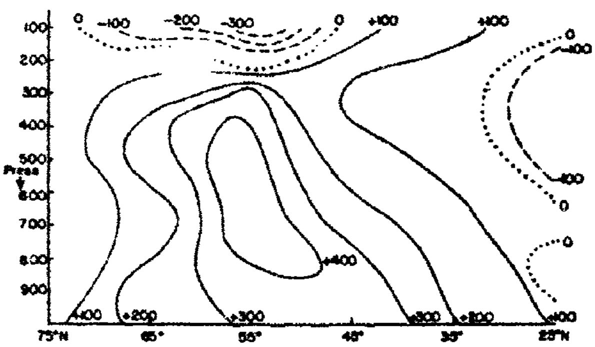

Figure 6. Transport of energy as the sum of the latent energy and sensible heat computed from soundings for February 1949, 1500 GMT + August 1949, 1500 GMT.

← Latitude

Figs. 4-6. The mean eddy latent-energy transport and the mean eddy transport of the sum of the latent energy and sensible heat in the atmosphere over regions of North America. Isolines represent eddy energy transport in units of cal/sec x 10^7 per degree of longitude per mb pressure difference in the vertical. Solid isolines indicate a northward transport ; dashed lines a southward transport. The isoline of zero transport is represented by dots.

MERIDIONAL EDDY FLUX OF ENERGY

The eddy energy transports are presented in the form of diagrams of isolines of transport in Figs. 1-6. These diagrams, drawn in a coordinate system in which the vertical coordinate is the pressure on a linear scale, and in which the isolines of transport are in cal sec^{-1} *per mb pressure difference in the vertical* per degree of longitude at the given pressure levels. These diagrams have the advantage that the eddy energy transports are presented as a function of the atmospheric mass. To prevent erroneous interpretations of such diagrams, a single sample diagram Fig. 7, is presented in which the vertical coordinate is geometrical height on a linear scale, and the isolines of transport are in cal sec^{-1} *per cm of height* per degree of longitude at specific geometric heights.

4. Interpretation of results

The pictures of the eddy energy transports computed from actual wind data over regions of North America, as outlined above, reveal many interesting features (see Figs. 1, 2, and 3).

The sensible-heat transport during February 1949, which has been taken as typical of a winter season, is northward within the troposphere everywhere north of 25°N., and southward in the stratosphere in mid-latitudes. This is well brought out in Fig. 2 where the height of the mean tropopause for February 1949 based upon temperature observations has been sketched in. It can be seen that there is remarkable agreement between the tropopause position and the level of reversal of the direction of the eddy transport of sensible heat.

A noticeable feature of the eddy transport of sensible heat during this month is the location of the latitude of maximum transport at 55°N. One may not draw the conclusion that this represents the winter mean latitude of maximum transport because of the limited amount of data treated.

At the latitude of maximum intensity, the transport per mb of pressure difference reaches its maximum at some level above the ground, in this case at 500 mb. In the regions south of the latitude of maximum transport, the eddy transport per mb of pressure difference is at a maximum in the lowest layers of the atmosphere.

Some mention should be made about the indications of a southward eddy transport of sensible heat in the middle troposphere near 25°N. However, it should be remembered that at such low latitudes only a small section of the latitude circle has been sampled and the standing eddy-flux term may not be adequately represented in the data from a limited region south of the United States.

During August 1949 (see Fig. 1) the same characteristics evident during the winter season are again present. The transports are northward but much weaker through most of the mid-latitude troposphere, reversing in the stratosphere. Again the coincidence of the tropopause with the level of the reversal of the transport direction may be noted. The latitude of maximum transport is at 55°N. with the level of maximum intensity again located at the 500 mb level. In lower middle-latitudes the transport is greatest near the ground. At low latitudes there is marked difference from winter to summer, in that the region of southward transport of sensible heat extends farther to the north.

With the present data the closest approximation to a picture of a typical yearly mean may be computed by combining the eddy transport for August, 1500 GMT

and February, 1500 GMT, Fig. 3. The characteristics of such a mean picture are again :

(a) Northward eddy flux of sensible heat in the troposphere.
(b) Latitude of maximum transport at 55°N.
(c) Level of maximum transport at 500 mb.
(d) Southward transport in the stratosphere.
(e) Southward transport at lowest latitudes.

In our discussion to this point, the eddy transports of sensible heat have been referred to constant pressure levels and discussed in terms of an eddy transport per mb of pressure difference. Though this method of description has many advantages in that it permits easy association of such transports within given masses of the atmosphere, it is of interest to relate such transports to a more readily pictured geometrical coordinate system with the transports given per unit height. It is, of course, obvious that such a conversion to a geometrical coordinate system will greatly emphasize the transports at the lower geometrical levels at the expense of the transports at the higher geometrical levels.

To accomplish this transformation, the eddy transport of sensible heat at each level was assigned to the geometric height above sea level as given by the NACA standard atmosphere. Although it is realized that the elevation of an isobaric surface can in no way be considered constant, the transformation is of sufficient accuracy to depict the desired features. In converting the eddy transports of sensible heat per mb of pressure difference in the vertical into the eddy transport per unit height, the mean temperature at each pressure level was used.

The diagram of the eddy transport of sensible heat in the new coordinate system for August 1949, 1500 GMT plus February 1949, 1500 GMT is shown in Fig. 7. This may be compared with Fig. 3 in which the same data are presented in the former coordinate system. The major difference between the two figures is the

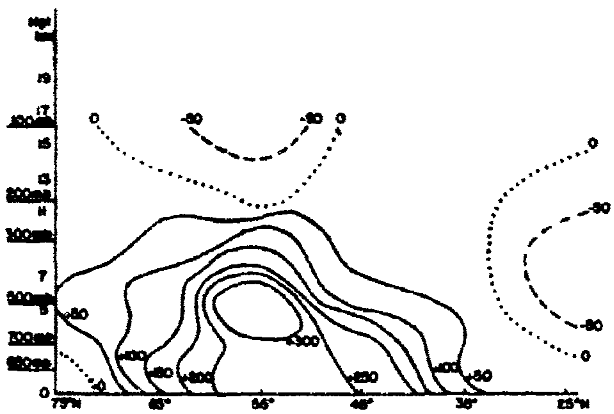

Figure 7. The mean eddy transport of sensible heat in the atmosphere over regions of North America for August 1949, 1500 GMT and February 1949, 1500 GMT described by means of isolines of transport in a geometrical coordinate system with latitude and elevation as the coordinates. The eddy transport is in units of 10^4 cal/sec per degree of longitude per unit height. Solid isolines indicate a northward transport, dashed isolines a southward transport. The isoline of zero transport is indicated by dots. The preponderance of the eddy transport in the lowest layers of the troposphere is emphasized.

concentration of the eddy transport of sensible heat in the lowest geometrical levels of the atmosphere in Fig. 7. The strong maximum of transport per mb of pressure difference at the 500 mb surface in Fig. 3 is considerably reduced in magnitude relative to the transports at lower layers in Fig. 7.

The eddy transport of latent-heat has a maximum in the lowest layers and decreases with height at all latitudes. During the winter season, as typified by the February data (see Fig. 5), the latent-heat transport has a slight double maximum at 35° and 55°N. and in the lowest layers. The reality of this double maximum as a mean hemispherical condition is questioned and is probably attributable to the location of the selected stations. During the summer, as typified by the August data (see Fig. 4), the maximum is to the north and in the lowest layers. An interesting feature is the southward flow in the very low latitudes.

In the final analysis, the sum of the transports of latent energy and sensible heat must be of fundamental significance for the global energy balance. The typical yearly-mean picture of the transport of the sum of these energy quantities, as computed from the 1500 GMT data for February and August 1949, appears in Fig. 6. The major characteristics may be stated as follows :

(a) Latitude of maximum eddy flux — 55°N.
(b) Level of maximum eddy flux at latitude of maximum eddy flux — 500 mb.
(c) Northward transport in mid-latitude troposphere.
(d) Southward transport in mid-latitude stratosphere.
(e) Southward transport in low-latitude troposphere.
(f) Decrease of transport with height in lower mid-latitudes.

The eddy transports of the various forms of energy may be integrated through height, under the assumption that such transports computed for regions of North America are representative of the transport across a complete circle of latitude. The latitude of maximum eddy transport of sensible heat during both winter and summer, as typified by the North American observed wind data, Table I, occurs at 55°N.

TABLE I. The total transport of (a) sensible heat and (b) latent heat, integrated vertically to the top of the atmosphere, transformed into the total transport across a latitude circle under the assumption that the data for North America are representative for the entire hemisphere. Values given for various latitudes and times of year. Units in cal/sec $\times 10^{11}$. F = Feb., A = Aug.

Lat. °N.	F0300	F1500	A1500	F0300 + F1500	F1500 + A1500
			(a) Sensible heat		
25	+ 399·3	− 3305·1	− 47·2	− 1452·9	− 677·1
35	+ 4875·7	+ 4303·4	+ 134·5	+ 4578·9	+ 2230·4
45	+ 3885·6	+ 5590·6	+ 3130·0	+ 4738·8	+ 4704·9
55	+ 21756·1	+ 10452·7	+ 4051·0	+ 16192·6	+ 7352·0
65	+ 3300·6	+ 5095·4	+ 1967·2	+ 5191·7	+ 3841·4
75	+ 743·6	+ 1048·3	+ 1033·2	+ 667·2	+ 1039·5
			(b) Latent heat		
25	+ 1074·5	+ 1764·2	− 2337·7	+ 1401·2	− 290·4
35	+ 3332·5	+ 2381·3	+ 1095·5	+ 2853·6	+ 1745·0
45	+ 1450·0	+ 1318·8	+ 1630·1	+ 1384·4	+ 1477·3
55	+ 2825·9	+ 2097·6	+ 1534·3	+ 2464·0	+ 1818·3
65	+ 588·1	+ 919·4	+ 1024·1	+ 757·1	+ 868·7
75	+ 145.6	+ 16·6	+ 1354·1	+ 108·2	+ 634·4

The average of the data for February 1949 and August 1949 reveals what may be considered an approximation to of the yearly mean total eddy transport of sensible heat. There exists a maximum transport across 55°N. with a marked and continuous divergence of the transport south of 55°N. and a marked and continuous convergence of the transport north of this latitude.

The total eddy transport of latent heat, Table I, differs slightly from winter to summer. During the winter time the total eddy transport of latent heat exhibits the double maximum at latitudes 35° and 55°N. and the transport decreases sharply to the north of 65°N. Contrasted with these conditions, the total eddy transport during the summer has a slight single maximum at 45°N. but is relatively constant between 35°N. and 75°N. Although the transport in the summer time is generally smaller in middle latitudes than in the same region during the winter, it is much larger at higher latitudes. An additional feature is the southward transport of latent heat across 25°N. in the summer time as contrasted with a winter-time northward transport across the same latitude.

When one combines the transport of energy in latent form and as sensible heat from the data for February and August 1949 a picture of the total eddy energy-transport as the sum of these energy forms is obtained. These figures should approximate the terms $\int\int(c_pT + ql)(\rho v)'dxdz$ of Eq. (2). This combined total transport, Table II, last column, shows a maximum at 55°N. decreasing both south and north of this latitude. A southward transport exists across 25°N.

TABLE II. The total eddy transport of sensible + latent heat, integrated vertically to the top of the atmosphere, transformed into the total transport across a latitude circle under the assumption that the data for North America are representative for the entire hemisphere. Values given for various latitudes and times of year. Units in cal/sec $\times 10^{11}$. F = Feb., A = Aug.

Lat. °N.	F0300	F1500	A1500	F0300 + F1500	F1500 + A1500
25	+ 1473·8	− 1540·9	− 2384·9	− 51·7	− 1987·5
35	+ 8208·2	+ 6684·7	+ 1230·0	+ 7432·5	+ 3975·4
45	+ 5335·6	+ 6909·4	+ 4760·1	+ 6122·5	+ 6182·2
55	+ 24582·0	+ 12550·3	+ 5585·3	+ 18656·6	+ 9170·3
65	+ 3888·7	+ 6014·8	+ 2991·3	+ 5948·8	+ 4710·1
75	+ 889·2	+ 1064·9	+ 2387·3	+ 775·4	+ 1673·9

In a rough manner it is possible to make a comparison of the magnitudes of the total eddy transport of energy as the sum of sensible and latent heat with published values for the total energy transport required by the radiation balance indicating that horizontal eddy-transport mechanisms accomplish a major portion of the required energy transport. Preliminary hemispherical determinations of long-term means of all the quantities entering into the global energy-balance and the significance of the eddy-energy flux in the total scheme have been completed by the author and will be reported in a forthcoming paper. However, in view of the fact that only a small portion of the hemisphere has been sampled for a short time-period in the investigation reported here, the magnitudes should not be considered as approaching true means. On the other hand, considerable weight must be attached to the information which such data give us regarding the variation of the eddy-transport processes in both time and space.

It is of great interest to determine what portion of the eddy transport of sensible heat is accomplished in the lower half of the atmosphere below 500 mb. It is also of interest to determine the difference in the magnitude of the transport of sensible heat at various latitudes from summer to winter. The analysis was carried out for the mid-latitude regions 35°N. to 65°N. The percentages of the total transport occurring below 500 mb based upon the data for February 1949 (0300 GMT + 1500 GMT) are as follows:

35°N.	45°N.	55°N.	65°N.
62	88	68	73

It can be seen that there is a slight variation in the percentages but not greater than might be expected from the nature of the data.

In comparing the rate of transport during the winter with the mean yearly transport, it is found that the ratio of the transport as given by the February data to the mean transport as given by the February and August data is:

35°N.	45°N.	55°N.	65°N.
1·93	1·19	1·42	1·34

It was also possible with the data at hand to show more clearly the relative magnitudes of the local eddy flux of sensible heat and the standing eddy flux. For this purpose a selection of five stations along latitude 45°N. was made. These included Spokane, Wash.; Tatoosh, Wash.; Bismark, N.D.; Int. Falls, Minn.; and Caribou, Maine.

The local eddy flux was computed for February 1949 for each of these stations in a manner similar to Priestley's. The standing eddy flux was then obtained from a consideration of the space correlation of the time means for each of these stations. The mean local eddy flux and the standing eddy flux for these stations are given in Table III. It can be seen that both terms are of the same order of magnitude.

TABLE III. A comparison of the magnitude of the mean local eddy flux and standing eddy flux of sensible heat at the various levels in the atmosphere over regions of the United States at latitude 45°N. Units in °A × m/sec per cm of longitude for a vertical distance equal to one mb of pressure difference.

mb	Mean local eddy flux	Standing eddy flux
850	+ 10·02	+ 14·41
700	+ 10·29	+ 7·43
500	+ 11·39	+ 2·52
300	+ 1·19	− 1·25
200	− 12·08	− 12·96
100	+ 1·34	− 0·27

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The Meridional Flux of Sensible Heat over the Northern Hemisphere

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(Manuscript received 22 November 1950)

Abstract

The meridional eddy flux of sensible heat is examined by means of finite difference integration methods from northern hemisphere charts for the winter 1945—46. The eddy sensible heat flux is shown to occur principally in the lowest layers of the atmosphere and to decrease with elevation. The magnitude of the eddy sensible heat transports are compared with the total poleward energy flux required by the radiation balance, and are shown to comprise a very large fraction of this flux.

Introduction

The difference in the net heating of equatorial and polar latitudes necessitates a meridional transport of total energy directed from equatorial to polar regions. This transport may be expressed to a high degree of accuracy as

$$T^* = h + \int_s (c_p\tau + ql + \Phi)\, \varrho v ds \qquad (1)$$

where h is the contribution to the poleward energy flux by the hydrosphere, c_p is the specific heat of air at constant pressure, τ is the temperature, q is the specific humidity, l is the latent heat of water vapor, Φ is the geopotential energy per unit mass, ϱ is the density, v is the northward component of the wind velocity and ds is an element of the conical surface above a given latitude. The integration is to be performed over the entire conical surface. Equation (1) may be written down directly from considerations of the conservation of energy. It may also be derived from the general physical energy equation and the equations of motion as shown by STARR and collaborators (1949).

Equation (1) simply states that the poleward flux of energy may be accomplished essentially as (a) an energy flux within the oceans, (b) a flux of sensible heat, (c) a flux of latent heat, (d) and a flux of potential energy. The term $\int_s c_p \tau \varrho v ds$ results from the combination of the flux of internal energy of dry air and the flux of kinetic energy transferred as work done by the pressure forces. These two energy forms are always transferred in the constant ratio c_v/R where R is the gas constant for dry air and c_v is the specific heat of air at constant volume. The transport of kinetic energy accomplished by the flow of existing kinetic energy has been neglected because of its smaller order of magnitude (see STARR 1949).

The partition of the poleward energy flux among the various terms in equation (1) is a problem of great importance for a better understanding of the mode of operation of the general circulation. The magnitude and variation of the flux of sensible heat across various latitudes upon a hemispherical basis will be examined below.

Computation procedure

Because of the lack of adequate observed wind data upon a hemispherical basis it

[1] This investigation was made possible through support extended by the Geophysical Research Directorate of the U.S.A.F. Cambridge Research Laboratories, A.M.C., under Contract No. AF 19-122-153.

becomes necessary to compute the meridional flux of sensible heat using geostrophic winds. Use of such a measure of the wind velocity to calculate this term introduces modifications into its meaning which are not without advantages.

The integrated flux of sensible heat across a latitude circle at a given geometrical level in the atmosphere is given by the expression

$$T_E = \oint c_p \tau (\varrho v)\, dx \qquad (2)$$

where dx is an element of distance longitudinally and the integration is performed around a complete circle of latitude. If we place

$$\tau = \bar{\tau} + \tau' \qquad (3)$$

$$\varrho v = \overline{\varrho v} + (\varrho v)' \qquad (4)$$

where the primes denote deviations of the quantities from the mean around the latitude circle at the given level, and take the product of equations (3) and (4), then equation (2) may be written as

$$T_E = c_p \oint \bar{\tau}\, \overline{\varrho v}\, dx + c_p \oint \tau' (\varrho v)'\, dx \qquad (5)$$

in which each of the terms has a distinct physical significance. The first term in the right hand member of equation (5) is the portion of the flux of sensible heat due to the non-zero value of $\overline{\varrho v}$ at the given level, i.e., the portion transported by virtue of a mass flux at the level and hence accomplished by mean meridional cells of the Hadley type.

The second term in equation (5) is that portion of the flux due to the correlation between τ and (ϱv) along the latitude circle. As such it represents the transport of sensible heat by large scale horizontal eddy processes associated with eddies of the size of migratory cyclones and anticyclones of middle latitudes.

If the terms in equation (5) are computed geostrophically, the first term must vanish and the only portion of the flux which does not vanish is that part due to the correlation between the temperature and the meridional component of the geostrophic wind. The question arises immediately as to the nature of the difference between the geostrophic eddy transport and the actual eddy transport. This point will be discussed in greater detail in a later part of this paper.

Estimates were made of the geostrophic eddy transport of sensible heat based upon complete northern hemisphere data by finite difference integration methods. The data which were used extended only up to 500 mb and were available in a mixed coordinate system. The surface data were available at the geometric height $z = 0$, whereas the upper air data were available along constant pressure surfaces.

The computation procedure is greatly simplified if the eddy flux term which depends solely on the correlation between the temperature and (ϱv) is calculated along constant pressure surfaces. The assumption which is introduced by this procedure is that the correlation between the temperature and the geostrophic mass flux along a constant level can be approximated by the correlation between the temperature and the geostrophic wind along some corresponding pressure surface. It is probable that the eddy flux computed from constant pressure charts resembles closely a similar flux calculated from constant level charts. For purposes of this investigation this condition is assumed to hold. In any case we may state that the eddy flux of sensible heat which has been calculated is that given by computations based upon constant pressure surfaces.

The data which are available for computational purposes are in the form of the sea level pressure distribution, and the height distributions of the 700 and 500 mb surfaces. In view of this it becomes more convenient to use the approximation that

$$\int_{z=0}^{z=z_1} \oint c_p \tau \varrho v_g\, dx\, dz \approx c_p/g \int_{500\,\text{mb}}^{p_0} \oint \tau v_g\, dx\, dp \qquad (6)$$

where $z = 0$ is sea level, z_1 is equal to some constant height at approximately the 500 mb level, p_0 is taken as the sea level pressure and the element ds has been approximated by $dx\,dz$ where dx is an element of distance longitudially and dz is an element of distance in the vertical. Substituting the geostrophic relation for v_g we find

$$c_p/g \int_{500}^{p_0} \oint \tau v_g\, dx\, dp = c_p/f \int_{500\,\text{mb}}^{p_0} \oint \tau \frac{\partial z}{\partial x}\, dx\, dp \qquad (7)$$

where f is the coriolis parameter.

Since temperature observations are not available within these layers over a wide network of stations it is necessary to substitute the mean virtual temperature of the layer in question for the actual temperature. Because the magnitude of the eddy transport across any given latitude circle depends not on the absolute value of the temperature but upon the correlation between the wind direction and the temperature and the variance of these quantities when integrated around a latitude circle, this substitution probably introduces only a slight error, the mean virtual temperature and the actual mean temperature of the layer being very highly correlated, and the variance of the mean virtual temperature being about equal to the variance of the actual mean temperature.

The sea level pressure was reduced to the height above sea level of the 1013 mb surface, assuming the NACA standard atmosphere. This reduction introduces a slight error into the calculations. We may write that the eddy transport of sensible heat evaluated up to 500 mb is composed of three terms as follows:

$$T_E = \frac{c_p}{f}\left[\int_{500\,\mathrm{mb}}^{700\,\mathrm{mb}} \oint \bar{\tau}_v \frac{\partial z}{\partial x}\,dx\,dp + \int_{700\,\mathrm{mb}}^{1013\,\mathrm{mb}} \oint \bar{\tau}_v \frac{\partial z}{\partial x}\,dx\,dp + \int_{1013\,\mathrm{mb}}^{p_0} \oint \bar{\tau}_v \frac{\partial z}{\partial x}\,dx\,dp\right] \qquad (8)$$

where $\bar{\tau}_v$ is the mean virtual temperature appropriate to the given layer. The integration of the first two terms with respect to pressure in the vertical is between two constant pressure surfaces and hence presents no problem in evaluation. The third term however has a variable upper limit of integration because the sea level pressure is not constant along a latitude circle. It seems highly probable that the transport given by the third term is no larger than the transport between two constant pressure surfaces separated by an amount equal to the average pressure difference between sea level and 1013 mb., i.e., by perhaps 30 mb, so that the third term is perhaps a tenth as large as the second. We may state then that in all probability the errors introduced by the conversion of the surface pressure to the height of the 1013 mb surface are minor in nature. The eddy transport of sensible heat was computed for the layers, 1013 to 700 mb, and 700 mb to 500 mb. The mean geostrophic wind for each layer defined as the mean of the geostrophic wind at the top and at the bottom of the layer in question was used. The product of the mean virtual temperature and the geostrophic wind was formed at 72 evenly spaced points at each circle of latitude and then the integration of equation (7) was accomplished by summing the 72 products so derived for each latitude circle and each layer for each day. The computations of the eddy transport of sensible heat were carried out by the General Circulation Project, M.I.T. partly by hand and partly by punch card methods.

The question may legitimately be raised as to the nature of the difference between the eddy transport of sensible heat by actual winds versus that by geostrophic winds. These eddy transports were compared over regions of North America, where sufficient observed upper wind data are available. Data for the months of January 1949 and December 1948 were selected over regions of the United States between latitudes 43° N and 50° N. Wherever possible, rawinsonde stations were selected to prevent bias toward fair weather synoptic patterns. The study was carried out only at 700 mb. The sources of the data were as follows:

(a) 700 mb actual winds and temperatures were gathered from the United States Weather Bureau Daily Upper Air Bulletin.[1]

(b) 700 mb geostrophic winds were computed from Daily United States Weather Bureau 700 mb charts. The ratio

$$\frac{\overline{\tau v_g} - \bar{\tau}\bar{v}_g}{\overline{\tau v_a} - \bar{\tau}\bar{v}_a} = \frac{\overline{\tau' v_g'}}{\overline{\tau' v_a'}} \qquad (9)$$

was examined. Here τ is the temperature in degrees centigrade, v_g is the geostrophic wind velocity, v_a is the observed wind velocity. The bars denote means with respect to time and space, and the primes denote deviations

[1] The stations selected for this study were: Bismark N.D. 764; Buffalo, N.Y. 528; Caribou, Me. 712: Glasgow, Mont. 758; International Falls, Minn. 747; Lander, Wyo. 576; Rapid City, S.D. 662; St. Cloud, Minn. 644; Sault Ste. Marie, Mich. 735; Spokane, Wash. 785: Tatoosh, Wash. 798; Great Falls, Mont. 775.

Table I

The mean geostrophic eddy transport of sensible heat across various latitudes for the winter season 1945—46 for layers and periods specified. Units in cals $sec^{-1} \times 10^{+12}$

Month	Layer	Lat. 75° N	65° N	55° N	45° N	35° N
Nov. 1945	1013—700 mb	— 38.0	+ 147.0	+ 447.1	+ 371.0	+ 193.9
	700—500 mb	+ 17.9	+ 91.2	+ 189.9	+ 168,3	+ 74.3
	1013—500 mb	— 20.2	+ 238.2	+ 636.9	+ 539.3	+ 268.1
Dec. 1945	1013—700 mb	— 2.4	+ 172.9	+ 670.7	+ 781.9	+ 361.2
	700—500 mb	+ 23.3	+ 84.1	+ 221.5	+ 168.6	+ 107.6
	1013—500 mb	+ 20.9	+ 257.0	+ 892.3	+ 950.5	+ 468.8
Jan. 1946	1013—700 mb		+ 335.4	+ 745.8	+ 580.1	+ 177.0
	700—500 mb		+ 120.8	+ 198.7	+ 232.7	+ 122.5
	1013—500 mb		+ 487.1	+ 941.7	+ 812.7	+ 279.2
Feb. 1946	1013—700 mb	+ 41.1	+ 269.4	+ 683.7	+ 713.0	+ 320.0
	700—500 mb	+ 1.1	+ 60.2	+ 178.3	+ 153.7	+ 841.7
	1013—500 mb	+ 42.1	+ 329.6	+ 862.1	+ 866.8	+ 404.1
Mean	1013—700 mb	— 0.7	+ 226.4	+ 632.5	+ 612.3	+ 261.2
	700—500 mb	+ 14.5	+ 88.4	+ 197.6	+ 179,4	+ 96.6
	1013—500 mb	+ 13.7	+ 328.7	+ 834.2	+ 792.6	+ 354.6

from the means of these quantities in both time and space. The quantities $\overline{\tau' v_g'}$ and $\overline{\tau' v_a'}$ are proportional to the mean eddy flux of sensible heat over the limited length of the latitude circle. $\overline{\tau v_g}$ and $\overline{\tau v_a}$ are proportional to the total flux of sensible heat including the flux due to the correlations between τ and v and the flux due to a non-zero $\overline{v} \cdot \overline{\tau} \overline{v_g}$ and $\overline{\tau} \overline{v_a}$ represent the component of the flux due to the non-zero $\overline{v}$ over the limited length of the latitude circle and that component which we wish to subtract out.

The data were examined separately by months and also as a single sample. The total number of cases was 472. The value of this ratio for the entire sample was 0.82. Thus in this sample, the eddy transport by actual winds is about 20 per cent greater toward the north than the transport by geostrophic winds. For the individual months, this ratio was essentially constant, being 0.83 during December 1948, and 0.79 during January 1949. These results indicate that the geostrophic eddy transport of sensible heat is a good measure of the observed eddy transport of sensible heat.

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Results

The mean monthly values of the geostrophic transport of sensible heat across various latitude circles are presented in Table I. One feature which is apparent is the concentration of the eddy transport of sensible heat in the layer between 1013 and 700 mb. Approximately 70 per cent of the eddy transport of sensible heat which is accomplished below 500 mb takes place in the lowest layer. This is in contrast to the horizontal meridional eddy transport of angular momentum which appears to be at a maximum in the high tropospheric layers (see Starr and White 1951).

It should be pointed out that because of the irregularities of the earth's surface, and because artificially reduced sea level pressures were used, the eddy flux of sensible heat between 1013 and 700 mb includes a fictitious component. This component is probably largest in the latitudes of the Himalaya mountains where the mountain barriers are oriented in an east-west direction and extend to great heights. It is not possible with the data at hand to determine the magnitude of this

fictitious component but it is felt to be of minor importance in view of the small portion of the latitude circle which is covered by such mountain ranges.

In all months the eddy transport of sensible heat reveals a maximum somewhere in middle latitudes, decreasing both poleward and equatorward. The location and intensity of the maximum eddy transport of sensible heat between 1013 and 500 mb varies from month to month. It is of interest to compare the mean monthly eddy transport with the character of the mean monthly northern hemisphere pressure distribution, Table II. However in view of the small number of months of data which have been examined, the results can only be considered as suggestive. Several factors must be taken into account in any attempt at such a comparison. The seasonal trend to stronger poleward transports from early to late winter must be considered. Thus the transport for November is somewhat smaller than those of other months..

As regards the three mid-winter months it appears that December 1945 which possesses the lowest sea-level zonal westerly index also possesses the most intense maximum (950.5×10^{12} cals sec^{-1}) which occurs at the relatively low latitude of 45° N.

Table II

The mean monthly meridional sea-level pressure profiles for the months as indicated. Units in mb

Month	Nov. 1945	Dec. 1945	Jan. 1946	Feb. 1946
Lat.				
80° N	1019.2	1022.3	1018.9	1026.9
75° N	1014.8	1019.8	1014.1	1021.2
70° N	1013.8	1019.7	1012.0	1016.9
65° N	1013.7	1020.1	1010.8	1013.5
60° N	1013.0	1018.7	1010.1	1011.3
55° N	1013.9	1016.6	1011.7	1011.0
50° N	1016.8	1014.1	1015.2	1012.9
45° N	1018.7	1012.7	1018.8	1015.7
40° N	1018.9	1014.4	1020.6	1017.8
35° N	1018.5	1016.8	1021.1	1018.7
30° N	1017.5	1018.1	1020.6	1019.1
25° N	1016.2	1018.0	1019.2	1018.4
20° N	1014.4	1016.1	1016.8	1016.7
15° N	1012.0	1013.4	1014.0	1013.7
10° N	1010.0	1011.2	1011.5	1011.2

On the other hand January 1946 has an abnormally high sea-level zonal westerly index with a comparable maximum (941.7×10^{12} cals sec^{-1}) which is displaced to a much higher latitude at 55° N. February 1946 which is a more normal month as regards the sealevel zonal westerlies has a less intense maximum (866.8×10^{12} cals sec^{-1}) located at 45° N. In February however the value of the transport across 55° N is almost equal to it in magnitude (862.1×10^{12} cals sec^{-1}) indicating that the maximum may lie somewhere between 45° N and 55° N.

It is of interest also to compare the actual magnitudes of the mean eddy transport of sensible heat between 1013 and 500 mb for the four months which have been investigated with estimates of the total energy transport which is required by the radiation balance of the earth and atmosphere. Such estimates are crude and vary within wide limits. The required yearly normal poleward total energy transport based upon the radiation data of Albrecht (1931) and another such preliminary estimate based upon data given by Gabites (1950) differ quite substantially but may be used as indicating possible limits of the required total energy transport. These data are given in Table III. Comparison with Table I where the mean geostrophic eddy transport of sensible heat for the four months November 1945 to February 1946 is given may be made.

Table III

The yearly normal total poleward energy flux across various latitudes required by the radiation balance to maintain the temperature difference between polar and equatorial regions based upon (a) radiation data according to Albrecht and (b) according to Gabites. Units in cals sec$^{-1} \times 10^{12}$

Lat.	(a)	(b)
30° N	+ 620	+ 903
40° N	+ 660	+ 1,044
50° N	+ 564	+ 917
60° N	+ 399	+ 670
70° N	+ 199	+ 364
80° N	+ 53	+ 105
90° N	0	0

Although rigid comparison of these two sets of figures is ruled out, because the eddy transports given in Table I have been computed for only one winter season and only up to 500 mb, and the transports in Table III are

Table IV

The mean monthly divergence of the eddy transport of sensible heat for the four months of the winter 1945—46 as indicated, in the layer from the surface to 500 mb within selected latitude belts. Unit in cals $sec^{-1} \times 10^{+11}$

Month	Lat. Belt				
	35—45° N	45—55° N	55—65° N	65—75° N	75—90° N
Nov.	+ 271.2	+ 97.6	— 398.7	— 258.4	+ 20.2
Dec.	+ 481.7	— 58.5	— 635.3	— 236.1	— 20.9
Jan.	+ 533.5	+ 129.0	— 454.6	— 487.1	← (65°—90° N)
Feb.	+ 462.7	— 4.7	— 532.5	— 287.5	— 42.1
Mean	+ 438.0	+ 41.6	— 505.3	— 315.0	— 13.7

yearly normals, some information about the efficiency of the eddy transport processes relative to other modes of meridional energy flux may be obtained because of the following considerations. The mean geostrophic eddy flux of sensible heat for the winter of 1945—46 has been computed up to 500 mb and if used as a typical yearly value will be considerably in underestimate because of the neglect of the transport above 500 mb, but on the other hand will be considerably in overestimate because only the winter season has been sampled. The extent to which these two factors tend to cancel will determine the appropriateness of the use of these mean geostrophic eddy transports of sensible heat in conjunction with the total energy transport which has been estimated on a yearly normal basis.[1]

It appears then from comparisons of Tables I and III that eddy sensible heat transport processes account for a major portion of the required total energy flux.

The relation between the mean monthly pressure profiles and the distribution of the divergence of the transport of sensible heat may be examined. The magnitude of the divergence of the eddy transport of sensble heat is proportional to the generation of kinetic energy, to the extent that horizontal eddy transport processes comprise the principal mode of transport in the atmosphere. The mean monthly divergence of the transport of sensible heat by latitude belts is given in Table IV. The divergence of the transport of sensible heat is represented by the difference between the rate of inflow of sensible heat into a given region and the rate of outflow from the same region. When a divergence of this transport exists, i.e., when there is more going out of a given latitude belt than is coming in, then there must exist a generation of kinetic energy within this belt. On the other hand a convergence of the transport of sensible heat signifies a disappearance of kinetic energy.

According to the present supposition it may be inferred from an examination of Table IV that in all months there exists a region of dissipation in more northerly latitudes. This systematic arrangement of the regions of kinetic energy generation and dissipation agrees with recent suggestions made by STARR and colloborators (1949) that a primary source of kinetic energy for the atmosphere lies in the subtropics.

[1] The extent to which the factors mentioned above actually do cancel was studied for regions of North America from rawin and rason observations which reach to great heights. It was found that in middle latitudes approximately 70 per cent of the total eddy transport of sensible heat takes place below 500 mb. On the other hand a measure of the wintertime rate of transport in middle latitudes was approximately 45 per cent greater than a measure of the yearly mean rate of transport. This study suggested that the geostrophic eddy transport of sensible heat computed during the winter months and only up to 500 mb may be a good approximation to the yearly mean. For further details of this study see WHITE (1950).

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A Note on the Kinetic Energy Balance of the Zonal Wind Systems

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Broadly speaking, there are two processes that may produce the needed transport of angular momentum from the zones of surface easterlies to those of surface westerlies. One is a large-scale mean convective process, with ascending motions in lower latitudes and descending motions in higher latitudes. The other is the large-scale eddy motions. Although some types of mean meridional circulations might be able to account for the necessary transport, the existence of such circulations is hard to establish observationally. On the other hand, the investigations conducted at the Massachusetts Institute of Technology and at the University of California have demonstrated that the eddy process not only produces enough angular momentum transfer but also produces sufficient transfer of energy to maintain the atmosphere in radiational equilibrium. This suggests that the general circulation is maintained mainly by the large-scale eddy process.

However, before any definite conclusion is drawn, one must also investigate the relative efficiency of the two processes as regards the kinetic energy balance of the mean zonal flow. For this purpose each velocity component may be decomposed into a mean value, defined as the average along a latitude circle, and a deviation from this mean so that $u = \bar{u} + u'$, $v = \bar{v} + v'$, $w = \bar{w} + w'$, where u, v, w are respectively the eastward, northward and upward components, and u', v', w' are the eddy components. In spherical coordinates the zonal momentum equation may be written

$$\frac{\partial \varrho u}{\partial t} = -\frac{\partial(\varrho u v \cos^2 \Phi)}{a \cos^2 \Phi \, \partial \Phi} - \frac{\partial(\varrho u w)}{\partial z} + f \varrho v - \frac{\partial(p + \varrho u^2)}{a \cos \Phi \, \partial \lambda}$$

where Φ is latitude, λ longitude, z elevation, f the coriolis parameter, a earth's radius, ϱ density, p pressure and t time. The effect of the vertical component of the deflecting force is small and is neglected. This equation may be multiplied by the mean zonal velocity $\bar{u}$ and integrated over the volume V bounded by two complete latitude circles and over the entire column of air in the vertical. For this particular problem the effect of the variation of density along latitude circles and with time is small so that $\int \bar{u} \frac{\partial \varrho u}{\partial t} d\lambda = \pi \partial \overline{\varrho u^2} / \partial t$. It follows that

$$\frac{\partial E_x}{\partial t} = -2 \pi a^2 \iint \left[\bar{\bar{\lambda}} \frac{\partial(\overline{\varrho u v} \cos^2 \Phi)}{\partial \Phi} + \right.$$

$$\left. + \bar{u} \cos \Phi \frac{\partial(\overline{\varrho u w})}{\partial z} - f \bar{u} \overline{\varrho v} \cos \Phi \right] d\Phi \, dz =$$

$$= \int \left[\overline{\varrho u' v'} \cos \Phi \frac{\partial \bar{\bar{\lambda}}}{\partial \Phi} + \overline{\varrho u' w'} \frac{\partial \bar{u}}{\partial z} \right] dV +$$

$$+ \int \left[\overline{\varrho w} \bar{u} \frac{\partial \bar{u}}{\partial z} + \left(f + \cos \Phi \frac{\partial \bar{\bar{\lambda}}}{\partial \Phi} \right) \overline{\varrho v} \bar{u} \right] dV +$$

$$+ \int \overline{\varrho u v_n} \cdot \bar{u} \, ds_1 + \int (\overline{\varrho u w})_0 \bar{u}_0 \, ds_0$$

where $E_x = \frac{1}{2} \int \bar{\varrho} \bar{u}^2 d V$ is the total kinetic energy of the mean zonal motion, $\bar{\lambda}$ is the mean zonal current in angular measure ds_1 and ds_0 are the area elements of the lateral walls and bottom respectively, v_n is inward meridional velocity and the limits of integration are as stated. This equation is of basic importance for the study of the general circulation. It is the writer's desire to call attention to the integral requirement which it expresses. It may be noted that this equation is similar to the Reynold's energy equation for the mean motion, except for the term involving f due to earth's rotation. However, it is derived without the assumptions of incompressible motion and the vanishing of the velocities along the boundaries. The pressure force does not appear in this equation. According to this equation, the kinetic energy of the mean zonal flow can change not only by the presence of mean meridional circulations, as represented by the terms containing $\overline{\varrho v}$, $\overline{\varrho w}$, but also by a *conversion of the eddy kinetic energy into that of mean zonal flow*. This rate of conversion is given by the products of the eddy stresses $\overline{\varrho u' v'}$ and $\overline{\varrho u' w'}$ into the horizontal and vertical shears of the mean zonal wind. Because the vertical velocity w generally differs from zero at the level where surface wind is measured, we also retained the last term, where the subscript zero denotes surface values. This term may be interpreted as representing the dissipation of the kinetic energy of mean zonal flow through ground friction. The dissipation of mean zonal kinetic energy directly through molecular viscosity in the free atmosphere is too small and is neglected. The equation obtained may also be looked upon as a continuity equation expressing the balance of mean zonal kinetic energy for the atmosphere. Thus the volume integrals can be looked upon as sources (or sinks) and the surface integrals as net inward transports of this energy across the boundaries.

To the extent that data are available it is possible to examine whether the eddy process can provide sufficient kinetic energy to account for the frictional loss. This is given by the first two terms containing $\overline{\varrho u' v'}$ and $\overline{\varrho u' w'}$ in the last member of the equation. These terms as they stand include the effects of small scale eddies as well as of large scale. It is probable that the small scale eddies lead to dissipation so that only large eddies need be considered.

To estimate the order of magnitude of the first term arising from the large eddies, we may make use of the values of $\overline{\varrho u' v'}$ obtained by STARR (1951) over the American continent. Although these values may not be very representative for the hemisphere, the order of magnitude and the trend of the meridional distribution are probably correct. In Starr's data, the meridional shear of the zonal wind, $\partial \bar{\lambda} / \partial \Phi$ varies very little with height above 700 mb, therefore one may compute this rate of conversion by using the integrated values of the angular momentum transfer $\tau = 2 \pi a^2 \cos^2 \Phi \int \overline{\varrho u' v'} dz$ (actually up to 100 mb level), and take the meridional wind shear at the 500 mb level as the mean wind shear. Such values are given in table 1.

Integrating from 25° N to 70° N, the result is that $\int \tau \, \partial \bar{\lambda} / \partial \Phi \, d\Phi \sim 125 \times 10^{19}$ gm cm² sec⁻³. There is also a net influx of about 100×10^{19} units through the latitude walls. A zero value of τ at around 10° N is suggested by an as yet unfinished study. Integration from 10° N to 70° N gives a total of about 200×10^{19} gm cm² sec⁻³. Each of these computations gives an average rate of conversion of about 10^{-4} watts per cm².

As for the second term, we still lack any reliable determinations. However, since almost all the large disturbances in middle latitudes have their axes inclined toward west with increasing elevation (one reason for this inclination is the thermal asymmetry), the correlation $\overline{u' w'}$ is mostly negative in middle latitudes. This conclusion has been confirmed by a few indirect computations by WHITE (1950). In low latitudes the correlation $\overline{u' w'}$ is probably positive. Since $\partial \bar{u} / \partial z$ is either negative or small there, the total contribution of the second term from the large eddies is probably negative. Thus this

Table 1

Values of τ in units of 10^{25} gm cm² sec⁻² and of $\partial \bar{\lambda} / \partial \Phi$ in 10^{-5} sec⁻¹

Lat. (deg. N)	10°	25°	35°	45°	55°	63°	76°
τ	(0)	+ 56.0	+ 41.5	+ 8.40	— 6.20	— 2.00	+ 0.60
$\partial \bar{\lambda} / \partial \Phi$	+ 0.63	+ 1.2	+ 0.57	+ 0.17	— 0.23	— 2.10	— 2.80

term may be combined with the last term and the parts of the first term arising from small eddies and looked upon as constituting the total dissipation through skin and turbulent friction.

According to BRUNT (1941) and others, the rate of dissipation of the total kinetic energy in the surface layer and in the free atmosphere is about 5×10^{-4} watts per cm^2. Since less than one-third of this total dissipation can be attributed to the loss of the kinetic energy of the mean zonal flow, and since the use of long period averages of $\overline{u'v'}$ and $\partial\overline{\lambda}/\partial\Phi$ gives an underestimation, it seems that the rate of conversion obtained is of the right order of magnitude and is sufficient to account for the frictional loss.

This seems to suggest that the eddy process in the atmosphere is a self consistent and self sufficient mechanism in maintaining the zonal wind systems. One or two consequences of the energy equation may be noted. Although this equation itself does not depend upon the scale of the motion, as already mentioned, the properties of the eddy stresses may be quite different for the small scale turbulent motions and for the large-scale disturbances. In the ordinary turbulence regime, the momentum flux is generally in the direction of the gradient of the mean velocity so that a virtual coefficient of viscosity K can be defined such that $\overline{\varrho u'v'} = - K\varrho\partial\overline{u}/\partial y$, where y is in the direction of increasing $\overline{u}$. However, this is not necessarily true for large-scale eddies. When the flow is stable and the disturbances are sustained by some other processes, the reverse may happen and the disturbances may add their momentum and energy to the main flow. The values of $\overline{\varrho u'v'}$ and $\partial\overline{\lambda}/\partial\Phi$ cited above show that this is really the case for the large-scale disturbances in the atmosphere. This is not only true for momentum, but also true for other properties. Thus even when the absolute vorticity of an element of air is nearly conserved during its motion, it does not necessarily follow that vorticity will be transported in the direction of the gradient; rather the reverse is what should be expected when the flow is relatively stable for horizontal disturbances. Only when the flow is quite unstable so that disturbances develop at the expense of the mean flow and a thorough mixing is produced, the momentum and vorticity will be transported along the gradient. One must not therefore use the concept of virtual viscosity in discussing the effect of the large-scale eddy motions in the atmosphere (unless the possibility of a negative viscosity coefficient is admitted).

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LARGE-SCALE ATMOSPHERIC EXCHANGE PROCESSES AS DIFFUSION PHENOMENA

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(Manuscript received 11 May 1951)

Introduction.—Consideration of the large-scale processes of the atmosphere as gross-turbulence phenomena was first introduced into the literature by Defant [1]. His paper stimulated considerable research into methods of characterizing the nature of the general circulation from this point of view. Any analogy between turbulent motions and the large-scale motions of the atmosphere must necessarily be crude; nevertheless, as a working hypothesis, this concept has proven of interest in certain phases of meteorology, and considerable effort has been expended in attempting to define the turbulent characteristics of the system.

In such investigations, it has been customary to treat the large-scale exchange processes by analogy with small-scale diffusion. It is thus supposed that a parcel of air, moving from one environment to another, maintains the characteristics of its original environment while moving a distance called the "mixing length," whereupon it mixes completely with its new surroundings. Thus, the diffusion of certain properties of the fluid may be characterized by use of a diffusion coefficient. In such a scheme, the flux of any quantity is necessarily directed opposite to the gradient of the quantity.

Many investigations have sought to determine the large-scale diffusion or gross-austausch coefficient as a measure of the intensity of the large-scale exchange processes and, hence, as a measure of the intensity of the general circulation. This gross-austausch coefficient has sometimes been assumed to be fairly constant and equally applicable to the flux of momentum, sensible heat and other quantities, save for the use of an appropriate dimensional constant. For the most part, these determinations were made directly from observed wind fluctuations, according to the formula which is used in studies of small-scale turbulence.

Lettau [3] determined the magnitude of the local gross-austausch coefficient for a grid of points over the northern hemisphere, by considering the character of the local fluctuations of the surface wind; the duration of the fluctuations was used to determine the mixing length. More recently, to determine the meridional sensible-heat flux, Elliot and Smith [2] made use of this method. Lettau [3] has presented values of the mean local austausch coefficient by latitudes, for the entire northern hemisphere for several periods. Values of the mean meridional gross-austausch coefficient, for the polar year 1932–33 as computed by Lettau, are given in table 1.

The gross-austausch coefficient has also been computed from considerations of the intensity of the meridional mass exchange. By this method, a mean austausch coefficient for each latitude is derived directly. This type of determination of the gross-austausch coefficient has been used largely to characterize the intensity of the general circulation, as a determination of the mean coefficient of large-scale mixing for a given synoptic chart. In this method, the mixing length is determined by noting the average number of disturbances along a latitude circle and then determining the average length of the disturbances. The results of such an investigation by Wagner [5], for March 1931, are also shown in table 1.

It is now becoming possible to measure the various exchange processes directly. It is of considerable interest to investigate the nature of the gross-austausch coefficient with such exchange data, because of the light which can thus be thrown on the validity of the gross-austausch concept.

TABLE 1. Magnitudes of mean gross-austausch coefficient (10^7 g cm^{-1} sec^{-1}) by latitude according to Lettau [3] and Wagner [5].

	Latitude (deg N) 30	40	50	60	70
Lettau	—	6.2	8.2	7.0	6.8
Wagner	5.9	6.5	7.0	6.2	5.0

[1] Mr. Jung's work was sponsored by the Geophysical Research Directorate of the Air Force Cambridge Research Laboratories under Contract No. AF 19(122)-153.

Large-scale exchange of sensible heat.—The validity of the application of the diffusion analogy to the large-scale exchange of sensible heat in the atmosphere can be tested in the following way. The flux of sensible heat, per unit area of surface erected at a given latitude in the atmosphere, is given, by analogy with the diffusion concepts, as

$$S = c_p \bar{A}_\phi R^{-1} \partial \bar{T}/\partial \phi, \tag{1}$$

where c_p is the specific heat of air at constant pressure, $\bar{A}_\phi$ the gross-austausch coefficient, R the radius of the earth, and $\bar{T}$ the mean temperature at latitude ϕ.

The mean horizontal flux of sensible heat per unit area and unit time may be determined directly in another manner, since the flux is given also by

$$S = c_p L_\phi^{-1} \oint \rho v T \, dx, \tag{2}$$

where ρ is the density, v the meridional velocity, L_ϕ the length of the latitude circle, dx an element of west-east distance, and the integration is performed around a complete latitude circle. Thus, we have at our disposal a means of examining the variability of $\bar{A}_\phi$ and, thereby, of testing this application of the diffusion concept to the flux of sensible heat.

One of the basic assumptions underlying (1) is that there exists no net mass flux through the surface across which the flux of sensible heat is to be determined. On the other hand, (2) includes a component of the flow of sensible heat due to a net mass flux across the surface. However, this component is eliminated if use is made of the geostrophic approximation to evaluate this integral, since, at a given level in the atmosphere,

$$\oint \rho v_g \, dx = \oint f^{-1}(\partial p/\partial x) \, dx = 0, \tag{3}$$

where v_g is the geostrophic wind, f the Coriolis parameter, p the pressure, and the integration is performed around a complete latitude circle.[2]

The geostrophic evaluation of (2) was undertaken by White [6] for four winter months, November 1945–February 1946, from daily synoptic charts for 0300 GCT. Finite-difference integration methods were used. The geostrophic wind was evaluated from charts of the northern hemisphere at four latitudes, extending from 35 to 65°N. It was found convenient to make use of the mean virtual temperature and to evaluate (2) by layers. The flux of sensible heat in the layers 1013–700 and 700–500 mb was evaluated by use of a grid of 72 points, evenly spaced around a latitude circle. It was not possible to evaluate (2) above the 500-mb level, because of lack of data. The meridional temperature gradient which enters into (1) is easily determined from the daily hemispherical meridional pressure profiles.

The resulting data consist of 120 instantaneous values of the sensible heat flux spaced 24 hr apart as evaluated by (2), and 120 values of the meridional gradient of the mean virtual temperature in the layer 1013–500 mb, at the four latitudes. In all cases, the temperature gradients were measured over a distance of 10 deg lat, extending 5 deg lat on each side of the latitude circle involved. With these data, the values of $\bar{A}_\phi$ for each of the layers and latitudes may be determined by equating (1) to (2).

Results.—The mean values of $\bar{A}_\phi$ are given in table 2, for the 4-month period in the layer 1013–500 mb for varying modes of time averaging, together with the ratio of the standard deviation to the mean $\bar{A}_\phi$, which is sometimes called the coefficient of variation V. V may be used as a crude measure of the variability of $\bar{A}_\phi$; it is zero when the coefficient is constant and unity when the standard deviation is as large as the mean.

The austausch coefficients shown in table 2 have the same order of magnitude as those computed by previous investigators (table 1) from considerations of the mass flux, but are somewhat smaller. The latitudinal variation agrees well with that found by other investigators, the coefficient reaching its maximum value in the vicinity of 55°N, indicating this region as the zone of most intense mixing in the atmosphere. The values of $\bar{A}_\phi$ were found to be consistently smaller in the upper layer that in the lower one (data not shown).

The dependence of V on latitude and mode of time averaging gives considerable information concerning the constancy of $\bar{A}_\phi$. The coefficient of variation of the daily values is extremely large, exceeding 0.50 in all cases, indicating that a very high degree of variability exists in the daily values. One may conclude that the

TABLE 2. Mean meridional gross-austausch coefficients (10^7 g cm^{-1} sec^{-1}) and coefficients of variation at various latitudes, for layer between 1013 and 500 mb, for four-month period November 1945–February 1946, for various modes of time averaging.

		Latitude (deg N) 35	45	55	65
Daily values	$\bar{A}_\phi$:	1.04	3.03	6.56	4.82
	V:	0.58	0.50	0.69	1.52
3-day means	$\bar{A}_\phi$:	1.03	2.96	6.13	4.17
	V:	0.39	0.31	0.49	0.91
6-day means	$\bar{A}_\phi$:	1.04	2.94	5.92	3.92
	V:	0.33	0.28	0.35	0.76
12-day means	$\bar{A}_\phi$:	1.03	2.93	5.74	3.58
	V:	0.18	0.26	0.22	0.60

[2] Thus, if the temperature is expressed as $T = \bar{T}(\phi) + T'$, where T' is a deviation from the mean value for the latitude circle, the geostrophic flux of sensible heat, according to (2), becomes

$$\oint \rho v_g T \, dx = \oint f^{-1} (\partial p/\partial x)(\bar{T} + T') \, dx = 0 + \oint \rho v_g T' \, dx.$$

For actual winds, on the other hand,

$$\oint \rho v T \, dx = \oint \rho v \bar{T} \, dx + \oint \rho v T' \, dx.$$

diffusion concept applied › the large-scale features of the atmosphere is completely invalid over short time periods whose length is of the order of one day. There appears to be a systematic increase in V with latitude, suggesting that the diffusion concept becomes even less applicable as higher latitudes are approached. This is true for all modes of time averaging.

The dependence of $\bar{A}_{\phi}$ on the mode of time averaging is rather interesting, but not entirely unexpected. There is a systematic decrease in $\bar{A}_{\phi}$ as longer time periods are considered. When the austausch coefficients are derived on the basis of averaging over a period of 12 days, V begins to approach a level which would indicate a very small degree of variability except at 65°N. This suggests that the diffusion concept, at least insofar as the variability of the austausch coefficient is concerned, begins to be valid as longer time periods (of the order of several weeks) are considered. It should be pointed out, however, that the very process of averaging any series of data will tend to reduce the variability of the resulting series.

An important feature of the diffusion concept, which is verified by the data, is that the sensible heat flux is almost always directed from high to low temperatures. Only eight of the 480 daily cases at the four latitudes considered reveal a sensible heat flux in the opposite direction. As the time averaging proceeds to longer periods, the flux is invariably opposite to the gradient.

A closer analysis of the relationship between the flux and the gradient is possible. According to the diffusion concept, there should exist a relation between the two such that, all other things being equal, there is an increase in the sensible heat flux when there is an increase in the temperature gradient. A measure of the relationship of this kind may be found in use of the coefficient of linear correlation between these quantities. The contemporaneous correlation coefficients between the gradient and the flux are given in table 3.

TABLE 3. Coefficient of linear correlation between geostrophic sensible-heat flux and temperature gradient at various latitudes, for layer between 1013 and 500 mb, for the four-month period November 1945–February 1946, for various modes of time averaging.

	Latitude (deg N)			
	35	45	55	65
Daily values	0.12	−0.18	−0.09	−0.23
3-day mean	0.17	−0.11	−0.26	−0.45
6-day mean	0.10	−0.08	−0.24	−0.66
12-day mean	0.20	−0.05	−0.37	−0.43

There is little statistical significance in the correlation coefficients, except perhaps at 65°N for 3-day and 6-day means, where the correlations reach the 1 per cent significance level. The data suggest that increased sensible-heat transports do not necessarily occur with increased temperature gradients; if anything, the contrary is true in high middle latitudes. These results agree with those of Rossby and Willett [4]; they find that it is during periods of high zonal index (and little meridional mixing) that the greatest temperature gradients are found.

It is suggested that the sensible heat flux over short periods may, to a large degree, be controlled by physical characteristics of the general circulation other than the meridional temperature gradient.

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NOTE ON THE MERIDIONAL TRANSPORT OF ENERGY BY THE OCEANS[1]

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ABSTRACT

The role of the oceans in providing poleward energy flux within the earth's fluid envelope is re-examined in regard to effective transport mechanisms. A hypothetical model near 30 degrees north latitude is found to produce a sizeable poleward energy transport when effects of closed vertical circulations operating in meridional planes are considered. Available data for the Atlantic do not conflict with this result. It is suggested that the importance of such meridional circulations should not be disregarded in computations of the global energy balance.

INTRODUCTION

As is well known, in polar regions there is a net radiation of energy into space from the earth and from its fluid envelope, the oceans and atmosphere. In tropical regions these receive a net excess of solar radiation. However, temperatures show neither a corresponding progressive cooling in the polar cap nor a progressive warming in the more tropical regions. There must exist, therefore, a poleward transport of energy through the earth's fluid envelope in order to maintain the observed temperature equilibrium, since conduction through the earth proper is of negligible importance.

From qualitative considerations the poleward energy flux through the oceans has been assumed small compared with the transport through the atmosphere (see von Bezold, 1906; Bjerknes, 1933). Sverdrup (1942) made a quantitative estimate for ocean energy transported by the Gulf Stream System in the North Atlantic Ocean. This estimate gives a small percentage of the radiation requirements, suggesting that ocean currents transport no more than 10% of the total poleward energy flux. It is to be noted that this estimate, necessarily did not include energy transported by means other than the standing horizontal eddy comprised of the Gulf Stream System and associated return currents.

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Let us consider first some possible energy transport mechanisms within the ocean which may be important. As Sverdrup has noted, standing horizontal eddies provide one such mechanism. In addition, it is conceivable that closed vertical mean circulations in meridional planes might transport large amounts of energy, even though the average velocities involved are extremely small.

Since it would seem improper to neglect this second possibility *a priori*, some estimate of the possible order of magnitude of the transport produced by it should be made. Furthermore, if this result is significant, then the actual presence of mean meridional circulations should be investigated from oceanographic data in so far as this is possible. It is hoped that the material which follows may serve as a beginning in the study of these two questions.

HYPOTHETICAL VERTICAL CIRCULATION

Let us consider the energy transported by a simple closed vertical circulation that operates within a hypothetical model. The model used is illustrated in Fig. 1. As shown in *B* of Fig. 1, a northward velocity of about 0.51 cm sec^{-1} is assumed to be present from the surface to a depth of 950 m. Mass continuity then gives a southward flow of about 0.14 cm sec^{-1} in the layer below 950 m which extends to an assumed bottom depth of 4,250 m.

Reasonable values of the vertical distribution for temperature (T), salinity (S), and density (ρ), as given by Fuglister (1947), were employed in the hypothetical model. These values are for approximately 30 degrees north latitude. The temperature distribution is given in C of Fig. 1. The width of the model (extent of ocean in the west-east direction) was chosen to be 5,000 km, i. e., comparable to the width of the Atlantic at 30 degrees north latitude.

Within the earth's fluid envelope, the total energy flux across a latitude wall can be represented by the following expression used by Starr (1951):

$$F = \int_A (P + \rho c_v T + \rho g Z + \rho C^2/2) v_n \, dA \, . \tag{1}$$

P is pressure and T is temperature. Applied only to the ocean, Z is vertical distance counted upward, ρ is the density, and c_v is the specific heat of the ocean water. C is the ocean current speed and v_n is the northward component of velocity. dA represents an element of area of the latitude wall within the ocean.

The term containing P may be regarded as the rate at which pressure forces do work across the boundary surface. It is reasonable to assume that the term containing T is a close approximation to the internal

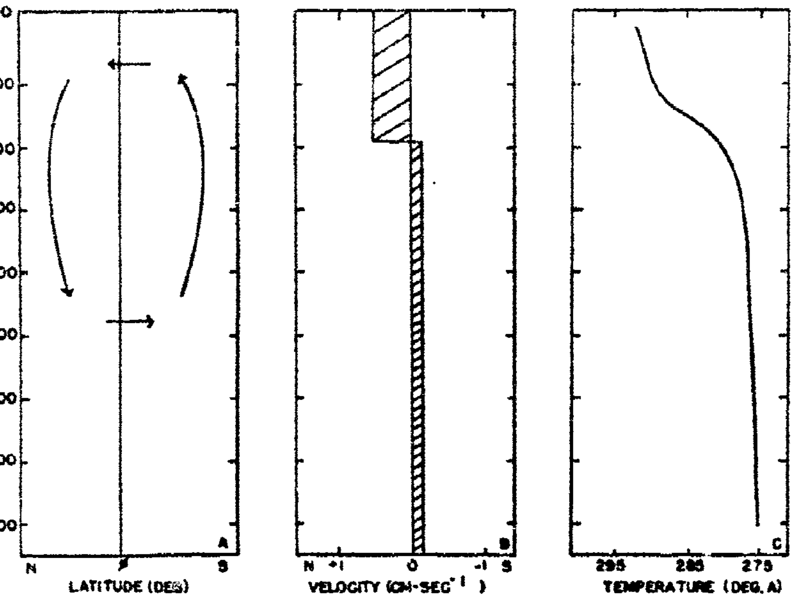

Figure 1. Hypothetical model of a closed vertical circulation in a meridional plane. *A.* Schematic representation. *B.* Assumed velocity distribution used for computations. *C.* Temperature distribution in vicinity of 30 degrees north latitude in the North Atlantic Ocean after Fuglister (1947).

energy transport. The terms containing Z and $C^2/2$ represent transports of potential and kinetic energy, respectively.

Under conditions of hydrostatic equilibrium the term in (1) involving pressure nearly cancels the term involving potential energy, so that the sum gives a negligible contribution. This cancellation would become exact in the case of uniform density.

Using any reasonable assumed values for the magnitude of C, it can be shown easily that the advection of kinetic energy is several orders of magnitude smaller than the transport of internal energy.

The transport of internal energy thus determines the order of magnitude of the total energy flux across the latitude wall. For the model, this transport is of the order of 3×10^{14} gm-cal sec^{-1}. In order to include the effects of all oceans in the northern hemisphere, this figure should be multiplied by a factor of 3.

The result is of the same order of magnitude as the total poleward energy transport required by radiation data for the entire fluid envelope. The latter figure is about 10^{15} gm-cal sec^{-1} at 30 degrees north latitude according to estimates discussed below. It appears extremely

important, therefore, to determine whether or not such net meridional circulations do actually exist within the oceans. If so, their effect cannot be neglected in any complete study of the energy balance of the globe.

COMPUTATIONS FROM ACTUAL DATA

From the model discussed, it is evident that measurements of velocity entering into ocean energy flux computations of the present type must be extremely accurate. Moreover, direct ocean velocity measurements are not sufficiently plentiful for this purpose. However, it is possible that new methods for measuring velocities, such as the one developed by von Arx (1950), may ultimately provide a solution to this problem after a large enough volume of data has been obtained.

It is interesting to note that a vertical meridional cell within an ocean differs essentially from a corresponding cell in the atmosphere. The latter must be nongeostrophic in order to effect a net meridional mass transfer at a given level, and thus it represents a motion in which the Coriolis forces are not balanced by pressure forces. In the ocean, on the other hand, due to the presence of continental barriers, this is not necessarily true. A net meridional transfer at a given level in an ocean may be associated with motion in which pressure forces balance Coriolis forces.

With appropriate reservations about possible motions other than those given by dynamic velocity measurements, we may use data compiled by Riley (1951) for the North Atlantic Ocean. At present these data represent the most comprehensive estimation of ocean volume transport available and results derived from them are at least of some interest.

The distribution with depth of the average meridional velocities so obtained across 27 degrees north latitude is shown in Fig. 2. Since Riley points out that his estimates do not account for Gulf Stream transport adequately, additional data from Sverdrup, et al. (1942) were obtained for this region and are incorporated into the averages shown in Fig. 2[3].

This velocity distribution gives a small net mass flow northward across the latitude section of the Atlantic Ocean. It is possible that a compensating outflow occurs on the opposite side of the hemisphere. Such an outflow would have to take place through the narrow and shallow Bering Strait into the Pacific Ocean with a velocity of about 100 cm sec^{-1}. However, such large ocean current velocities are im-

[3] The velocity at 4,000 m estimated from Riley's data agrees reasonably well with preliminary results of a carbon 14 dating technique reported by Kulp (1951).

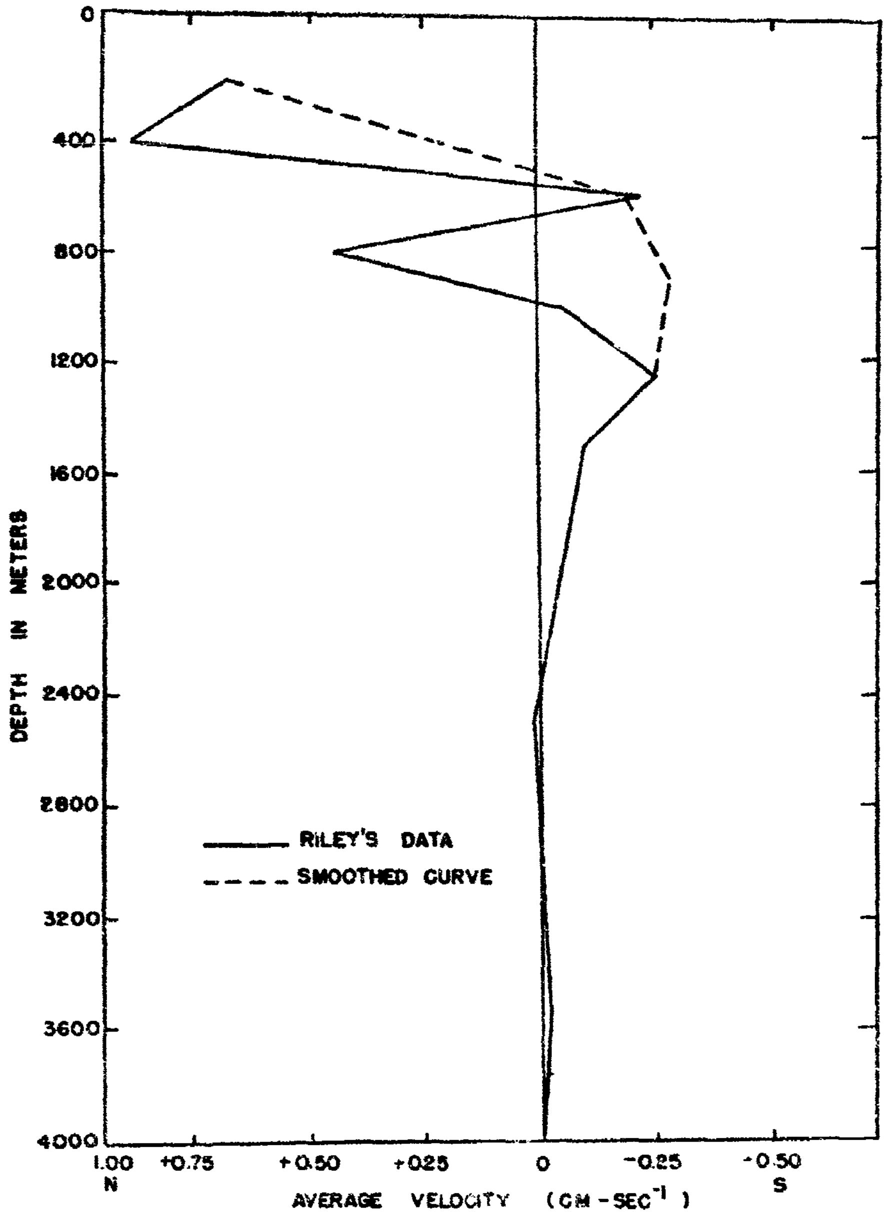

Figure 2. Vertical distribution of average velocities across 27 degrees north latitude in the North Atlantic Ocean. Solid curve is from data of Riley (1951). Dashed velocities give zero resultant net mass transport across this latitude.

probable in this region. In the event that such a mass outflow actually does exist, negative heat transport across 27 degrees north latitude in the Pacific Ocean must occur. This would bring about a cancellation of the heat energy transported northward by the net mass flow in the Atlantic Ocean, when the hemisphere as a whole is considered. Thus, whether or not this effect is real, it is perhaps best to eliminate it. This may be accomplished by one of several procedures.

The velocity profile may be shifted so that no net mass flux occurs. This may be accomplished by moving the profile a small amount in the negative velocity direction. As an alternate method, the ragged upper portion of the velocity profile may be appropriately smoothed, i. e., as shown in Fig. 2.

Each of these procedures was used together with an average sounding of oceanographic variables from the METEOR Atlas (Wüst and Defant, 1936) to obtain estimates of internal energy transport by meridional cells within the Atlantic Ocean. The METEOR sounding, representing averages over the entire Atlantic section, is quite similar to the Fuglister sounding used in the hypothetical model derived from a small region of the Atlantic Ocean. The Fuglister temperature curve is about 4° C warmer than the METEOR curve at 500 m depth, the level of the greatest difference between the two.

The estimate of the magnitude of the internal energy transported is about 3×10^{14} gm-cal sec^{-1} in each of these cases. This value is almost the same as that obtained from the hypothetical model discussed above.

COMMENTS

Baur and Phillips (1935) gave radiation values which can be used to obtain net radiation excess or deficit values for the earth averaged over latitude belts for the northern hemisphere (see Haurwitz and Austin, 1944: 18). From these, estimates may be made of the energy required for poleward transport in order to maintain the observed temperature equilibrium. The amount required for transport by the entire fluid envelope across 30 degrees north latitude is approximately 10^{15} gm-cal sec^{-1}.

In this latitude, oceans occupy approximately three times the horizontal extent assumed in our hypothetical model or occupied by the Atlantic Ocean. Assuming that the results may be considered typical of all oceans in the northern hemisphere, the hypothetical model ocean could account for 9×10^{14} gm-cal sec^{-1}, while from actual data the oceans could account for $9 - 10 \times 10^{14}$ gm-cal sec^{-1}. Of course these results depend upon dynamic velocity measurements, and therefore they must be treated with caution.

However, if such small vertical meridional cells in the ocean are capable of transporting energy in the amounts estimated here, then this possibility should not be neglected in energy balance relationships of the fluid envelope.

According to Wexler (1944), the atmospheric layer from the surface to 10,000 feet gains a maximum of heat energy in the region off the New England coast in the vicinity of 48 degrees north latitude for the month of February. This is located in a region of net radiation deficit (see Baur and Phillips' data), and hence such heating must depend on sources other than radiation. Wexler proposes that these sources may be adiabatic heating from subsidence in polar air masses over Canada as well as heat conducted upward from the ocean surface. The present estimates indicate that the ocean could indeed act as such a heat source for the atmosphere within this region, giving up the large amounts of energy transported across latitudes farther south.

ACKNOWLEDGMENTS

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Note Concerning the Nature of the Large-Scale Eddies in the Atmosphere

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(Manuscript received August 10, 1953)

Abstract

The rate of generation of kinetic energy of mean zonal motions by the large-scale horizontal eddies in the atmosphere is computed from three independent sets of hemispheric wind data. The results are larger than certain analogous dissipation estimates would require. Meteorological implications of the material are discussed.

General discussion

The aim of this brief article is to discuss further, with the aid of new additional data, some essential properties of the large-scale horizontal eddies in the atmosphere, which are involved in their relation to the mean zonal motions present. The particular physical principle to be reviewed has been treated previously mainly by my colleague Dr. H. L. Kuo (1950, 1951 a, 1953) both theoretically and observationally, although it appears that certain additional remarks and graphic illustrations might be of value, since some residual misunderstandings seem to be prevalent.

The mean zonal motions (i. e., averaged with respect to longitude) of the atmosphere may be pictured as comprising a large vortex about the polar axis in which the angular velocity is a function of elevation, latitude and time. As an example the solid curve in (a) of the figure gives the distribution with latitude of this angular velocity relative to the earth, averaged also with respect to pressure in the vertical from 150 to 900 mb and with respect to time over the six-month period January to June 1950 inclusive (N. H.). Most of what is to be said concerning this profile can be made to apply equally to an instantaneous profile at some given level. One may now ask what the effect of genuine lateral friction alone on such a flow might be. Quite obviously the effect of such viscosity would be to retard the zones of most rapid rotation and to increase the angular velocity of the less rapidly rotating ones, so as to cause the whole to assume a more nearly uniform angular velocity. That is to say, lateral friction would cause a flow of angular momentum northward and also southward away from the zone of most rapid rotation.

One may inquire next as to whether the large-scale eddies in the atmosphere act upon the mean zonal motions in a manner analogous to friction and hence may be regarded as merely giving rise to an eddy viscosity. This is outstandingly not the case, for according to any compilation of data at all adequate for the purpose (and there are a number of such independent ones now available) there exists a strong northward eddy transport of angular momentum to the south of the jet of westerlies (N. H.) and therefore *a fortiori* to the south of the zone of maximum angular velocity, on the average. This is exactly contrary to a frictional effect and would be compatible only with a negative virtual viscosity coefficient, if it were desirable even to use such terminology. The mere existence of this state of affairs is in itself most remarkable and by no means to be

lightly dismissed, for regardless of further considerations it is a manifestation of a fundamental process which requires explanation ultimately. The associated angular momentum transport by eddies is given by the dashed line in (a) of Fig. 1.

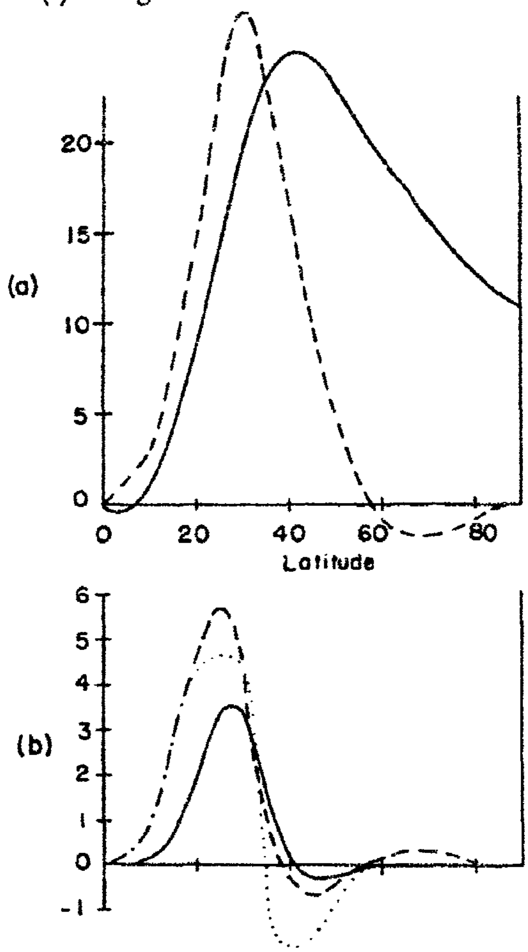

Fig. 1. Relative angular velocity in 10^{-7} sec^{-1} as function of latitude given by full curve in (a). Total angular momentum transport by eddies in 10^{25} c.g.s. units given by dashed curve in (a). Production of zonal kinetic energy using mean wind and total transport (full curve), using 300 mb wind and transport (dashed curve) and using 200 mb wind and transport (dotted curve) shown in (b) in terms of arbitrary units.

Kinetic energy considerations

It is convenient and instructive to study these effects of the eddies upon the *kinetic energy* of the mean zonal motions. Considering the northern hemisphere as acting independently, the net influence on this total kinetic energy is to increase it, if the (positive) action to the south predominates and to decrease it if conditions are the reverse. The rate of production may be written as

$$2\pi r^2 \iint \varrho [u'v'] \cos^2 \Phi \frac{\partial}{\partial \Phi}\left(\frac{[u]}{r \cos \Phi}\right) d\Phi dz \quad (1)$$

if small approximations such as the horizontal uniformity of density are allowed in the definition of mean zonal kinetic energy. The notation is that of STARR and WHITE (1951, 1952), u, v, being eastward and northward wind components, ϱ density, r earth's radius, Φ latitude, z elevation, the square brackets denoting averaging along complete latitude circles, and the primes indicating deviations from such averages. This expression may either be set up directly from a consideration of the work done by net forces due to the eddy stresses on elementary rings of air, or it may be obtained by writing the proper form of the balance equation for zonal kinetic energy as done, for example, by KUO (1951 a).

Data and computations

Among the available data for investigating expression (1) the following may be listed.

(1) The first six months of the year 1950 already referred to. These data are based upon (daily) actual wind observations similar to those of Starr and White in the references already given, for Latitudes 13°, 31°, 42°, 55°, and 70° N. The integrand of (1) evaluated from the six-month averages of $[u]$ and $[u'v']$ is given for the 300 and 200-mb levels by the dashed and dotted lines respectively in (b) of the figure. The full line results from using the vertical pressure averages of wind and eddy transport (i.e., corresponding to the curves given in (a) already discussed). Since in (b) the area under each curve measures the production of zonal kinetic energy, it is to be noted that in each case this area is positive by a wide margin.

(2) The second six months of the year 1950. These data are exactly analogous to those for the first six months and were treated in the same way. The integrands (not shown) were again positive without question, i.e., do not represent small differences between large quantities.

(3) One month of data (Jan. 1949) published by MINTZ (1951). These represent geostrophic evaluations obtained by punched card methods. As in the case of the curves (b), here also there is no ambiguity as to the sign of the areas beneath the curves (not shown) in all three instances.

Using the vertically averaged winds and transports the data (1) give 4.2×10^{20} ergs per second for the production over the hemisphere. The data (2) give 4.6×10^{20} units. Data (3) give 10.5×10^{20} units. These magnitudes are to be compared with appropriately defined and properly estimated measures of the dissipation. When the vertical averages are used one deals essentially with the kinetic energy of the mean rotation of the atmosphere, and the dissipation becomes merely the work done against the zonal component of ground stresses in virtue of this mean relative rotation, save for minor other effects. Further discussion of this subject is given by STARR and WHITE (1953). The zonal ground stresses may be obtained in the form of empirical climatological estimates as given by PRIESTLEY (1951), for example. The stresses deduced from the angular momentum balance considerations for the particular period in question as calculated for example by MINTZ (1951) would not give an independent check for the present purpose. Use of Priestley's stresses with the data (1) gives 3.6×10^{20} units for the hemisphere. The data (2) give 1.9×10^{20} units, while data (3) give 4.0×10^{20} units.

If the angular velocity curve in (a) of the figure is assumed to be representative of the entire depth of the atmosphere, we may compute from it the corresponding kinetic energy of mean rotation for the northern hemisphere, which turns out to be about 6.8×10^{26} ergs. Upon appeal to the figure given for the mean rate of production from data (1), it is seen that this amount is normally generated in about 19 days. The corresponding period for data (2) is 12 days and for data (3) it is 17 days. Presumably if the production were suddenly to cease while the dissipation were to maintain its normal rate, the motions of the air here involved would be abolished in about two weeks.

Discussion of results

Relegating certain further technical matters and qualifying remarks to the next section, we may enumerate certain important inferences which may be drawn from the general character of the results, if these latter are correct in main outline, as seems most probable. These are:

1. The large disturbances in the atmosphere produce kinetic energy of mean zonal motions, and not the reverse as has been with few exceptions generally supposed in the past.

2. Since the magnitudes of this generation are of the order of the estimated dissipation, and could regenerate the motions involved in about two weeks, it may be stated that any theory for the general circulation of the atmosphere which omits this effect is crucially deficient, lacking an essential correspondence to reality, however interesting it may otherwise be.

3. Through the years there has been current in meteorology an assortment of general circulation theories which do not presuppose this action of the eddies. The singular sterility and lack of promise which have characterized these can perhaps be ascribed largely to the shortcoming under discussion.

4. It is increasingly clear that a valid formal theory of the general circulation showing how the incoming radiational energy supply results in the observed motions (rather than others) will probably be evolved only slowly because of its inherent complexity.

The problem of the mean zonal motions and the large-scale disturbances is one and inseparable, because of the now demonstrated interdependence of these circulations; hence there exists a natural and direct connection to such other modern developments as those in cyclone theory and similar topics, which latter will first have to be not only better understood but also fitted into the more general picture. For the time being it may be necessary to content oneself with the theoretical examination of only portions or phases of the total process. Indeed much of what now may appear as composed of separate and distinct subjects in theoretical meteorology must ultimately form a coordinated and harmonious whole, displaying in true light the proper and natural relationships among these parts.

5. The theoretical efforts of my esteemed friend Dr. H. L. KUO may be mentioned specifically as an example of what has just been said (see KUO, 1950, 1951 b, 1953). These

studies are directed toward the elucidation of the transformation of eddy kinetic energy into mean zonal kinetic energy through the application of the vorticity equation. This procedure may be interpreted also as an effort to apply the general principles, if not the specific techniques, used in what is now called numerical forecasting to the problem of the general circulation. It is to be noted that in this connection, and also from the general results stated in this paper, the seat of the kinetic energy processes is placed in the cyclonic and anticyclonic systems in the atmosphere, which directly or indirectly account for the continued replenishment of the eddy kinetic energy.

6. It may with profit be again contemplated that the positive nature of the production process as measured from data is not contingent upon the attainment of some petty accuracy, but is at once fixed by very gross aspects of the zonal-wind and angular momentum transport profiles. The end result could, in fact, hardly be otherwise if the maximum northward angular momentum transport is found in the vicinity of the mean latitude of the jet stream.

7. Most of the so-to-speak simple theories of the general circulation start from the assumption that the mean zonal motions are maintained by mean meridional circulations. From time to time thinking men have expressed dissatisfaction with such a framework to various degrees, e. g., JEFFREYS (1926), ROSSBY (1947, 1949), although the direct measurement of the mean meridional circulations cannot be carried out with great enough confidence for a direct refutation which would be reliable. Once, however, very extensive independent compilations of data consistently point to the existence of a sufficiently potent alternative, it is quite unscientific to concentrate merely on the older approach. The writer's opinion in this regard does not exclude the possibility that sufficiently extensive observational studies may show that mean meridional circulations perform certain other important functions, or even modify to some extent the eddy-produced zonal flows. In any case it would be highly improper not to give recognition to those indications and suggestions deriving most clearly and unambiguously from purely objective data as already expounded.

Critique and further computations

During the course of the work performed a number of added questions and matters of thoroughness were examined and treated in much greater detail than can be fully entered into for the purpose of this note. Some critical statements and results of corresponding checks in some instances may nevertheless be made for purposes of better orientation.

1. The use of vertical averages for the zonal wind profile is a rather specialized device which introduces valuable simplifications, but general interest still demands that the results be compared with what would be obtained from calculations at individual levels which are then integrated vertically as a final step. Such alternative figures were calculated giving for data (1) 3.9×10^{20}, for data (2) 5.5×10^{20}, and for data (3) 9.8×10^{20} erg sec^{-1}.

2. A second check carried out was to calculate the production using a vertically integrated zonal-wind profile but applying the technique to the instantaneous daily conditions, the long term mean being then formed by averaging the daily figures for the production.

By this procedure an opportunity is also afforded to compute confidence limits defined as twice the standard error of the mean of the daily values, indicating roughly the 95 per cent confidence range. The data (1) were subjected to such a treatment, although at the time this work was done the observations for 70° N latitude had as yet not been tabulated and hence were not included. The figure obtained was $(4.5 \pm 1.0) \times 10^{20}$ erg sec^{-1}.

3. A still further elaboration consists of the use of the analysis by levels, not with the long period averages, but with instantaneous daily data. The long term mean may then again be formed by averaging the daily production figures. The data (1) with the 70° N latitude circle omitted were used also for this purpose. The figure which resulted was 9.8×10^{20} erg sec^{-1}.

4. In the computations it was arbitrarily assumed that $[u]$ and $[u'v']$ are zero at the equator. What the exact values of the momentum transport are at the equator is not known, but the use of more proper negative values of $[u]$ would tend to increase the anticyclonic wind shear immediately to the north where the transport is positive and hence would probably increase the production.

5. Selectivity in the availability of wind observations favoring lower speeds probably leads to an underestimate of the production. It is to be noted that the integrand of expression (1) involves a triple product of quantities in the nature of wind velocity components, hence underestimates of the latter could produce a larger percentual underestimate here than in such quantities as, for example, the kinetic energy itself.

6. It should be most carefully observed that if the effect of the eddy processes on the mean zonal kinetic energy is studied for a narrower belt of latitude, terms representing the meridional transport of this energy across the bounding latitudes should be considered, and the mere adjustment of the limits of integration in (1) does not provide for this requirement. In the case of the whole hemisphere it is doubtful, however, that such a transport across the equator is anything more than a slight correction, which is here neglected (and would necessarily vanish for the entire atmosphere).

7. The surface zonal stresses given by Priestley are according to most proper interpretation values for the oceans. Doubtless the extensive land masses present in the northern hemisphere give rise to larger such stresses as suggested by the angular momentum convergence study of MINTZ (1951), thus helping to bridge the great disparity found here between the eddy production and the dissipation of the energy of mean rotation.

8. The expression (1) is actually nothing more than the spherical polar form of the corresponding term in the familiar theory of OSBORN REYNOLDS (1895) concerning the nature of eddy processes. A review of this material is given also by LAMB (1932) in his textbook on hydrodynamics. Indications of the importance of the process here measured by expression (1) have been obtained also by BLACKADAR (1950) and by VAN MIEGHEM (1953).

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The Counter-Gradient Flux of Sensible Heat in the Lower Stratosphere

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Introduction

Several recent investigations, KUO (1951), VAN MEIGHEM (1953), STARR (1953) and ARAKAWA (1953), have been concerned with examining the physical processes which account for the maintenance of the kinetic energy of zonal motion in the atmosphere. These studies have pointed up the important role played by the large scale horizontal eddies. The fundamental point made by all these investigators centers on the existence within the free atmosphere of a systematic organization of the large scale eddy momentum exchange processes in such a manner that momentum flows from regions of low to regions of high momentum. The reflection of this condition is most prominent equatorward of the jet stream where for example observational studies of STARR and WHITE (1952) show that the flux of momentum by the large scale horizontal eddy processes is poleward into the jet. STARR (1953) and ARAKAWA (1953) have actually evaluated the work done by such large scale eddy stresses and indicate that the rate of generation of kinetic energy of zonal motion by such processes seems to be sufficient to maintain the westerlies of middle and high latitudes. It is of interest, therefore, to inquire whether similar conditions prevail in the atmosphere with regard to the transfer of other atmospheric properties and in particular with respect to the transfer of sensible heat. Sufficient data are now at hand to show that similar counter-gradient sensible heat flows do occur on a systematic basis.

Data

In connection with a more extensive study of the atmospheric energy balance, computations of the large scale horizontal eddy flux of sensible heat were undertaken. Except for a constant factor, the large scale meridional horizontal eddy flux of sensible heat is measured by the term $[\overline{v' T'}]$, where the brackets represent a mean with respect to longitude, the bar a mean with respect to time, the primes represent deviations from the longitudinal means, v is the northward component of the wind velocity, and T the absolute temperature. The evaluation of this term at various latitudes and at the standard pressure levels in the atmosphere, was made using observed wind and radiosonde observations available in the data compilations of the Northern Hemisphere Historical Weather Map Series, prepared by the U. S. Weather Bureau, in cooperation with the Army, Navy and Air Force, for the full year 1950. The procedure was similar in many respects to that used in connection with investigations of the angular momentum balance reported by STARR and WHITE (1951),

Table 1. Numerical analysis of sensible heat flux data for the entire year 1950. Temperature in degrees absolute, velocities in m sec⁻¹ and N is the total number of observations. The 95 % confidence intervals defined as twice the standard error of the time means is also given.

Level mb	70° N			55° N			42.5° N			31° N		
	$[\bar{T}]$	$[\overline{V'T'}]$	N	$[\bar{T}]$	$[\overline{V'T'}]$	N	$[\bar{T}]$	$[\overline{V'T'}]$	N	$[\bar{T}]$	$[\overline{V'T'}]$	N
100	225	+0.4 ±2.9	288	220	+5.6 ±2.7	763	215	+4.8 ±3.8	546	206	+3.2 ±2.9	1253
200	223	+1.4 ±4.0	919	220	+5.2 ±4.0	1601	219	+8.3 ±4.5	1684	218	+7.4 ±2.7	2859
300	222	+0.0 ±3.2	1302	225	+2.5 ±2.7	2361	231	+3.0 ±3.3	2463	236	+3.0 ±1.8	4003
500	244	+5.2 ±3.1	2047	249	+8.5 ±2.4	3295	256	+1.9 ±1.9	3625	262	+1.5 ±1.1	5181
700	259	+9.2 ±2.1	2266	264	+12.7 ±2.0	3599	272	+6.6 ±1.3	4319	278	+2.3 ±0.9	5756
850	265	+9.6 ±2.0	2374	271	+17.5 ±2.0	3684	279	+10.8 ±1.5	4425	285	+5.0 ±1.1	5218
1000	267	+7.6 ±2.3	1716	275	+11.8 ±2.0	2207	284	+5.5 ±1.2	2494	292	+2.8 ±0.7	4507

in which strings of stations in the vicinity of certain latitudes were selected from which observations were collected. A complete listing of these stations together with the frequency distribution of the observations will be forthcoming in a paper soon to be published by Starr and White.

For purposes at hand it is sufficient to present the relevant data in Table 1, where the mean temperature, magnitude of the eddy transfer of sensible heat, and total number of observations entering the evaluations, are shown. From this table it will be noticed that throughout the troposphere, at all latitudes up to the 200 mb level the eddy-flux of sensible heat is poleward from regions of high to regions of low temperature as might normally be expected. At and above this level the reverse is true. There is an eddy flux of sensible heat from the cold tropical lower stratosphere poleward to the warm polar regions at these levels. This condition was also recognized by PRIESTLEY (1949) and MINTZ (1951) from data compilations which were restricted in space or time. The 95 % confidence limits of the time means of the eddy-transfer term are also indicated and in almost all cases exclude zero, indicating that at least a necessary statistical condition is satisfied by these data.

Meteorological implications

a. The existence of conditions illustrated by these data, and similar ones indicated by STARR (1953) point up the fact that meteorologists should recognize not only the possible existence of these counter-gradient eddy transfer processes but also the actual prevalence of such conditions in certain regions of the atmosphere.

b. The flux of sensible heat from cold to warm regions on the scale of the general circulation at and above the jet-stream-tropopause level indicates that the eddy processes are acting to build up rather than dissipate the existing temperature gradient.

c. Certain fundamental questions concerning the location of the height of the tropopause, and the height of the jet are inseparable from the questions concerning the reversed meridional temperature gradient in the lower stratosphere. Hence a process such as that indicated by these data which obviously acts to build up this reversed gradient may be of considerable importance.

d. Since the full significance of these observational findings is not clear, the data are presented primarily in the hope that they will encourage other interested investigators to consider the implications of such conditions.

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Commentaries Concerning Research on the General Circulation

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Abstract

Several phases of research significant for the general circulation are reviewed briefly. The effect of rotation on convective regimes is discussed. An example is given showing that mean meridional circulations are not essential for the release of potential energy.

During the past several years the present writer has been conducting observational studies on an extensive scale with the aim of securing measures of various quantities chosen on the basis of their direct dynamic significance for the primary mechanism of the general circulation.[1] An outcome of central importance which emerges from this work is that the required meridional transports of angular momentum in the atmosphere are due primarily to horizontal eddy exchange processes, together with the closely allied concept that the kinetic energy of the mean zonal motions is maintained against friction to a large extent by a transference of kinetic energy from the large-scale horizontal eddy kinetic energy to these mean motions.

This action of the eddies is so marked and unambiguous that probably no amount of additional data, if properly brought to bear upon the question, would controvert it.[2] Furthermore, this link in the operation of the general circulation as depicted by the data is so dissimilar to the picture given by most classical schemes which ascribe the drive for the mean zonal motions to the Coriolis force on net meridional flow of air at individual levels, that no really successful manipulation of language or logic can identify them as being, after all, one and the same. We are thus confronted by a state of affairs which, although not completely unsuspected before, is relatively novel in character and hence intriguing—and in many ways puzzling. We must, however, be on the qui vive in our thinking in order to eliminate such difficulties as are merely apparent and not real, and free ourselves of incorrect prepossessions.

In order to show that this behavior of the atmosphere is not unique we may call attention to recent developments of paramount importance for the resolution of the general circulation problem, in the field of experimental hydrodynamics due principally to the original work of D. Fultz and R. Long at the University of Chicago. The particular experiments in question involve the generation of flow patterns in a rotating shallow cylindrical vessel of water resulting from differential heating, which under specified circumstances

[1] It would now appear that the long range planning originally involved in formulating this protracted program has already borne sufficient fruits not only to justify it, but also to suggest rather forcefully the desirability of placing such activities on a firmer footing than has hitherto been possible. Such steps have been advocated before by the writer elsewhere (see STARR 1951), on the basis of preliminary results.

[2] See Appendix.

assume forms similar with respect to a variety of detail to those comprising the atmospheric general circulation (see FULTZ 1952). Some of this work has recently been repeated by A. Faller at the Massachusetts Institute of Technology.

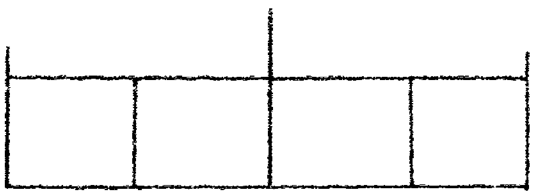
For the purposes of this discussion (and, it should be added, in this writer's opinion) the outstanding new concept made available through these experiments is that the otherwise simple and familiar regime of convective motions is profoundly altered as soon as the rate of rotation exceeds a definite value determined by external parameters such as the intensity of the differential heating. The more intense the heating, the higher becomes this critical rate of rotation. For slow rotation, below the limiting rate, the convection proceeds in an axially symmetric manner with zonal motions developing in much the same fashion as presupposed in the classic explanations of the general circulation. In this case the mean meridional circulations are quite apparent. For rotations higher than the critical one similarity to the atmosphere is obtained as to the following properties (among others): (a) The mean meridional circulations are no longer obvious except perhaps in the form of weak remnants as boundary layer phenomena. (b) Horizontal exchange processes appear to be dominant in the meridional transport of momentum and no doubt other properties as well (see STARR and LONG 1953). The eddies involved are again "waves in the westerlies" as in the atmosphere. (c) The motions appear to be quasi-geostrophic except in the boundary layers. (d) Some evidence of structures within the fluid having the characteristics of occluding cyclones with attendant frontal phenomena has been noted.

To the statement of these experimental findings one should add here a remark concerning some theoretical analyses suggested by them. The symmetrical one is of course the more tractable of the two regimes to study thus, and here the subject has received attention notably from T. V. DAVIES (1952). Still more recently, through the inclusion of the effects of heat advection, H. L. KUO (1953, 1954) has been able to state a necessary condition for the existence of an axially symmetric Hadley type convection in this so-called dishpan experiment. It turns out that a non-dimensional number P involving (among other parameters) the rate of rotation and the axis-to-rim density contrast, is the governing quantity which discriminates between the two regimes and assumes a critical value P_c at the turn-over point. The actual experiments seem to fit in with the theoretical value of P_c in a very satisfactory manner. Preliminary estimates of an analogous parameter for the atmosphere indicate the presence of unmistakable high rotation conditions.

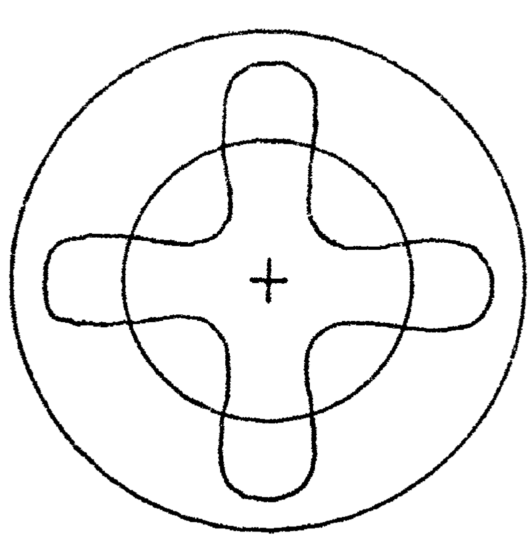
To summarize the situation, it appears that the effect of rotation, if it be rapid enough, is to inhibit large scale convection as ordinarily conceived, and to substitute for it a process which produces the necessary radial transfer of heat and of certain other properties through the agency of quasi-horizontal eddy motions of which cyclones, anticyclones and their attendant upper troughs and ridges are the prime example.

In discussing these matters with other meteorologists the writer has noted frequent skepticism. These reactions seem to imply that any scheme which does not contain some form of the classic convective motions as a starting point is in a general sense too unnatural to be valid, although it is usually not easy to proceed further in order to discover wherein precisely the alleged difficulty lies. It turns out, however, that these attitudes spring mainly from a belief, whether articulate or not, that mean meridional circulations of the toroidal type are indispensable for the utilization of the potential energy inherent in the equator-to-pole temperature gradient in the atmosphere. It is to be sure first necessary to define terms more accurately, but speaking generally this claim is but a delusion, which in actuality the atmosphere circumvents without any real difficulty.

In order to exhibit the nature of the fallacy involved, it suffices to treat a very simple case in the dishpan. Let it be supposed that in the initial state there are two fluids of differing density separated by a vertical discontinuity concentric with the rim as shown in cross section in (a) of the figure and by the inner circle in (b) which is a view seen from the top. It is of course clear that if one of the fluids, say the inner one, is denser the system possesses a certain available potential energy, and the question now is whether some or perhaps all of it can be released without resorting to mean



(a)



(b)

Fig. 1. Diagrams showing a possible rearrangement of two fluids in a cylindrical vessel, prior to the release of potential energy.

meridional circulations, or whether this is precluded through some topological constraint of the continuity principle or other kinematic consideration. It is here assumed that

$$\frac{d\varrho}{dt} = 0; \ \nabla \cdot \vec{C} = 0; \ [v] = 0$$

during the readjustment. In these relations ϱ is density, $\vec{C}$ is the vector velocity, v is the inward radial component of velocity, t is time and the square brackets denote zonal space averaging completely around the pan at a given radius and given depth so that the third relation constitutes the stipulation of no mean meridional circulations.

The answer is rather obvious. The total displacements may be divided into two stages so that in the first the vertical velocity w everywhere vanishes and $[v]$ is therefore identically zero because of the continuity requirement. The vertical discontinuity may therefore be deformed by purely two-dimensional motion into the lobed form shown. In the second stage vertical motions are supposed to take place in such a way as to release potential energy but still so as to make $[v]$ zero, say by having $v \equiv 0$, and allowing each lobe to diverge zonally near the bottom and converge zonally near the top (i.e., by having a downward eddy transport of mass).

With a little thought it becomes evident that the simplified model used here may be generalized to include much more complicated initial states involving continuous density distributions of various types, and that the separate stages of motion may be suitably combined. Also one could without difficulty modify the motions so as to liberate potential energy even in spite of a reverse mean meridional circulation, if the downward eddy transport of mass is vigorous enough.

The outlines of the analogous processes in the actual atmosphere are easily recognized on synoptic charts and are very basic meteorological phenomena, appealing as such even to a casual observer.[1] They are embodied in the large-scale outbreaks of cold and warm air so typical of weather conditions over a wide range of latitude. Viewed from the present standpoint these facts suggest anew the importance of such endeavors as those of Rossby (1949) and Phillips (1949) to study the mechanism for the release of potential energy by the sinking of cold air domes.

[1] We here make the tacit (but probably correct) assumption that the release of geopotential energy is necessary for the maintainance of the general circulation. Since the total geopotential energy of the atmosphere must on hydrostatic principles bear a fixed ratio to its total internal heat energy, and futhermore since the sum is maintained at a more or less fixed value when radiational and other effects are included, it becomes a philosophical problem of considerable complexity to ascertain whether the generation of kinetic energy takes place more directly from one of the forms of energy mentioned rather than the other.

Appendix

As pointed out by STARR (1953) the transfer of kinetic energy from the eddies to the mean zonal motions is a deep-seated process intense enough to generate in a short period of time an amount of such energy as is normally present - even according to measurements which are probably underestimates of its true potency. The inquiring reader may nevertheless propose that in spite of the fact that it is difficult to secure accurate measurements of the mean meridional circulations in the atmosphere due to their smallness, as an experiment a direct calculation should be made of their efficacy in generating mean zonal kinetic energy through the action of the Coriolis force, using the best data at hand. Formally this can be done with the aid of the compilations of actual wind statistics for the calender year of 1950 given by STARR and WHITE (1954) and used in the reference given above. However, as has been stated frequently by these authors, much is left to be desired in such use of the data, and the calculations made here can indeed be viewed as an experiment only.

The hemispheric wind statistics were compiled from daily data for 13°, 31°, 42°, 55° and 70° N latitude for levels from 1000 to 100 mb. The first consideration is that due most probably to a bias in the selection of the station network so as to favor unduly conditions either to the front of troughs on the one hand, or of ridges on the other, small net mean velocities were obtained across the several latitude walls. These, in order, were −22, −03, −17, +37 and +64 cm sec^{-1} (velocities taken positive northward). Only the last two are really serious in regard to magnitude and would imply a mean rising motion in the polar regions at practically all levels in the troposphere and into the stratosphere, if taken literally. In view of this situation, these mean values of the meridional velocities were subtracted out from the data at all latitudes in an effort to minimize spurious effects. All mean meridional circulations were taken to be zero at the equator.

The effect of the Coriolis forces associated with the mean meridional circulations in producing kinetic energy of mean zonal motions is an amount $\varrho f [\bar{u}] [\bar{v}]$ per unit volume (see, e.g., KUO 1951). Here ϱ is density, f the Coriolis parameter, u the eastward and v the northward component of wind velocity; brackets signify zonal averages and bars time averages. The (volume) integrals of this quantity by latitude belts and also for the whole northern hemisphere (up to 100 mb) are given in the table below in units of 10^{20} ergs per second.

No measures of statistical significance were calculated for these figures, although this could have been done, let us say by computing the daily results and then finding the standard error of the mean. It was not deemed worth the while to do this.

Although it is true that not much reliance can be placed upon the particular outcome here obtained, it should be carefully observed that there exists no valid reason which would necessarily exclude the possibility of such a small negative value of the integral for the hemisphere. The contribution of the energy transfer from the eddies is sufficiently large to allow for this (see STARR 1953 and ARAKAWA 1953). Also one may note that it is the (indirect) cell in middle latitudes which is most effective for the present problem, because an equally intense one at low latitudes is rendered much less potent due to the smallness of the Coriolis parameter, while nearer to the pole the length of latitude circles and hence the amount of air involved becomes small. A net destruction of kinetic energy of mean zonal motions by mean meridional circulations would imply as suggested by CHARNEY (1951), that these latter are merely secondary effects, or perhaps associated with surface friction as discussed by FLOHN (1953), for example.

It is doubtful whether the technique of subtracting out the over all mean velocities across latitude walls really eliminates all the unwholesome consequences of any appreciable bias in the sense of sampling of conditions to the east of troughs unduly, as probably was the case at 55° and 70° N. In the forward portions of troughs at high latitudes there usually is found an increase with elevation in the troposphere of the northward component of the wind. Mere subtraction of the over all mean velocity still leaves an abnormally large sampling of this vertical shear in the data, which then manifests itself as a spuriously strong direct mean meridional cell at a northerly location. The positive contributions shown in the table north of 55° N may owe their origin in part to such an artificially introduced circumstance.

In the computations made above, only the effect of the Coriolis term in generating kinetic energy of mean motions was evaluated. This term, however, occurs in combination with others which likewise represent means through which the mean meridional circulations can contribute to the generation of this kinetic energy. For example the term $[\varrho w] [u] \frac{\partial}{\partial z} [u]$ also enters the problem (see KUO 1951). Here w is the vertical component of velocity and z is height. This quantity could be evaluated from the data already used, although the work was not actually done. Since in the troposphere the term normally takes the algebraic sign of the vertical motion, and since this motion is apt to be downward in the latitudes of the strongest vertical shear in the westerlies, it is quite possible that the total effect is negative.

Many findings will no doubt be reported in the course of time relative to direct evidences of mean meridional circulations. What has been said here makes it evident that such claims must not only be scrutinized carefully as to their reliability, but also it is essential that their importance for maintaining the mean zonal motions be properly assessed from a sufficiently realistic viewpoint. For we have reached such a stage in the development of our subject, that the halcyon, free-wheeling days when investigators could propound mean meridional cells in a rough and ready manner to explain each feature of the mean zonal wind distribution, are irretrievably behind us.

Table 1. Volume integral up to the 100 mb level of quantity $\varrho \cdot f [\bar{u}] [\bar{v}]$ in 10^{20} erg sec^{-1}.

Lat. Belt	0—13°	13—31°	31—42°	42—55°	55—70°	70—90°	Hemisphere
Integral	+ 0.01	— 0.35	— 3.55	— 2.65	+ 2.69	+ 1.76	— 2.09

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Available Potential Energy and the Maintenance of the General Circulation

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Abstract

The available potential energy of the atmosphere may be defined as the difference between the total potential energy and the minimum total potential energy which could result from any adiabatic redistribution of mass. It vanishes if the density stratification is horizontal and statically stable everywhere, and is positive otherwise. It is measured approximately by a weighted vertical average of the horizontal variance of temperature. In magnitude it is generally about ten times the total kinetic energy, but less than one per cent of the total potential energy.

Under adiabatic flow the sum of the available potential energy and the kinetic energy is conserved, but large increases in available potential energy are usually accompanied by increases in kinetic energy, and therefore involve nonadiabatic effects.

Available potential energy may be partitioned into zonal and eddy energy by an analysis of variance of the temperature field. The zonal form may be converted into the eddy form by an eddy-transport of sensible heat toward colder latitudes, while each form may be converted into the corresponding form of kinetic energy. The general circulation is characterized by a conversion of zonal available potential energy, which is generated by low-latitude heating and high-latitude cooling, to eddy available potential energy, to eddy kinetic energy, to zonal kinetic energy.

1. The concept of available potential energy

The strengths of the cyclones, anticyclones, and other systems which form the weather pattern are often measured in terms of the kinetic energy which they possess. Intensifying and weakening systems are then regarded as those which are gaining or losing kinetic energy. When such gains or losses occur, the source or sink of kinetic energy is a matter of importance.

Under adiabatic motion, the total energy of the whole atmosphere would remain constant. The only sources or sinks for the kinetic energy of the whole atmosphere would then be potential energy and internal energy.

In general the motion of the atmosphere is not adiabatic. The only nonadiabatic process which directly alters kinetic energy is friction, which ordinarily generates internal energy while it destroys kinetic energy, but which may also, under suitable circumstances, change atmospheric kinetic energy into the kinetic and potential energy of ocean currents and ocean waves. The remaining nonadiabatic processes, including the release of latent energy, alter only the internal energy directly. Hence the only sources for the kinetic energy of the whole atmosphere are atmospheric potential energy and internal energy, while the environment may also act as a sink.

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It is easily shown (cf. HAURWITZ 1941) that the potential and internal energies within a column extending to the top of the atmosphere bear a constant ratio to each other, to the extent that hydrostatic equilibrium prevails. Hence, net gains of kinetic energy occur in general at the expense of both potential and internal energy, in this same ratio. It is therefore convenient to treat potential and internal energy as if they were a single form of energy. The sum of the potential and internal energy has been called *total potential energy* by Margules (1903).

Evidently the total potential energy is not a good measure of the amount of energy available for conversion into kinetic energy under adiabatic flow. Some simple cases will serve to illustrate this point. Consider first an atmosphere whose density stratification is everywhere horizontal. In this case, although total potential energy is plentiful, none at all is available for conversion into kinetic energy. Next suppose that a horizontally stratified atmosphere becomes heated in a restricted region. This heating adds total potential energy to the system, and also disturbs the stratification, thus creating horizontal pressure forces which may convert total potential energy into kinetic energy. But next suppose that a horizontally stratified atmosphere becomes cooled rather than heated. The cooling removes total potential energy from the system, but it still disturbs the stratification, thus creating horizontal pressure forces which may convert total potential energy into kinetic energy. Evidently removal of energy is sometimes as effective as addition of energy in making more energy available.

We therefore desire a quantity which measures the energy available for conversion into kinetic energy under adiabatic flow. A quantity of this sort was discussed by MARGULES (1903) in his famous paper concerning the energy of storms. Margules considered a closed system possessing a certain distribution of mass. Under adiabatic flow, the mass may become redistributed, with an accompanying change in total potential energy, and an equal and opposite change in kinetic energy. It the stratification becomes horizontal and statically stable, the total potential energy reaches its minimum possible value, and the gain of kinetic energy thus reaches its maximum. This maximum gain of kinetic energy equals the maximum amount of total potential energy available for conversion into kinetic energy under any adiabatic redistribution of mass, and as such may be called the *available potential energy*.[1]

Available potential energy in this sense can be defined only for a fixed mass of atmosphere which becomes redistributed within a fixed region. The storms with which Margules was primarily concerned do not consist of fixed masses within fixed regions, nor do any other systems having the approximate size of storms. It is perhaps for this reason that available potential energy has not become a more familiar quantity.

It is in considering the general circulation that we deal with a fixed mass within a fixed region—the whole atmosphere. It is thus possible to define the available potential energy of the whole atmosphere as the difference between the total potential energy of the whole atmosphere and the total potential energy which would exist if the mass were redistributed under conservation of potential temperature to yield a horizontal stable stratification.

The available potential energy so defined possesses these important properties:

(1) The sum of the available potential energy and the kinetic energy is conserved under adiabatic flow.

(2) The available potential energy is completely determined by the distribution of mass.

(3) The available potential energy is zero if the stratification is horizontal and statically stable.

It seems fairly obvious that the available potential energy so defined is the only quantity possessing these properties, although a rigorous proof would be somewhat involved. Moreover, it possesses the further property:

(4) The available potential energy is positive if the stratification is not both horizontal and statically stable.

It follows from property (1) that available potential energy is the only source for kinetic

[1] This quantity is called *available kinetic energy* by Margules, since it represents an amount of kinetic energy attainable. From the point of view of this discussion the term *available potential energy* is preferable, since it represents a part of the existing total potential energy.

energy. On the other hand, it is not the only sink. When friction destroys kinetic energy it creates internal energy, but in doing so it increases the minimum total potential energy as well as the existing total potential energy. Thus the loss of kinetic energy exceeds the gain of available potential energy.

There is no assurance in any individual case that all the available potential energy will be converted into kinetic energy. For example, if the flow is purely zonal, and the mass and momentum distributions are in dynamically stable equilibrium, no kinetic energy at all can be realized. It might seem desirable to redefine available potential energy, so that, in particular, it will be zero in the above example. But the available potential energy so defined would depend upon both the mass and momentum distributions. If it is desired to define available potential energy as a quantity determined by the mass distribution, the definition already introduced must be retained.

2. Analytic expressions and approximations

Θ	:	potential temperature
p	:	pressure
T	:	temperature
z	:	elevation
$\mathbf{v}$	:	horizontal velocity vector
g	:	acceleration of gravity
c_v, c_p	:	specific heats of air at constant volume and constant pressure
R	:	gas constant for air, equal to $c_p - c_v$
λ	:	ratio of specific heats, c_p/c_v, approximately $^7/_5$
$\varkappa$	:	ratio R/c_p, equal to λ-$1/\lambda$, approximately $^2/_7$

Under adiabatic flow, the statistical distribution of Θ is conserved. More precisely, if $g(\Theta_1)d\Theta$ is the probability that a unit mass of atmosphere selected at random has a value of Θ between Θ_1 and $\Theta_1 + d\Theta$, the probability function $g(\Theta)$ is conserved. If

$$\bar{p}(\Theta_1) = \bar{p}_0 \int_{\Theta_1}^{\infty} g(\Theta)\, d\Theta \qquad (1)$$

where $\bar{p}_0$ is the average value of the pressure p_0 at the earth's surface, regarded as horizontal, $\bar{p}(\Theta)$ is also conserved under adiabatic flow. If the particles for which $\Theta = \Theta_1$ form a continuous surface which intersects every vertical column exactly once, $\bar{p}(\Theta_1)$ is the average pressure on the isentropic surface $\Theta = \Theta_1$, with respect to the area of the horizontal projection of this surface. Equation (1) also defines the average pressure over isentropic surfaces which intersect the ground, or which lie entirely "underground", if along each vertical we define $p(\Theta) = p_0$ if $\Theta < \Theta_0$, where Θ_0 is the value of Θ at the earth's surface.

To express the minimum total potential energy in terms of the invariant $\bar{p}\,(\Theta)$ it is sufficient to express the total potential energy in terms of $p(\Theta)$. The potential and internal energies per unit mass are gz and c_vT, respectively. Since, as mentioned previously, the potential and internal energies P and I of a vertical column above a unit area bear the ratio $P/I = (c_p - c_v)/c_v$, and since an element of mass per unit area is $g^{-1}\,dp$,

$$P + I = c_p g^{-1} \int_0^{p_0} T dp \qquad (2)$$

Upon substituting $T = \Theta p^{\varkappa} p_{00}^{-\varkappa}$, where $p_{00} = 1{,}000$ mb, and integrating by parts, we find that

$$P + I = (1+\varkappa)^{-1} c_p g^{-1} p_{00}^{-\varkappa} \int_0^{\infty} p^{1+\varkappa} d\Theta \quad (3)$$

The minimum total potential energy which can result from adiabatic rearrangement occurs when $p = \bar{p}$ everywhere, and is obtained by setting $p = \bar{p}$ in (3). Thus the average available potential energy per unit area of the earth's surface is

$$\bar{A} = (1+\varkappa)^{-1} c_p g^{-1} p_{00}^{-\varkappa} \int_0^{\infty} (\overline{p^{1+\varkappa}} - \bar{p}^{1+\varkappa}) d\Theta \quad (4)$$

where the bar over $p^{1+\varkappa}$ again denotes an average over an isentropic surface.

Since p is always positive and $1 + \varkappa > 1$, it is readily shown that $\overline{p^{1+\varkappa}} - \bar{p}^{1+\varkappa} > 0$ unless $p = \bar{p}$. The precise magnitude of $\bar{A}$, particularly as compared to the average total potential energy per unit area, $\bar{P} + \bar{I}$, is less obvious. Expansion in a power series will aid the comparison. Thus, if $p = \bar{p} + p'$, it follows from the binomial theorem, applied to $p^{1+\varkappa}$, that

$$\bar{A} = (1+\varkappa)^{-1} c_p g^{-1} p_{00}^{-\varkappa} \int_0^\infty \bar{p}^{1+\varkappa} \cdot \left[\frac{\varkappa(1+\varkappa)}{2!} \overline{\left(\frac{p'}{\bar{p}}\right)^2} - \frac{(1-\varkappa)\varkappa(1+\varkappa)}{3!} \overline{\left(\frac{p'}{\bar{p}}\right)^3} + \ldots \right] d\Theta \quad (5)$$

The ratio of $\bar{A}$ to $\bar{P}+\bar{I}$ is a suitable mean value of the quantity enclosed in square brackets.

The power series (5) must converge if $p' < \bar{p}$ everywere, but the rapidity with which it converges depends upon typical values of $p'/\bar{p}$. The distribution of p' is in general far from normal, since tropospheric isentropic surfaces tend to be nearly horizontal in the tropics, so that p' is close to its maximum value over about half the area of the earth. Suppose that on a particular isentropic surface $p = 1{,}000$ mb over half the area, and p decreases linearly from 1,000 mb to 300 mb over the remaining half. In this case $\bar{p} = 825$ mb, $\overline{p'^2}/\bar{p}^2 = 0.075$, and $\overline{p'^3}/\bar{p}^3 = -0.019$. The ratio of the second to the first term in the power series is therefore—0.06, so that even in this rather extreme case, the power series is well represented by its leading term.

Therefore, approximately

$$\bar{A} = \frac{1}{2} \varkappa c_p g^{-1} p_{00}^{\varkappa} \int_0^\infty \bar{p}^{1+\varkappa} \overline{\left(\frac{p'}{\bar{p}}\right)^2} d\Theta \quad (6)$$

and $\bar{A}$ depends upon the *variance* of pressure over the isentropic surfaces.

This variance is closely related to the variance of potential temperature on an isobaric surface, which in turn resembles the variance of temperature on an isobaric or horizontal surface. If $\bar{\Theta}$ and $\bar{T}$ are the average values of Θ and T on a isobaric surface, and Θ' and T' are the departures of Θ and T from $\bar{\Theta}$ and $\bar{T}$, the function $\bar{\Theta}(p)$ is not completely determined by the function $\bar{p}(\Theta)$, but approximately $p = \bar{p}(\bar{\Theta}(p))$, so that

$$p' = \bar{p}(\Theta - \Theta') - \bar{p}(\Theta) \sim -\Theta' \partial \bar{p}/\partial \Theta \quad (7)$$

Thus

$$\bar{A} = \frac{1}{2} \varkappa c_p g^{-1} p_{00}^{-\varkappa} \int_0^{p_0} \bar{\Theta}^2 p^{-(1-\varkappa)} \left(-\frac{\partial \bar{\Theta}}{\partial p}\right)^{-1} \cdot \overline{\left(\frac{\Theta'}{\bar{\Theta}}\right)^2} dp \quad (8)$$

From the hydrostatic equation, it follows that

$$\partial \Theta/\partial p = -\varkappa \Theta p^{-1} (\Gamma_d - \Gamma) \Gamma_d^{-1} \quad (9)$$

where $\Gamma = -\partial T/\partial z$ is the lapse rate of temperature and $\Gamma_d = g c_p^{-1}$ is the dry-adiabatic lapse rate. Since $\Theta'/\Theta = T'/T$,

$$\bar{A} = \frac{1}{2} \int_0^\infty \bar{T} (\Gamma_d - \bar{\Gamma})^{-1} \overline{\left(\frac{T'}{\bar{T}}\right)^2} dp \quad (10)$$

Expression (10) is suitable for estimating the ratio $A/(P+I)$. This ratio evidently equals suitable average value of $\frac{1}{2} \Gamma_d (\Gamma_d - \Gamma)^{-1} \overline{T'^2} \bar{T}^{-2}$. The maximum values of $(\Gamma_d - \bar{\Gamma})^{-1}$ and probably also of $\overline{T'^2}$ occur in the troposphere. If $\bar{\Gamma} = \frac{2}{3} \Gamma_d$ and $\overline{T'^2} = (15°)^2$ are taken as typical values,

$$\bar{A}/(\bar{P}+\bar{I}) \sim 1/200$$

Hence less than one per cent of the total potential energy is generally available for conversion into kinetic energy.

3. Available potential energy and kinetic energy

It is a familiar observation that the total potential energy of the atmosphere greatly exceeds the kinetic energy. In considering the possible release of kinetic energy, however, we should compare the kinetic energy with the available potential energy.

The average kinetic energy per unit area of the earth's surface is approximately

$$\bar{K} = \frac{1}{2} g^{-1} \int_0^{p_0} \overline{V^2} dp \quad (11)$$

In this expression we have neglected horizontal variations of p_0. From (2) it follows that

$$\overline{P} + \overline{I} = \frac{1}{\lambda - 1} g^{-1} \int_0^{\overline{p_0}} \overline{c^2}\, dp \qquad (12)$$

where $c^2 = \lambda RT$ is the square of the speed of sound. If we assume that a typical average wind speed is $^1/_{20}$ of the speed of sound, which lies between 300 m sec^{-1} and 350 m sec^{-1}, we find that

$$\overline{K}/(\overline{P} + \overline{I}) \sim 1/2000$$

If we also assume that our result

$$\overline{A}/(\overline{P} + \overline{I}) \sim 1/200$$

is typical, we find that

$$\overline{K}/\overline{A} \sim 1/10$$

Evidently, if kinetic energy is not released, it is not because a supply of available potential energy is lacking.

Let us see next how $\overline{K}$ and $\overline{A}$ vary. From the equation of continuity

$$\nabla \cdot \mathbf{v} + \partial \omega / \partial p = 0$$

where $\omega = \dot{p} = dp/dt$ is the individual pressure change, determined in the free atmosphere primarily by the vertical speed, and the thermodynamic equation

$$\partial \Theta / \partial t + \mathbf{v} \cdot \Delta \Theta + \omega \partial \Theta / \partial p = c_p^{-1} \Theta T^{-1} Q \quad (14)$$

where Q is the rate of addition of heat, per unit mass, we find that

$$\frac{1}{2} \partial \overline{\Theta'^2} / \partial t = - \overline{\Theta \omega}\, \partial \overline{\Theta} / \partial p - \frac{1}{2} \partial \overline{\Theta'^2 \omega} / \partial p + $$
$$+ C_p^{-1} \overline{\Theta}\, \overline{T}^{-1} \overline{\Theta' Q'} \qquad (15)$$

The second term on the right of (15) involves the space average of the product of three quantities, each of which is itself a departure from a space average. Such "triple correlations" are often negligible. In this case the term arises because another triple correlation, namely the term involving $\overline{p'^3}$ in (5), has been omitted in deriving expressions (6), (8), and (10) for $\overline{A}$ from (5). If we neglect the term involving $\overline{\Theta'^2 \omega}$, we find, since $\Theta'/\overline{\Theta} = T'/\overline{T}$, that

$$\partial \overline{A} / \partial t = - C + G \qquad (16)$$

where

$$C = - R g^{-1} \int_0^{\overline{p_0}} p^{-1} \overline{T \omega}\, dp = - \int_0^{\overline{p_0}} \overline{V \cdot \nabla z}\, dp \quad (17)$$

and

$$G = g^{-1} \int_0^{\overline{p_0}} \Gamma_d (\Gamma_d - \overline{\Gamma})^{-1} \overline{T}^{-1} \overline{T' Q'}\, dp \quad (18)$$

The latter integral in (17) is obtained from the former through the hydrostatic equation and the equation of continuity.

Likewise, from the equation of continuity and the equations of horizontal motion

$$\partial \mathbf{v} / \partial t + (\mathbf{v} \cdot \nabla) \mathbf{v} + \omega \partial \mathbf{v} / \partial p =$$
$$= 2\, \Omega \cdot \mathbf{v} - g \nabla z + \mathbf{F} \qquad (19)$$

where Ω is the vector angular velocity of the earth, and F is the horizontal force of friction, per unit mass, we find that

$$\partial \overline{K} / \partial t = C - D \qquad (20)$$

where

$$D = - g^{-1} \int_0^{\overline{p_0}} \overline{\mathbf{v} \cdot \mathbf{F}}\, dp \qquad (21)$$

Under adiabatic frictionless flow the *generation* G and the *dissipation* D vanish, so that the sum of $\overline{A}$ and $\overline{K}$ is conserved.

We have seen that the available potential energy depends upon the departure of the density stratification from horizontal. If the wind were exactly geostrophic everywhere, the kinetic energy of the whole atmosphere would be zero if and only if the available potential energy were zero. Since the actual wind tends to be nearly geostrophic throughout much of the atmosphere, it still follows that the kinetic energy is generally small or large according to whether the available potential energy is small or large. Large increases in available potential energy and kinetic energy should in general accompany each other.

We have seen, however, that under adiabatic flow increases in available potential energy and decreases in kinetic energy must accompany each other. It follows that when both forms of energy increase together, non-adiabatic effects are involved. Likewise, since

$\overline{K}$ is usually about one tenth of $\overline{A}$, any increase in $\overline{A}$ by more than about ten percent must involve nonadiabatic processes. Such processes are certainly responsible for the large increase in $\overline{A}$ from summer to winter, which is accompanied by a proportionately large increase in $\overline{K}$. This situation, which involves a decrease in $\overline{P}+\overline{I}$, has been discussed by SPAR (1949).

We may ask at this point whether any appreciable changes in the ratio $\overline{K}/\overline{A}$ are possible under adiabatic flow, if the winds remain nearly geostrophic. That such changes are possible may be seen from the approximate expressions

$$\overline{A} = \frac{1}{2} K^{-1} g R^{-1} \int_0^{\overline{p}_0} \Gamma_d (\Gamma_d - \overline{\Gamma})^{-1} \overline{T}^{-1} \cdot p^2 \overline{(\partial z'/\partial p)^2}\, dp \qquad (22)$$

$$\overline{K} \sim \frac{1}{2} g \int_0^{\overline{p}_0} \overline{f^{-2} (\nabla z)^2}\, dp \qquad (23)$$

obtained from (10) and (11) by substituting the hydrostatic and geostrophic relations, f being the Coriolis parameter. Both $\overline{A}$ and $\overline{K}$ depend upon the distribution of z' throughout the atmosphere, but, for a given variance of z', $\overline{K}$ is larger the more the fluctuations of z' in the horizontal, while $\overline{A}$ is larger the more the fluctuations of z' in the vertical. If the vertical variance spectrum of z' is relatively constant, the ratio $\overline{K}/\overline{A}$ will be larger when shorter wave lengths predominate in the horizontal variance spectrum of z'. It is thus possible for $\overline{K}$ and $\overline{A}$ to vary under adiabatic quasi-geostrophic flow. A familiar hypothetical example is the increase of kinetic energy which accompanies the exponential growth of short-wave pertubations superposed upon an unstable zonal current.

4. Zonal and eddy energy

An approach to the general circulation which has currently found much favor consists of resolving the field of motion into the mean zonal motion and the eddies superposed upon it. This resolution partitions the kinetic energy of the whole atmosphere into two types, which may be called *zonal kinetic energy* and *eddy kinetic energy*, and which represent the kinetic energies of the two types of motion. The maintenence of each type of kinetic energy is then considered. In addition to other forms of energy, each type of kinetic energy is a possible source or sink for the other type.

This partitioning of kinetic energy is essentially an analysis of variance of the wind field, and is possible because, aside from the contribution of the average wind, kinetic energy is the sum of the variances of the wind components. A similar analysis of variance of the temperature field is possible. To the extent that available potential energy is measured by the variance of temperature, this analysis partitions the available potential energy into two types, one due to the variance of zonally averaged temperature, and one due to the variance of temperature within latitude circles. These types may be called *zonal available potential energy* and *eddy available potential energy*. In addition to other forms of energy, each type of available potential energy may be a source or a sink for the other type.

Analytic expressions for $\overline{A}_Z$ and $\overline{A}_E$, the zonal and eddy available potential energies, and $\overline{K}_Z$ and $\overline{K}_E$, the zonal and eddy kinetic energies, per unit area, may be obtained from expressions (11) and (12) for $\overline{A}$ and $\overline{K}$ by replacing T and V by their zonal averages, or their departures from their zonal averages. Thus

$$\left.\begin{aligned}
\overline{A}_Z &= \frac{1}{2} \int_0^{\overline{p}_0} (\Gamma_d - \overline{\Gamma})^{-1} \overline{T}^{-1} \overline{[T]'^2}\, dp \\
\overline{A}_E &= \frac{1}{2} \int_0^{\overline{p}_0} (\Gamma_d - \overline{\Gamma})^{-1} \overline{T}^{-1} \overline{T^{*2}}\, dp \\
\overline{K}_Z &= \frac{1}{2} g^{-1} \int_0^{\overline{p}_0} \overline{[V]^2}\, dp \\
\overline{K}_E &= \frac{1}{2} g^{-1} \int_0^{\overline{p}_0} \overline{V^{*2}}\, dp
\end{aligned}\right\} \qquad (24)$$

Here square brackets denote an average with respect to longitude, at constant latitude and constant pressure, while an asterisk denotes

a departure from an average denoted by square brackets.

The time derivatives of these quantities may by applying suitable averaging processes to the continuity equation (13), the thermodynamic equation (14), and the equations of motion (19). Thus, if we allow the same sort of approximations which we used in the expressions for $\partial\overline{A}/\partial t$ and $\partial\overline{K}/\partial t$,

$$\left.\begin{aligned} \partial\overline{A}_Z/\partial t &= -C_Z - C_A + G_Z \\ \partial\overline{A}_E/\partial t &= -C_E + C_A + G_E \\ \partial\overline{K}_Z/\partial t &= C_Z - C_K - D_Z \\ \partial\overline{K}_E/\partial t &= C_E + C_K - D_E \end{aligned}\right\} \quad (25)$$

where

$$\left.\begin{aligned} C_Z &= -Rg^{-1}\int_0^{\overline{p_0}} p^{-1}\overline{[T][\omega]}\,dp = \\ &= -\int_0^{\overline{p_0}} \overline{[\mathbf{v}]\cdot\nabla[Z]}\,dp \\ C_E &= -Rg^{-1}\int_0^{\overline{p_0}} p^{-1}\overline{T^\star\omega^\star}\,dp = \\ &= -\int_0^{\overline{p_0}} \overline{\mathbf{v}^\star\cdot\nabla Z^\star}\,dp \\ C_A &= -\frac{R}{g}\int_0^{\overline{p_0}} \frac{\overline{\Theta}}{\overline{\overline{T}}}\overline{\left([T^\star v^\star]\frac{\partial}{\partial y} + [T^\star\omega^\star]'\frac{\partial}{\partial p}\right)}\cdot \\ &\qquad \overline{\cdot\left(\frac{\Gamma_d}{\Gamma_d - \overline{\Gamma}}\frac{1}{\overline{\Theta}}[T]'\right)}\,dp \\ C_K &= -\frac{1}{g}\int_0^{\overline{p_0}} \cos\phi\,\overline{\left([u^\star v^\star]\frac{\partial}{\partial y} + \right.} \\ &\qquad \overline{\left. + [u^\star\omega^\star]\frac{\partial}{\partial p}\right)\left(\frac{[u]}{\cos\phi}\right)}\,dp \\ &\quad -\frac{1}{g}\int_0^{\overline{p_0}} \overline{\left([v^{\star 2}]\frac{\partial}{\partial y} - \sin\phi\,[V^{\star 2}] + \right.} \\ &\qquad \overline{\left. + [v^\star\omega^\star]\frac{\partial}{\partial p}\right)[v]}\,dp \end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned} G_Z &= g^{-1}\int_0^{\overline{p_0}} \Gamma_d(\Gamma_d - \overline{\Gamma})^{-1}\,\overline{T}^{-1}\,\overline{[T]'[Q]'}\,dp \\ G_E &= g^{-1}\int_0^{\overline{p_0}} \Gamma_d(\Gamma_d - \overline{\Gamma})^{-1}\,\overline{\overline{T}}^{-1}\,\overline{T^\star Q^\star}\,dp \\ D_E &= -g^{-1}\int_0^{\overline{p_0}} \overline{[\mathbf{v}]\cdot[\mathbf{F}]}\,dp \\ D_Z &= -g^{-1}\int_0^{\overline{p_0}} \overline{V^\star\cdot F^\star}\,dp \end{aligned}\right\} \quad (27)$$

In (26), u and v are the eastward and northward components of V, ϕ is latitude, and $\partial/\partial y$ is the derivative with respect to distance northward. The alternative forms for C_Z and C_E are analogous to the alternative forms (17) for C.

In equations (25) we observe that each of the quantities C_Z, C_E, C_A, and C_K occurs twice, with opposite signs. It is then tempting to say, for example, that C_K represents the rate of conversion from zonal to eddy kinetic energy, and to draw analogous conclusions about the other "C's", i.e., to interpret the C's as energy transformation functions, as described by MILLER (1950). We must note, therefore, that the C's are not uniquely defined by the time derivatives of the various forms of energy, since, for example, if all the C's were altered by the additions of the same quantity, equations (25) would still be valid. To justify the interpretation of the C's as conversions from one form of energy to another, we must examine the physical processes which they describe. The necessity for considering physical processes when interpreting energy equations has recently been emphasized by LETTAU (1954).

We note first the quantities G_Z and G_E, which may be called the *zonal generation* and the *eddy generation*, represent the generation (or destruction) of available potential energy by nonadiabatic processes, and do not involve conversion from one form of atmospheric energy to another. Similarly, the quantities D_Z and D_E, which may be called the *zonal dissipation* and the *eddy dissipation*, represent the dissipation of kinetic energy by friction,

and do not involve conversions of energy. Here we have regarded friction as involving a simultaneous destruction of kinetic energy and generation of potential energy, rather than a process of conversion from kinetic to potential energy, since very little available potential energy is generated by frictional heating.

It follows that the sum of the C's in the change of any one form of energy must equal the sum of the conversions to that form of energy from all other forms.

We next note that the C's all involve horizontal or vertical transports of momentum or sensible heat. These transports may be resolved into separate modes of transport; for example, the vertical transport of sensible heat, represented by $[T\omega]$, may be resolved into a transport by meridional circulations, an eddy-transport whose value per unit area is independent of latitude, and an eddy-transport whose value vanishes when averaged over latitude. Each of these modes of transport enters only one of the terms in the relation

$$[T\omega] = [T][\omega] + [T^{\star}\omega^{\star}] + [T^{\star}\omega^{\star}]' \quad (28)$$

Let us agree to regard the separate modes of transport as separate physical processes.

We then observe the following situation: Horizontal eddy-transports of sensible heat, and vertical eddy-transports whose values vanish when averaged over latitude, enter the expression for C_A, but not C_K, C_Z, nor C_E. They therefore affect A_Z and A_E by altering the analysis of variance of temperature, but they do not affect K_Z, K_E, nor the sum $A_Z + A_E$.

Eddy-transports of momentum enter the expression for C_K, but not C_A, C_Z, nor C_E. They therefore affect K_Z and K_E by altering the analysis of variance of wind, but they do not affect A_Z, A_E, nor the sum $K_Z + K_E$.

Transports of sensible heat by meridional circulations, and accelerations due to horizontal displacements by meridional circulations, enter the equivalent expressions for C_Z, but not C_E, C_A, nor C_K. They therefore affect A_Z and K_Z by altering the variance of zonally averaged temperature and wind, but they do not affect A_E, K_E, nor the sum $A_Z + K_Z$.

Vertical eddy-transports of sensible heat whose value per unit area is independent of latitude, and accelerations due to horizontal displacements by eddies, enter the equivalent expressions for C_E, but not C_Z, C_A, nor C_K. They therefore affect A_E and K_E by altering the variance of temperature and wind within latitude circles, but they do not affect A_Z, K_Z, nor the sum $A_E + K_E$.

It follows that C_A, C_K, C_Z, and C_E are energy transformation functions, which involve respectively only available potential energy, only kinetic energy, only zonal forms of energy, and only eddy forms of energy.

The energy transformation function C_K has appeared frequently in recent works. It is a modification of an expression derived by REYNOLDS (1894) in connection with turbulent flow. It has been presented in nearly the same form by VAN MIEGHEM (1952), while the first integral in expression (25) for C_K, which is the dominating term, has been discussed by KUO (1951) and STARR (1953).

The energy transformation function C_A bears nearly the same relation to temperature which C_K bears to wind. It depends upon the transport of sensible heat along the gradient of temperature in much the same way in which C_K depends upon the transport of angular momentum along the gradient of angular velocity.

The two possible remaining energy transformation functions—the conversions from A_Z to K_E and from A_E to K_Z, do not enter equations (25). Moreover, if we regard the separate modes of transport as separate physical processes, there is no process which affects both A_Z, and K_E, or both A_E and K_Z. These remaining energy transformation functions therefore vanish identically.

It must be remembered that this conclusion depends upon our regarding the separate modes of transport as separate physical processes. Without the distinction between the two modes of eddy transport, it would be impossible to say whether or not a direct conversion of zonal available potential energy to eddy kinetic energy is possible, although conversion from any form of available potential energy to eddy kinetic energy, which involves nonvanishing values of $T^{\star}$, would still require the *presence* of eddy available potential energy. The distinction between the modes of eddy-transport is probably as logical, if not as familiar, as the

distinction between eddy transport and transport by meridional circulations. Without the latter distinction none of the energy transformation functions involving zonal or eddy available potential energy could be defined.

5. The maintenance of the energy of the general circulation

The zonal winds and the superposed eddy motions are not identical with the meridional pressure gradient and the superposed pressure perturbations, since the former are features of the distribution of momentum, which possesses kinetic energy, and the latter are features of the distribution of mass, which possesses available potential energy. It is therefore a legitimate problem to study exchanges of kinetic energy between the zonal winds and the eddies, without considering similar exchanges of available potential energy.

Nevertheless the zonal winds are often identified with the meridional pressure gradient, and cyclonic and anticyclonic circulations are often identified with the low and high pressure systems which almost always accompany them. Indeed the wind systems could not long maintain their identities without the accompanying pressure systems, and vice versa. It may therefore be possible to achieve a better understanding of the general circulation by regarding exchanges of kinetic energy and exchanges of available potential energy as features of a single problem.

The conversion C_K of zonal to eddy kinetic energy depends primarily upon the transport of angular momentum horizontally and vertically by eddies along the gradient of angular velocity. Recent computations by STARR (1953), based upon wind observations over the northern hemisphere (see STARR and WHITE 1954) confirm the earlier suspicions of some meteorologists that the horizontal transport is predominantly against the gradient of angular velocity, and yield the approximate value -10×10^{20} ergs per second for the integral of C_K over the northern hemisphere, so that the eddies appear to supply sufficient kinetic energy to the zonal flow to maintain it against frictional dissipation.

At this point we must become more specific and state just what sort of dissipation we are considering. The dissipation of zonal kinetic energy by molecular friction is probably very small. The principal "frictional" dissipation of zonal kinetic energy is instead due to small-scale turbulent eddies, and it is principally the kinetic energy of these eddies which is dissipated by molecular friction. It is therefore not correct to say that the eddies on the whole supply kinetic energy to the zonal flow, if all scales of eddies are included.

Instead, we must make a distinction between large-scale eddies, which, roughly speaking, are the eddies large enough to appear on synoptic weather maps, and the remaining small-scale eddies. It is then correct to say that the large-scale eddies supply enough kinetic energy to the zonal flow to maintain it against the dissipative effects of small-scale eddies. This important feature of the general circulation would be obscured if eddies of all scales were included in a single category.

Again, if only molecular friction and conduction were considered, the skin friction and surface heating would have nearly infinite values, per unit mass, throughout nearly infinitesimal depths. The generation and dissipation functions G_Z and D_Z would then depend largely upon the usually unmeasured temperatures and winds in a thin layer next to the ground. This difficulty is overovercome if eddy viscosity and conductivity replace molecular viscosity and conductivity, so that the skin friction and surface heating have moderate values throughout moderate depths. Let us agree, therefore, to regard only the large-scale eddies as eddies, and to include the small-scale eddies in a category with molecular motions.

The conversion C_A of zonal to eddy available potential energy depends primarily upon the transport of sensible heat horizontally and vertically across the gradient of temperature T'. The studies of STARR and WHITE (1954) confirm the generally accepted idea that the horizontal transport is with the temperature gradient, and computations based upon the results of this study yield the approximate value 200×10^{20} ergs per second for the integral of C_A over the northern hemisphere, so that C_A is about twenty times as large as C_K.

It follows that if the zonal winds and the meridional pressure gradient are regarded as separate manifestations of the same zonal pattern, and if the wind and pressure variations

within latitude circles are regarded as separate manifestations of the same eddies, it is not possible to say that the eddies maintain the zonal circulation. All that can be said is that the zonal pressure field maintains the eddy pressure variations, but the eddy motion maintains the zonal motion.

To understand the maintenance of the energy of the general circulation, it is therefore not sufficient to know the exchange of energy between the zonal circulation and the eddies. A knowledge of all the energy transformation functions and the generation and dissipation functions is required.

The presence of net heating in low latitudes and net cooling in high latitudes is a familiar feature of the general circulation; it leads to a generally positive value of $\overline{[T]'[Q]'}$, so that the zonal generation G_Z is positive, and indeed seems to represent the primary source of the energy of the general circulation. Crude estimates of G_Z, based upon radiation-balance figures of Albrecht (see HAURWITZ 1941), and neglecting the release of latent energy, yield the approximate figure $G_Z = 200 \times 10^{20}$ ergs per second so that G_Z and C_A are about equal.

Less obvious is the sign of the eddy generation G_E, which depends upon $\overline{T^*Q^*}$, and hence upon the correlation between temperature and heating within latitude circles. Presumably it is negative, in view of the probable warming of cold air masses and cooling of warm air masses in middle latitudes, but the possible preference of warm longitudes for the release of latent energy may suppress this negative value.

The dissipation functions D_Z and D_E may safely be regarded as positive. We have just seen that C_K is negative, while C_A is positive. It follows by continuity that C_E must be positive, since it represents the only remaining source for eddy kinetic energy. This positive value must be associated with sinking of colder air and rising of warmer air at the same latitude. Hemispheric data for the direct compu ation of C_E are unfortunately not available.

The sign of C_Z cannot be inferred by continuity, but the magnitude seems to be small, in view of the failure of hemispheric wind observations to reveal strong meridional cells. A value of -2×10^{20} ergs per second for the integral of C_Z over the northern hemisphere has been estimated by STARR (1954) from the data available. The negative sign occurs because the middle-latitude indirect cell occupies the zone of maximum temperature gradient.

We are thus led to the following picture of the maintenance of the energy of the general circulation: The net heating of the atmosphere by its environment in low latitudes and the net cooling in high latitudes result in a continual generation of *zonal available potential energy*. Virtually all of this energy is converted into *eddy available potential energy* by the eddies. Some of this energy may be dissipated through heating of the colder portions of the eddies and cooling of the warmer portions; the remainder is converted into *eddy kinetic energy* by sinking of the colder portions of the eddies and rising of the warmer portions. Some of this energy is dissipated by friction; the remainder is converted into *zonal kinetic energy* by the eddies. Most of this energy is dissipated by friction; a small residual is converted into zonal available potential energy again by an indirect meridional circulation.

In conclusion, let us see how our picture of the energy transformation compares with earlier descriptions of the general circulation. Certainly it bears little resemblance to any theory which attributes the conversion of potential into kinetic energy to a general rising motion in low latitudes and sinking in high latitudes. However, discussions of some of the alternative processes which we have described have been appearing with increasing frequency in recent meteorological literature.

The idea that eddy kinetic energy is the immediate source of zonal kinetic energy is closely related to the idea expressed by ROSSBY (1947, 1949) that large-scale mixing processes (eddies) may account for the distribution of zonal winds. EADY (1950) has described the inequality of the mean zonal angular velocity as a result of turbulence (eddies). Computations of a positive rate of conversion of eddy into zonal kinetic energy, based upon wind observations, have been presented by KUO (1951) and STARR (1953).

An idealized quantitative model in which

the motions associated with waves in the westerlies (eddies) are responsible for the generation of kinetic energy has been presented by MINTZ (1947). The importance of sinking cold air masses and rising warm air masses was pointed out by ROSSBY (1949). VAN MIEGHEM (1952) describes the conversion of potential energy into *eddy* kinetic energy as one of the two most important energy transformations on the scale of the general circulation, the other being the conversion of eddy into zonal kinetic energy.

The importance of a poleward eddy-transport of sensible heat, in conjunction with the excess of radiational heating in low latitudes and the deficit in high latitudes, has long been recognized (cf. HAURWITZ 1941), but the inevitable other effect of this transport—an increasing of the variance of temperature within latitude circles—seems to have been generally overlooked. Very recently STARR (1954) has described an idealized two-stage process of conversion of potential into kinetic energy, the first step consisting of a deformation of zonally oriented isotherms. This stage is essentially a conversion from zonal into eddy available potential energy. But regardless of whether one wishes to think in terms of available potential energy, a variance of temperature within latitude circles is a prerequisite for the conversion of potential energy into eddy kinetic energy, which involves a correlation within latitude circles between temperature and vertical motion (cf. VAN MIEGHEM 1952). If non-adiabatic heating creates primarily a cross-latitude variance of temperature, a poleward eddy-transport of sensible heat is necessary to maintain the variance of temperature within latitude circles. The conversion of zonal to eddy available potential energy may therefore be regarded as a third important energy transformation on the scale of the general circulation.

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On Conversions between Potential and Kinetic Energy in the Atmosphere

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From a consideration of the large-scale horizontal variations of individual pressure change and 500 mb temperature in a mid-latitude sector of the Northern Hemisphere, computations are made of the required mean conversion of potential energy into the kinetic energy of the horizontal wind systems. The order of magnitude of the estimate obtained is in agreement with that obtained by Brunt from considerations of the frictional dissipation of kinetic energy. In addition, the role of organized overturnings is investigated. It is indicated that overturnings in east-west vertical planes associated with the large-scale disturbances are of primary importance in effecting the release of potential energy.

1. Introduction

In recent years increasing attention has been given to basic questions relating to the manner in which the energy provided by solar heating is utilized in maintaining the large-scale atmospheric wind systems. In this connection it has been convenient to think in terms of a "cycle" which traces the transformation and flow of energy in the atmosphere. In such an energy cycle, conversions between the potential, internal, and kinetic forms play a vital role. Aspects of this conversion process have been discussed by many authors including EADY (1949), CHARNEY (1951), FJØRTOFT (1951), VAN MIEGHEM (1952, 1955), KUO (1954), LORENZ (1954), PHILLIPS (1954, 1955), PISHAROTY (1954), and STARR (1954).

Although the stages of the energy cycle which involve the partition of available potential energy and kinetic energy into eddy and zonal components have been measured by direct observations (see, for example, STARR and WHITE, 1954), little quantitative observational evidence has been obtained thus far concerning the conversion between potential and kinetic energy. This has been largely due to the difficulties in computing the field of vertical motion, or divergence, on which such measurements critically depend.

It is the purpose of this article to present some observational estimates of the conversion process based on data recently computed by means of a two-parameter model of atmospheric flow, for a grid network covering a portion of the North American continent.

2. The nature of the conversion process

In accordance with the ideas originally expressed by MARGULES (1903) and recently extended by LORENZ (1954), one may conceive of the primary effect of solar heating as being the creation of horizontal temperature differences which represent an "available" potential energy in the atmosphere. Through the action of vertical motions, under conditions closely approximating hydrostatic equilibrium, this available potential energy is released and converted largely into the kinetic energy of the horizontal wind (see KUO, 1954).

In order to obtain a mathematical expression which represents this energy conversion process we may proceed as follows. Taking the

pressure, p, as the vertical coordinate and using the hydrostatic equation we may write the equation of motion for the horizontal wind in the form,

$$\frac{d\mathbf{v}}{dt} + f\mathbf{k} \times \mathbf{v} = -\nabla\phi - \mathbf{F}, \quad (1)$$

where $\mathbf{v}$ is the horizontal vector wind, f is the Coriolis parameter, $\mathbf{k}$ is the unit vector in the vertical, $\phi = gz$ is the geopotential of an isobaric surface, ∇ is the two-dimensional del-operator in a pressure surface and $\mathbf{F}$ is the vector frictional force per unit mass. If we scalar-multiply this equation by $\mathbf{v}$, and expand the total derivative, we obtain the energy equation for the horizontal motions in the form,

$$\frac{\partial k}{\partial t} + \mathbf{v} \cdot \nabla k + \omega \frac{\partial k}{\partial p} = -\mathbf{v} \cdot \nabla\phi - D, \quad (2)$$

where $k = \mathbf{v}^2/2$ is the kinetic energy of horizontal motion per unit mass, $D = \mathbf{v} \cdot \mathbf{F}$ is the rate of frictional dissipation of kinetic energy per unit mass, and $\omega = dp/dt$. With the use of the hydrostatic equation,

$$\frac{\partial \phi}{\partial p} = -\alpha, \quad (3)$$

where $\alpha = 1/\varrho$ is the specific volume, and the continuity equation,

$$\frac{\partial \omega}{\partial p} = -\nabla \cdot \mathbf{v}, \quad (4)$$

we may rewrite equation (2) in the form,

$$\frac{\partial k}{\partial t} + \nabla \cdot (k + \phi)\mathbf{v} + \frac{\partial}{\partial p}(k + \phi)\omega = -\omega\alpha - D. \quad (5)$$

Finally, if this equation is integrated over the entire mass of the atmosphere, M, we obtain,

$$\frac{\partial}{\partial t}\int_M k\,dm = -\int_M \omega\alpha\,dm - \int_M D\,dm, \quad (6)$$

where $dm = g^{-1}\,dx\,dy\,dp$ (x is distance eastward and y is distance northward).

This equation states that for the entire mass of the atmosphere the kinetic energy of the horizontal wind can vary as a result of a frictional dissipation and an effect depending on the product of ω and the specific volume throughout the mass of the fluid.

That this latter effect actually represents a conversion of potential and internal energy may be demonstrated as follows. The first law of thermodynamics may be written in the form,

$$\frac{dQ}{dt} = c_p \frac{dT}{dt} - \omega\alpha \quad (7)$$

where c_p is the specific heat at constant pressure, T is temperature, and dQ/dt is the rate of heat addition per unit mass (including the generation of heat by friction). With the use of (4), this equation becomes

$$c_p \frac{\partial T}{\partial t} + \nabla \cdot c_p T\mathbf{v} + \frac{\partial}{\partial p} c_p T\omega = \omega\alpha + \frac{dQ}{dt}. \quad (8)$$

Integrating this equation over the entire atmosphere and recalling that for a vertical column of the atmosphere in hydrostatic equilibrium $c_p g^{-1}\int_0^{p_0} T\,dp$ is equal to the sum of the potential and internal energy of the column,[1] we find that the expression for the time-rate of change of total potential and internal energy, $\int_M (\Phi + I)\,dm$, takes the form,

$$\frac{\partial}{\partial t}\int_M (\Phi + I)\,dm = \int_M \omega\alpha\,dm + \int_M \frac{dQ}{dt}\,dm. \quad (9)$$

The appearance of $\int_M \omega\alpha\,dm$ with opposite sign in both equations (6) and (9) indicates that this term represents a conversion between potential and kinetic energy, a loss in one representing a gain in the other.

By virtue of the strong relation between ω and the vertical motion it is clear that the effect represented by this term is associated with the process of rising of warm air masses and sinking of cold air masses, which has been connected with the conversion process since the time of Margules.

Since over a long period of time there is no observed change in the total kinetic energy of

[1] Owing to the fact that in a hydrostatic atmosphere the potential energy in a given column bears a fixed ratio to the internal energy, we shall henceforth include both of these forms under "potential" energy.

the horizontal wind, it follows from equation (6) that the conversion integral, $-\int \omega\alpha\, dm$, must be positive in the mean to balance the frictional dissipation. From another point of view, it has been noted by EADY (1949), PHILLIPS (1954), and KUO (1954) that the vertical transport of heat (of which this integral is a close measure) should, in the long-time mean, be positive, in order to maintain the observed stable stratification against radiational losses in the upper levels of the atmosphere.

It may be noted also, that by comparing the long-time averages of equations (6) and (9), the well-known result that the mean rate of heat addition equals the mean rate of frictional dissipation is obtained.

3. Measurement of the conversion term

An evaluation of the conversion term requires a knowledge of the spatial distribution of ω over the entire atmosphere. Although it is impossible at present to obtain such extensive data, computations of ω for limited regions of the atmosphere now being made in connection with numerical weather prediction experiments are becoming available. The reliability and coverage of these new computations are in many respects superior to those provided by other methods. It seems desirable therefore to make use of such data for the present problem.

In the process of carrying out a series of numerical forecasting experiments for each day of January 1953, values of the vertically-averaged individual pressure change were computed by the joint Geophysics Research Directorate—Air Weather Service numerical weather prediction project (THOMPSON and GATES, 1956). These computations were based upon a simple 2-parameter, quasi-geostrophic, baroclinic model developed by THOMPSON (1953). The results of these computations were kindly made available to the authors for the present study.

More specifically, the vertically-averaged individual pressure change, $\tilde{\omega}$, was computed by means of the adiabatic energy equation in the form,

$$\tilde{\omega} = -\frac{g}{f\tilde{\varkappa}}\left[\frac{\partial h}{\partial t} + \frac{g}{f}J(Z, h)\right] \qquad (10)$$

where $\tilde{(\ )} = 1/p_0 \int_0^{p_0} (\ )\, dp$, $\varkappa = RTf^{-1}\Theta^{-1}\partial\Theta/\partial p$ (Θ = potential temperature), and $J(Z, h)$ is the two-dimensional Jacobian of the 500 mb height, Z, and the thickness between 1,000 and 500 mb, h. $\tilde{\varkappa}$, which is a measure of the static stability through the depth of the atmosphere, was estimated from a set of observational data for the period 1—10 January, 1953.

The values of the tendency, $\partial h/\partial t$, computed from the thermal vorticity equation for the initial time $t = 0$, and the corresponding values of $J(Z, h)$ were used to evaluate $\tilde{\omega}$ at a network of 204 grid points covering a large part of the North American sector. In the solution of the thermal vorticity equation it is necessary to specify the thickness tendency along the boundaries of the region. This was approximated by assuming that the tendency at the initial time is proportional to the known 24-hour thickness change centered on time $t = 0$.

Since these numerical computations provide *vertically-averaged* values of ω only, it is not possible in the present study to consider the multi-level effects of the horizontal variations of α. Synoptic evidence suggests, however, that the general mid-tropospheric variations of α are of considerable importance in the conversion process. Accordingly, as a simple assumption the horizontal variations of α were estimated by measuring the field of temperature along the 500 mb surface. It is recognized that such an estimate does not necessarily give a representative measure of effects in the upper levels of the atmosphere where the horizontal field of α tends to be somewhat out-of-phase with that in the troposphere.

The 500 mb temperature fields and the corresponding $\tilde{\omega}$-fields were analyzed on separate charts, from which the values of temperature and $\tilde{\omega}$ were extracted at every 5 degree latitude-longitude intersection over the area between latitudes 35° N to 60° N and longitudes 70° W to 120° W. An example of the type of distribution of $\tilde{\omega}$ and temperature which was found is shown in Fig. 1.

It is reasonable to expect that, in the mean, the conditions in the area sampled in this study are similar in their essential features to other middle-latitude areas of the hemisphere. The geographical extent of this area is sufficiently

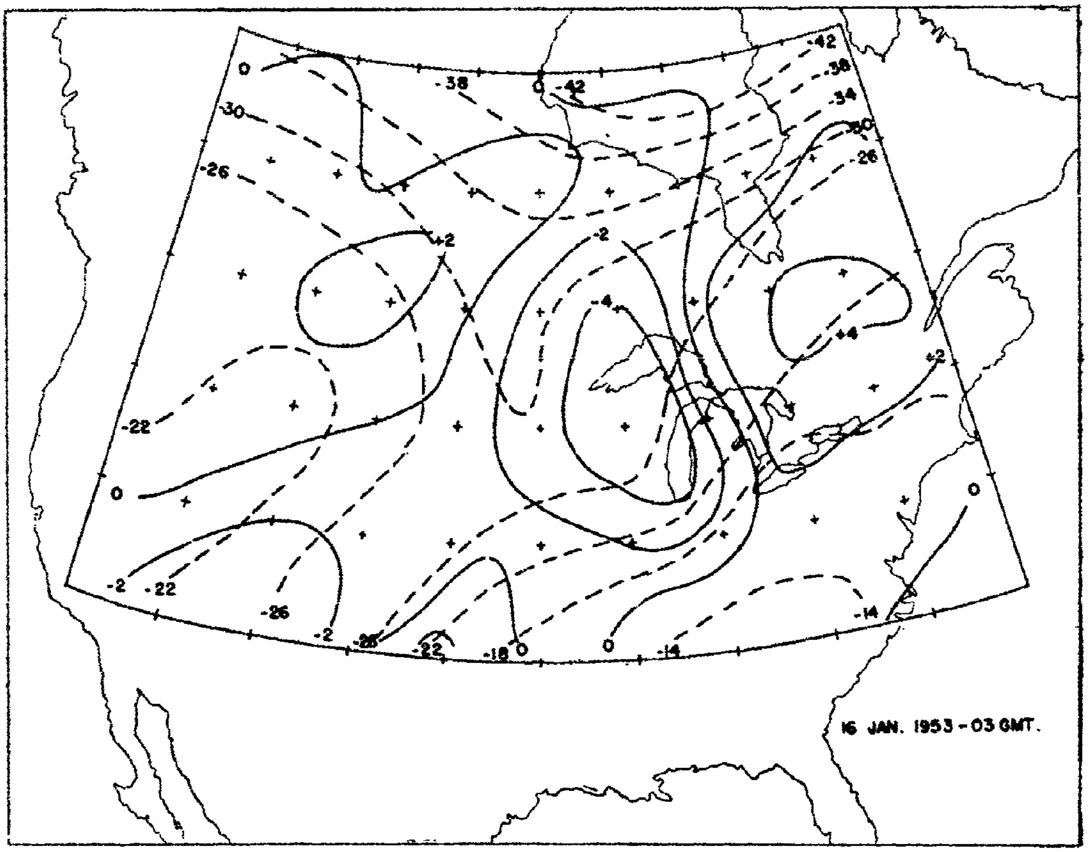

Fig. 1. Map for 16 January 1953. The values of $\tilde{\omega}$, converted to estimates of the vertical velocity (cm sec $^{-1}$) by the formula $w = -\varrho^{-1} g^{-1} \tilde{\omega}$, are represented by the solid lines. The dashed lines represent the 500 mb isotherms in degrees Centigrade.

large so that it always includes at least one synoptic-scale disturbance with associated temperature and $\tilde{\omega}$-fields.

In view of the approximations outlined above and the limited geographical area treated, the computations based on these data can be taken only as a rough measure of the actual conversion process.

4. Analysis of the data

As outlined in the preceding section, estimates of the horizontal variation of the individual pressure change and specific volume were obtained in this study. We henceforth denote these estimates by $\omega^\star(x, y)$ $(=\tilde{\omega})$ and $\alpha^\star(x, y)$ ($\sim$ 500 mb temperature) respectively.

We may write the time mean of the space average of the product $\omega^\star\alpha^\star$, in the form, $\overline{\{[\omega^\star\alpha^\star]\}}$, where the brackets, brace and bar represent averages with respect to x, y and t respectively. This quantity may be further resolved into components representing the contributions due to vertical overturnings in $x-p$ "planes", overturnings in the $y-p$ "plane", and the time variations of the space averaged individual pressure change and specific volume. Such a resolution is accomplished by the following expansion,

$$\overline{\{[\omega^\star\alpha^\star]\}} - \overline{\{[\omega^\star]\}}\;\overline{\{[\alpha^\star]\}} = \overline{\{[\omega^\star]''[\alpha^\star]''\}} + \\ + \overline{\{[\omega^{\star\prime}\alpha^{\star\prime}]\}} + \overline{\{[\omega^\star]\}'''\{[\alpha^\star]\}'''}, \qquad (11)$$

where the single, double and triple primes denote deviations from the x-, y- and t-means respectively.

It can be shown that, in the long-time mean, the integral of ω over the entire mass of the atmosphere must vanish. Hence, the second term on the left, which involves $\overline{\{[\omega^\star]\}}$, is identically zero if the entire atmosphere is considered. Since only a limited region of the atmosphere is treated in the present study this term can have a significant non-zero value. It is desireable, therefore, to take as a more representative measure of the *hemispheric* conversion process the left-hand side of equation (11) rather than $\overline{\{[\omega^\star\alpha^\star]\}}$ alone. If the vertical motions associated with the large-scale synoptic features in mid-latitudes are the important ones in effecting the required conversion,[1] and if the region sampled is somewhat representative of other regions of the hemisphere, one would expect this quantity to be negative in accordance with the theory of section 2.

The first term on the right-hand side of (11) represents the contribution resulting from the correlation of $[\omega^\star]$ and $[\alpha^\star]$ in the north-south direction, associated with overturnings in the $y-p$ "plane". The second term on the right depends on the correlation between $\omega^\star$ and $\alpha^\star$ along a latitude circle and results from overturnings in $x-p$ "planes". The third term depends on the time correlation between the space averages of $\omega^\star$ and $\alpha^\star$.

More meaningful interpretations of these expressions can be made in terms of the energy cycle of the general circulation conceived by LORENZ (1954). In accordance with his scheme, $y-p$ overturnings are associated with transformations between mean zonal kinetic energy and the "*zonal* available potential energy", while the $x-p$ overturnings represent a transformation between eddy kinetic energy and "*eddy* available potential energy".

[1] It is possible that phenomena of a smaller scale than the grid network used in this study, such as cumulus convection, may be of some importance in the global conversion process.

5. Results

Each of the terms in equation (11) were computed from the data, and their values are given in table 1.

If we consider that the mass of a column of the atmosphere of unit cross-sectional area is approximately 10^3 gm, then an estimate of the net rate of energy conversion for the entire depth of the atmosphere may be obtained from the first column. In accordance with the remarks of section 2, this quantity also represents an estimate of the mean rate of frictional dissipation of kinetic energy per unit area. In the present case the value obtained is 5.0×10^3 ergs cm^{-2} sec^{-1}, representing about two per cent of the effective solar radiation. Considering the approximations made in this study, this estimate agrees well with that obtained by BRUNT (1941) from other considerations. This result lends some support to the assumption implicit in this study that the gross-scale horizontal tropospheric variations of ω and α in mid-latitudes are of primary importance in the conversion process.

As previously noted, the total energy conversion is brought about by various kinds of organized atmospheric circulations. The processes represented by the second and third columns of table 1 are those associated with vertical overturnings in north-south and east-west vertical "planes" respectively. On the basis of these data it appears that in middle latitudes the $x-p$ overturnings consistently act to increase the kinetic energy, with $y-p$ overturnings acting in the opposite sense. Thus, in terms of the Lorenz energy cycle, the overturnings result in the conversion of eddy available potential energy into the kinetic energy of the large scale atmospheric disturbances while the $y-p$ overturnings give rise to an increase in the zonal available potential energy at the expense of the zonal kinetic energy.

A more detailed description of the $x-p$ overturning process is given in Fig. 2 where

Table 1. Rate of conversion between potential and kinetic energy based on data for 2—31 January, 1953. A minus sign indicates a conversion from potential to kinetic energy. Units in ergs gm^{-1} sec^{-1}.

$\overline{\{[\omega^\star\alpha^\star]\}} - \overline{\{[\omega^\star]\}}\ \overline{\{[\alpha^\star]\}}$	$\overline{\{[\omega^\star]''[\alpha^\star]''\}}$	$\overline{\{[\omega^{\star\prime}\alpha^{\star\prime}]\}}$	$\overline{\{[\omega^\star]\}'''\{[\alpha^\star]\}'''}$
— 5.0	+ 2.0	— 6.8	— 0.2

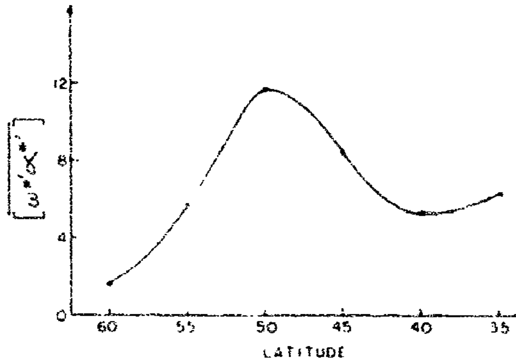

Fig. 2. Latitudinal variation of the mean rate of conversion of eddy available energy into eddy kinetic energy as measured by $[\overline{\alpha\star'\ \omega\star'}]$, in units of ergs cm^{-2} sec^{-1} $\times$ 10^{3}.

the latitudinal variation of the mean rate of conversion by this process is shown. It can be seen that kinetic energy conversion from eddy available potential energy due to this process is most intense in the vicinity of 50° N. Interestingly, this is also the region where the horizontal poleward flux of sensible heat, which is a measure of the generation of eddy available potential energy, is a maximum (see, for example, STARR and WHITE, 1954).

The negative contribution of the $y-p$ overturnings result largely from the existence of an indirect mean meridional circulation over the latitudinal belt which has been sampled in this study. It should be noted that observational evidence provided by STARR and WHITE (1954), MINTZ and LANG (1955) and PALMEN (1955) suggest the existence of weak direct meridional circulations in tropical and polar regions. Such circulations convert zonal available potential energy into zonal kinetic energy, tending therefore to counteract the negative contribution from the middle-latitude indirect cell. One would not expect this effect to be appreciable, however, since observations have already revealed that the kinetic energy of the zonal wind is maintained primarily by a transfer of kinetic energy from the disturbances through the action of horizontal eddy stresses (e.g. KUO 1951).

The energy conversion associated with the term in the fourth column of table 1, which depends on the time variations of the space means of $\omega^\star$ and $\alpha^\star$, appears to be negligibly small.

6. Conclusions

From a consideration of synoptic-scale horizontal variations in a middle-latitude sector of the atmosphere, an estimate of the required mean conversion of potential to kinetic energy has been obtained which appears to be of the correct order of magnitude.

The empirical study has suggested that this total conversion is accomplished mainly by overturnings in east-west vertical planes, in general agreement with the proposals of STARR (1954) and the descriptions of the energy cycle of the general circulation proposed by LORENZ (1954) and PHILLIPS (1955).

As noted earlier, it is recognized that, due to the limitations in data, the results of this study can only be taken as a rough measure of the true global conversion process. When sufficient data become available it would, accordingly, be desirable to extend computations of the type presented here to the entire hemisphere on a multi-level basis.

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The authors wish to thank Drs. P. D. Thompson and L. W. Gates and Mr. L. Berkofsky of the joint Geophysics Research Directorate—Air Weather Service numerical weather prediction project for permission to use their computations of the field of individual pressure change. We would also like to acknowledge the considerable aid given by Miss F. Seaver who supervised the computations.

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EQUATIONS GOVERNING THE ENERGETICS OF THE LARGER SCALES OF ATMOSPHERIC TURBULENCE IN THE DOMAIN OF WAVE NUMBER

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ABSTRACT

By considering the Fourier analysis of the planetary field of motion in the atmosphere, it is possible to define "scales" of motion and to write equations which govern the behavior of these separate scales of motion. Specifically, equations for the rate of change of the kinetic and available potential energy of a disturbance of a given wave number are presented. Such equations, which include the effects of the generation and release of potential energy, friction, and the transfer of energy among the various scales of eddies and the mean flow, can serve as a basis for studying the day-to-day variations of the spectral distribution of kinetic energy and for computing the "steady-state" atmospheric energy cycle in the domain of wave number, with the use of daily hemispheric data.

1. Introduction

It is conventional in considering the planetary atmospheric flow to think in terms of a *mean* motion obtained by averaging the flow along latitude circles, and an irregularly-varying *eddy* or *disturbed* motion which represents a turbulent departure from this mean condition. In the atmosphere, the turbulence elements which comprise this eddy motion vary in scale over a wide spectrum, ranging from the minute fluctuations recorded by sensitive micrometeorological instruments to the very large-scale irregularities in the flow observed on hemispheric synoptic charts. The propriety of considering the larger-scale eddies as turbulence elements was first recognized by Defant (1921).

By considering the physical processes which bring about the eddy motion, it is possible to distinguish between two basic regimes of turbulence which occur in the atmosphere. In particular, we may distinguish between a DIRECT turbulence in which kinetic energy of the eddy motion is maintained primarily by a direct *conversion* from other forms of energy, and an INDIRECT turbulence in which the eddy motion arises as a result of a *transfer* of kinetic energy from the laminar-type motions associated with a larger scale of flow. As an example of direct turbulence, we may cite the cyclone-scale disturbances in mid-latitudes which grow largely at the expense of potential energy due to baroclinic instability. The frictionally-induced, gusty wind variations near the ground surface, which represent the degradation of the energy of the larger-scale global motions, are an example of the indirect type.

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Most of the classical treatments of turbulence [see Sutton (1953)] have been concerned primarily with the latter case, and consequently are not generally applicable to the larger-scale meteorological turbulence which tends to be of the direct type. Past attempts to make such an application have, accordingly, resulted in many paradoxical inconsistencies, exemplified by the requirement for an imaginary mixing-length. In recent years, however, the essential energetical differences between the two regimes of turbulence have been recognized more widely, and several writers have suggested a more general viewpoint in which the existence of both types is considered (*e.g.*, Blackadar, 1950; Kuo, 1951; Starr, 1953; Lettau, 1954; and Hutchings, 1955). The purpose of this article is to present equations for the study of the statistical and dynamical properties of the larger-scale atmospheric turbulence from this broader viewpoint, with special regard for the energy-flow characteristics in the domain of wave number.

A customary approach to the study of turbulent fluid motion is to consider the stability[2] properties of the disturbances in the flow; accordingly, much attention has been given recently to the examination of the stability characteristics of the larger-scale atmospheric disturbances. These studies have depended largely on idealizations of the flow, usually based on the method of small perturbations proposed originally by Helmholtz (1888). In this connection we may cite, as examples, studies by Charney (1947), Eady (1949), Fjörtoft (1950), Kuo (1949; 1951; 1952; 1953), Phillips (1951), and Thompson (1953). The "stability" of flows initially containing *finite* disturbances has been treated by Fjörtoft (1950; 1951), Starr (1950), Platz-

[2] The term "stability" is used here in its broadest sense, being applied to disturbances of large as well as small amplitude.

man (1952), Kuo (1953), and Lorenz (1953). The flows dealt with in these latter studies were constrained by simple boundary and initial conditions and simplified dynamical relationships such as the barotropic vorticity equation.

In the well-known formulation of the problem from the energy standpoint by Reynolds (1894), the basis is provided for studying, in complete generality, the stability properties of the large, finite disturbances which are observed in any turbulent fluid. A serious drawback of this approach, however, is the fact that no information is given as to the behavior of the separate scales of eddies, as distinct from the growth and decay of *total* eddy kinetic energy.

In this article, methods similar to those used in the modern statistical theory of turbulence based on Fourier analysis [see, for example, Batchelor (1953)] are used to extend the Reynolds formulation of the turbulence problem into the domain of *scale*. Such an approach makes it possible to investigate the energetics of the atmospheric disturbances directly from daily observational data. Whereas, for reasons of mathematical expediency, most past studies have treated either the "barotropic problem" in which the flow has a horizontal shear or the "baroclinic problem" in which the flow has a vertical shear, the present formulation includes both effects simultaneously. [Recently Pocinki (1955) and Phillips (1956) have made first attempts to unify the barotropic and baroclinic problems, the former with the framework of the perturbation method and the latter using numerical integration techniques.]

Specifically, it is possible to derive from the fundamental equations a relation for the time rate of change of the kinetic energy of a disturbance of any given wave number as a function of the latitudinal spectra of several meteorological quantities. This equation contains terms representing the transfer of energy among the various-wavelength disturbances and the mean flow, and terms representing the conversions from potential and internal energy. In accordance with the common usage, one would speak of a given-wavelength disturbance as being *barotropically unstable* if its kinetic energy is increasing at the expense of the kinetic energy of the mean flow and of other disturbances, and *baroclinically unstable* if it tends to grow at the expense of potential and internal energy. The evaluation of such an equation over a sufficiently long period of time provides a basis for determining the "steady-state" cycle of energy conversion and transfer in the domain of scale.

The development of this equation is presented in section 5. As a preliminary, the basic equations to be used in this study, a review of the more conventional equations for zonal and total eddy kinetic energy, and a review of the basic concepts of Fourier analysis are presented in the following three sections.

2. Fundamental equations

In this article we shall make use of the fact that, to a high degree of accuracy, the atmosphere is in a state of hydrostatic equilibrium, so that we may take the pressure, p, as the vertical coordinate. Thus, if we neglect the Coriolis-force terms involving the vertical component of the wind, we may express the equations of motion in spherical coordinates as follows:

$$\frac{\partial u}{\partial t} + V\cdot\nabla u + \omega\frac{\partial u}{\partial p} = v\left(f + \frac{u\tan\phi}{a}\right) - \frac{g}{a\cos\phi}\frac{\partial z}{\partial\lambda} - X, \quad (1)$$

$$\frac{\partial v}{\partial t} + V\cdot\nabla v + \omega\frac{\partial v}{\partial p} = -u\left(f + \frac{u\tan\phi}{a}\right) - \frac{g}{a}\frac{\partial z}{\partial\phi} - Y, \quad (2)$$

and

$$0 = -g\,\partial z/\partial p - \alpha. \quad (3)$$

In these equations, λ is longitude; ϕ latitude; u and v are the eastward and northward components of the wind, respectively; $V = ui + vj$ (i and j are the unit vectors in the eastward and northward directions, respectively) is the two-dimensional vector wind in a pressure surface; $\omega = dp/dt$, $\nabla = i(a\cos\phi)^{-1}\partial/\partial\lambda + ja^{-1}\,\partial/\partial\phi$; a is the radius of the earth; z the height of an isobaric surface; $\alpha = 1/\rho$ is specific volume; X and Y are the eastward and northward components, respectively, of the frictional force per unit mass; $f = 2\Omega\sin\phi$ is the Coriolis parameter; and t is time.

In this (λ, ϕ, p, t) coordinate system the continuity equation takes the following simple form:

$$\partial\omega/\partial p = -\nabla\cdot V = -\left(\frac{1}{a\cos\phi}\frac{\partial u}{\partial\lambda} + \frac{1}{a}\frac{\partial v}{\partial\phi} - \frac{v\tan\phi}{a}\right). \quad (4)$$

The thermodynamical energy equation may be written in the form,

$$h = C_p\,dT/dt - \omega\alpha, \quad (5)$$

where h is the rate of heat addition per unit mass (including the generation of heat by friction), C_p the specific heat at constant pressure, and T is temperature.

For completeness, we have also the equation of state,

$$\alpha = RT/p, \quad (6)$$

in which R is the gas constant.

The relationships (1) to (6) constitute the system of equations which we shall use throughout this article. In addition, we shall hereafter consider the earth's surface to be perfectly spherical. As a result of this simplification of the lower boundary, it will, of course, be impossible to describe the direct effects of orography on the energetics of the atmosphere.

3. Conventional equations for total, mean and eddy kinetic energy

If we multiply (1) and (2) by u and v, respectively, we obtain the mechanical energy equations for the "horizontal" wind components in the form

$$\frac{\partial}{\partial t}\left(\frac{u^2}{2}\right) = -V\cdot\nabla\left(\frac{u^2}{2}\right) - \omega\frac{\partial}{\partial p}\left(\frac{u^2}{2}\right) + uv\left(f + \frac{u\tan\phi}{a}\right) - \frac{gu}{a\cos\phi}\frac{\partial z}{\partial\lambda} - uX, \quad (7)$$

and

$$\frac{\partial}{\partial t}\left(\frac{v^2}{2}\right) = -V\cdot\nabla\left(\frac{v^2}{2}\right) - \omega\frac{\partial}{\partial p}\left(\frac{v^2}{2}\right) - uv\left(f + \frac{u\tan\phi}{a}\right) - \frac{gv}{a}\frac{\partial z}{\partial\phi} - vY. \quad (8)$$

It may be seen that the effect represented by the term $uv[f + (u\tan\phi)/a]$ is to transfer kinetic energy between the zonal and meridional components of the wind. By adding (7) and (8), this term is eliminated, yielding the following relation for the rate of change of the total kinetic energy of the "horizontal" wind, $k = \frac{1}{2}(u^2 + v^2)$:

$$\partial k/\partial t = -V\cdot\nabla k - \omega\,\partial k/\partial p - gV\cdot\nabla z - V\cdot F, \quad (9)$$

where $F = Xi + Yj$.

Equations (7), (8) and (9) may be averaged with respect to longitude to give the following relations:

$$\frac{\partial}{\partial t}\left(\frac{\overline{u^2}}{2}\right) = -\overline{V\cdot\nabla\left(\frac{u^2}{2}\right)} - \overline{\omega\frac{\partial}{\partial p}\left(\frac{u^2}{2}\right)} + \overline{u^2v}\,\frac{\tan\phi}{a} + f\,\overline{uv} - \frac{g}{a\cos\phi}\overline{u\frac{\partial z}{\partial\lambda}} - \overline{uX}, \quad (10)$$

$$\frac{\partial}{\partial t}\left(\frac{\overline{v^2}}{2}\right) = -\overline{V\cdot\nabla\left(\frac{v^2}{2}\right)} - \overline{\omega\frac{\partial}{\partial p}\left(\frac{v^2}{2}\right)} - \overline{u^2v}\,\frac{\tan\phi}{a} - f\,\overline{uv} - \frac{g}{a}\overline{v\frac{\partial z}{\partial\phi}} - \overline{vY}, \quad (11)$$

and

$$\frac{\partial \bar{k}}{\partial t} = -\overline{V\cdot\nabla k} - \overline{\omega\frac{\partial k}{\partial p}} - g\,\overline{V\cdot\nabla z} - \overline{V\cdot F}, \quad (12)$$

where the bar, defined by $\overline{(\ )} = (1/2\pi)\int_0^{2\pi}(\ )\,d\lambda$, denotes the zonal average.

The kinetic energy averaged around a latitude circle may be further resolved into components representing the kinetic energy of the zonally averaged (mean) wind and the mean eddy kinetic energy, according to the equations

$$\overline{u^2} = \bar{u}^2 + \overline{u'^2}, \quad (13)$$

$$\overline{v^2} = \bar{v}^2 + \overline{v'^2}, \quad (14)$$

and

$$\bar{k} = \tfrac{1}{2}(\bar{u}^2 + \bar{v}^2) + \tfrac{1}{2}(\overline{u'^2} + \overline{v'^2}) = \tfrac{1}{2}(\bar{V}^2 + \overline{V'^2}). \quad (15)$$

In these relations, the primes denote deviations from the zonal mean so that, for example, $u = \bar{u} + u'$. We shall now present equations for the time rate of change of these separate components of $\bar{k}$.

If we multiply (1) and (2) by $\bar{u}$ and $\bar{v}$, respectively, apply the continuity equation (3), and average the resulting equations along a latitude circle, we obtain the relations

$$\frac{\partial}{\partial t}\left(\frac{\bar{u}^2}{2}\right) = -\frac{\bar{u}}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{uv}\cos\phi) - \bar{u}\frac{\partial}{\partial p}\overline{u\omega} + \bar{u}\left(f\bar{v} + \overline{uv}\,\frac{\tan\phi}{a}\right) - \bar{u}\bar{X}, \quad (16)$$

and

$$\frac{\partial}{\partial t}\left(\frac{\bar{v}^2}{2}\right) = -\frac{\bar{v}}{a\cos\phi}\frac{\partial}{\partial\phi}(\overline{v^2}\cos\phi) - \bar{v}\frac{\partial}{\partial p}\overline{v\omega} - \bar{v}\left(f\bar{u} + \overline{u^2}\,\frac{\tan\phi}{a}\right) - \frac{g\bar{v}}{a}\frac{\partial\bar{z}}{\partial\phi} - \bar{v}\bar{Y}. \quad (17)$$

These equations may be expanded further to read

$$\frac{\partial}{\partial t}\left(\frac{\bar{u}^2}{2}\right) = \left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\bar{v}\frac{\bar{u}^2}{2} - \bar{u}\,\overline{uv}\right)\cos\phi + \frac{\partial}{\partial p}\left(\bar{\omega}\frac{\bar{u}^2}{2} - \bar{u}\,\overline{u\omega}\right)\right] + \overline{u'v'}\,\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right) + \overline{u'\omega'}\,\frac{\partial\bar{u}}{\partial p} + \bar{u}\,\bar{v}\left(f + \bar{u}\frac{\tan\phi}{a}\right) - \bar{u}\,\bar{X}, \quad (18)$$

and

$$\frac{\partial}{\partial t}\left(\frac{\bar{v}^2}{2}\right) = \left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\bar{v}\frac{\bar{v}^2}{2} - \bar{v}\,\overline{vv}\right)\cos\phi + \frac{\partial}{\partial p}\left(\bar{\omega}\frac{\bar{v}^2}{2} - \bar{v}\,\overline{v\omega}\right)\right] + \overline{v'v'}\,\frac{1}{a}\frac{\partial\bar{v}}{\partial\phi} + \overline{v'\omega'}\,\frac{\partial\bar{v}}{\partial p} - \overline{u'u'}\,\bar{v}\,\frac{\tan\phi}{a} - \bar{u}\,\bar{v}\left(f + \bar{u}\frac{\tan\phi}{a}\right) - g\frac{\bar{v}}{a}\frac{\partial\bar{z}}{\partial\phi} - \bar{v}\,\bar{Y}. \quad (19)$$

Adding (18) and (19), we obtain for the rate of change of the kinetic energy of the zonally-averaged flow,

$$\frac{\partial}{\partial t}\left(\frac{\bar{V}^2}{2}\right) = \left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\bar{v}\frac{\bar{V}^2}{2} - \bar{u}\,\overline{uv} - \bar{v}\,\overline{vv}\right)\cos\phi + \frac{\partial}{\partial p}\left(\bar{\omega}\frac{\bar{V}^2}{2} - \bar{u}\,\overline{u\omega} - \bar{v}\,\overline{v\omega}\right)\right] - \overline{u'u'}\,\bar{v}\frac{\tan\phi}{a} + \overline{v'v'}\frac{1}{a}\frac{\partial\bar{v}}{\partial\phi} + \overline{u'v'}\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right) + \overline{u'\omega'}\frac{\partial\bar{u}}{\partial p} + \overline{v'\omega'}\frac{\partial\bar{v}}{\partial p} - g\frac{\bar{v}}{a}\frac{\partial\bar{z}}{\partial\phi} - \bar{d}, \quad (20)$$

where $\bar{d} = \bar{V}\cdot\bar{F} = (\bar{u}\bar{X} + \bar{v}\bar{Y})$ is the rate of frictional dissipation of the mean flow.

Equations for the rate of change of the mean eddy kinetic energy may be obtained by subtracting (18) and (19) from (10) and (11), respectively, in accordance with the relations (13), (14) and (15). The resulting equations may be expressed in the form

$$\frac{\partial}{\partial t}\left(\frac{\overline{u'^2}}{2}\right) = -\left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\overline{v\frac{u'^2}{2}}\cos\phi + \frac{\partial}{\partial p}\overline{\omega\frac{u'^2}{2}}\right] - \overline{u'v'}\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right) - \overline{u'\omega'}\frac{\partial\bar{u}}{\partial p} + \overline{u'v'}\left(f + \bar{u}\frac{\tan\phi}{a}\right) + \frac{\tan\phi}{a}\overline{uu'v'} + \frac{\tan\phi}{a}\bar{v}\,\overline{u'u'} - \frac{g}{a\cos\phi}\overline{u'\frac{\partial z'}{\partial\lambda}} - \overline{u'X'}, \quad (21)$$

$$\frac{\partial}{\partial t}\left(\frac{\overline{v'^2}}{2}\right) = -\left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\overline{v\frac{v'^2}{2}}\cos\phi + \frac{\partial}{\partial p}\overline{\omega\frac{v'^2}{2}}\right] - \overline{v'v'}\frac{1}{a}\frac{\partial\bar{v}}{\partial\phi} - \overline{v'\omega'}\frac{\partial\bar{v}}{\partial p} - \overline{u'v'}\left(f + \frac{\bar{u}}{a}\tan\phi\right) - \frac{\tan\phi}{a}\overline{uu'v'} - \frac{g}{a}\overline{v'\frac{\partial z'}{\partial\phi}} - \overline{v'Y'}, \quad (22)$$

and

$$\frac{\partial}{\partial t}\left(\frac{\overline{V'^2}}{2}\right) = -\left[\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\overline{v\frac{V'^2}{2}}\cos\phi + \frac{\partial}{\partial p}\overline{\omega\frac{V'^2}{2}}\right] - \overline{u'v'}\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right) - \overline{v'v'}\frac{1}{a}\frac{\partial\bar{v}}{\partial\phi} - \overline{u'\omega'}\frac{\partial\bar{u}}{\partial p} - \overline{v'\omega'}\frac{\partial\bar{v}}{\partial p} + \overline{u'u'}\,\bar{v}\frac{\tan\phi}{a} - g\,\overline{V'\cdot\nabla z'} - \bar{d}', \quad (23)$$

where $\bar{d}' = \overline{V'\cdot F'}$ is the rate of frictional dissipation of the eddy flow.

If we integrate (20) and (23) over a closed mass of fluid (*e.g.*, the entire atmosphere), neglecting the generally small effects of variations of surface pressure, we obtain

$$\frac{\partial}{\partial t}\int_M \frac{\bar{V}^2}{2}\,dm = \int_M\left[\overline{u'v'}\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right) + \overline{v'v'}\frac{1}{a}\frac{\partial\bar{v}}{\partial\phi} - \overline{u'u'}\,\bar{v}\frac{\tan\phi}{a} + \overline{u'\omega'}\frac{\partial\bar{u}}{\partial p} + \overline{v'\omega'}\frac{\partial\bar{v}}{\partial p}\right]dm - \int_M g\frac{\bar{v}}{a}\frac{\partial\bar{z}}{\partial\phi}\,dm - \int_M \bar{d}\,dm, \quad (24)$$

and

$$\frac{\partial}{\partial t}\int_M \frac{\overline{V'^2}}{2}\,dm = -\int_M\left[\overline{u'v'}\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right) + \overline{v'v'}\frac{1}{a}\frac{\partial\bar{v}}{\partial\phi} - \overline{u'u'}\frac{\bar{v}}{a}\tan\phi + \overline{u'\omega'}\frac{\partial\bar{u}}{\partial p} + \overline{v'\omega'}\frac{\partial\bar{v}}{\partial p}\right]dm - \int_M g\,\overline{V'\cdot\nabla z'}\,dm - \int_M \bar{d}'\,dm, \quad (25),$$

where $dm = g^{-1}a^2\cos\phi\,d\lambda\,d\phi\,dp$, and M indicates that the integration is over the whole atmosphere.

Adding (24) and (25), we obtain

$$\frac{\partial}{\partial t}\int_M \bar{k}\,dm = -\int_M g\,\overline{V\cdot\nabla z}\,dm - \int_M \overline{V\cdot F}\,dm, \quad (26)$$

which could have been written directly from (12) with the use of the continuity equation.

The appearance of the first integral in (24) with opposite sign in (25) indicates that this term measures the transfer of energy between the mean flow and eddy flow. It depends on the product of the so-called Reynolds eddy stresses and the shear of the mean velocity throughout the fluid.

The second integrals of (24) and (25) represent conversions between potential energy and the kinetic energy of the mean and eddy components of the flow, respectively. This may be demonstrated by considering the thermodynamical energy equation (5). Using the continuity equation (4) and the hydrostatic equation (3), and integrating over M, we obtain [see White and Saltzman (1956)]

$$\frac{\partial}{\partial t}\int_M C_pT\,dm = \int_M \omega\alpha\,dm + \int_M h\,dm = \int_M gV\cdot\nabla z\,dm + \int_M h\,dm, \quad (27)$$

where $\int_M C_p T\,dm$ is the total potential and internal energy of the atmosphere. (Under hydrostatic conditions, the potential energy bears a fixed ratio to the internal energy, so that we may henceforth speak of "potential" energy alone.) If, now, we apply the bar operator to (27), we obtain

$$\frac{\partial}{\partial t}\int_M C_p\bar{T}\,dm = \int_M g\,\overline{V\cdot\nabla z}\,dm + \int_M \bar{h}\,dm$$

$$= \int_M g\frac{\bar{v}}{a}\frac{\partial\bar{z}}{\partial\phi}\,dm$$

$$+ \int_M g\,\overline{V'\cdot\nabla z'}\,dm + \int_M \bar{h}\,dm. \quad (28)$$

From this equation it may be seen that the second integrals in (24) and (25), *taken together*, represent the conversion between total kinetic energy and total potential energy; *taken separately*, they represent the conversions between "zonal available potential energy" and the kinetic energy of the mean flow, and between "eddy available potential energy" and the kinetic energy of the eddy flow, respectively, according to the terminology introduced by Lorenz (1955).

The last terms in (24) and (25) measure the rate of frictional dissipation of the kinetic energy of the mean and eddy flows, respectively.

The equations discussed in this section are well known and essentially date back to the classical paper of Reynolds (1894) [see, also, Van Mieghem (1952) and Arakawa (1953)]. In accordance with the introductory remarks, in the following sections we shall use the methods of Fourier analysis to define *scales* of eddies, and we shall derive energy equations corresponding to (24) and (25) which govern the behavior of these individual scales of motion.

4. Basic concepts from the theory of Fourier analysis

Any real, single-valued function $f(\lambda)$, which is piecewise differentiable in the interval $(0, 2\pi)$, may be written in terms of a Fourier representation,

$$f(\lambda) = \sum_{n=-\infty}^{\infty} F(n)\,e^{in\lambda}, \quad (29)$$

where the complex coefficients, $F(n)$, are given by

$$F(n) = \frac{1}{2\pi}\int_0^{2\pi} f(\lambda)\,e^{-in\lambda}\,d\lambda. \quad (30)$$

For the purposes of this discussion, we shall consider the Fourier representation of meteorological quantities specified along a given latitude circle. Thus, in (29) and (30), λ is taken as longitude, and n is the number of waves around the latitude circle. The functions $f(\lambda)$ and $F(n)$ to be considered here are listed in table 1.

TABLE 1. Fourier transform pairs considered in this study.

$f(\lambda)$:	u	v	ω	z	T	h	X	Y
$F(n)$:	U	V	Ω	A	B	H	P	Q

The quantity $F(n)$ is the representation of $f(\lambda)$ in the domain of wave number and is called the *spectral function* of f. The set of equations, (29) and (30), is often referred to as a Fourier transform pair.

Using (29) and (30), we may write the Fourier transform pairs for the derivatives of $f(\lambda, \phi, p, t)$. Specifically, we have

$$\frac{\partial f}{\partial\lambda} = \sum_{n=-\infty}^{\infty} in\,F(n)\,e^{in\lambda}, \quad (31)$$

and

$$[in\,F(n)] = \frac{1}{2\pi}\int_0^{2\pi}\frac{\partial f}{\partial\lambda}e^{-in\lambda}\,d\lambda; \quad (32)$$

and

$$\frac{\partial f}{\partial\xi} = \sum_{n=-\infty}^{\infty} F_\xi(n)\,e^{in\lambda}, \quad (33)$$

and

$$F_\xi(n) = \frac{1}{2\pi}\int_0^{2\pi}\frac{\partial f}{\partial\xi}e^{-in\lambda}\,d\lambda, \quad (34)$$

where ξ may be ϕ, p or t, and the use of subscript denotes a partial differentiation (*i.e.*, $F_\xi(n) = \partial F(n)/\partial\xi$).

We now consider the product of two functions, $f(\lambda)$ and $g(\lambda)$, whose spectral functions defined by (30) are $F(n)$ and $G(n)$, respectively. For these functions, we may write

$$\frac{1}{2\pi}\int_0^{2\pi}[f(\lambda)\,g(\lambda)]e^{-in\lambda}\,d\lambda$$

$$= \frac{1}{2\pi}\int_0^{2\pi} f(\lambda)\left[\sum_{m=-\infty}^{\infty} G(m)\,e^{im\lambda}\right]e^{-in\lambda}\,d\lambda. \quad (35)$$

If we assume that $g(\lambda)$ is uniformly convergent, so that the order of summation and integration may be interchanged, (35) can be written in the form

$$\frac{1}{2\pi}\int_0^{2\pi}[f(\lambda)\,g(\lambda)]e^{-in\lambda}\,d\lambda$$

$$= \sum_{m=-\infty}^{\infty} G(m)\frac{1}{2\pi}\int_0^{2\pi} f(\lambda)\,e^{-i(n-m)\lambda}\,d\lambda, \quad (36)$$

or, finally,

$$\frac{1}{2\pi}\int_0^{2\pi}[f(\lambda)\,g(\lambda)]e^{-in\lambda}\,d\lambda = \sum_{m=-\infty}^{\infty} G(m)\,F(n-m). \quad (37)$$

This expression, which gives the spectral function for the product of two variables, is often called the ***multiplication theorem***. As a special case, we may obtain Parseval's theorem by setting $n = 0$ in (37):

$$\frac{1}{2\pi}\int_0^{2\pi} f(\lambda)\,g(\lambda)\,d\lambda = \sum_{m=-\infty}^{\infty} G(m)\,F(-m). \quad (38)$$

If, further, $f = g$, we have

$$\frac{1}{2\pi}\int_0^{2\pi} f^2(\lambda)\,d\lambda = \sum_{m=-\infty}^{\infty} |F(m)|^2. \quad (39)$$

In (39), use has been made of the fact that $F(-m)$ is the complex conjugate of $F(m)$, which implies that $F(m)\,F(-m) = |F(m)|^2$.

It may be noted, also, from (30) that $F(0) = \bar{f}$.

5. Equations in the domain of wave number

Transformation of fundamental equations.—With use of the relations presented above, we may now transform the basic equations (1) to (5) from the space domain to the domain of wave number. These transformations are effected explicitly by multiplying the basic equations by $(2\pi)^{-1}e^{-in\lambda}$, integrating around a latitude circle, and applying (29) to (34) and (37).

Thus, the equations of motions for the horizontal components of the wind take the form (see table 1)

$$\begin{aligned}\frac{\partial}{\partial t}U(n) = &-\sum_{m=-\infty}^{\infty}\Big[\frac{im}{a\cos\phi}U(m)\,U(n-m) + \frac{1}{a}U_\phi(m)\,V(n-m) + U_p(m)\,\Omega(n-m) - \frac{\tan\phi}{a}U(m)\,V(n-m)\Big] \\ &- \frac{gni}{a\cos\phi}A(n) + fV(n) - P(n),\end{aligned} \quad (40)$$

and

$$\begin{aligned}\frac{\partial}{\partial t}V(n) = &-\sum_{m=-\infty}^{\infty}\Big[\frac{im}{a\cos\phi}V(m)\,U(n-m) + \frac{1}{a}V_\phi(m)\,V(n-m) + V_p(m)\,\Omega(n-m) + \frac{\tan\phi}{a}U(m)\,U(n-m)\Big] \\ &- \frac{g}{a}A_\phi(n) - fU(n) - Q(n).\end{aligned} \quad (41)$$

The hydrostatic equation (3) takes the form

$$A_p(n) = -(R/gp)\,B(n). \quad (42)$$

The continuity equation (4) takes the form

$$\Omega_p(n) = -\Big[\frac{in}{a\cos\phi}U(n) + \frac{1}{a}V_\phi(n) - \frac{\tan\phi}{a}V(n)\Big]. \quad (43)$$

Finally, the thermodynamical energy equation (5) takes the form

$$\begin{aligned}\frac{\partial}{\partial t}B(n) = &-\sum_{m=-\infty}^{\infty}\Big[\frac{im}{a\cos\phi}B(m)\,U(n-m) + \frac{1}{a}B_\phi(m)\,V(n-m) + B_p(m)\,\Omega(n-m) \\ &- \frac{R}{C_p p}B(m)\,\Omega(n-m)\Big] + \frac{1}{C_p}H(n).\end{aligned} \quad (44)$$

Equations (40) to (44) represent a closed system of equations governing the five dependent variables U, V, Ω, A and B as functions of n, ϕ, p and t, provided that the heating distribution $H(n)$ is specified. Although this aspect will not be treated here, it is worth noting that such a system of equations can serve as the basis for a hemispheric numerical prediction scheme.

Mechanical energy equation.—We shall now derive the equations for the rate of change of kinetic energy of disturbances of given scale. We may conceive of a disturbance of a scale proportional to $1/n$ as the harmonic component of the complete flow whose wave number is n, wave number zero corresponding to the mean flow

From the Parseval theorem (39), we may write

$$\bar{k} = \frac{1}{2\pi}\int_0^{2\pi}\frac{V^2}{2}\,d\lambda = \frac{\bar{V}^2}{2} + \sum_{n=1}^{\infty}K(n), \quad (45)$$

where

$$K(n) = |U(n)|^2 + |V(n)|^2 \quad (46)$$

is the spectral function for the zonally-averaged eddy kinetic energy per unit mass.

By multiplying (40) and (41) by $U(-n)$ and $V(-n)$, respectively, and applying (43), we obtain

the following expressions for the rate of change of the separate components of $K(n)$:

$$
\begin{aligned}
\frac{\partial}{\partial t}|U(n)|^2 = &-[U(-n)\,V(n) + U(n)\,V(-n)]\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar u}{\cos\phi}\right) - [U(-n)\,\Omega(n) + U(n)\,\Omega(-n)]\,\partial\bar u/\partial p \\
&+ [U(-n)\,U(n) + U(n)\,U(-n)]\bar v(\tan\phi)/a - \sum_{\substack{m=-\infty\\ \neq 0}}^{\infty}\left\{\frac{in}{a\cos\phi}U(m)[v(-n)\,U(n-m)\right. \\
&- U(n)\,U(-n-m)] + \frac{1}{a\cos\phi}[U(-n)\,\{U(m)\,V(n-m)\cos\phi\}_\phi \\
&+ U(n)\,\{U(m)\,V(-n-m)\cos\phi\}_\phi] + [U(-n)\,\{U(m)\,\Omega(n-m)\}_p \\
&\left.+ U(n)\,\{U(m)\,\Omega(-n-m)\}_p] - \frac{\tan\phi}{a}V(m)[U(-n)\,U(n-m) + U(n)\,U(-n-m)]\right\} \\
&- \frac{gni}{a\cos\phi}[A(n)\,U(-n) - A(-n)\,U(n)] + \left(f + \bar u\frac{\tan\phi}{a}\right)[U(-n)\,V(n) + U(n)\,V(-n)] \\
&- [U(-n)\,P(n) + U(n)\,P(-n)],
\end{aligned}
\tag{47}
$$

and

$$
\begin{aligned}
\frac{\partial}{\partial t}|V(n)|^2 = &-[V(-n)\,V(n) + V(n)\,V(-n)]\frac{1}{a}\frac{\partial\bar v}{\partial\phi} - [V(-n)\,\Omega(n) + V(n)\,\Omega(-n)]\,\partial\bar v/\partial p \\
&- \sum_{\substack{m=-\infty\\ \neq 0}}^{\infty}\left\{\frac{in}{a\cos\phi}V(m)\,[V(-n)\,U(n-m) - V(n)\,U(-n-m)]\right. \\
&+ \frac{1}{a\cos\phi}[V(-n)\,\{V(m)\,V(n-m)\cos\phi\}_\phi + V(n)\,\{V(m)\,V(-n-m)\cos\phi\}_\phi] \\
&+ [V(-n)\,\{V(m)\,\Omega(n-m)\}_p + V(n)\,\{V(m)\,\Omega(-n-m)\}_p] \\
&\left.+ \frac{\tan\phi}{a}U(m)[V(-n)\,U(n-m) + V(n)\,U(-n-m)]\right\} \\
&- \frac{g}{a}[A_\phi(n)\,V(-n) + A_\phi(-n)\,V(n)] - \left(f + \bar u\frac{\tan\phi}{a}\right)[U(n)\,V(-n) + U(-n)\,V(n)] \\
&- [V(-n)\,Q(n) + V(n)\,Q(-n)].
\end{aligned}
\tag{48}
$$

The desired equation for the rate of change of the total kinetic energy of a given wave number may now be obtained by using (46) and integrating over the entire mass of the atmosphere. In this integration, we neglect the terms which arise as a result of variations of surface pressure, as was done in the derivation of (24). Thus, we finally obtain

$$
\begin{aligned}
\frac{\partial}{\partial t}\int_M K(n)\,dm = &-\int_M\left[\Phi_{uv}(n)\frac{\cos\phi}{a}\frac{\partial}{\partial\phi}\left(\frac{\bar u}{\cos\phi}\right) + \Phi_{vv}(n)\frac{1}{a}\frac{\partial\bar v}{\partial\phi}\right. \\
&\left.+ \Phi_{u\omega}(n)\frac{\partial\bar u}{\partial p} + \Phi_{v\omega}(n)\frac{\partial\bar v}{\partial p} - \Phi_{uu}(n)\,\bar v\frac{\tan\phi}{a}\right]dm \\
&+ \int_M\sum_{\substack{m=-\infty\\ \neq 0}}^{\infty}\left\{U(m)\left[\frac{1}{a\cos\phi}\Psi_{uu_\lambda}(m,n) + \frac{1}{a}\Psi_{vu_\phi}(m,n) + \Psi_{\omega u_p}(m,n) - \frac{\tan\phi}{a}\Psi_{uv}(m,n)\right]\right. \\
&\left.+ V(m)\left[\frac{1}{a\cos\phi}\Psi_{uv_\lambda}(m,n) + \frac{1}{a}\Psi_{vv_\phi}(m,n) + \Psi_{\omega v_p}(m,n) + \frac{\tan\phi}{a}\Psi_{uu}(m,n)\right]\right\}dm \\
&- \int_M g\left[\frac{1}{a\cos\phi}\Phi_{uz_\lambda}(n) + \frac{1}{a}\Phi_{vz_\phi}(n)\right]dm - \int_M[\Phi_{uX}(n) + \Phi_{vY}(n)]\,dm,
\end{aligned}
\tag{49}
$$

where[8]

$$\Phi_{fg}(n) = [F(n)\, G(-n) + F(-n)\, G(n)], \quad (50)$$

and

$$\Psi_{fg}(m, n) = [F(n - m)\, G(-n) + F(-n - m)\, G(n)]. \quad (51)$$

As a special case, we may obtain the expression for the time rate of change of the kinetic energy of the mean flow by noting that $\bar{V}^2 = K(0)$. Thus, setting $n = 0$ in (49) and using the continuity equation (43), we obtain

$$\frac{\partial}{\partial t}\int_M \frac{\bar{V}^2}{2}\, dm = \int_M \sum_{n=1}^{\infty} \Big[\Phi_{uv}(n) \frac{\cos\phi}{a} \frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right) + \Phi_{vv}(n) \frac{1}{a}\frac{\partial\bar{v}}{\partial\phi} + \Phi_{u\omega}(n)\frac{\partial\bar{u}}{\partial p} + \Phi_{v\omega}(n)\frac{\partial\bar{v}}{\partial p} - \Phi_{uu}(n)\, \bar{v}\,\frac{\tan\phi}{a} \Big] dm - \int_M \frac{g}{a}\,\bar{v}\,\frac{\partial\bar{z}}{\partial\phi}\, dm - \int_M \bar{d}\, dm. \quad (52)$$

Equations (49) and (52) represent the transforms of (25) and (24) in the wave-number "space." The terms of these equations may be interpreted as follows.

The terms in the first integral on the right-hand side of (49) depend on the products of the shear of the mean flow and the transport of momentum by the separate scales of eddies. It seems plausible to regard this transport of momentum by an individual eddy scale as a separate physical process, which is distinct from the transport of momentum by other eddy scales and distinct from the physical processes represented by quadratic functions of the spectra appearing in the second integral. These latter functions all involve more than one scale of disturbance and can best be described as representing an "interaction" process.

On this basis, we may regard the first integral as a "transformation function" measuring the transfer of energy between the individual scales of disturbances and the mean flow; and we may regard the second integral, which vanishes when summed over all wave numbers, as a measure of the transfer of energy, due to non-linear interaction, between a disturbance of a given wave number and disturbances of all other wave numbers. Some insight into the behavior of these terms has recently been given by Fjörtoft (1953), who demonstrated that, for the two-dimensional non-divergent case, a change in the kinetic energy of one scale of motion is accompanied by changes in the kinetic energy of disturbances of both smaller and larger scale.

With the use of the hydrostatic equation (42) and the continuity equation (43), we may write the third integral in the successive forms

$$\int_M g\left[\frac{1}{a\cos\phi}\,\Phi_{uz_\lambda}(n) + \frac{1}{a}\,\Phi_{vz_\phi}(n)\right] dm = -\int_M g\,\Phi_{\omega z_p}(n)\, dm = \int_M \frac{R}{p}\,\Phi_{\omega T}(n)\, dm. \quad (53)$$

If we regard the process represented by (53) as physically distinct for each wave number, we may, in accordance with the discussion at the end of section 3, regard this integral as a measure of the conversion between eddy available potential energy and the eddy kinetic energy of the individual wave components which comprise the flow. In the following part of this section, the available potential energy equation applying in the wave-number domain is presented, and it may be verified that (53) appears with opposite sign in this equation. Qualitatively, the last integral of (53) demonstrates that the baroclinic growth of a disturbance of a given wave number depends on the degree to which the variations in the vertical motion of that wave number are in phase with the variations of temperature.

The last integral in each of (49) and (52) represents the frictional dissipation of the different scales of disturbances and of the mean flow, respectively.

Available potential energy equation.—Following the procedures introduced by Lorenz (1955) and Phillips (1956), we may use the first law of thermodynamics to derive equations for the rate of change of the total, zonal and eddy potential energy available for conversion into kinetic energy. These equations take the following forms, respectively, differing only in minor respects from those derived by Lorenz (1955):

$$\frac{\partial}{\partial t}\int_M \frac{C_p\gamma}{2}\,\overline{T^{*2}}\, dm = \int_M \frac{R}{p}\,\omega T\, dm + \int_M \gamma\,\overline{T^* h^*}\, dm, \quad (54)$$

$$\frac{\partial}{\partial t}\int_M \frac{C_p\gamma}{2}\,\{\bar{T}''^2\}\, dm = \int_M \frac{R}{p}\,\bar{\omega}\bar{T}\, dm + \int_M \gamma\{\bar{T}''\bar{h}''\}\, dm + \int_M \Big[\frac{C_p\gamma}{a}\,\overline{v'T'}\,\frac{\partial\bar{T}}{\partial\phi} \quad \gamma\,\frac{p^\mu}{\mu}\Big\{(\overline{\omega'T'})''\,\frac{\partial\bar{\theta}''}{\partial p}\Big\}\Big] dm, \quad (55)$$

[8] It may be verified from (38) that $\sum_{n=1}^{\infty} \Phi_{fg}(n) = \overline{f'g'}$. Accordingly, the general expression for the spectrum of the meridional transport of any physical quantity f (*e.g.*, momentum, sensible heat, water vapor) is given by $\Phi_{fv}(n)$. The evaluation of such spectra from hemispheric data will demonstrate the relative importance of the various scales of eddies in effecting the horizontal transport processes in the general circulation. Computations of this type have recently been performed by Van Isacker and Van Mieghem (1956) and Kubota and Iida (1955).

and

$$\frac{\partial}{\partial t}\int_M \frac{C_p\gamma}{2}\overline{T'^2}\,dm$$
$$= \int_M \frac{R}{p}\overline{\omega' T'}\,dm + \int_M \gamma\,\overline{T'h'}\,dm$$
$$- \int_M \left[\frac{C_p\gamma}{a}\overline{v'T'}\frac{\partial \bar{T}}{\partial \phi} + \gamma\frac{p^\mu}{\mu}\left\{ (\overline{\omega' T'})''\frac{\partial \bar{\theta}''}{\partial p} \right\} \right] dm, \quad (56)$$

where the brackets denote a cosine-weighted average with respect to ϕ, $(\ ) = \{(\ )\} + (\ )''$; the wavy bar is defined by $\widetilde{(\ )} = \{\widetilde{(\ )}\}$; the asterisk denotes a deviation from this average, $(\ ) = \widetilde{(\ )} + (\ )^*$; $\mu = R/C_p$; θ = potential temperature; and

$$\gamma = -\mu R/p^{1+\mu}\,(\partial\tilde{\theta}/\partial p)^{-1}.$$

Applying the same procedure used to obtain (49) from the equations of motion to the thermodynamical energy equation (44), we may obtain an equation describing the variations of the separate scales of eddy available potential energy. This equation, which is the Fourier transform of (56), may be written in the following form, with use of the notation defined by (50) and (51):

$$\frac{\partial}{\partial t}\int_M C_p\gamma\,|B(n)|^2\,dm$$
$$= -\int_M \left[\frac{C_p\gamma}{a}\Phi_{T_v}(n)\frac{\partial \tilde{T}}{\partial \phi} + \frac{\gamma p^\mu}{\mu}\left\{ \Phi_{T_\omega}(n)''\frac{\partial \bar{\theta}''}{\partial p} \right\} \right] dm$$
$$+ \int_M \frac{R}{p}\Phi_{T_\omega}(n)\,dm + \int_M \gamma\,\Phi_{T_h}(n)\,dm$$
$$+ \int_M C_p\gamma \sum_{\substack{m=-\infty \\ \neq 0}}^{\infty} B(m)\left[\frac{1}{a\cos\phi}\Psi_{uT_\lambda}(m,n) + \frac{1}{a}\Psi_{vT_\phi}(m,n) + \Psi_{\omega T_p}(m,n) + \frac{R}{C_p p}\Psi_{\omega T}(m,n) \right] dm. \quad (57)$$

The integral $\int_M C_p\gamma\,|B(n)|^2\,dm$ is the spectral function for eddy available potential energy.

The first integral on the right measures the contribution from *zonal* available potential energy to the growth of eddy available potential energy of a given scale. It may be seen that these terms depend essentially on the spectrum of the eddy transport of sensible heat.

The second integral, which has an equal and opposite counterpart in the kinetic energy equation (49), measures the conversion between eddy available potential energy of a given wavelength and eddy kinetic energy of the same wavelength.

The third integral measures the generation of eddy available potential energy of a given wavelength through differential heating.

The final integral represents the transfer of eddy available potential energy among its separate scales due to the rearrangement of the temperature field arising from the non-linear interaction of the wind and temperature eddies.

For completeness, we may write down the transform of the *zonal* available potential energy equation (53) in the form

$$\frac{\partial}{\partial t}\int_M \frac{C_p\gamma}{2}\{\tilde{T}''^2\}\,dm$$
$$= \int_M \sum_{n=1}^{\infty}\left[\frac{C_p\gamma}{a}\Phi_{T_v}(n)\frac{\partial \tilde{T}}{\partial \phi} + \frac{\gamma p^\mu}{\mu}\left\{ \Phi_{T_\omega}(n)''\frac{\partial \bar{\theta}''}{\partial p} \right\} \right] dm$$
$$+ \int_M \frac{R}{p}\tilde{\omega}\tilde{T}\,dm + \int_M \gamma\{\tilde{T}''\tilde{h}''\}\,dm. \quad (58)$$

Discussion.—The equations presented in this section govern the behavior of the finite disturbances which are observed in the atmosphere. Through the measurement of the terms of (49) and (57) from observational data, many aspects of the cycle of energy transformation involving these disturbances can be examined quantitatively. At this point it is of interest to speculate, on the basis of our present meteorological intelligence, about the nature of such a cycle.

In accordance with the ideas expressed by Lorenz (1955), the radiational heating tends to create a "*zonal* available potential energy" which is transformed into "*eddy* available potential energy" largely through the horizontal transport of heat by the existing eddies. By means of the baroclinic processes represented by (53), it appears that cyclone-scale disturbances of intermediate wave numbers (*e.g.*, wave numbers seven to ten) grow at the expense of this available potential energy. It is known from studies of the angular momentum transport in the atmosphere [see, for examples, Kuo (1951), Starr (1953), and Starr and White (1954)] that the eddies tend to transfer their energy to the mean flow; and, in particular, synoptic evidence indicates that it is the largest eddies located at about 30 deg lat which are responsible for this transfer of energy to the mean flow. This has been supported quantitatively by a case study by Kao (1954) and a more recent study by Van Isacker and Van Mieghem (1956). Thus, one might expect that energy is being fed from the intermediate wavelengths not only to smaller eddies but also to the longer wavelengths which correspond in part to the so-called "semi-permanent" centers. (It is recognized that the

orography is also of importance in maintaining the stationary long waves.) In connection with the introductory remarks, the verification of such a picture from observations would demonstrate that the large-scale atmospheric disturbances behave in a different manner from the supposed behavior of smaller-scale turbulence. For this latter case, according to the currently accepted theory of Kolmogoroff (1941), the mean flow feeds its energy stepwise *down* the scale of disturbances until, finally, the energy is dissipated by viscosity.

6. Simplifications

The computations required to evaluate the integrals in (49) and (57) from observational data are of an extremely laborious nature, and it is only with the advent of high-speed computing facilities that such computations have become practicable. Even with these high-speed machines, it is desirable to make certain simplifications to make the computational problem more tractable.

Use of a discrete number of harmonics.—First, it is, of course, necessary to replace the summations over infinity by finite sums. In view of the limitations in the data coverage over the hemisphere, it seems desirable at present to use 36 points at 10 deg long intervals around a latitude circle as the basis for the Fourier expansions. In this case, one could not compute more than about twelve wave numbers with accuracy. It would appear, however, that enough of the variance of the hemispheric chart would be captured by this limited number of waves to make it feasible to use this approximation for the present problem. As one possibility, one could assume that all wave numbers larger than twelve act according to the Kolmogoroff theory and are included as frictional effects.

Use of geostrophic winds.—Direct wind observations for the hemisphere are not available in sufficient quantities to compute the spectra of the wind field directly. Observations of the angular momentum transport [*i.e.*, Mintz (1951), and Widger (1949)] have already revealed, however, that horizontal variations in the general circulation are gross enough so that geostrophic winds are suitable for computations of the type required to measure the terms in the first two integrals in (49) except those involving $\bar{v}$. It is not permissible to use geostrophic winds to evaluate the third integral in (49), since this term would vanish identically under this assumption. When geostrophic winds are used, the spectra for u and v may be obtained from the spectral function for the contour height by means of the relations

$$U(n) = -\frac{g}{fa} A_{\phi}(n), \tag{59}$$

and

$$V(n) = \frac{gni}{fa\cos\phi} A(n). \tag{60}$$

Computation of $\Omega(n)$.—With the use of the adiabatic form of the thermodynamical energy equation, it is possible to obtain an estimate of $\Omega(n)$ for the mid-troposphere which may be used in evaluating the important baroclinic term (53).

The adiabatic energy equation may be written in the form

$$\omega = -\frac{1}{\left(\dfrac{\partial T}{\partial p} - \dfrac{\alpha}{C_p}\right)} \left[\frac{\partial T}{\partial t} + V \cdot \nabla T\right]. \tag{61}$$

Assuming that the stability factor,

$$[(\partial T/\partial p) - (\alpha/C_p)] = \kappa,$$

can be assigned some horizontally uniform value, we may apply equations (30) to (34) and (37) to obtain the transform

$$\Omega(n) = -\frac{1}{\kappa}\left\{\frac{\partial}{\partial t} B(n) + \sum_{m=-\infty}^{\infty}\left[\frac{im}{a\cos\phi} B(m)\, U(n-m) + \frac{1}{a} B_{\phi}(n-m)\, V(m)\right]\right\}, \tag{62}$$

or, in terms of "thickness,"

$$\Omega(n) = \frac{g}{\left(\dfrac{\alpha}{\theta}\dfrac{\partial\theta}{\partial p}\right)}\left\{\frac{\partial}{\partial t} A_p(n) + \sum_{m=-\infty}^{\infty}\left[\frac{im}{a\cos\phi} A_p(m)\, U(n-m) + \frac{1}{a} A_{\phi p}(n-m)\, V(m)\right]\right\}. \tag{63}$$

It may be seen that, to compute $\Omega(n)$ at a given instant in time, it is necessary to evaluate $\partial B(n)/\partial t$, or alternately $\partial A_p(n)/\partial t$, at that time. This may be accomplished by solving the so-called "thermal vorticity" equation used in simple baroclinic models of the atmosphere.

Remarks concerning frictional effects.—Most of the frictional dissipation takes place near the ground surface, below the level of the atmosphere where the kinetic energy is primarily located. In this connection, it is pertinent to note that the effects measured by the terms involving Ω in the first integral on the right side of (49) probably act in the sense of true eddy viscosity (see Kuo, 1951), and, as such, it may be possible to include them in the friction term along with the effects

of the small-scale disturbances. A direct calculation of these combined dissipative effects from observational data is extremely difficult, and, therefore, it is probably best to estimate them as a residual of computing the other terms in the energy equation.

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SOME HEMISPHERIC SPECTRAL STATISTICS

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ABSTRACT

The results of computations of the spectrum of the eddy kinetic energy, the angular momentum transport, and the transfer of kinetic energy between the eddies and the mean motion, for the month of January 1949, are presented. The calculations are based on geostrophic winds measured at the 500-mb level. Of greatest interest is the finding that, for the month, disturbances of wave-number three were of singular importance as a source of the energy of the mean zonal motion.

1. Introduction

The achievement of an "understanding" of the behavior of so complicated a physical system as the atmosphere depends to a large extent on satisfying an innate desire to reduce the system to a chain or cycle of events involving smaller parts which still retain a physical identity. The bases for the construction of this kind of sequence are the so-called *conservation* principles which express the fundamental laws in such a manner that they place constraints on certain integral properties of the system (mass, momentum, and energy). In the case of the general circulation it has been convenient to consider a breakdown into the two Reynolds components, the *eddy* and *mean* flow, and, indeed, a knowledge of the role of these components in the momentum and energy cycles has proved to be the rallying point for a mechanistic description of the behavior of the system.

It is recognized, however, that the eddy component, in itself, is a very complicated part of the general circulation, which implies that a description of the mechanism of the planetary flow will remain incomplete as long as this eddy component is not subdivided into still smaller parts whose respective roles in satisfying the conservation requirements are determined. In this connection, the wave-like character of the disturbances in the atmosphere suggests the use of Fourier analysis as a natural method for distinguishing between different "scales" of eddies. From a mathematical point of view such a resolution seems particularly desirable since the orthogonality property of sinusoidal functions permits one to write energy equations for these separate components in a relatively straightforward manner. A presentation of equations of this type has been given by the writer (1957). In order to obtain a description of the "steady-state" cycle of energy transformation in this *domain of wave-number* it is necessary to compute from a long record of daily observations (at least of the order of a year) the integrals which appear in these equations representing, respectively, the transfer of energy among the various scales of eddies and the mean flow, the conversions between potential and kinetic energy, the generation of available potential energy and the frictional dissipation of kinetic energy.

With the advent of the modern high-speed computer the execution of such a computational task has come within the realm of possibility, although it would require a very large effort in terms of both human and financial resources. It seems desirable, accordingly, to make a less ambitious start by investigating the spectral properties of those dynamically-significant quantities which are at present most amenable to calculation. To this end, the present paper describes the results of computations for the 31 day period, January 1949, of the mean kinetic energy spectra and momentum transport spectra for three latitude circles (27.5 deg N, 47.5 deg N, and 67.5 deg N) and, of most interest from the energetical point of view, the mean spectrum of the transfer of energy between various scales of eddies and the mean motion due to horizontal processes. These computations are based solely on geostrophic winds measured at the 500 mb level, which to some extent are representative of general conditions in the mid-troposphere. Thus, no attempt is made here to compute the transfer of energy among the individual wave components due to non-linear interaction or to compute the conversions between available potential energy and the kinetic energy of the various wave components, both of which are of crucial importance in establishing a complete energy cycle.

2. Formulae

As a preliminary we shall present briefly the mathematical expressions for the quantities computed in this study.

[1] This research has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center under Contract No. AF 19(604)-1000.

Any regularly behaving function of longitude, $f(\lambda)$, which is specified along a given latitude ϕ, and pressure surface p, may be expressed in terms of a Fourier expansion of the form,

$$f(\lambda) = \sum_{n=-\infty}^{\infty} F(n)e^{in\lambda}, \tag{1}$$

where the complex coefficients, $F(n)$ are given by,

$$F(n) = \frac{1}{2\pi}\int_0^{2\pi} f(\lambda)e^{-in\lambda}d\lambda \tag{2}$$

(n = wave number, $i = \sqrt{-1}$). From this Fourier transform pair it is easy to demonstrate that for two functions, say $f(\lambda)$ and $g(\lambda)$, having the complex Fourier coefficients $F(n)$ and $G(n)$, respectively, the following theorem of Parseval applies:

$$\begin{aligned}\overline{fg} &= \frac{1}{2\pi}\int_0^{2\pi} fg d\lambda \\ &= \sum_{n=-\infty}^{\infty} F(n)G(-n).\end{aligned} \tag{3}$$

If we let $U(n)$ and $V(n)$ represent the complex spectral functions for the zonal and meridional components of the wind (u and v, respectively) we may use (3) to write the following expression for the zonally-averaged eddy kinetic energy per unit mass:

$$\frac{\overline{u'^2 + v'^2}}{2} = \sum_{n=1}^{\infty} K(n), \tag{4}$$

where

$$K(n) = |U(n)|^2 + |V(n)|^2 \tag{5}$$

is the spectral function for the zonally-averaged kinetic energy per unit mass, $|U|^2$ and $|V|^2$ being the spectral functions for the kinetic energy of the individual components.

Similarly, from (3) we may write for the meridional eddy transport of angular momentum across latitude ϕ, per unit pressure difference per unit time, the expression,

$$\left(\frac{2\pi a^2 \cos^2\phi}{g}\right)\overline{u'v'} = \sum_{n=1}^{\infty} J_M(n), \tag{6}$$

where

$$J_M(n) = \left(\frac{2\pi a^2 \cos^2\phi}{g}\right) \times [U(n)V(-n) + U(-n)V(n)] \tag{7}$$

is the spectral function for the eddy momentum transport (a = radius of earth, g = acceleration of gravity). Expressions similar to (7) have been presented by Lorenz (1951), Kao (1954), Kubota and Iida (1955) and Van Isacker and Van Meighem (1956).

The spectral function for the transfer of energy between the mean motion and the eddy motion due to the action of horizontal Reynolds stresses on the zonal current is given (see Saltzman, 1957) by the expression,

$$M^*(n) = \int_0^{p_0}\int_{-\pi/2}^{\pi/2} J_M(n)\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{a\cos\phi}\right)d\phi dp \tag{8}$$

which satisfies the relation,

$$\frac{2\pi a}{g}\int_0^{p_0}\int_{-\pi/2}^{\pi/2}\overline{u'v'}\cos^2\phi\frac{\partial}{\partial\phi}\left(\frac{\bar{u}}{\cos\phi}\right)d\phi dp = \sum_{n=1}^{\infty} M^*(n) \tag{9}$$

(p_0 = surface pressure).

In the present study geostrophic winds were used to estimate $K(n)$, $J_M(n)$ and $M^*(n)$ at the 500-mb level. These estimates were obtained from the complex spectra of the contour height at this level, denoted by $A(n)$, with the use of the relationships,

$$U(n) = -\frac{g}{fa}\frac{\partial}{\partial\phi}A(n) \tag{10}$$

and

$$V(n) = \frac{ing}{fa\cos\phi}A(n), \tag{11}$$

where $f = 2\Omega\sin\phi$ is the Coriolis parameter.

Harmonics up to wave number 12 were computed, based on information at 36 points around a latitude circle.

3. Kinetic energy spectra

The means of the daily kinetic energy spectra for January 1949, computed at three latitudes, are presented in fig. 1. It should be noted that these and all subsequent spectra are discrete "line-spectra" and, hence, have meaning only for the integral values of wave-number. To aid visually in tracing the fluctuations of the spectra, however, lines have been drawn between points on the diagram.

In summary of this figure we may note several salient features. Notably, the spectra reveal a tendency for the variations of the zonal wind around a latitude circle to be of longer wave-length than the variations of the meridional velocity component. In particular, wave-numbers 1 to 3 in the zonal kinetic energy are dominant at all three latitudes. The maximum meridional kinetic energy resides in wave-number 6 at 27.5 deg N, shifting to a double maximum at wave-numbers 6 and 3 at 47.5 deg N. At 67.5 deg N the variations of meridional kinetic energy are of lower wave-number, with maxima at n = 2 and 3.

TABLE 1. Standard deviations of spectral functions in units of corresponding figures (±).

Wave-number	(n)	1	2	3	4	5	6	7	8	9	10	11	12
	ϕ												
$\sigma(\lvert U\rvert^2)$	67.5	1.07	0.79	0.89	0.25	0.24	0.13	0.10	0.07	0.04	0.04	0.03	0.04
	47.5	1.91	1.05	1.01	0.54	0.55	0.43	0.24	0.11	0.19	0.10	0.13	0.09
	27.5	1.24	0.78	0.75	0.56	0.35	0.48	0.44	0.25	0.22	0.19	0.15	0.11
$\sigma(\lvert V\rvert^2)$	67.5	0.13	0.95	0.97	0.81	0.42	0.32	0.28	0.24	0.13	0.10	0.07	0.03
	47.5	0.10	0.45	0.95	0.85	0.64	1.28	0.71	0.35	0.46	0.27	0.25	0.14
	27.5	0.02	0.05	0.18	0.16	0.68	1.19	0.70	0.20	0.30	0.39	0.20	0.24
$\sigma(J_M)$	67.5	2.70	5.32	3.86	2.49	1.31	0.85	0.65	0.62	0.32	0.26	0.22	0.14
	47.5	5.36	11.23	18.06	11.37	9.53	11.28	6.90	4.01	3.96	3.43	2.42	2.07
	27.5	3.41	4.41	8.54	7.46	12.55	16.72	13.79	5.81	5.01	7.02	4.05	4.73
$\sigma(M^*)$		1.64	2.63	2.98	1.14	1.78	2.32	1.68	0.81	0.72	1.16	0.57	0.44

The greatest "spread" of energy over the various wave numbers occurs at 47.5 deg N. This result is not surprising in view of the fact that the mid-latitudes are the seat of the transient cyclones which display great variations in size and intensity depending on the stage of development. The low and high latitudes, on the other hand, are generally the locations of the more regular "semi-permanent" centers.

In the zonal kinetic energy spectra there is a striking occurrence of energy in wave-number 1 which indicates that the zonal current tends to be more intense on one side of the hemisphere than the other. Related aspects of this asymmetry of the polar vortex have been studied in detail by La Seur (1954).

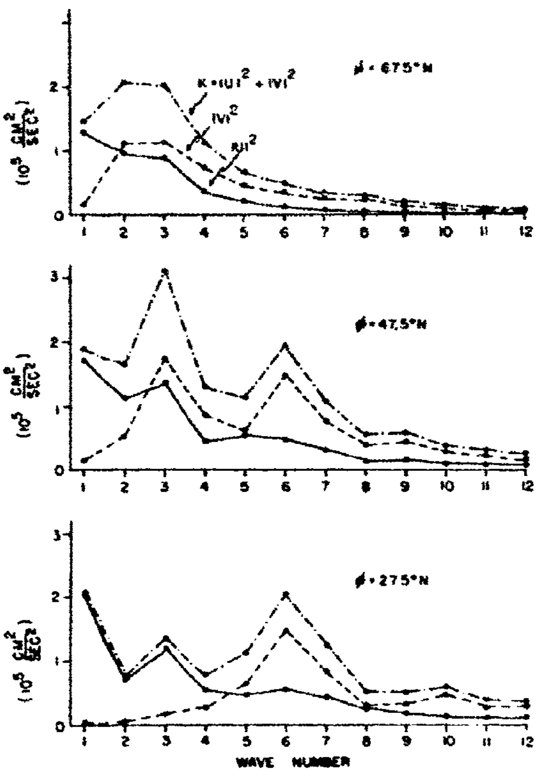

FIG. 1. Mean spectra of the 500-mb geostrophic kinetic energy per unit mass for the 31-dy period, January 1949. Latitudes from bottom to top are 27.5 deg N, 47.5 deg N and 67.5 deg N. $|U|^2$ denoted by solid line, $|V|^2$ by dashed line, $K = |U|^2 + |V|^2$ by dotted line. Corresponding standard deviations are given in table 1.

Kinetic energy spectra for the meridional component of the wind have been computed by other investigators (*e.g.*, Charney (1951), White and Cooley (1956)). In particular, White and Cooley present a graph of the 500-mb values of $|V(n)|^2$ for January 1950 at 45 deg N which may be compared with the 47.5 deg N values shown in fig. 1. The general characters of the two spectra are similar except that the White and Cooley results for 1950 show maxima at $n = 4$ and 7 whereas the present 1949 study shows maxima at $n = 3$ and 6. This is illustrative of the type of variability which may be expected in a monthly mean spectrum from one year to another.

In order to measure the departure of the *daily* spectral distributions from the mean condition shown in the figure, standard deviations were computed and are tabulated in table 1.

4. Momentum transport spectra

In addition to the kinetic energy, which depends on the *square* of the magnitudes of the individual wind components, there is another important quadratic function of the wind which depends on the *cross-product* of the wind components—the momentum transport. In particular, the cross-product of u and v is a measure of the northward transport of angular momentum relative to the earth's axis, and, in the wave-number domain, the *cross-spectrum* of u and v, with the appropriate weighting factor given in (7), is the measure of the contribution of the individual scales of eddies to this transport at a particular latitude.

In fig. 2 the mean of the daily values of $J_M(n)$ for January, 1949, again computed at 27.5 deg N, 47.5 deg N and 67.5 deg N, are presented.

It may be seen that the net transport is northward at 27.5 deg N

$$\left(\sum_{n=1}^{12} J_M(n) = 35.7 \times 10^{22} \text{ gm cm}^2 \text{ sec}^{-2} \text{ mb}^{-1}\right)$$

with the major contributions coming from wave numbers three and five.

At 47.5 deg N the net mean transfer of momentum is still of almost the same magnitude, northward (35.8×10^{22} gm cm² sec⁻² mb⁻¹), a figure which is probably higher than normal for this latitude. Disturbances of wave-number three are clearly the important agents for effecting this transport.

A small net northward transport occurs at 67.5 deg N (0.49×10^{22} gm cm² sec⁻² mb⁻¹) despite a relatively strong mean southward transport by disturbances of wave-number two.

These results indicate that, for the month, variations of wave-number three in the zonal and meridional components of the wind tended to be in phase over a broad latitude band; an effect which undoubtedly reflects the existence of a "tilt" in the large-scale semipermanent pressure systems (see Starr, 1948). Another probable consequence of this tilting of disturbances of wave-number three is the coincident occurrence of high zonal kinetic energy at $n = 3$ and high meridional kinetic energy at $n = 6$ (see fig. 1).

The standard deviations corresponding to the values plotted in fig. 2 are presented in table 1. It is indicated that the variability about the mean is large, especially for the intermediate wave numbers. The peak mean values of $J_M(n)$ at $n = 3$ (27.5 deg N and 47.5 deg N) seem to be significantly different from zero however, whereas the significance of the maxima at $n = 5$ (27.5 deg N) and $n = 2$ (67.5 deg N) are in question.

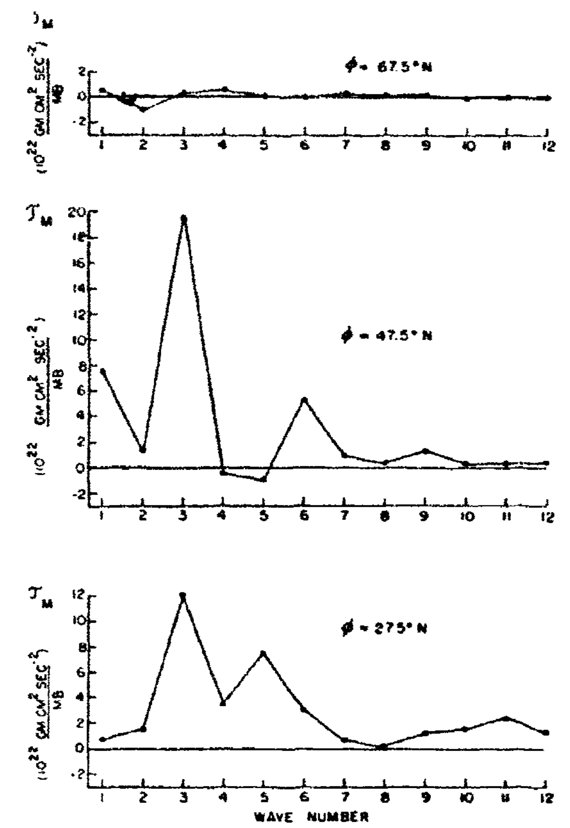

FIG. 2. Mean (January 1949) spectra of the 500-mb geostrophic meridional eddy transport of angular momentum, J_M, across latitudes 27.5 deg N, 47.5 deg N and 67.5 deg N, per unit pressure difference and per unit time. A positive sign indicates a northward transport. Corresponding standard deviations are given in table 1.

5. Estimate of the energy transfer between the individual eddy scales and the mean flow

From (8) it may be seen that an evaluation of $M^*(n)$ depends on computations of the product of the spectrum of the momentum transport and the shear of the relative angular velocity, measured throughout the depth of the atmosphere, for latitudes ranging between the two poles or, if it is assumed that the hemisphere can be treated as a "closed" system, between the equator and the pole. (Only for a closed system can M^* be taken as a measure of the transfer of energy between the eddies and the mean flow.)

In this study, however, we have measured the integrand of (8) only at 500 mb in the hope that the results for this level are somewhat representative of conditions throughout the depth of the atmosphere. In addition, due to inadequacies of data coverage and the inapplicability of the geostrophic assumption at low latitudes, we can compute only the *contribution* to the complete integral from a limited latitude band in the Northern Hemisphere. If the latitude band is sufficiently wide, however, it may be expected that the character of the energy transfers implied by this measured contribution will not be significantly altered by the contribution from the remaining portion of the fluid. In the present case the integral, M^*, is measured over a latitude belt ranging from 20 deg N to 75 deg N (62 per cent of the surface of the hemisphere), the integrand being computed at every 5 deg of latitude. The neglect of the contribution from below 20 deg N probably leads to an underestimate of the overall transfer of energy from the eddies to the mean flow for the hemisphere since both the shear of the mean

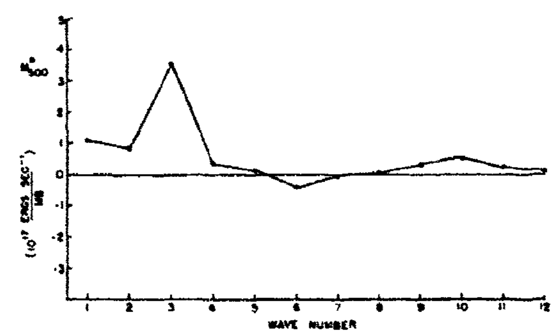

FIG. 3. Mean contribution to the energy transfer spectrum, M^*_{500}, from the region between 20 deg N and 75 deg N for the 31 dy period, January 1949. A positive sign indicates a transfer of kinetic energy from the disturbance to the mean motion. Corresponding standard deviations are given in table 1.

flow and the eddy momentum transport tend to be positive, though of diminished magnitude in the latter case, below this latitude (see Starr and White, 1954). The contribution of the region above 75 deg N is probably of minor importance.

The results of the computations are shown in fig. 3. This graph gives the mean spectral distribution of M_{500}^* for the month studied, computed with the above-mentioned restrictions. A positive sign indicates that the transfer of kinetic energy is directed from the disturbance to the mean motion.

It is pertinent to note, first, that the total transfer of energy due to horizontal processes, obtained by summing $M_{500}^*(n)$ over all twelve wave-numbers in accordance with (9), is in the positive sense (from the eddies to the mean flow), the magnitude being 6.86×10^{20} ergs per second if this 500 mb computation is extended to apply to a 1000-mb depth. In this respect the month is representative of the condition which prevails in the long-time mean as reported by Starr (1953).

The most striking feature of the spectral distribution is the prominent role played by disturbances of wave-number three in supporting the mean motion. From a statistical viewpoint, it is significant that on only three days of the 31-dy period did disturbances of this wave-number *draw* energy from the mean motion (note the comparatively small standard deviation for $n = 3$ in table 1). This result is a consequence of the fact that, on a daily basis, the momentum which was transported in large quantities by eddies of wave-number three (see section 4) was usually directed to those portions of the zonal current which were of highest velocity.

The high standard deviations displayed by wave number 6 and other intermediate wave numbers (*e.g.*, $n = 5$–8) reflect the fact that, during the month, disturbances of these wave numbers were alternately great drainers and great suppliers of the kinetic energy of the mean flow. In this sense, the intermediate wave-lengths, as a group, showed a greater capability for amplification at the expense of the mean motion than disturbances in the other portions of the spectrum. Aside from the somewhat erratic behavior of wave number 2, the longer and shorter waves tended to be stable (*i.e.*, damp their energy to the mean motion), wave number 3 being particularly active in this regard.

It is of interest, finally, to note that these particular observations are in qualitative agreement with Kuo's (1953) theoretical results regarding the barotropic stability properties of large amplitude disturbances. According to these theoretical results, disturbances of all wave lengths are generally damped except under conditions of a sharply developed jet-like horizontal mean profile, in which case a disturbance of roughly intermediate wave length is most likely to become unstable and amplify. It may be remarked that the results of the *linear* barotropic studies (Kuo; 1949, 1951) also indicate similar stability properties for small amplitude disturbances.

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On the Maintenance of the Large-Scale Quasi-Permanent Disturbances in the Atmosphere

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Abstract

It is proposed that large-scale quasi-permanent flow systems are maintained against dissipative effects by a transfer of kinetic energy from smaller, cyclone-scale, disturbances which have baroclinic energy sources, the processes involved being non-linear and barotropic. To illustrate how such processes can operate, an example is given based on an idealized flow consisting of a limited number of double-Fourier components.

1. Introduction

In Fig. 1 a 500 mb synoptic map for a region covering the North American Continent and the North Atlantic Ocean is presented. The most prominent features of this map—namely, the large cyclonic area to the north sloping from northwest to southeast and the large, more diffuse, anticyclonic area to the south sloping from northeast to southwest—are typical of the quasi-steady winter conditions observed in this region. The largest scale systems shown on this map correspond roughly to disturbances of wave number three around the hemisphere and are so oriented that they transfer their kinetic energy to the zonally-averaged mean current which has its maximum velocity somewhere between the two main pressure centers. As such, these large systems constitute a representative example of those disturbances which, in the winter average, are most effective in maintaining the mean zonal current (see SALTZMAN 1957, 1958; SALTZMAN and FLEISHER 1960).

In viewing the persistence of flow patterns of this type over long time intervals (see, e.g., monthly mean maps presented by LAHEY, et al, 1958) one is led to pose the following question: How is the kinetic energy of these large-scale systems maintained in the face of a steady drain of their energy through viscous effects and through transfer to the zonal current? There are two possible answers to this question: Either the kinetic energy of the large disturbances is supplied by a direct *conversion* of potential and internal energy (i.e., by risings of warm air and sinkings of cold air on this large scale) or the kinetic energy is supplied by *transfer* from disturbances of other scales which themselves grow at the expense of potential and internal energy (e.g., from the smaller transient cyclone waves which tend to develop in the baroclinic zone south of the large quasi-stationary cyclonic vortex, such as the shorter wave length disturbances clearly evident in Fig. 1).

If, as presently indicated in studies by REED and TANK (1956), and WHITE and SALTZMAN (1956), for example, the indirect circulations in mid-latitudes, whose upward branches are

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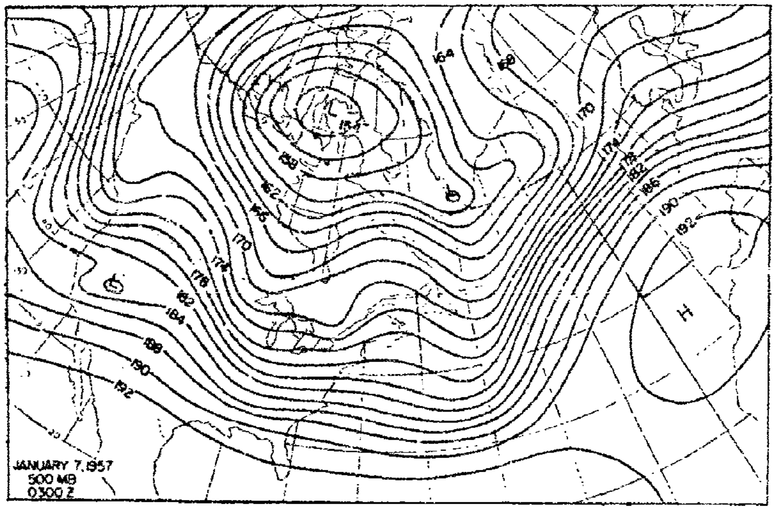

Fig. 1. Contour map of the 500 mb pressure surface for 7 Jan. 1957.

represented by rising motion in the large cold vortices to the north and whose downward branches are represented by sinking motion in the large warm highs to the south, are predominant over direct circulations on this scale, then one is forced to conclude that the large-scale features such as those shown in Fig. 1 are, in fact, steadily damped rather than amplified by direct conversion processes. In this event the maintenance of the kinetic energy of these features must rest entirely on the transfer processes mentioned above as the second alternative. In any event, however, it will be of interest to illustrate the operation of this transfer process,—by nature, a *non-linear* process—with a simple theoretical case. It is to this end that the present discussion is directed.

2. Governing equations

We assume that the transfer processes important for the maintenance of the large-scale disturbances are, as in the case of the maintenance of the zonal westerlies, of an essentially horizontal character and are governed by the barotropic vorticity equation. For simplicity, we shall apply this equation to a plane rather than a spherical surface, in which case the equation takes the form

$$\frac{\partial}{\partial t}\nabla^2\psi = -\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}\nabla^2\psi + \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\nabla^2\psi - \beta\frac{\partial\psi}{\partial x} \quad (1)$$

where x and y are Cartesian coordinates pointing eastward and northward respectively, t is time, ψ is a stream function proportional to the contour height of a pressure surface, $\nabla^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$, and β is the derivative with respect to y of the Coriolis parameter. Following procedures similar to those employed by KAMPÉ DE FÉRIET (1948), GAMBO, et al (1955), WIPPERMANN (1956) and LORENZ (1957), we express ψ over a fundamental region whose dimensions in x and y are K and L, respectively, in terms of a double Fourier expansion of the form

$$\psi(x, y, t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \Psi(m, n, t)\, e^{i(kmx + lny)}, \quad (2)$$

where $k = 2\pi/K$, $l = 2\pi/L$, m and n denote the wave number in the x and y directions respectively, and the complex Fourier coefficients, $\Psi(m, n, t)$, are given by the relation

$$\Psi(m, n, t) = = \frac{1}{KL}\int_0^K\int_0^L \psi(x, y, t)\, e^{-i(kmx + lny)} dx dy \quad (3)$$

To simplify the boundary conditions we take ψ to be periodic of wave lengths K and L in x and y, respectively.

By multiplying both sides of (1) by $(KL)^{-1}$ $exp\,[-i(kmx + lny)]$, integrating over the fundamental region, and applying (3) we obtain the

transformed vorticity equation governing the Fourier coefficients of the stream function. This equation has the form

$$\frac{d}{dt}\Psi(m,n) =$$

$$= \frac{kl}{(k^2m^2+l^2n^2)}\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}$$

$$[(mq-np)(p^2k^2+q^2l^2)\Psi(p,q)\Psi(m-p,n-q)]$$

$$-\frac{\beta ikm}{(m^2k^2+n^2l^2)}\Psi(m,n) \qquad (4)$$

The total kinetic energy integrated over the fundamental region

$$\bar{E} = \frac{1}{KL}\int_0^K\int_0^L \frac{1}{2}\left[\left(\frac{\partial\psi}{\partial x}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right]dx\,dy \quad (5)$$

may be expanded in terms of a Fourier spectrum of the form

$$\bar{E} = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{1}{2}K(m,n) \qquad (6)$$

$$= \sum_{n=1}^{\infty}K(o,n) +$$

$$+\sum_{m=1}^{\infty}\Bigl\{K(m,o) +$$

$$+\sum_{n=1}^{\infty}\left[K(m,n)+K(m,-n)\right]\Bigr\} \qquad (7)$$

where $K(m,n) = (k^2m^2+l^2n^2)|\Psi(m,n)|^2$. The first term on the right of (7) gives the kinetic energy of the mean zonal wind, while the term in curled brackets represents the spectral function for the mean disturbance kinetic energy superimposed on the mean zonal field. For the purpose of this study we are interested in obtaining an equation for the rate of change of kinetic energy of a given double Fourier component as a function of the transfer of kinetic energy to or from the other double Fourier components which comprise the complete stream field. Such an equation is obtained by multiplying both sides of (4) by $\Psi(-m,-n)$ and using the fact that $\Psi(-a,-b)$ is the complex conjugate of $\Psi(a,b)$. Thus we find,

$$\frac{d}{dt}K(m,n) = kl\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}(p^2k^2+q^2l^2)(mq-np)$$

$$[\Psi(-m,-n)\Psi(p,q)\Psi(m-p,n-q)-$$

$$-\Psi(m,n)\Psi(p,q)\Psi(-m-p,-n-q)] \quad (8)$$

By means of this last relation we are now able to calculate the instantaneous barotropic energy transfers due to the non-linear interactions among the Fourier components which comprise a field of flow in the fundamental region. It may be verified from (6) and (8) that the total kinetic energy is conserved (i.e., $d\bar{E}/dt = 0$), as is implicit in the barotropic system treated.

3. An example

In this section we shall discuss the energetical properties of a simple flow system having the general features of the actual case shown in Fig. 1. This flow system will be composed of a few selected double Fourier components, grouped to represent, respectively, the mean zonal current, the long-wave disturbance, and the smaller-scale disturbance. We shall see that in such a system kinetic energy is transferred from the small-scale disturbance (which may be viewed as baroclinically-generated) to the long-wave component and, simultaneously, from the long-wave component to the mean zonal current.

For this example we take the fundamental region to be a square so that $l = k$. The stream function, ψ, is taken to be of the form

$$\psi(x,y,t) = Z(y,t) + L(x,y,t) + S(x,y,t) \quad (9)$$

where

$$Z(y,t) = A(t)\sin ky \qquad (10)$$

represents the zonal current (wave number zero in x),

$$L(x,y,t) = B(t)[\cos k(x+y) +$$

$$+\cos k(x-y)] + C(t)\sin kx \qquad (11)$$

represents the long-wave component (wave number one in x), and

$$S(x,y,t) = D(t)\cos k(2x-y) +$$

$$+E(t)\cos 3kx \qquad (12)$$

represents the short-wave component (a combination of wave numbers two and three in x). Here $A = 2Im\,\Psi(0,1)$, $B = 2Re\,\Psi(1,1) = 2Re\,\Psi\cdot(1,-1)$, $C = 2Im\,\Psi(1,0)$, $D = 2Re\,\Psi(2,-1)$ and $E = 2Re\,\Psi(3,0)$. The dynamical properties of a system consisting of Z and L alone has already been studied in some detail by LORENZ (1957).

The components Z, L, and S are shown in igures 2, 3, and 4 with the following values of the Fourier coefficients:

$$\begin{aligned} A &= +3.00 \\ B &= -0.75 \\ C &= -1.50 \\ D &= +1.00 \\ E &= +1.00 \end{aligned} \tag{13}$$

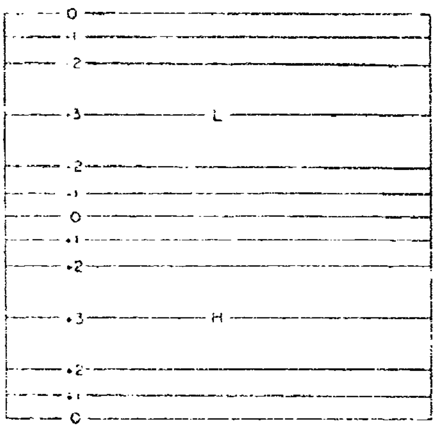

Fig 2. Stream function pattern, Z, representing the zonal motion: $Z(y) = 3.00 \sin ky$.

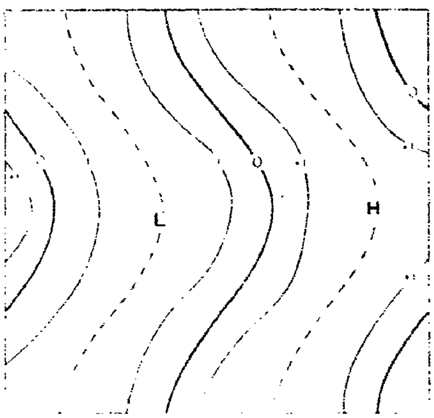

Fig. 3. Stream function pattern, L, representing the large-scale disturbances: $L(x, y) = -.75\,[\cos k\,(x+y) + \cos k\,(x-y)] - 1.50 \sin kx$.

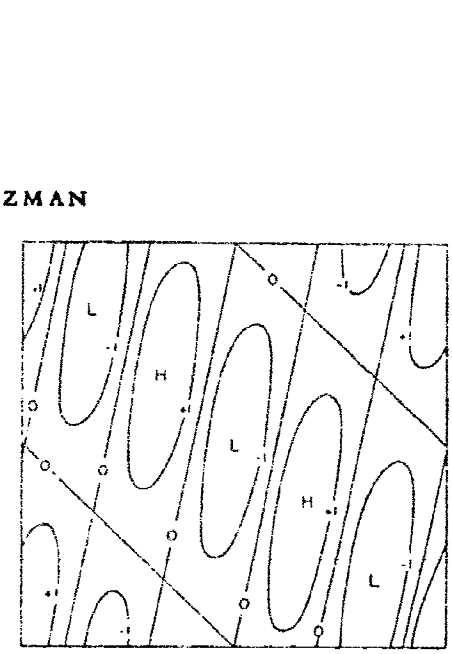

Fig. 4. Stream function pattern, S, representing the small-scale disturbances: $S(x, y) = \cos k\,(2x - y) + \cos 3\,kx$.

In Fig. 5 the sum of Z and L, representing a composite of the largest scales of motion, is

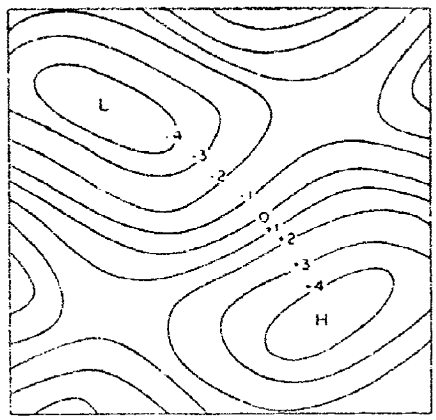

Fig. 5. Composite stream function pattern, $(Z + L)$, representing the largest scales of flow.

shown. In Fig. 6 the complete stream field, $(Z + L + S)$, obtained by superimposing the small-scale disturbance field, S, on $(Z + L)$, is shown. This stream field has many of the characteristics of the actual case shown in Fig. 1, the most obvious differences being the intensity of the large high pressure region and the strength of the easterlies.

The kinetic energy in each of these components is

$$\overline{E}_Z = .25\,k^2A^2 \tag{14}$$

$$\overline{E}_L = .25\,k^2(4B^2 + C^2) \tag{15}$$

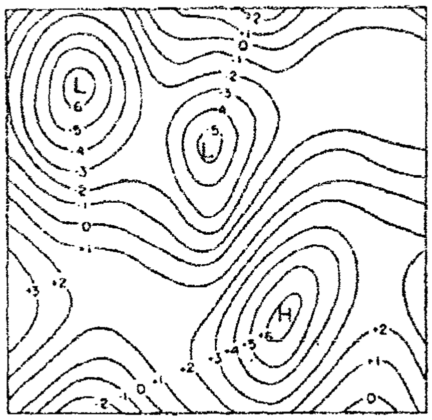

Fig. 6. Composite stream function pattern, $(Z + L + S)$, representing the complete field of flow.

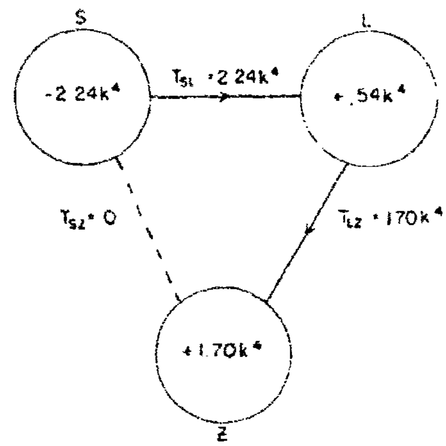

Fig. 7. Kinetic energy flow diagram for the example treated. Numbers within circles indicate the instantaneous rate of energy loss or gain due to the barotropic transfer.

$$\overline{E}_S = .25\, k^2(5D^2 + 9E^2) \tag{16}$$

and, given the values listed in (13), $\overline{E}_Z = 2.25\, k^2$, $\overline{E}_L = 1.14\, k^2$, and $\overline{E}_S = 3.50\, k^2$.

By applying (8) we obtain the following expressions for the rate of change of $\overline{E}_Z$, $\overline{E}_L$ and $\overline{E}_S$:

$$\frac{d}{dt}\overline{E}_Z = T_{LZ} \tag{17}$$

$$\frac{d}{dt}\overline{E}_L = -T_{LZ} + T_{SL} \tag{18}$$

$$\frac{d}{dt}\overline{E}_S = -T_{SL} \tag{19}$$

where

$$T_{LZ} = .50\, k^4 ABC \tag{20}$$

measures the transfer of kinetic energy between the long wave and the mean zonal motion (i.e., between L and Z), and

$$T_{SL} = -3.00\, k^4 BDE \tag{21}$$

measures the transfer between the short and long waves (i.e., between S and L). For this system there can be no kinetic energy exchange between the short waves and the mean flow, (i.e., $T_{SZ} = 0$). With the numbers given in (13) these energy transfer functions have the values $T_{LZ} = +1.70\, k^4$ and $T_{SL} = +2.24\, k^4$, implying an instantaneous kinetic energy flow simultaneously from the smallest scale component, S, to the larger scale component L, and to a lesser degree, from the component L to the mean zonal component Z (see Fig. 7). If we take $K = 10^9$ cm (corresponding to the dimensions of a comparable region of the atmosphere) and let the units of ψ be 10^{11} cm^2/sec (corresponding to a maximum mean zonal westerly velocity of 19 m/sec at $y = K/2$), the energy transfers are of sufficient strength to regenerate the existing kinetic energy of L in six days and that of Z in four days. These figures are of the order required by estimates of atmospheric frictional dissipation. Thus we have constructed one example in which the energy of the larger synoptic features is being maintained against dissipative effects by a transfer of energy from the smaller scale features. The latter may be presumed to grow baroclinically and hence have an independent energy source.

It will be recalled (see eq. 12) that $S(x, y)$ consists of two double-Fourier components, one of wave numbers $(2, -1)$ and the other of wave numbers $(3, 0)$. In agreement with the results of FJØRTOFT (1953), the loss of energy by the component, $(2, -1)$, is balanced by a small gain of energy by the shortest wave-length component, $(3, 0)$, and a larger gain of energy by the long-wave component, $L(x, y)$, the net effect being the transfer from S to L described above. It may be remarked, also, that kinetic energy of disturbances of higher

wave numbers than are included initially in our example would tend to be generated as *second-order* effects. Since we have been concerned here with the problem of the maintenance of quasi-steady features we have restricted attention only to the first order transfers. If potential energy conversions are not adequate to maintain the large-scale disturbances directly, these first-order transfers must, in the long-time average, be predominantly of the sign illustrated here. On specific days, however, first-order energy transfers of opposite sign (i.e., transfers from the larger to smaller scale components) may also occur and perhaps are of great actual importance in the sudden deepening of individual intermediate-scale waves observed on weather charts. An example of this reverse transfer may be obtained simply by changing the sign of either B, D or E. Clearly, many other examples illustrating transfers in either direction may be constructed using different Fourier components. It has been our purpose here to demonstrate at least one case which has the energetical properties mentioned in the introductory discussion, and, at the same time, is somewhat realistic in its correspondence with observed phenomena.

In connection with the above remarks it is pertinent to note that for our case the sign of the kinetic energy transfer between S and L, or between S and $(L + Z)$, is independent of the phase of the small-scale disturbances along the diagonal from northwest to southeast in Fig. 4. Thus the small-scale disturbances will continue to feed energy into the larger features at the same rate even if the highs and lows in Fig. 4 were interchanged, as might result from the motions of the small systems relative to the larger ones.

4. Further remarks

a) The view concerning the role of the smaller scale disturbances expressed here undoubtedly has certain connections with the experience of synoptic forecasters who regard the large features, particularly the large cold vortex to the north, as a sort of 'graveyard' for the smaller scale disturbances. The process of energy transfer from the smaller to larger waves is probably related to the familiar 'occlusion' process.

b) According to this discussion, the large scale features of the mean pressure map (i.e., of the average of the ensemble of many maps of the type shown in Fig. 1) are decisively influenced by transient smaller scale phenomena which are entirely absent from mean maps. If this view is correct, it is suggested that a linear, steady-state, theory will not be completely adequate in explaining the large-scale mean features. It would appear that a primary influence of land-sea thermal differences is to create favored areas for the baroclinic development of small-scale disturbances which, in turn, damp by non-linear barotropic processes, to maintain the larger systems. The discussion by SUTCLIFFE (1951) is of pertinence in this connection.

c) Computations from actual atmospheric data of the energy transfers occurring in the wave number domain can be performed feasibly with the use of the high speed computer. Such computations, based on equations presented by the writer (1957), are now under way at M.I.T.

Acknowledgement

The writer wishes to express his thanks to Professor V. P. Starr for his interest in this work.

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A Preliminary Study of the Potential to Kinetic Energy Conversion Process in the Stratosphere

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Abstract

The potential to kinetic energy conversion process in the lower stratosphere associated with the vertical exchange of warm and cold air is evaluated using adiabatically derived vertical velocities for the North American region for a five-day period. Preliminary results suggest the possibility that on the average the kinetic energy of stratospheric motions may not result from a conversion of potential energy within the stratosphere by this process. The further implication is that stratospheric motions are maintained by the motions in the adjacent layers of the atmosphere.

1. Introduction

A fundamental question about stratospheric motions refers to the manner of the maintenance of their kinetic energy. It seems clear that the kinetic energy of the motions of the troposphere is maintained by the conversion from available potential energy as described and verified by many investigators, most recently by LORENZ (1955) and WHITE and SALTZMAN (1956), and that this process is associated with the rising of warm and sinking of cold air. On the other hand, it has not been established that a similar process operates in the stratosphere. This preliminary study represents an attempt to evaluate from observations the nature of this potential to kinetic conversion process in the stratosphere.

2. Procedure

Following WHITE and SALTZMAN (1956) we may write the equations expressing the time rate of change of the kinetic energy of the horizontal wind and the total potential and internal energy of the entire mass of the atmosphere as follows:

$$\frac{\partial}{\partial t}\int_M k\,dm = -\int_M \omega\alpha\,dm - \int_M D\,dm \quad (1)$$

$$\frac{\partial}{\partial t}\int_M (\Phi + I)\,dm = \int_M \omega\alpha\,dm + \int_M \frac{\partial Q}{\partial t} \quad (2)$$

where dm is the element of mass and the integration is carried out over the entire mass of the atmosphere, k is the kinetic energy of the horizontal wind, $\omega = \frac{dp}{dt}$ is the individual time rate of change of pressure, α is the specific volume, D is the rate of frictional dissipation of kinetic energy, Φ is the geopotential, I is the internal energy, and $\frac{dQ}{dt}$ is the net rate of heat addition. The appearance of the integral

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$\int\limits_M \omega\alpha dm$ with opposite signs in both equations represents the process of potential to kinetic energy conversion in the atmosphere which is familiarly associated with the vertical exchange of warm and cold air.

The critical problem in evaluating this integral lies in the determination of ω which is closely related to the field of vertical velocity. In this investigation ω was evaluated by the adiabatic relation

$$\omega = -\frac{\dfrac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T}{\dfrac{\partial T}{\partial p} - \dfrac{\alpha}{c_p}} \tag{3}$$

where T is the temperature, $\mathbf{v}$ is the horizontal wind vector on an isobaric surface, and C_p is the specific heat at constant pressure.

The finite difference form of equation (3) was evaluated from constant pressure charts at 200, 100, 50 and 25 mb over the region of North America extending from 30° to 60°N and from 70° to 120°W. These charts had been carefully analyzed for other purposes by Mr. S. Muench of the Atmospheric Circulations Laboratory, Air Force Cambridge Research Center, and kindly provided us by [illegible] W. Hering and Mr. S. Muench for this inve[illegible]ion. The available data consisted of seven 200, 100, and 50 mb and six 25 mb charts covering the period 28 January to 3 February, 1957, a period of intense stratospheric warming as descibed by CRAIG and HERING (1959). The wind velocities were evaluated geostrophically and the temperatures hydrostatically. Contour height values were abstracted at a grid system of 273 points spaced 2.5 degrees apart and the ω's were obtained at a grid system of 60 points spaced 5 degrees apart. The time derivatives were approximated by finite differences over 24 hour intervals. The computation yielded fields of the 24-hour average values of ω and α for the layers 200—50 mb and 100—25 mb which were taken to be representative of these fields at the 100 and 50 mb levels.

Using the values of ω and α thus derived it was possible to sample the integrand for this region of the hemisphere for the short period of time as indicated, and for the layers of the atmosphere at approximately the height of the 100 and 50 mb levels. Average values of the integrand $\omega\alpha$ over space and time were computed and an analysis of the covariance of $\omega\alpha$ was performed in a manner entirely analogous to that of WHITE and SALTZMAN (1956). Following their procedure we may write:

$$\begin{aligned}\overline{\{[\omega\alpha]\}} - \overline{\{[\omega]\}}\,\overline{\{[\alpha]\}} &= \overline{\{[\omega]''[\alpha]''\}} \\ &+ \overline{\{[\omega'\alpha']\}} + \overline{\{[\omega]\}'''\{[\alpha]\}'''}\end{aligned} \tag{4}$$

where the single, double and triple primes denote deviations from east-west, north-south, and time averages respectively, the brackets indicate an average in the east-west direction, the braces an average in the north-south direction and the bars an average in time.

The left side of equation (4) is a more representative measure of the integrand average than the term $\{[\omega\alpha]\}$ alone, since the second term on the left side must vanish when the average is taken over the entire hemisphere and will give rise to spurious non-zero values if not subtracted out because of the limited area treated. The first term on the right is a measure of the conversion process due to overturnings in the north-south direction, the second is a measure of the process due to overturnings in the east-west direction. The last term is associated with temporal pulsations of the space average values of ω and α.

3. Results

The values of ω obtained are reasonable. When transformed by means of the hydrostatic equation to equivalent vertical velocities, the standard deviation is 0.86 cm sec^{-1} at 100 mb and 1.35 cm sec^{-1} at 50 mb. A typical example of the distribution of the vertical velocity in relation to the temperature and contour height fields at 50 mb is shown in Fig. 1 for 29 January, 1957.

The numerical values of the potential to kinetic energy conversion processes are shown in Table 1.

Table 1. The rate of conversion between potential and kinetic energy. A minus sign indicates a conversion from potential to kinetic energy. Units in ergs gm^{-1} sec^{-1}.

	1	2	3	4
	$\overline{\{[\omega\alpha]\}} - \overline{\{[\omega]\}}\,\overline{\{[\alpha]\}}$	$\overline{\{[\omega]''[\alpha]''\}}$	$\overline{\{[\omega'\alpha']\}}$	$\overline{\{[\omega]\}'''\{[\alpha]\}'''}$
50 mb	7.5	— 0.1	7.5	0.1
100 mb	— 0.5	— 0.3	— 0.4	0.2

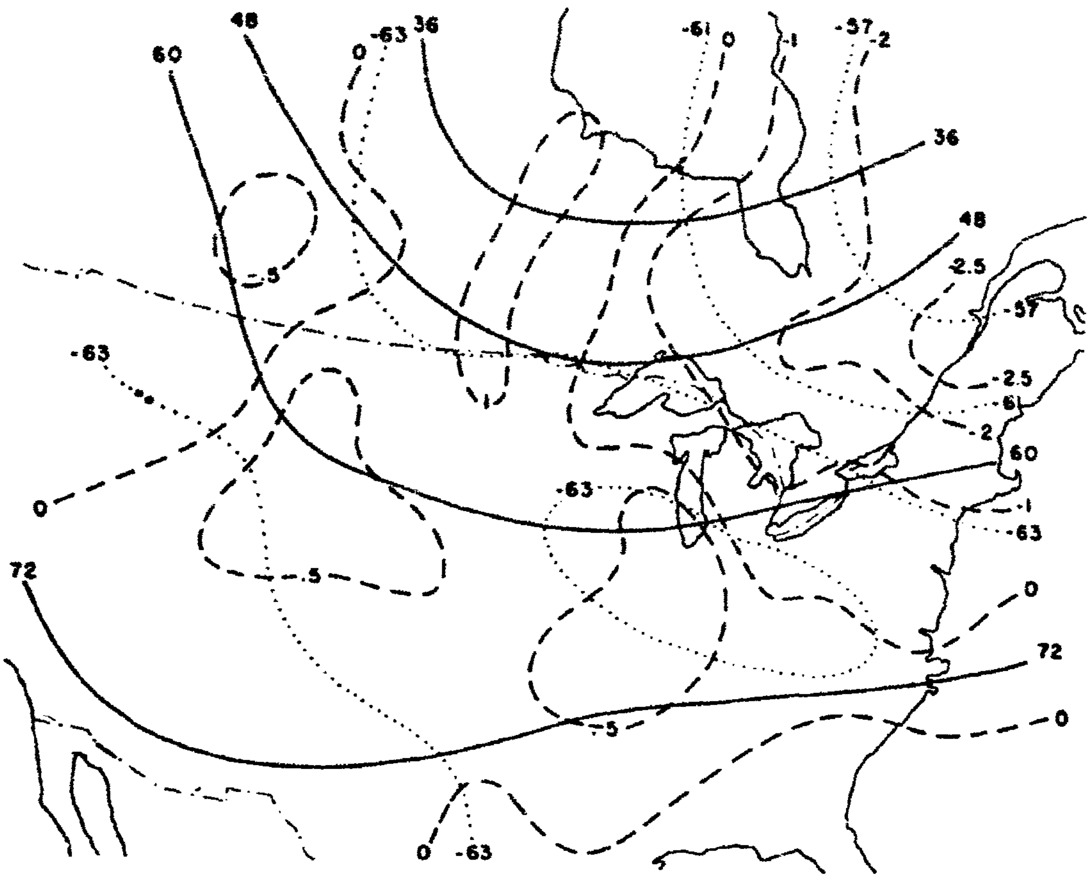

Fig. 1. The vertical velocity, temperature, and contour height distribution at 50 mb for January 29, 1957, 03 GMT. Vertical velocity units in cm sec^{-1}.

The first column of Table 1 indicates that at 50 mb the conversion process is opposite to that found in the troposphere being from kinetic to potential energy in this region of the stratosphere. The sense of this conversion process was the same on each of the four individual days examined. At 100 mb the magnitude of the conversion term is not essentially different from zero and the six individual daily values on which the mean is based vary in sign.

A more detailed analysis of this conversion process is given in columns 2, 3, and 4 of table 1. At 50 mb the principal contribution to the conversion process is associated with overturnings in the east-west direction as indicated in column 3. The contributions of the other two terms in columns 2 and 4 are negligible. At 100 mb the conversion rates associated with each of the individual terms are small.

Since the 100 mb data are representative of the layer between 200 and 50 mb, this layer will be wholly contained in the stratosphere only at high latitudes and will be partially within the troposphere at low latitudes. It should be possible upon examination of the latitudinal variation of the term $[\overline{\omega'\alpha'}]$ to detect whether

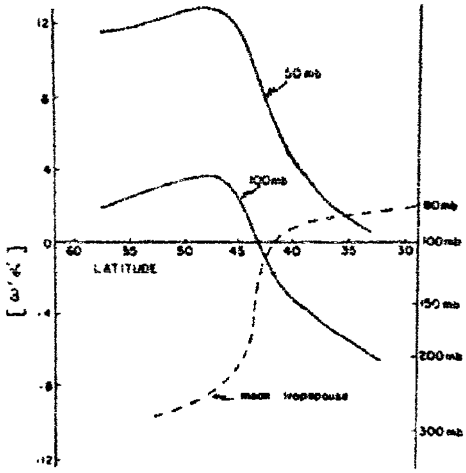

Fig. 2. The latitudinal variation of the potential to kinetic energy conversion process due to east-west overturnings in units of ergs gm^{-1} sec^{-1}. Also shown is the latitudinal variation of the mean tropopause height as a function of pressure.

there is any systematic difference in the sense of the conversion process as one proceeds from troposphere to stratosphere. The latitudinal variation of this term at both 100 and 50 mb together with a plot of the mean tropopause height is shown in Fig. 2.

It can be seen that the sign of the conversion process changes at approximately that latitude (43°N) where the mean tropopause crosses the 100 mb level, being from kinetic to potential to the north and from potential to kinetic to the south of this latitude. This observational feature agrees with the concept of a reversal of the potential to kinetic energy conversion process from troposphere to stratosphere..

If these observational findings are further substantiated by more extensive investigations on a hemispherical basis and over longer periods of time then the implications seem far-reaching. If the kinetic energy of the stratospheric motions is not maintained by conversion from potential energy within the stratosphere by the processes associated with the vertical exchange of warm and cold air then it must be maintained by boundary interactions with the adjacent layers above and below. Such processes involve the vertical transport of existing kinetic energy through the top and bottom boundaries of the layer as pointed out by STARR (1959) and other boundary processes associated with variations in the height of the bounding pressure surfaces. Should general verification of these concepts be obtained, one possible implication would be that stratospheric motions must to a large extent vary in response to tropospheric changes, and that the explanation for many observed characteristics of the stratosphere, such as the size and motion of circulation systems and the seasonal and latitudinal distribution of ozone, may very well lie in a better understanding of the linkages between the stratosphere and troposphere.

4. Critical Remarks

As in all such limited investigations care must be exercised in generalization before confirmation on the basis of more extensive data. This particular study suffers from the following deficiencies:

a. The results are based on a sample of data from a small area of the hemisphere and for a very short period of time.
b. The period of time studied was one of abnormal high level temperature changes, and the results may not be typical of more normal conditions.
c. The vertical velocity computations on which the results fundamentally depend are based upon the adiabatic assumption.

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Further Statistics Concerning the General Circulation

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During the past few years it has become increasingly clear that the newer outlook upon the fundamental processes of the general circulation continues to flourish and prosper. It bids fair to become the essential starting point for all rational discussion of the large motions of the atmosphere, as indeed it must if its principal ideas are correct. This stage has been reached not without much debate about logical matters, much effort to reinstate older schemes through secondary adjustments, and much examination of evidence. Also, any number of theoretical solutions, most of which lend support to the new views, have been advanced to illustrate the workings of the general circulation.

It likewise is worth our while to note that the trends of thought which have led us to what seems a more just view of this large subject do not find their entire application here, but have shown relevance also for smaller structures present in synoptic flow patterns. Here again consideration of the flow of momentum and the associated transfer of kinetic energy from one scale of motion to another seems to be a primary concept in the rational investigation of the behavior of synoptic systems. This is shown in a recent paper by SALTZMAN (1959). Another recent paper by KUO (1959) purports to show that, just as is the case for the circumpolar vortex, certain other large circulation systems may not release convectively their solenoidal energy. They presumably can be maintained in the atmosphere only through the action of still smaller eddies which do have associated baroclinic releases of energy in them. A general interpretation of all these developments has been given by STARR (1959).

Let us return to the larger problem, however. The writer (STARR, 1948) has long ago pointed out that the processes of gathering statistics concerning the new view of the general circulation would require much time and effort. We are still thus occupied, but since the results are quite gratifying, these labors are not so much of a burden as one might suppose judging merely from the amount of time and drudgery which is entailed. On the other hand, such work must be done, if we are to build solidly, because no other source of information can be equivalent to it (see STARR 1958 in this regard).

One activity which has its aim in this general direction is the work of SALTZMAN (1957, 1958). This consists of a plan to study the transfers of kinetic energy from the eddies to the mean zonal motions by use of the technique of spectral analysis, in order to examine the role played in this action by eddies corresponding to various wave numbers around the hemisphere. In addition, the process of convective kinetic energy release

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through vertical motions may also be brought within the ambit of the scheme. Aside from the partial results already presented earlier, new findings in this area, obtained from the analysis of a complete year of geostrophic data are now being reported by SALTZMAN and FLEISHER (1959). Still more extensive results are expected at a later date.

Since they serve as an interesting comparison to the Saltzman and Fleisher statistics, the writer wishes to make available the outcome of computations from actual wind data for an additional year, similar in an over-all fashion to those published several years ago (STARR, 1953) for the year 1950. The new findings are for the calendar year 1951—the same period as was used by SALTZMAN and FLEISHER.

The 1950 study concerned itself with the evaluation of an integral over the atmosphere in the northern hemisphere of the form

$$2\pi r^2 \int\int \varrho\,[u'v']\, coz^2 \phi \frac{\partial}{\partial \phi}\left(\frac{[u]}{r\cos\Phi}\right) d\phi dz \quad (1)$$

u, v being the eastward and northward wind components, ϱ density, r earth's radius, Φ latitude, z elevation, the square brackets denoting averages along complete latitude circles, and the primes indicating deviations from such averages. Since the integrand contains, in proper form, the product of the eddy stress into the meridional shear of angular velocity of the mean flow, (1) gives the instantaneous rate of conversion of eddy kinetic energy into mean flow kinetic energy through the action of the horizontal wind. In almost all details the procedures used for the 1951 study are a duplication of those for the 1950 study (to which the reader is referred). The chief differences are due to the somewhat better quality and quantity of the 1951 data, so that it seemed desirable to make use of compilations for the 100- and 1,000-mb levels which were omitted previously. The following are the results.

(1) The expression above was evaluated on a daily basis. The average of these 365 values was $(5.8 \pm 1.0) \times 10^{20}$ erg sec^{-1}. All individual monthly values were positive. Since these calculations were made by levels and by days, this figure is comparable to that obtained in the previous study in paragraph (3) under »Critique and further computations», for the first half of 1950, namely 9.8×10^{20} erg sec^{-1}. The confidence limits are twice the standard error of the mean. The 70° N latitude data were included for 1951 in all studies.

(2) The use of a vertically-averaged momentum transport and a vertically-averaged shear (on a daily basis) in evaluating (1) resulted in a value of $(4.8 \pm 0.7) \times 10^{20}$ erg sec^{-1} for the year. All individual monthly values were again positive. This is comparable to the value $(4.5 \pm 1.0) \times 10^{20}$ erg sec^{-1} reported previously in paragraph (2) under "Critique, etc.", for the first half of 1950.

(3) When one resorts to computations based upon mean profiles for the year averaged vertically as well as with respect to time (comparable to those shown in Fig. 1 of the previous study), one gets the value of 4.6×10^{20} erg sec^{-1}. This is in good agreement with the value of 4.4×10^{20} erg sec^{-1} calculated for the entire year of 1950.

(4) Use of the 500-mb data alone for the momentum transport and shear, assuming these to be representative of the average for the atmosphere (on a daily basis) resulted in the quantity $(6.2 \pm 1.2) \times 10^{20}$ erg sec^{-1} for the year (all monthly values positive). A figure to be compared with this, based upon 500-mb geostrophic data, was obtained by SALTZMAN and FLEISHER, also for the year 1951. Their figure is $(3.8 \pm 1.2) \times 10^{20}$ erg sec^{-1} with all months positive. It is tempting to assume that the lower figure for the geostrophic calculations is mainly a result of a smoothing process in the drawing of maps of the contours from which gridpoint data were read.

It must not be lost sight of, that the present calculations are subject to all the strictures and qualifications enumerated in the discussion of the previous study. Such features as the probable underestimation of the total effects computed here, due to selectivity favoring more frequent observation of lighter winds at higher levels, is as true for the present data as for the previous compilations. Added to these, it should be borne in mind that individual years may vary greatly one from another in regard to the properties here studied. Shorter periods may of course show even more striking differences.

The data compiled for the year 1951 afforded an opportunity to repeat another calculation which the writer had the privilege to communicate some years ago in the pages of this

journal, again using an assemblage of statistics for the calendar year 1950 (see STARR 1954). These two studies pertain to the measurement from actual wind data of the efficacy in the atmosphere of the classical mechanism for the maintenance of the kinetic energy of the mean zonal motions. As explained in the former paper such efforts are afflicted with great difficulties, but since a new set of extremely extensive and in many ways better observations offered another independent occasion to execute the computation, the resulting figures are given herewith.

For our present purpose it is needed to evaluate the effect of the mean meridional circulations in producing kinetic energy of the mean zonal motions through the action of Coriolis forces. This action is given by

$$\varrho f [\bar{u}] [\bar{v}[\qquad (2)$$

per unit volume of air. Here ϱ is density, f the Coriolis parameter, u the eastward and v the northward component of velocity, while the brackets denote zonal averages and bars time averages. The reader is directed to the former paper for added details. The volume integrals by latitude belts and for the hemisphere are given in the accompanying table together with a reproduction of the corresponding numbers for 1950. The figures for the hemisphere for both years show a net destruction of kinetic energy of mean zonal motions.

Since the kinetic energy stored in the mean meridional circulations themselves is always extremely small, and since the mean zonal motions are sensibly geostrophic, the negative value of the figures given above for the hemisphere possess the implication that the mean meridional circulations convert kinetic energy back into potential and internal. This has been commented upon, for example, by LORENZ (1955).

Excluding the figure quoted from Saltzman and Fleisher's study, the amount of data on which the results given for 1950 and 1951 are based is enormous. About a quarter million winds were processed—a task which has required the sustained efforts of a crew of skilled computers for several years. In view of the fact that much of the labor in this type of study consists in the tabulating of the initial observations, not much is gained through the use of electronic calculators, unless the input can be secured in a form already recorded on magnetic tape. Fortunately this is the case for the International Geophysical Year observations which are now being processed so as to obtain further information from data which no doubt are significantly better than either those for 1950 or 1951.

TABLE 1.

Volume integral for the atmosphere from 1 000 to 100 mb of the quantity $\varrho f [\bar{u}] [\bar{v}]$ in 10^{20} erg sec^{-1}.

Lat. Belt	0—13°	13—31°	31—42°	42—55°	55—70°	70—90°	Hemisphere
Int. 1950	+ 0.01	— 0.35	— 3.55	— 2.65	+ 2.69	+ 1.76	— 2.09
Int. 1951	+ 0.27	— 0.00	— 4.79	— 3.03	+ 3.61	+ 1.68	— 3.16

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SHORTER CONTRIBUTION

Spectrum of Kinetic Energy Transfer due to Large-Scale Horizontal Reynolds Stresses

By BARRY SALTZMAN and AARON FLEISHER, Massachusetts Institute of Technology[1]

(Manuscript received April 6, 1959)

All whirls do trade their velocity—but,
The biggest whirl exacts its cut.

We have made estimates of the integral,

$$M(n) = \frac{2\pi a}{g} \iint \Big[U(n, \phi, p) V(-n, \phi, p) + U(-n, \phi, p) V(n, \phi, p) \Big] \cos^2\phi \frac{\partial}{\partial \phi} \left[\frac{\bar{u}(\phi, p)}{\cos \phi} \right] d\phi \, dp,$$

which measures the rate of transfer of kinetic energy between the mean zonal flow and the harmonic components of the eddy flow. Only the action of the horizontal Reynolds stresses on the zonal current is considered. n is the wave number around latitude circles; ϕ is the latitude; p is the pressure; U and V are the complex Fourier coefficients respectively of the zonal and meridional components of the wind; $\bar{u}$ is the mean zonal wind; a is the radius of the earth; g is the acceleration of gravity; and the integration spans the entire atmosphere. Further details are contained in two papers by B. SALTZMAN (1957, 1958).

The estimates were made for each day of the year 1951 using geostrophic winds at 500 mb and were based on a fifteen wave-number Fourier resolution along every five degrees of latitude from 15 deg N to 80 deg N of contour heights spaced every ten degrees of longitude. To estimate the contribution from the unsampled portions of the hemisphere we assumed that the integrand fades to zero linearly from 22.5 deg N to 10 deg N, and from 72.5 deg N to the pole. We hope this extrapolation is conservative.

The results are shown in the figure and

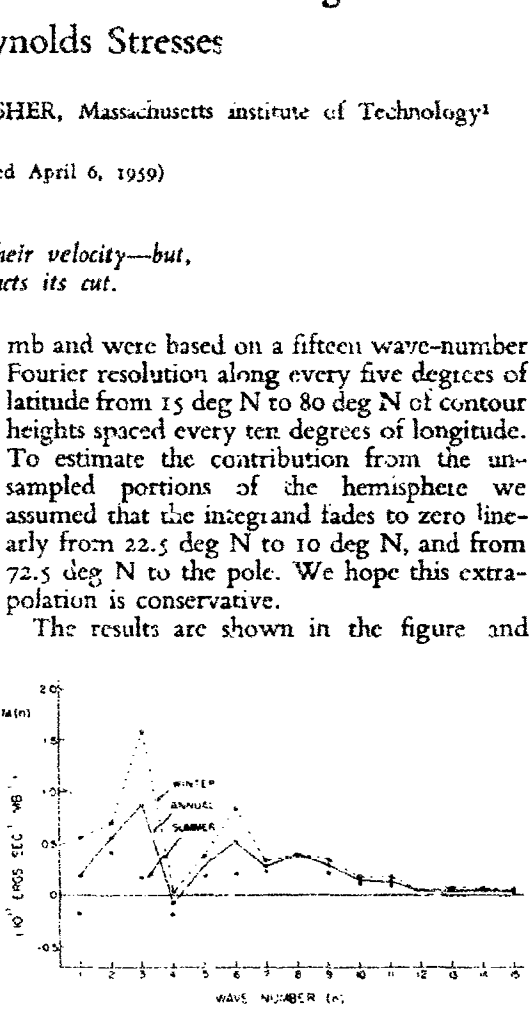

Fig. 1. Average values of $M(n)$ for the entire year 1951, for the winter half-year (January—March, October—December) and for the summer half-year (April—September). A positive value signifies a transfer of kinetic energy from disturbances of wave number n to the mean zonal current.

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Table 1. Standard deviations of the daily values of *M(n)* from the averages shown in fig. 1, in units of (±) 10^{-17} ergs sec^{-1} mb^{-1}. The last column gives the standard deviation of the net daily transfer.

Wave number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15} M(n)$
Winter ..	1.54	2.59	2.76	1.90	2.20	2.41	1.63	1.47	.91	.79	.48	.47	.34	.33	.34	9.10
Summer .	.73	1.22	1.52	1.08	1.02	.99	1.03	.92	.62	.45	.41	.34	[illegible]	.30	.23	3.66
Annual ..	1.26	2.03	2.33	1.55	1.72	1.87	1.37	1.23	.78	.65	.45	.41	.29	.32	.29	6.98

the accompanying table. These place in perspective the spectrum obtained for the single month, January 1949, previously reported (SALTZMAN, 1958).

The net transfer, i.e., the sum over the fifteen wave numbers, is directed from the eddies to the zonal flow. If we take this 500-mb estimate to be representative of the entire depth of the atmosphere then the annual average is 3.8×10^{20} ergs sec^{-1}, the winter average is 5.8×10^{20} ergs sec^{-1} and the summer average is 1.8×10^{20} ergs sec^{-1}, which are smaller but still of the same order as obtained by V. P. STARR (1959) for the same year using observed winds (see, also, STARR 1953).

We are indebted to the people of the Computation Center at the Massachusetts Institute of Technology for machine time and for running our program.

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The Exchange of Kinetic Energy between Larger Scales of Atmospheric Motion

By BARRY SALTZMAN and AARON FLEISHER,[1] Massachusetts Institute of Technology

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Abstract

The rate of transfer of kinetic energy between the different scales of eddies of the 500 mb geostrophic wind has been measured for each day of 1951. In the mean, there is a net transfer from intermediate (cyclone) scales to both longer and shorter scales.

The rate, per unit area and per unit pressure difference, at which kinetic energy is transferred from eddies of wave number n to eddies of all other wave numbers is (SALTZMAN 1957)

$$L^*(n) = L^{I}(n) + L^{V}(n) \tag{1}$$

$$\begin{aligned}
L^{I}(n) = -\frac{1}{2gp_0}\int_0^{p_0}\int_{-\pi/2}^{\pi/2}\sum_{\substack{m=-\infty \\ \neq 0}}^{\infty} \Bigg\{ & \frac{in}{a\cos\phi}\{U(m)[U(-n)\,U(n-m) - U(n)\,U(-n-m)] \\
& + V(m)[V(-n)\,U(n-m) - V(n)U(-n-m)]\} \\
& + \frac{1}{a\cos\phi}\{U(-n)[U(m)\,V(n-m)\cos\phi]_\phi + U(n)[U(m)\,V(-n-m)\cos\phi]_\phi \\
& + V(-n)[V(m)V(n-m)\cos\phi]_\phi + V(n)[V(m)V(-n-m)\cos\phi]_\phi\} \\
& - \frac{\tan\phi}{a}\{V(m)[U(-n)\,U(n-m) + U(n)\,U(-n-m)] \\
& - U(m)[V(-n)\,U(n-m) + V(n)\,U(-n-m)]\}\Bigg\}\cos\phi\, d\phi\, dp
\end{aligned}$$

$$\begin{aligned}
L^{V}(n) = -\frac{1}{2gp_0}\int_0^{p_0}\int_{-\pi/2}^{\pi/2}\sum_{\substack{m=-\infty \\ \neq 0}}^{\infty} \Big\{ & U(-n)[U(m)\,\Omega(n-m)]_p + U(n)[U(m)\,\Omega(-n-m)]_p \\
& + V(-n)[V(m)\,\Omega(n-m)]_p + V(n)[V(m)\,\Omega(-n-m)]_p\Big\} \\
& \cos\phi\, d\phi\, dp
\end{aligned}$$

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λ = longitude

ϕ = latitude

p = pressure

t = time

m, n = wave number around a latitude circle

$u\ (\lambda, \phi, p, t) = (a \cos \phi)\, d\lambda/dt$ = zonal wind component

$v\ (\lambda, \phi, p, t) = a\, d\phi/dt$ = meridional wind component

$\omega\ (\lambda, \phi, p, t) = dp/dt$ = individual pressure change

$F(x) = (2\pi)^{-1} \int_0^{2\pi} x(\lambda, \phi, p, t)\, e^{-in\lambda} d\lambda$ = Fourier transform of x

$U(n, \phi, p, t) = F(u)$

$V(n, \phi, p, t) = F(v)$

$\Omega\ (n, \phi, p, t) = F(\omega)$

p_0 = pressure at the earth's surface

a = radius of the earth

g = acceleration of gravity,

$(x)\xi = \partial x/\partial \xi$.

Since $\sum_{n=1}^{\infty} L^*(n) = 0$ we can think of $L^*(n)$ as the rate at which the existing kinetic energy is being redistributed among all the scales of atmospheric eddies, this process being independent of other processes which can alter the total eddy kinetic energy—i.e., transfers to or from the zonal motion $(n = 0)$, viscous dissipation, and conversions to or from potential and internal energy.

We are concerned now with estimating $L'(n)$, the contribution to $L^*(n)$ from interactions of the horizontal winds, for the scales represented on hemispheric charts. For this purpose we apply the geostrophic approximation. We do not think these charts justify a resolution in more than 15 wave numbers and therefore take all scales of motion smaller than wave number 15 to be zero. We label the result of this approximation $L(n)$. Then,

$$\sum_{n=1}^{15} L(n) = 0 \tag{2}$$

which is to say that $L(n)$ measures the geostrophic redistribution of kinetic energy among only the fifteen wave numbers. Transfers due to non-geostrophic components and transfers between the wave group $n = 1$ to 15 and shorter waves are considered to be separate processes and are not treated here.

Our data are the 500 mb northern hemisphere charts for every day of 1951. Contour heights are given at every ten degrees of longitude and every five degrees of latitude from 15° N to 80° N. These are the data from which we obtained the spectrum of the rate of transfer of kinetic energy between the zonal current and the eddies (SALTZMAN and FLEISHER, 1960).

We assume that events at 500 mb are representative of the vertical average, at least in regard to sign.

Our finite difference scheme limits the integration to the region between 22.5° N and 72.5° N. For these new boundaries,

$$\sum_{n=1}^{15} L(n) = \frac{2\pi a \cos\phi}{g} \left[v' \left(\frac{u'^2 + v'^2}{2} \right) \right] \Bigg|_{\phi = 72.5}^{\phi = 22.5}$$

(brackets denote a zonal average and primes the deviation therefrom). However, if our

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results are to be representative of the globe it is necessary that (2) be satisfied, which means that the net eddy transport of eddy kinetic energy into this region must be zero. We have measured this and found it quite small. In place of a sufficient condition we assume that the region we treat is large enough to represent the interactions over the globe.

The integral involves a summation of triple products of Fourier components—effectively a triple correlation. The variability of the atmosphere and the quality of the data hardly encourages such an enterprise. It should be expected, therefore, that only a long-time mean will have a variance sufficiently small to be worth quoting. We present the annual mean and, in computing its error, take half of the 365 days to be independent. To obtain additional smoothing, we consider three groups of wave numbers: 'long' waves $1 \lesssim n \lesssim 5$, 'cyclone' waves $6 \lesssim n \lesssim 10$, 'short' waves $11 \lesssim n \lesssim 15$, and we denote the transfer function for these groups by L_i, $i = 1, 2, 3$, respectively; i.e.,

$$L_1 = \sum_{n=1}^{5} L(n)$$

$$L_2 = \sum_{n=6}^{10} L(n)$$

$$L_3 = \sum_{n=11}^{15} L(n)$$

The loss in resolution is appreciable, but physically less than might be supposed since the same wave number at different latitudes corresponds to different physical scales.

Results

We define

$\bar{x}$ = annual average of x

$\sigma(x) = (\overline{x^2} - \bar{x}^2)^{\frac{1}{2}}$ = standard deviation of x

$\dfrac{2\sigma(x)}{\sqrt{182}}$ = error of $\bar{x}$

The mean values obtained and their errors, in 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}, are

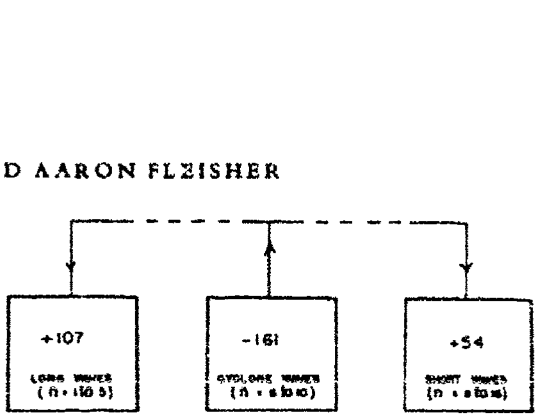

Fig. 1. Estimate of the mean rate of kinetic energy exchange between long waves ($n = 1$ to 5), cyclone waves ($n = 6$ to 10) and shorter waves ($n = 11$ to 15), based on daily 500 mb geostrophic measurements for the year 1951. Units are 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}.

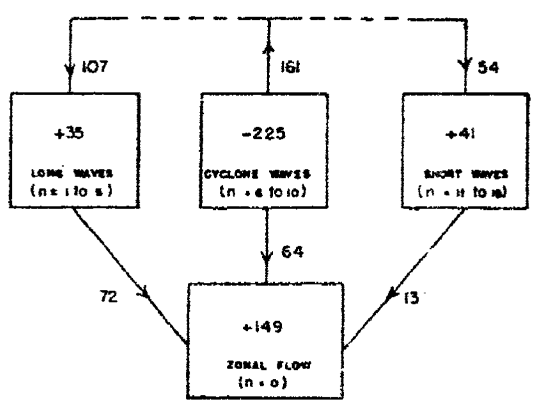

Fig. 2. Estimate of the mean rate of kinetic energy exchange between the zonal current ($n = 0$), long waves ($n = 1$ to 5), cyclone waves ($n = 6$ to 10) and shorter waves ($n = 11$ to 15), based on daily 500 mb geostrophic measurements for the year 1951. Units are 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}.

$$\overline{L}_1 = +124 \pm 81$$
$$\overline{L}_2 = -145 \pm 96$$
$$\overline{L}_3 = +\ 71 \pm 54$$

and

$$\sum_{i=1,2,3} \overline{L}_i = +\ 50 \pm 56$$

from which we conclude that $\overline{L}_1$ and $\overline{L}_3$ are significantly positive, $\overline{L}_2$ is significantly negative, and as required, $\sum_{i=1,2,3} \overline{L}_i$ is not significantly different from zero. In physical terms, there is, in the mean, a net transfer of kinetic energy from the 'cyclone' band ($n = 6$ to 10) to the long-wave band ($n = 1$ to 5) and to the short-wave band ($n = 11$ to 15). FJØRTOFT (1953) has demonstrated the possibility of transfers of this kind, and SALTZMAN (1959) has con-

structed a model which displays these energy flow characteristics.

To estimate the numerical magnitude of this effect we assume that $\sum_i \overline{L}_i$ is entirely in the nature of a computational noise which affects each of the three wave-groups equally. Thus, we subtract $1/3 \sum_i \overline{L}_i$ from $\overline{L}_1$, $\overline{L}_2$ and $\overline{L}_3$, to achieve the balance required by (2). The results are pictured in Figure 1. An arrow indicates a net loss or gain to or from the other two wave groups.

In Figure 2 we display the results shown in Figure 1 combined with those presented by SALTZMAN and FLEISHER (1960) for the interactions between the mean zonal current and the eddies. This is our estimate of the mean geostrophic energy flow in the domain $n = 0$ to 15. Estimates of the release of potential and internal energy in this domain are given by SALTZMAN and FLEISHER (1959) and WIN-NIELSEN (1959). There are no direct measurements of the transfer of kinetic energy between these scales of disturbances and scales of wave number larger than 15.

We are indebted to the people of the Computation Center of the Massachusetts Institute of Technology for time on the IBM 704 and for running our program. This research has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center under Contract No. AF 19(604)-2242.

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The Modes of Release of Available Potential Energy in the Atmosphere

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Abstract. We have obtained the space spectra of the fields of vertical motion and temperature in the free atmosphere for February 1959, and have estimated from these the conversions between available potential energy and kinetic energy as a function of wave number.

Introduction. Given the daily hemispheric fields of the thickness between 850 and 500 mb and the individual pressure change at 600 mb, we have calculated the following quantities: (*i*) the available potential energy as a function of wave number around latitude circles; (*ii*) the rate of conversion between potential and kinetic energy showing the effects of the mean meridional circulations and the eddy circulations of different scales.

The basic energy equations in which these quantities appear and discussions of their significance for the general circulation are given by *Lorenz* [1955], *White and Saltzman* [1956], and *Saltzman* [1957, 1958].

Formulas and data. We shall use these symbols:

λ = longitude
ϕ = latitude
p = pressure
t = time
n = wave number around a latitude circle
∇ = surface spherical gradient operator on an isobaric surface
ω = dp/dt = individual pressure change
w = vertical velocity
$\mathbf{v}$ = horizontal wind vector
T = temperature (deg Abs)
ρ = density
g = the acceleration of gravity
C_p = specific heat at constant pressure
R = gas constant

$$B(n, \phi, p, t) = (2\pi)^{-1} \cdot \int_0^{2\pi} T(\lambda, \phi, p, t)e^{-in\lambda}\, d\lambda$$

= complex Fourier coefficient for the temperature

$$\Omega(n, \phi, p, t) = (2\pi)^{-1} \cdot \int_0^{2\pi} \omega(\lambda, \phi, p, t)e^{-in\lambda}\, d\lambda$$

= complex Fourier coefficient for ω

$$[x] = (2\pi)^{-1} \int_0^{2\pi} x\, d\lambda$$

= zonal average of x

$$x' = x - [x]$$

= deviation from the zonal average

$$\{x\} = (\sin \phi_2 - \sin \phi_1)^{-1} \int_{\phi_1}^{\phi_2} x \cos \phi\, d\phi$$

= meridional average of x between latitudes ϕ_1 and ϕ_2

$$x'' = x - \{x\}$$

= deviation from the meridional average

$$\{[x]\}$$

= area average over a constant pressure surface

$$x^* = x - \{[x]\}$$

= deviation from the area average

$$\bar{x} = \tau^{-1} \int_0^{\tau} x\, dt$$

= time average (τ = length of record)

$$x''' = x - \bar{x}$$

= deviation from the time average

$$\sigma(x) = (\overline{x^2} - \bar{x}^2)^{1/2}$$

= standard deviation

$$\gamma = (\{[T]\} - p^{c}{}_{v}R^{-1}\, \partial\{[T]\}/\partial p)^{-1}$$

The total available potential energy per unit area per unit pressure difference is [*Lorenz*, 1955; *Saltzman*, 1957]

$$\mathbf{P} = \frac{C_P\gamma}{2g}\{[T^{*2}]\}$$

$$= P_Z + P_E$$

where

$$P_Z = \frac{C_P\gamma}{2g}\{[T]''^2\}$$

is the *zonal* available potential energy and

$$P_E = \frac{C_P\gamma}{2g}\{[T'^2]\}$$

is the total *eddy* available potential energy. The contribution to the total eddy available potential energy from temperature variations of wave number n is

$$\mathcal{P}(n) = \frac{C_P\gamma}{g}\{|B(n)|^2\}$$

which satisfies

$$P_E = \sum_{n=1}^{\infty}\mathcal{P}(n)$$

We call $\mathcal{P}(n)$ the 'spectral function' for eddy available potential energy.

The total rate of conversion from available potential energy to kinetic energy per unit area and per unit pressure difference is [*Lorenz*, 1955]

$$\mathbf{C} = -\frac{R}{pg}\{[\omega T]\}$$

The average is to be taken over a closed pressure surface (i.e., $\phi_1 = -\pi/2, \phi_2 = \pi/2$) and can be expressed as a sum of subaverages in the manner,

$$\mathbf{C} = C_E + C_Z + C_0$$

where

$$C_E = -\frac{R}{pg}\{[\omega' T']\}$$

is the rate of conversion from eddy available potential energy to eddy kinetic energy,

$$C_Z = -\frac{R}{pg}\{[\omega]''[T]''\}$$

is the rate of conversion from zonal available potential energy to zonal kinetic energy, and

$$C_0 = -\frac{R}{pg}\{[\omega]\}\{[T]\}$$

C_0 is identically zero, since

$$\{[\omega]\} = \int_0^p \{[\nabla\cdot\mathbf{v}]\}\,dp = 0.$$

In practice, we can measure these quantities over only a limited region of the globe (in our case, the belt between 20°N and 80°N). If, however, the region is large enough to capture most of the significant scales of variations of ω and T (as we believe is the case here) we can have confidence that the values obtained for C_E and C_Z are fairly representative of what would be obtained if the entire pressure surface were sampled. On the other hand, $\{[\omega]\}$ and hence C_0 need not vanish for such a limited region because of systematic variations of ω with latitude. Such a non-zero value of C_0, however valid for the particular region, cannot, a priori, be correct for the entire pressure surface. For these reasons we take as our estimate of $\mathbf{C}$ the measured value $(C_E + C_Z)$, which we denote by $\mathbf{C}_{EST}$. We shall take up the question of representativeness again.

The quantity C_E can be expanded in terms of a spectral function measuring the rate of conversion from eddy available potential energy of a given wave number to eddy kinetic energy of disturbances of the same wave number [*Saltzman*, 1957]. This spectral function is

$$\mathcal{C}(n) = -\frac{R}{pg}\{\Omega(n)B(-n) + \Omega(-n)B(n)\}$$

which satisfies

$$C_E = \sum_{n=1}^{\infty}\mathcal{C}(n)$$

TABLE 1. Mean values and standard deviations of the zonally averaged individual pressure change $[\overline{\omega}] = \Omega(0)$, in 10^{-4} g cm^{-1} sec^{-3}, and of the zonally averaged temperature $[\overline{T}] = B(0)$, in deg Abs., both as a function of latitude, for February 1959 (Fig. 1)

ϕ	80	75	70	65	60	55	50	45	40	35	30	25	20
$[\overline{\omega}]$	−344	+43	−325	−781	−403	−300	+425	+782	+565	+643	+204	−300	−406
σ (±)	1167	957	795	1032	809	1276	845	1158	785	718	460	397	348
$[\overline{T}]$	244.01	245.59	247.13	248.62	250.28	252.19	254.62	257.71	261.61	265.74	269.77	273.37	276.88
σ (±)	1.68	1.02	1.04	1.06	1.03	1.06	1.06	0.73	0.56	0.15	0.23	0.42	0.55

The functions **P**, **C**, and their components were evaluated for the month of February 1959. The data comprised the 850- to 500-mb thickness in centimeters, denoted here by h, and the 600-mb individual pressure change ω in g cm^{-1} sec^{-3}, computed from the two-parameter, quasi-geostrophic, adiabatic, frictionless model of the Joint Numerical Weather Prediction Unit. We are indebted to the JNWP Unit for these data and to Mr. B. Lewis and Mr. V. Murino of the Extended Forecast Section of the U. S. Weather Bureau for the computer program that interpolates these quantities from the JNWP grid to intersections at every 5° of longitude and 10° of latitude from 20°N to 80°N. The computations began with a Fourier analysis of h and ω in 15 wave numbers around latitude circles.

The fields of both ω and the mean temperature equivalent to h were taken to apply in the vicinity of 600 mb. The hydrostatic approximation to the equivalent temperature is

$$T = 0.64 \times 10^{-3} h$$

We have set γ equal to 10^{-2} deg Abs^{-1}, which corresponds roughly to the normal value of $\partial\{[\overline{T}]\}/\partial p$ at the midtroposphere.

The ω field and the available potential energy. (*a*) The ω field: The mean values and standard deviations of $[\overline{\omega}]$ are given in Table 1 and Figure 1 as a function of latitude (a sine scale is used in all graphs having latitude as the abscissa). This is a representation of the monthly mean zonally averaged meridional circulation. Two large cells in low through middle latitudes and the suggestion of a third cell in high latitudes are visible. The maximum vertical velocity of these cells, computed from the approximate formula $w = -(\rho g)^{-1}\omega = -1.3\omega$, is about 0.1 cm sec^{-1}. This pattern resembles the picture of the mean meridional circulation first suggested by *Ferrel* [1856] and later modified by *Rossby* [1941]. Since heat sources and friction are not included in the computation of ω, the meridional cells obtained can only be the result of imbalances caused by meridional eddy transports of heat and momentum. However, if the direct forcing effects of the zonally averaged heat sources and friction were included, the

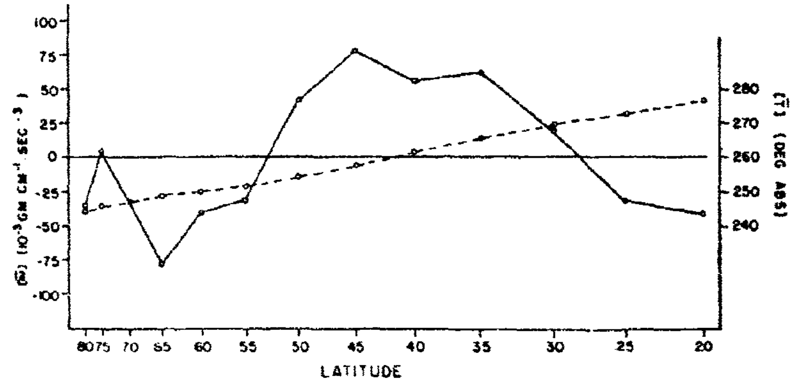

Fig. 1. Mean values of $[\overline{\omega}]$ (solid curve) and of $[\overline{T}]$ (dashed curve) for February 1959. See Table 1.

TABLE 2. Mean values and standard deviations of the Amplitude Spectrum of the individual pressure change $2\,|\,\overline{\Omega(n)}\,|$, in 10^{-4} g cm^{-1} sec^{-3}, and of the temperature $2\,|\,\overline{B(n)}\,|$, in deg, around the three latitude circles, 30, 50, and 70°N, for February 1959 (Figs. 2 and 4)

		Wave Number, *n*														
	ϕ	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	70	1661	1761	2189	2140	2034	1702	1322	1208	859	732	526	372	371	248	167
$2\,\vert\overline{\Omega(n)}\vert$	50	1526	1933	1669	2038	1502	2403	2660	3335	2892	2674	2198	1951	1606	1549	1413
	30	1163	1653	2098	1837	2613	2721	2588	2334	2690	2490	2113	2248	1969	1840	1998
	70	998	1069	976	1190	846	962	847	791	648	303	308	235	202	147	97
σ (±)	50	889	919	982	1264	1041	1112	1361	1888	1315	1888	1109	958	783	983	752
	30	662	924	968	848	1286	1112	1403	921	1218	1388	956	1201	964	1067	886
	70	4.03	6.21	1.78	1.45	1.22	0.78	0.49	0.39	0.33	0.22	0.14	0.13	0.08	0.07	0.05
$2\,\vert\overline{B(n)}\vert$	50	3.60	6.10	4.62	2.37	2.16	1.77	1.18	1.25	0.88	0.63	0.59	0.49	0.42	0.34	6.34
	30	2.86	2.25	1.71	1.53	1.04	1.59	1.07	0.95	0.69	0.67	0.48	0.43	0.29	0.37	0.26
	70	2.02	2.34	1.07	0.61	0.70	0.36	0.25	0.22	0.17	0.11	0.09	0.06	0.04	0.04	0.03
σ (±)	50	1.90	1.21	1.63	1.25	0.84	1.07	0.53	0.73	0.32	0.36	0.30	0.26	0.16	0.21	0.17
	30	0.92	1.08	0.73	0.53	0.53	0.65	0.41	0.39	0.33	0.39	0.23	0.22	0.15	0.20	0.13

results probably would not be changed very much [*Phillips*, 1954, 1956; *Kuo*, 1956].

The area under the curve in Figure 1 is proportional to $\{[\overline{\omega}]\}$, which we said should be zero for a closed pressure surface. Our value of $\{[\overline{\omega}]\}$ is slightly positive (+.0098 g cm^{-1} sec^{-3}) indicating a net mean sinking motion for the region of roughly 0.01 cm sec^{-1}. It would appear from Figure 1, however, that the systematic negative values which are likely to be found in tropical regions below 20°N would bring $\{[\overline{\omega}]\}$ closer to zero.

To obtain some idea of the eddy variations of the ω field the monthly mean amplitude spectrum of ω around three latitude circles 30, 50, and 70°N, and their standard deviations were computed and are listed in Table 2. A graphical representation is shown in Figure 2. Please observe that this and all subsequent spectra are line spectra. It is notable that in middle and low latitudes the contribution of the high wave numbers to the total zonal variability of ω is quite large. The mean zonal variance $\overline{[\omega'^2]}$ around all the latitude circles studied is given in Table 3 and Figure 3.

(*b*) The available potential energy: The mean values and standard deviations of the zonally averaged temperature around latitude circles are listed in Table 1. The graph of these data is shown in Figure 1. The monthly mean amplitude spectrum of T around three latitude circles, 30, 50, and 70°N, and their standard deviations are listed in Table 2, and a graphical representa-

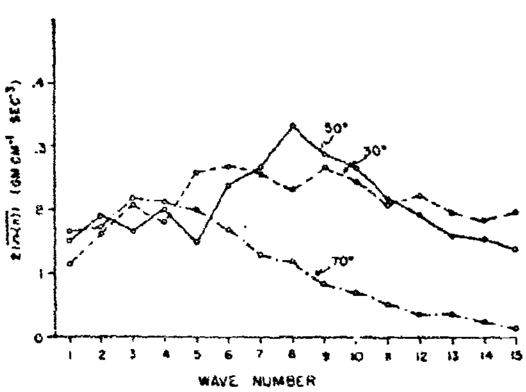

Fig. 2. Mean amplitude spectrum of the individual pressure change, $2|\overline{\Omega(n)}|$, at the three latitudes, 30, 50, and 70°N, for February 1959. See Table 2.

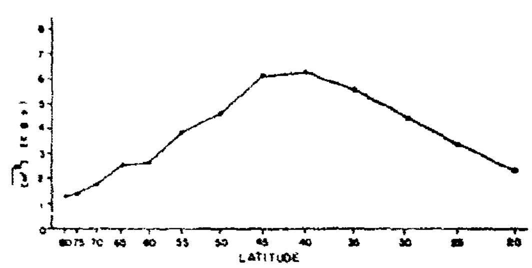

Fig. 3. Mean variance of the individual pressure change around latitude circles, $\overline{[\omega'^2]}$, for February 1959. See Table 3.

TABLE 3. Mean values and standard deviations of $[\overline{\omega'^2}]$ in 10^{-8} cgs units, $[\overline{T'^2}]$ in deg^2, and $[\overline{\omega' T'}]$ in 10^{-8} g cm^{-1} sec^{-3} deg, as a function of latitude, for February 1959 (Figs. 3, 5, and 8)

ϕ	80	75	70	65	60	55	50	45	40	35	30	25	20
$[\overline{\omega'^2}]$	128	140	178	252	264	392	459	615	639	563	449	344	237
σ $(\pm)$	144	78	120	158	226	186	199	275	258	170	157	115	87
$[\overline{T'^2}]$	16.8	27.0	37.5	47.0	52.4	55.6	51.3	40.3	26.9	18.9	15.0	9.7	4.1
σ $(\pm)$	10.2	13.5	14.1	16.9	13.3	9.1	8.3	8.6	6.2	4.6	4.1	3.5	1.6
$[\overline{\omega' T'}]$	−599	−530	−572	−700	−746	−1106	−854	−1156	−1105	−606	−337	−45	−41
σ $(\pm)$	995	482	706	457	540	687	648	568	576	418	291	210	167

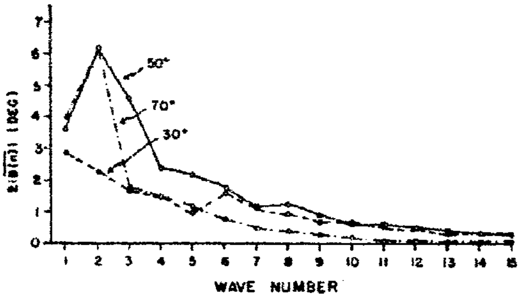

Fig. 4. Mean amplitude spectrum of the temperature, $2|\overline{B(n)}|$, at the three latitudes, 30, 50, and 70°N, for February 1959. See Table 2.

tion is shown in Figure 4. The maximum zonal variability occurs at the longer wavelengths and at higher latitudes. The latitude dependence of the zonal variability is more clearly evident in Table 3 and in Figure 5, which gives the meridional profile of $[\overline{T'^2}]$.

The monthly mean values of **P**, P_Z, P_E, and $\mathcal{P}(n)$ and their standard deviations are given in Table 4. Figure 6 is a graph of the mean spectral function $\overline{\mathcal{P}}(n)$. The largest store of available potential energy resides in the zonal component P_Z; the longer wavelengths (especially wave number 2, which is the scale of the major continents and oceans) contain most of the eddy available potential energy. We find that the mean total available potential energy is about 5 times as great as the total kinetic energy measured for the same region of the northern hemisphere [cf. *Lorenz*, 1955]. The largest contribution to $\bar{P}_E$ comes from variations of the temperature in middle to high latitudes (Fig. 5).

The energy conversions. The mean values of the conversion integrals C_Z, C_E, $\mathcal{C}(n)$, and $\mathbf{C}_{ZET}$ and their standard deviations are listed in Table 5. Figure 7 is a graph of the spectral function $\mathcal{C}(n)$, showing the modes of release of eddy available potential energy. We have

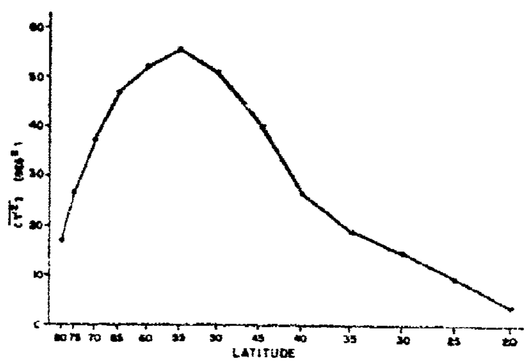

Fig. 5. Mean variance of temperature around latitude circles $[\overline{T'^2}]$, for February 1959. See Table 3.

TABLE 4. Mean values and standard deviations of the total, zonal, and eddy available potential energy and of the spectral function for eddy available potential energy, in units of 10^3 ergs cm^{-2} mb^{-1}, for February 1959 (Fig. 6)

	$\bar{P}$	$\bar{P}_Z$	$\bar{P}_E = \sum_{n=1}^{15} \overline{\mathcal{P}}(n)$
	6903	5280	1623
σ $(\pm)$	402	333	151

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\overline{\mathcal{P}}(n)$	331	622	269	113	82	87	35	36	17	12	7	6	4	3	2
σ $(\pm)$	104	208	99	74	46	57	15	21	10	8	4	3	2	1	1

TABLE 5. Mean values and standard deviations of the rate of conversion from available potential energy to kinetic energy, in units of 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}, for February 1959 (Fig. 7)

	$\bar{C}_Z$	$\bar{C}_E = \sum_{n=1}^{15} \bar{\mathfrak{C}}(n)$	$\bar{C}_{EST}$
	−196	+2878	+2682
σ (±)	567	715	...

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\bar{\mathfrak{C}}(n)$	+210	+355	+181	+297	+317	+509	+251	+309	+184	+140	+42	+36	+16	+20	+10
σ (±)	189	240	254	280	280	426	280	294	172	164	78	68	39	50	27

defined these quantities so that a positive value signifies a conversion from potential to kinetic energy.

In accordance with the remarks made above, we take $(C_Z + C_E)$, given in the last column of Table 5, as the estimate of the net rate of conversion. This value is 2.682 ergs sec^{-1} cm^{-2} mb^{-1}, which agrees in order of magnitude with estimates of the rate of frictional dissipation made, for example, by *Brunt* [1941]. (The net conversion rate, in the long-time average, must be equal to the net rate of dissipation; see, for example, *White and Saltzman* [1956].) Almost all of this net conversion is due to the release of eddy available potential energy (+2.878 ergs sec^{-1} cm^{-2} mb^{-1}), the conversion of zonal available potential energy being an order of magnitude smaller and of opposite sign. (−0.196 ergs sec^{-1} cm^{-2} mb^{-1}). We recognize that these measurements do not properly weight the processes in low latitudes. To estimate the effect of this omission we have assigned to $[\bar{\omega}]$ a constant value (12.9 × 10^{-3} g cm^{-1} sec^{-3}) from 20°N to the equator, which makes $\{[\bar{\omega}]\} = 0$, and have extended the mean temperature profile linearly to the equator with the slope observed at 20°N.

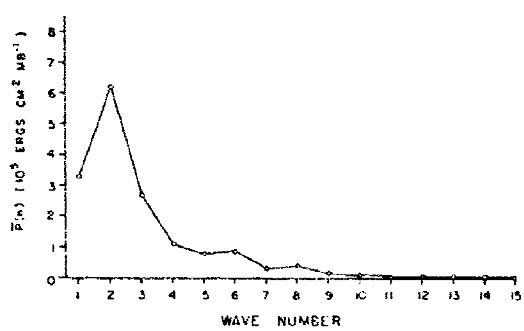

Fig. 6. Mean spectral function for eddy available potential energy for February 1959. See Table 4.

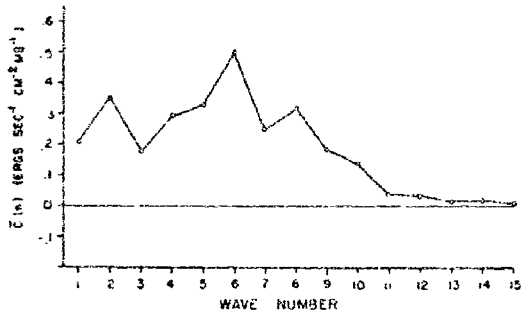

Fig. 7. Mean spectral function for rate of conversion from eddy available potential energy to eddy kinetic energy for February 1959. See Table 5.

The value of C_Z that results from this mean field is +0.294 ergs sec^{-1} cm^{-2} mb^{-1}, which is still 1 order of magnitude less than $\bar{C}_E$. This suggests that, over the whole hemisphere, it is primarily the large-scale eddy motions, rather than the zonally averaged motions, which grow directly at the expense of the available potential energy [cf. *Starr*, 1954].

These results are substantially the same as those obtained by *White and Saltzman* [1956] using data from a more restricted region (the sector of the North American continent between latitudes 35°N and 60°N and between longitudes 70°W and 120°W). Their values for $\bar{C}_Z$ and $\bar{C}_E$ were larger but of the same sign as obtained here, (−2000 × 10^{-3} and +6800 × 10^{-3} ergs sec^{-1} cm^{-2} mb^{-1}, respectively). The larger value of $\bar{C}_E$ is probably due to two factors: (1) compared with other sectors of the hemisphere, the North American sector is particularly active in the baroclinic development of eddies and (2) the data coverage for this particular sector is much better than for the hemisphere as a whole, permitting the use of 5° grid spacing in longitude and giving a more accurate representation of the

smaller-scale cyclogenetic processes which probably contribute to the over-all conversion rate. The larger negative value of $\bar{C}_Z$ is a consequence of the fact that the region is located entirely within middle latitudes where the Ferrel cell predominates, and hence includes little of the positive energy-releasing effects of the Hadley cell.

Although our coverage is much more extensive than was the case in the study by White and Saltzman, it is still deficient in the sense that the region is not 'closed,' as is evident from the non-zero value of $\{[\bar{\omega}]\}$. The simple extrapolation described above indicates that if latitudes down to the equator were sampled $\bar{C}_Z$ would become positive, owing to the fuller inclusion of the direct Hadley cell, but would remain small. The decrease of the eddy correlation between ω and T toward lower latitudes (Fig. 8) shows that $\bar{C}_E$ is slightly over-estimated on this account. However, the value of $\bar{C}_E$ may, on another account, be somewhat underestimated as a result of the failure to include as detailed a representation of smaller-scale phenomena as was done in the study by *White and Saltzman* [1956]. It is also possible that positive contributions could be made by scales of variations which are even smaller than those considered in either study (cumulus convection, for example).

We emphasize that our measurements apply only to a single level in the mid-troposphere. If these measurements are to be a fair estimate of the rate of conversion for the entire atmosphere, we must assume that the events occurring at all other levels, weighted by mass and integrated, would give very much the same result that we obtain by taking the 600-mb level to represent the entire atmosphere. We think we can assume that this is so. However, the differences at other levels are of interest. In the stratosphere there is some indication of a reversal in the correlation between ω and T which gives rise to a small negative value of $\bar{C}_E$ [*White and Nolan*, 1960]. At lower levels, on the other hand, the observed convergence into warm lows and divergence from cold highs may be reflected in a higher correlation between ω and T than is found in the mid-troposphere.

The spectral resolution of $\bar{C}_E$ is shown in Figure 7. For this month the kinetic energy of eddies of all scales grows at the expense of available potential energy. Eddies of wave number 6 are most active, in general agreement with the linear theory of baroclinic instability which predicts a maximum growth rate for waves of roughly intermediate scale. The maximum conversion occurs from scales which store comparatively little available potential energy (Fig. 6). In the long run the total store of available potential energy must be maintained through generation by diabatic processes. It is of interest to know the modes of this generation and the manner in which the available energy is exchanged among the modes. The expressions for these processes have been discussed by *Saltzman* [1957].

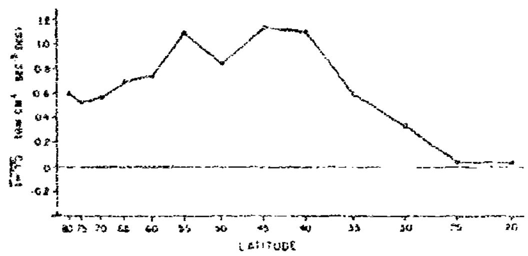

Fig. 8. Mean covariance between the individual pressure change and the temperature around latitude circles, $-[\overline{\omega' T'}]$, for February 1959. See Table 3.

The marked decrease in $\bar{C}(n)$ beyond wave number 10 may be due in part to lack of detail in the analysis of hemispheric charts. Concerning the long-wave end of the spectrum, Mr. A. Wiin-Nielsen has informed us that, when the diabatic component of the ω field is considered, a large reduction or even a reversal in the sign of $\bar{C}(n)$ probably occurs. If this is the case, these long waves must, on the average, obtain energy from the scales of motion which do have potential energy sources (for example, wave numbers 5 to 10) in order to overcome their losses through friction and by transfer to the zonal current [*Saltzman and Fleisher*, 1960]. A discussion of the nonlinear processes whereby such an energy transfer can occur was given by *Saltzman* [1959].

It is of interest to note the comparatively small standard deviation of the daily values of C_E from the monthly mean (Table 5). This is a reflection of the fact that the release of potential energy by the eddy circulations is a rather steady process, contrary to the often expressed

belief that this release is a more-or-less sporadic event after which the atmosphere 'coasts' barotropically for a comparatively long period. As it appears from these computations, some part of the atmosphere is continuously involved in overturnings which release energy at a fairly steady rate.

The computations reported here for the single month of February 1959 probably reveal most of the essential features of the long-time mean winter hemispheric fields of ω and T and of the conversion process. However, in order to establish greater significance for these statistics and to determine the seasonal variations, we plan to consider a longer record of data. It seems that most of the limitations of this study (the adiabatic method of computing ω, the incomplete sampling in the horizontal and the vertical, the exclusion of smaller-scale variations) will not be substantially removed in the very near future.

Acknowledgments. We thank the staff of the M.I.T. Computation Center for providing the IBM 704 computer time and for running our program, Lt. Col. P. D. Thompson, Mr. G. 'Arnason, Mr. L. Carstensen, Mr. A. Wiin-Nielsen, and Dr. G. Cressman of the Joint Numerical Weather Prediction Unit, and Mr. B. Lewis and Mr. V. Murino of the Extended Forecast Section, United States Weather Bureau, for their cooperation in securing the data and for supplying the information necessary for coding the problem.

During the time we were doing this work we learned that Mr. A. Wiin-Nielsen was engaged in a similar study. We are grateful to him for the opportunity to discuss the problem and to compare results.

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Questions Concerning the Energy of Stratospheric Motions[1]

By

V. P. Starr

Summary. In this article the theoretical conclusion is reached that the vertical transport of kinetic energy in the atmosphere from one horizontal layer to another is effected solely by an area integral over the internal boundary, of the kinetic energy per unit volume multiplied by the vertical velocity. Speculations are made as to whether the kinetic energy in stratospheric levels is maintained against friction through such a vertical transport from other levels, or whether the needed supply is derived from convective sources *in situ*.

Zusammenfassung. Auf Grund theoretischer Überlegungen wird der Schluß gezogen, daß der Vertikaltransport kinetischer Energie in der Atmosphäre von einer horizontalen Schicht zu einer anderen einzig durch ein Flächenintegral über die Grenzfläche bestimmt wird, dessen Integrand das Produkt aus kinetischer Energie pro Volumeinheit und der Vertikalgeschwindigkeit ist. Es werden Überlegungen angestellt, ob die kinetische Energie in Stratosphärenschichten gegenüber der Reibung durch einen solchen Vertikaltransport aufrechterhalten werden kann oder ob der benötigte Nachschub von konvektiven Quellen in der Schicht selbst stammt.

Résumé. Se fondant sur des considérations théoriques, l'auteur arrive à la conclusion que le transport vertical d'énergie cinétique dans l'atmosphère, d'une couche horizontale à une autre, est uniquement déterminée par une intégrale de surface étendue à la surface limite et dont l'intégrande est égale au produit de l'énergie cinétique par unité de volume par la vitesse verticale. Il discute la question de savoir si l'énergie cinétique d'une couche stratosphérique, en raison du frottement, est maintenue par un tel transport vertical ou si l'énergie d'entretien provient de sources convectives de la couche elle-même.

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1. Introduction

Meteorologists, when writing about the phenomena of the stratosphere are often wont to speculate concerning such things as the energy sources for the motions in the higher regions of the atmosphere. These ponderings are usually, and perhaps understandably, rather vague and diffuse—what with the lack of proper observational information and of a suitable theoretical framework within which observations might give expression to significant over-all physical facts. But more and more soundings reaching high altitudes are now being made on a routine basis, and it seems to me that the formulation of precise questions, grounded in the most reliable principles of physics, is the primary business of the research worker in this field who wishes eventually to secure some degree of rational understanding. The principles of physics might be such statements as those of the conservation of mass, of momentum and of energy, etc. The problem requiring resourcefulness is to formulate them in such a fashion as to make possible the observational study of the manner of their fulfilment, and to arrive thereby at some nontrivial physical conclusions pertaining to the real atmosphere. The eventual accumulation of such deductions should then lead, ideally at least, to the synthesis of quantitative models of various types (see, e. g., STARR [2]). What follows is an attempt to take one simple step along such a road.

2. Physical and Mathematical Considerations

We shall begin by considering the entire atmosphere as being divided into two horizontal layers by a closed constant level surface located at some appropriate fixed geodynamic height above sea level. Assuming that this constant elevation is plausibly chosen, we may for the purpose of our discussion name the entire upper region the "stratosphere," and the lower one the "troposphere." We realize, of course, that certain liberties are thus taken with standard terminology, although it may later prove that something less drastic is also amenable to precise treatment.

A number of years ago the writer had occasion to discuss the mechanical energy equation for the horizontal components of motion in the atmosphere (STARR [1]). Let us write this equation for the stratosphere in the form

$$\frac{d}{dt}\int \rho \frac{u^2+v^2}{2}\, d\tau = \int \rho \frac{u^2+v^2}{2}\, V_n\, ds + \int p \operatorname{div} \vec{V}_h\, d\tau - D \qquad (1)$$

Here u, v are the eastward and northward components of particle velocity, ρ is density, p pressure, t time, $d\tau$ a volume element, ds an element of surface of the volume taken, V_n the inward normal component of particle velocity across the boundary surface, $\vec{V}_h$ is the horizontal wind vector and D is the total rate of frictional dissipation of kinetic energy in the volume. The genesis of eq. (1) from the equations of motion for the horizontal directions and the general continuity equation is rather obvious

and will not be repeated here. It contains the statement for our problem that the amount of kinetic energy of horizontal motions in the stratosphere may change (left-hand side) as a result of advection of such energy across the boundaries of the stratosphere (first term on right), or as a result of sources of kinetic energy in the volume of the stratosphere (second term), or as a result of work done against friction (third term).

In the long term average the total kinetic energy in the stratosphere neither increases nor decreases, except for possible minute effects which reflect a changing climatic regime. On the other hand the dissipation must continuously be degrading the energy of stratospheric motions. Hence if A denotes the advection of kinetic energy into the stratosphere and S denotes the source term we may write that for the long term mean

$$D = A + S \tag{2}$$

that is, *the dissipation must be made good either by an advection effect or by a generation of kinetic energy in situ.* Our entire conception of the operation of the stratospheric circulation depends crucially upon the elucidation of how this equation or some equivalent of it is satisfied in actuality.

Let us examine first the advective effect A for the stratosphere. The upper surface may be eliminated because only negligible amounts of kinetic energy are exchanged with outer space (meteors and the like). This leaves only the horizontal internal boundary to be reckoned with. It follows therefore that A may be written as follows,

$$A = \int \frac{u^2 + v^2}{2} \rho\, w\, d\sigma \tag{3}$$

where $d\sigma$ is an element of area of the internal boundary and w is the upward component of particle velocity. In the average we may, with a high degree of precision, write that

$$\int \rho w\, d\sigma = 0 \tag{4}$$

from continuity considerations. Comparison of (4) with (3) now leads us to the purely mathematical conclusion that if A is to be non-zero, then there must exist a correlation between $(u^2 + v^2)/2$ on the one hand and ρw on the other. Thus if A were to be positive, ρw would have to be positive by and large over those regions where $(u^2 + v^2)/2$ is relatively large.

It is to be specifically stressed at this point that, due to the physical circumstances at *any* constant level in the atmosphere, the exchange of kinetic energy in the vertical takes place through the simple action specified by eq. (3), without any other significant *transport* mechanism.

The second term on the right side of eq. (1) which was designated as S in eq. (2) has been discussed previously by the writer in the reference given and also in other connections. It is a volume integral which may be written with sufficient accuracy for our problem in the form

1*

$$S = \int [\int p \operatorname{div} V_h \, d\sigma] \, dz \tag{5}$$

in which $d\sigma$ is again an element of horizontal area and dz is an element of vertical distance. Since we know that

$$\int \operatorname{div} V_h \, d\sigma \equiv 0 \tag{6}$$

we may reason about the eqs. (5) and (6) in the same manner as we did in regard to (3) and (4). We see that in order for S to have a non-zero value the pressure and the divergence should, at least at some level be correlated in a spacewise sense.

In our problem the value of the expression (5) for S is difficult to estimate from general information or from data, unless perhaps its form is first changed in order to eliminate the necessity of measuring the horizontal divergence. Let us try to do this. We may first write that

$$\int p \operatorname{div} \vec{V}_h \, d\sigma = -\int V_h \cdot \Delta_h \, p \, d\sigma \tag{7}$$

since the terms formally representing the difference between these expressions integrate to zero for our region. We now define

$$\omega \equiv \frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{V}_h \cdot \nabla_h p + w \frac{\partial p}{\partial z} \tag{8}$$

whereupon it is seen that

$$S = -\int \omega \, d\tau + \frac{d}{dt} \int p \, d\tau + \int w \frac{\partial p}{\partial z} \, d\tau. \tag{9}$$

The second integral on the right vanishes since the long term average of the pressure is nearly constant. So also does the third integral which, on hydrostatic principles, is nothing more than the rate of change of the total potential energy of the air in the region (or essentially its total vertical momentum multiplied by the acceleration of gravity).

Only the first term now remains. By use of the thermodynamic coordinate p in the place of the geometrical vertical coordinate z, hydrostatically, we have

$$S = -\int \omega \, d\tau = -\int_0^{p_0} \int \omega \alpha \, d\sigma \, dp. \tag{10}$$

In (10) α is the specific volume, p_0 is the pressure at the internal boundary and $d\sigma$ here again stands for an element of horizontal area but now following a given pressure surface. The last term is the same quantity as has been used, for example, by WHITE and SALTZMAN [5] to measure the generation of kinetic energy in the troposphere.

For a given closed isobaric surface in the stratosphere it may be shown that

$$\int \omega \, d\sigma = 0. \tag{11}$$

This follows directly through an integration of the continuity equation in the form

$$\frac{\partial w}{\partial p} + \nabla \cdot V_h = 0 \tag{12}$$

where the divergence of the wind is measured along an isobaric surface. From considerations which have recurred previously in this discussion, a comparison of the inner integral of the last term in (10) with eq. (11) shows that a given closed isobaric surface in the stratosphere can make a non-zero contribution to S, if and only if there exists a spacewise correlation of ω and α over the area of the isobaric surface. The fact that a few isobaric surfaces are not closed, but intersect the base of the stratosphere probably does not interfere much with this mode of thinking about the significance of the $\omega\,\alpha$ integral.

We observe that a correlation of α with ω as here discussed is a convenient means for the specification of a basic convective process. Thus if the negative values of ω are predominantly associated with large values of α, there exists a positive generation of kinetic energy. This presumably would signify a preponderance of rising motions of warmer air and a corresponding sinking of equal masses of colder air. It appears that for this form of S there is present some chance of a successful observational assessment of the energy producing action in the stratosphere.

3. Discussion of Conditions in the Real Atmosphere

I must confess that at least at present adequate stratospheric measurements of either A or of S are not at my disposal. Some small beginnings in this direction are, however, being made. WHITE and NOLAN [4] have been interested in measuring S, while certain other workers have, at my suggestion, become interested in appraising the observational possibilities of measuring A.

In a general sense the classical view has been that at least the lower stratosphere is a passive region wherein any tendency toward direct convective action is suppressed by the large hydrostatic stability present. If this is a dominant characteristic, then S should be zero or negative making the portion of the atmosphere above, let us say, 16 km a region of forced motion *on the average*. Such a view is strengthened by the circumstance that there exists a countergradient northward flow of heat at these levels (see, e. g., WHITE [4]).

If the notion of an inert, passive stratosphere corresponds to real fact, then we are at once brought to the concept that the continuance of motions in that region is to be explained by a vertical transport from other convectively more active layers above or below, through the action of a process represented by an integral like A. In that case although it cannot be gainsaid that still higher layers might contain sources of kinetic energy which could be transported downward, still the first consideration might be given to the hypothesis that the needed supply originates in the troposphere and is fed upward across levels such as 16 km or thereabouts.

4. Some General Comments

Due to the pivotal nature of the matters dealt with in the preceding discussions, several additional sidelights are not without justification. The following ones present themselves to the writer.

(a) Although our data still leave much to be desired, the time is coming shortly, if indeed it is not already here, when the evaluation of processes such as those dealt with in this paper should be given a high priority in meteorological research endeavors. Even though finally conclusive results may not appear immediately from such efforts, the partial insights gained would still be of no small importance, and should encourage thinking of an adequate scope and suitable perspective. We have seen somewhat comparable sequences of events during the past decade or two concerning other matters pertaining to the mechanics of the general circulation.

(b) In the manipulations the frictional term D is simply the work done by the fluid against frictional stresses arising from the viscosity of the fluid. No particular mathematical form for this action is assumed here, nor is any assumption made concerning the disposal of the energy involved—it may either remain in the fluid in the form of heat, potential energy etc., or it may be communicated to contiguous fluid masses by the frictional action itself. An example of the latter effect is the frictional transmission of kinetic energy from the lower atmosphere to the oceans which therefore *gain* energy thereby. This can of course happen only because the sea surface moves in response to the surface stresses which can therefore perform work upon the water.

It is an open question whether a similar action might not take place at the top of the troposphere, so that the stratosphere is dragged around by friction in the same manner as the oceans. Much here depends on what we might conceive as comprizing friction. If we limit the term to mean only molecular viscosity, then the drag due to it is no doubt much too small to cause concern and all other actions would be included in A. More usually meteorologists include as friction all rather small scale eddy effects such as those found in the so-called friction layer near the ground. With such a convention the frictional drag across levels such as 16 km would probably still be quite small, on the average. Any action from eddies of appreciable size would again be included in A.

The interpretation of the entire quantity A as a sort of gross friction is not very helpful, if for no other reason simply because we do not know even the sign of the viscosity coefficient which would be involved. Besides, we desire to know the details of the vertical velocity distribution and of its correlation with the kinetic energy.

(c) The primary convective actions in the troposphere take place at appreciable elevations above the earth's surface. In any event they do not take place in their entirety within the confines of the friction layer. Yet it is true that a disproportionately large fraction of the total dissipation of kinetic energy does take place in this bottom layer. It there-

fore follows that an integral of the form A acting across moderate elevations within and above the friction layer must represent an extremely important link in the total workings of the general circulation. The advisability of the detailed study of this anticipated phenomenon is manifest. Plans are being made currently to investigate it from data.

(d) It may be noted that the material treated is primarily of a mechanical nature. No equation of state has been used, and hence the conclusions are not dependent upon any such particular equation for the atmosphere which be assumed.

(e) Eq. (1) requires no assumption in regard to the presence or absence of hydrostatic balance, although the further manipulations of S do depend upon this condition being present. Likewise the validity of eq. (1) is in no way contingent upon the smallness of the vertical velocities in the atmosphere, although this latter is generally the case.

(f) As was stated in the discussion of 1948 by the writer, transports of kinetic energy horizontally across, let us say, vertical boundaries within the atmosphere may be effected through the work done by pressure forces in virtue of normal velocity components. Such a term arises in addition to the advective term in such a case.

Such a pressure-work term is absent in our equations for the transport of kinetic energy across horizontal surfaces. A pertinent consideration in this connection is that in our present case the pressure-work term would be capable of transferring kinetic energy of vertical motion only Even for this certain departures from hydrostatic balance would enter. This explains its absence from our discussions in this paper which deals with the kinetic energy of horizontal motions only.

(g) It must be realized, however, that when the foregoing analysis is performed in pressure coordinates instead of geometric ones, the simplicity and directness of the results are to some degree lost. Thus an additional boundary term involving the product ω and the geopotential appears. In view of this fact one may question the adequacy of the approximation made when a constant pressure surface is arbitrarily substituted for the constant level bottom boundary.

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The Meridional Eddy Transport of Kinetic Energy at 500 Mb

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(Manuscript received March 6, 1961)

The meridional transport of kinetic energy across latitude ϕ per unit time and per unit pressure difference is given by

$$T_k(\phi, p, t) = \frac{2\pi a \cos\phi}{g}\left[v\left(\frac{u^2+v^2}{2}\right)\right], \qquad (1)$$

where u is the zonal wind speed, v is the meridional wind speed, a is the radius of the earth, g is the acceleration of gravity and the brackets denote an average with respect to longitude, (cf., STARR 1948, and KAO 1954). If we let $x' \equiv x - [x]$, T_k can be resolved into components as follows:

$$T_k = T_{k0} + T_{k1} + T_{k2} \qquad (2)$$

$$T_{k0} = \frac{2\pi a \cos\phi}{g}[v]\left(\frac{[u^2]+[v^2]}{2} + [v'^2]\right)$$

$$T_{k1} = \frac{2\pi a \cos\phi}{g}[u]\,[u'v']$$

$$T_{k2} = \frac{2\pi a \cos\phi}{g}\left[v'\left(\frac{u'^2+v'^2}{2}\right)\right]$$

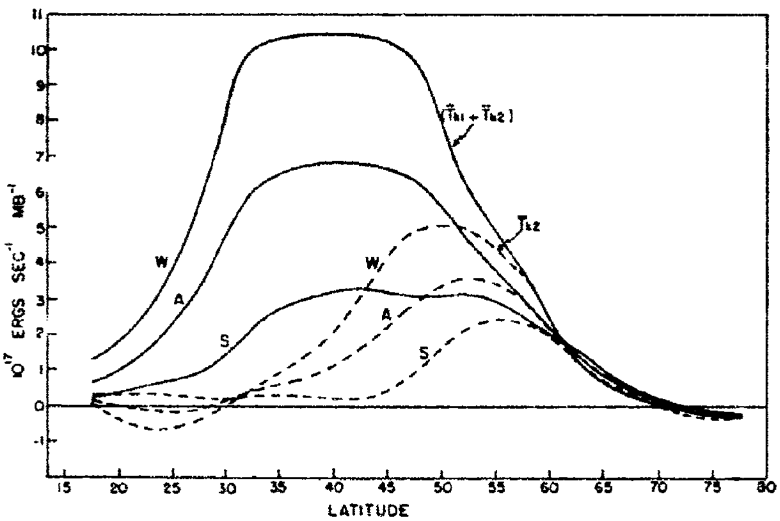

Fig. 1. Average rate of poleward eddy transport of kinetic energy at 500 mb. The solid line denotes $(\overline{T}_{k1}+\overline{T}_{k2})$ and the dashed line $\overline{T}_{k2}$, the difference being $\overline{T}_{k1}$. W, S, and A denote the winter, summer, and annual averages, respectively.

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Table 1. Average rate of northward eddy transport of kinetic energy at 500 mb. A bar denotes the average and $S(x) = \pm 2(\overline{x^2} - \overline{x}^2)^{1/2} M^{-1/2}$ the error of the average. M is the number of independent data, which we have taken to be half the number of days, N. Units are 10^{15} ergs sec^{-1}mb^{-1}. Graphical representation is given in Figure 1.

	ϕ (N)	17.5	22.5	27.5	32.5	37.5	42.5	47.5	52.5	57.5	62.5	67.5	72.5	77.5
$(\overline{T}_{k1} + \overline{T}_{k2})$	annual (365)	74	160	333	605	669	684	635	463	314	148	40	−10	−27
	winter (181)	26	263	578	998	1,039	1,043	968	608	382	133	31	−5	−17
	summer (184)	22	59	91	218	304	331	307	321	247	163	49	−15	−37
$S(T_{k1} + T_{k2})$	annual......	38	55	83	204	161	168	171	163	144	112	68	52	32
	winter......	59	100	147	390	287	300	307	286	259	200	116	86	52
	summer.....	30	31	40	72	96	115	122	154	130	100	71	58	38
$\overline{T}_{k2}$	annual......	17	−10	−7	39	84	160	294	358	305	143	54	−16	−27
	winter......	7	−58	−34	51	142	299	493	503	374	133	41	−18	−18
	summer.....	28	36	19	27	26	23	96	216	238	153	67	−13	−35
$S(T_{k2})$	annual......	27	31	42	81	103	126	130	140	129	93	55	40	28
	winter......	44	54	81	158	196	233	240	250	232	167	90	66	45
	summer.....	30	31	25	42	62	88	89	119	113	88	61	42	33

Since $[v]$ cannot be estimated reliably with existing wind data it is not feasible to measure T_{k0}. T_{k1} and T_{k2} can be measured with some confidence using geostrophic winds. The results of such measurements, based on hemispheric 500 mb data for each day of 1951, are summarized in Table 1 and Figure 1 in the form of averages for the entire year, the winter half-year (November to April) and the summer half-year (May to October). The average meridional convergence of kinetic energy, $C_k = \partial T_k / \partial \phi$, is given in Table 2 and Figure 2.

Since T_k is a cubic function of the wind we should expect that measurements at 500 mb,

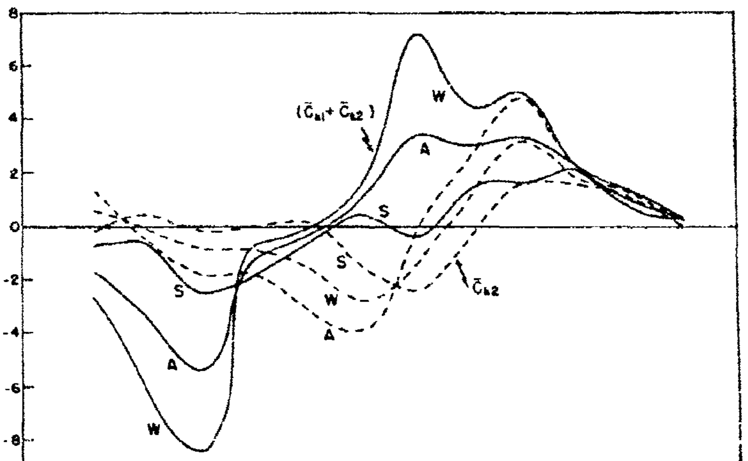

Fig. 2. Average convergencey of kinetic energy at 500 mb. The solid line denotes $(\overline{C}_{k1} + \overline{C}_{k2})$ and the dashed line $\overline{C}_{k2}$, the difference being C_{k1}. W, S, and A denote the winter, summer, and annual averages, respectively.

Table 2. Average meridional convergence of kinetic energy, $\overline{C}_k = \partial \overline{T}_k / \partial \phi$. Units are 10^{15} ergs sec^{-1} mb^{-1} deg. lat^{-1}. Graphical representation is given in Figure 2.

		20	25	30	35	40	45	50	55	60	65	70	75
$(\overline{C}_{k1} + \overline{C}_{k2})$	annual	−17	−35	−54	−13	−3	+10	+34	+30	+33	+22	+10	+4
	winter	−27	−63	−84	−8	−1	+15	+72	+45	+50	+21	+13	+3
	summer	−7	−6	−25	−17	−5	+5	−3	+15	+17	+23	+7	+5
$\overline{C}_{k2}$	annual	+6	−1	−9	−9	−15	−27	−13	+11	+32	+18	+14	+2
	winter	+13	−5	−17	−18	−31	−39	−2	+26	+48	+19	+12	0
	summer	−2	+4	−2	0	+1	−15	−24	−5	+17	+17	+16	+4

where the wind speeds are somewhat less than at jet stream levels, underestimate the average over the entire depth of the atmosphere This is in agreement with the results reported by MINTZ (1955), PISHAROTY (1955) and PALMÉN, RIEHL and VUORELA (1958), who discuss multi-level geostrophic computations of $(T_{k1} + T_{k2})$ for a four month period.

The transports of kinetic energy can be viewed as manifestations of the non-linear processes whereby energy is exchanged between harmonic components of the flow (cf., SALTZMAN and FLEISHER 1960). In lower latitudes transports involving interactions between the mean zonal current and the eddies, as measured by T_{k1}, are more effective than those involving the interactions between the eddies, as measured by T_{k2}. The reverse is true in higher latitudes.

The largest transports occur between 30° N and 50° N, leading to an average convergence of kinetic energy into the polar cap north of 40° N of the order of 10^{18} ergs sec^{-1} mb^{-1}. This is the same as the order of the eddy generation of kinetic energy within the cap measured by

$$\int_{\phi=40^\circ}^{\phi=90^\circ} \frac{2\pi a^2 \cos\phi}{g} [\omega'\alpha'] \, d\phi$$

($\omega = dp/dt$, and α = specific volume: see WHITE and SALTZMAN 1956, SALTZMAN and FLEISHER 1959, and WIIN-NIELSEN 1959).

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Energy Transformation and Vertical Flux Processes over the Northern Hemisphere[1]

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Abstract. Three integrals in the mechanical energy equation have been statistically evaluated as a function of a broad division of eddies in the atmosphere over the northern hemisphere for January and April 1958. The integral of the product of the instantaneous pressure change and the specific volume exhibits two significant positive (potential to kinetic energy) modes, one in the boundary region from 1000 to 850 millibars and the other in the middle troposphere from 700 to 500 millibars. The integral values of the product of the instantaneous pressure change and the potential energy are about an order of magnitude larger than the values computed for the integral representing the transport of kinetic energy. The three integral values were introduced into the mechanical energy equation and the residual was taken, ideally at least, to represent the dissipation rate for the horizontal kinetic energy within various regions of the atmosphere up to 50 millibars. These dissipation rates compare favorably with the appropriate estimates by Brunt.

Introduction. As was first suggested by *Margules* [1903], the significant action in the transformation process which produces kinetic energy in the atmosphere appears to be the simultaneous rising of warm air and sinking of cold air. One might suspect that the more vigorous energy transformation processes take place in the lower and middle troposphere where baroclinicity is more pronounced and where kinetic energy is produced at the expense of potential energy. In the upper troposphere and lower stratosphere there is some evidence [*White and Nolan*, 1959] of a reversal of this process; i.e., kinetic energy is transformed into potential and internal energy. Studies such as the one just cited and others by *White and Saltzman* [1956], *Saltzman and Fleisher* [1959], and *Wiin-Nielsen* [1959] have had the limitations of a small spatial sample and/or the use of values in the vertical motion term for only one surface or for only one layer.

In this paper we have evaluated the magnitude of the energy transformation processes on a hemispheric basis as a function of height and also as a function of a rather broad division of eddies, i.e., the so-called standing eddies and the transient eddies. In addition we have evaluated certain vertical flux processes involving potential and kinetic energy for regions of the atmosphere bounded by closed pressure surfaces. These measures of the vertical flux of energy have been introduced into the energy balance equation along with or as part of the values for the energy transformation process for the region in question. Any residual may be taken, ideally at least, as an approximation to the magnitude of the dissipative action within the same region.

The mechanical energy equation. To obtain the mathematical expression which represents the balance equation for mechanical energy, we follow a technique introduced by *Starr* [1951] with x, y, z, t coordinates and later adapted by *Phillips* [1954] to the x, y, p, t coordinate system. The horizontal vector equation of motion per unit mass, with hydrostatic equilibrium assumed and pressure used as the vertical coordinate, is

$$\frac{d\mathbf{V}}{dt} + f\mathbf{k} \times \mathbf{V} + \nabla_p \varphi + \mathbf{F} = 0 \qquad (1)$$

where $\mathbf{V}$ is the horizontal wind vector, f is the horizontal component of the Coriolis force, $\mathbf{k}$ is the unit vertical vector, $\varphi = gz$ is the geopotential of an isobaric surface, ∇_p is the horizontal 'del' operator along a constant pressure surface, and $\mathbf{F}$ is the horizontal friction force.

If we take the scalar product of (1) and $\mathbf{V}$

[1] This research was performed at the Massachusetts Institute of Technology in partial fulfillment of the requirements for the degree of Doctor of Philosophy. The author's current address is Headquarters 3rd Weather Wing, Offutt Air Force Base, Nebraska.

and expand the total derivative, letting $V^2/2 = K$ be the horizontal kinetic energy, $\varphi = gz = \Phi$ be the potential energy, and $V \cdot F = D$ be the rate of dissipation of kinetic energy, all per unit mass, we obtain

$$\frac{\partial K}{\partial t} + V \cdot \nabla_p K + \frac{dp}{dt}\frac{\partial K}{\partial p} + V \cdot \nabla_p \Phi + D = 0 \qquad (2)$$

If the hydrostatic equation in the form

$$\partial \Phi / \partial p + \alpha = 0 \qquad (3)$$

where $\alpha = 1/\rho$ is the specific volume, and the continuity equation

$$\partial \omega / \partial p + \nabla_p V = 0 \qquad (4)$$

where $\omega = dp/dt$ are used (ω is the instantaneous pressure change representing vertical motion in the coordinate system where pressure replaces height as the vertical axis), we may write (2), the mechanical energy equation for horizontal motion, as

$$\frac{\partial K}{\partial t} + \nabla_p \cdot (K + \Phi)V + \frac{\partial(\omega K)}{\partial p} + \frac{\partial(\omega \Phi)}{\partial p} + \omega\alpha + D = 0 \qquad (5)$$

Integrating (5) over the mass of the atmosphere between closed constant pressure surfaces, we obtain

$$\frac{\partial}{\partial t}\iiint K \frac{dp}{g}\, dx\, dy + \iint [(\omega K)_u - (\omega K)_l] \frac{dx}{g}\, dy + \iint [(\omega \Phi)_u - (\omega \Phi)_l] \frac{dx}{g}\, dy + \iiint \omega\alpha \frac{dp}{g}\, dx\, dy + \iiint D \frac{dp}{g}\, dx\, dy = 0 \qquad (6)$$

where the subscripts u and l refer to the upper (1000-mb) and lower (50-mb) boundary values, respectively.

Using the definition of the 'source' of kinetic energy given by *Starr* [1951] as the integral of the scalar product of the horizontal wind and the gradient of pressure in the x, y, z, t coordinate system, this integral may be transformed into the x, y, p, t coordinate system as follows:

$$\iiint V \cdot \nabla_h p\, dx\, dy\, dz = \iint [(\omega\Phi)_u - (\omega\Phi)_l] \frac{dx}{g}\, dy + \iiint \omega\alpha \frac{dp}{g}\, dx\, dy \qquad (7)$$

where the integration is taken between two closed pressure surfaces.

Thus, we define the following processes which are represented in (6):

$$\iint [(\omega\Phi)_u - (\omega\Phi)_l] \frac{dx}{g}\, dy + \iiint \omega\alpha \frac{dp}{g}\, dx\, dy = \text{transformation process}$$

$$\iint [(\omega K)_u - (\omega K)_l] \frac{dx}{g}\, dy = \text{transport process}$$

$$\iiint D \frac{dp}{g}\, dx\, dy = \text{dissipation process}$$

where, of course, the sum of the three processes make up the observed change in the horizontal kinetic energy of the portion of the atmosphere being sampled between two closed pressure surfaces. It is noteworthy that the integral involving the boundary values of $\omega\Phi$ represents in essence a transport of potential energy, and yet it more rigorously belongs with the energy transformation process and is a consequence of the slope of pressure surfaces relative to constant heights.

The vertical motion equation. In the present study an attempt is made to examine the integral of $\omega\alpha$ as well as the integrals of $\omega\Phi$ and ωK horizontally over the northern hemisphere north of 20°N and vertically by pressure increments within the layer from 1000 to 50 mb.

The crucial measurement is the one involving the instantaneous pressure change or the vertical motion term, ω. If we expand the total derivative of temperature with respect to time in the

equation which expresses the first law of thermodynamics for our atmosphere, and assume adiabatic changes, we obtain

$$-\frac{\omega}{\rho g} = -\frac{\partial T/\partial t + \mathbf{V}\cdot\nabla_p T}{g/c_p + \partial T/\partial z} \qquad (8)$$

where T is the temperature and c_p is the specific heat at constant pressure.

It is interesting to note that the left-hand side of (8) is the well-known approximate expression for the vertical velocity, w [see *Charney*, 1948, for example]. The expression on the right-hand side of (8) was evaluated for several pressure layers from actual upper-air data taken every 12 hours at approximately 100 stations over the northern hemisphere for the months of January and April 1958. All the individual components of the right-hand side of (8) except one are readily determined from available constant-pressure data. The one exception is the term $\mathbf{V}\cdot\nabla_p T$, which represents the mean temperature advection within the layer. The evaluation of this term was accomplished in a manner somewhat similar to the technique first presented by *Panofsky* [1944].

The final expression submitted in the finite-difference form for machine computational purposes and designed to give the 12-hour time-averaged vertical velocity (with equivalence to ω noted) for a specified isobaric layer and for a specified station, using the data from that station only, is as follows:

Statistical analysis. Computation of the vertical motion by means of (9) permits examination of the variability of this quantity itself and also of its statistical interrelationships with parameters such as α, Φ, and κ as a function of specified layers up to the top of the atmosphere. Seven layers were selected for this study, namely, 1000–850, 850–700, 700–500, 500–300, 300–200, 200–100, and 100–50 mb. The vertical-motion data computed by (9) and the associated 12-hour mean temperatures were tabulated by 12-hour periods for the months of January and April 1958 and for approximately 100 stations located north of 20°N. The mean temperature was used in lieu of the specific volume in the actual evaluation of that portion of the transformation process represented by the integral of $\omega\alpha$.

In the statistical treatment of the integrals containing $\omega\alpha$, $\omega\Phi$, and $\omega\kappa$, we follow closely the technique first applied to hemispheric data by *Starr and White* [1952] and adopt the bar notation for the time average of a quantity between specified limits, the bracket for the zonal average, and the brace for the meridional average. Further, the notation of a single prime denotes the eddy or turbulent fluctuations of the quantity about its time average, a double prime the deviation about its zonal average, and a triple prime the deviation about its meridional average.

Inasmuch as we do not integrate over the entire atmosphere nor do we perhaps take a long enough sample in time, it is possible for

$$w = -\frac{\omega}{\rho g} = -\frac{\frac{1}{2}(T_1 + T_2)\big|_{t=0} - \frac{1}{2}(T_1 + T_2)\big|_{t=-12}}{\frac{g}{c_p} + \frac{1}{2}\left[\frac{T_2 - T_1}{\Delta h}\Big|_{t=0} + \frac{T_2 - T_1}{\Delta h}\Big|_{t=-12}\right]}$$
$$+\frac{\frac{1}{2}\left[V_1 V_2 \frac{\Delta\sigma}{\Delta h}\Big|_{t=0} + V_1 V_2 \frac{\Delta\sigma}{\Delta h}\Big|_{t=-12}\right]\frac{1}{4}\left[(T_1 + T_2)\big|_{t=0} + (T_1 + T_2)\big|_{t=-12}\right]\frac{\pi f}{(980)(180)}}{\frac{g}{c_p} + \frac{1}{2}\left[\frac{T_2 - T_1}{\Delta h}\Big|_{t=0} + \frac{T_2 - T_1}{\Delta h}\Big|_{t=-12}\right]} \qquad (9)$$

where the subscripts 1 and 2 refer to the higher and the lower pressure surfaces, respectively, $\Delta\sigma$ is the change in wind direction in degrees of the compass computed as the wind direction at the lower pressure surface minus the wind direction at the higher pressure surface measured positive clockwise from north, Δh is the thickness between the two pressure surfaces, and v is the wind speed.

the mean-motion terms, such as $\{[\bar{\omega}]\}\{[\bar{T}]\}$, to have questionable nonzero values, which, however, we cannot hope to measure separately. Thus, to get a more representative value of the total contribution from the individual transformation and transport integrals, we subtract the possibly spurious mean-motion term from both sides of the eddy correlation equations,

TABLE 1. Mean Vertical Motion in Centimeters per Second by Pressure Layer and by Latitude for January 1958

Pressure Layer, mb	Latitude						
	20°	30°	40°	50°	60°	70°	80°
1000–850	0.281	0.728	1.301	1.392	0.678	0.567	0.347
850–700	−0.036	−0.128	0.317	0.711	0.469	0.186	0.011
700–500	−0.067	−0.214	0.125	0.503	0.080	−0.206	0.133
500–300	0.060	−0.061	0.008	0.411	0.197	−0.253	−0.044
300–200	−0.531	0.403	0.033	0.194	−0.117	−0.642	−0.306
200–100	−0.036	0.008	−0.081	−0.061	−0.106	−0.039	−0.081
100–50	0.007	−0.017	0.031	0.014	0.039	0.206	−0.139

with the result (shown only for the case of ω and T) that

$$\{[\overline{\omega T}]\} - \{[\bar{\omega}]\}\{[\bar{T}]\} = \{[\overline{\omega' T'}]\} + \{[\bar{\omega}'' \bar{T}'']\} + \{[\bar{\omega}]'''[\bar{T}]'''\} \qquad (10)$$

The following equivalences are noted relative to the mechanical energy equation (6), and we now introduce the vertical integration with respect to pressure as denoted by the parentheses and further assume that lower (50-mb) boundary values vanish in the integrals of $\omega\Phi$ and $\omega\kappa$:

$$\frac{R}{pg}\{([\overline{\omega T}])\} - \frac{R}{pg}\{([\bar{\omega}])\}\{([\bar{T}])\} = +\iiint \omega\alpha \frac{dp}{g}\,dx\,dy \qquad (11)$$

$$\frac{1}{g}\{([\overline{\omega\Phi}])\} - \frac{1}{g}\{([\bar{\omega}])\}\{([\bar{\Phi}])\} = +\iint \omega\Phi \frac{dx}{g}\,dy \qquad (12)$$

$$\frac{1}{g}\{([\overline{\omega \mathrm{K}}])\} - \frac{1}{g}\{([\bar{\omega}])\}\{([\bar{\mathrm{K}}])\} = +\iint \omega \mathrm{K} \frac{dx}{g}\,dy \qquad (13)$$

It is seen from (10) that the integrals of ωT, $\omega\Phi$, and $\omega\kappa$ can each be resolved into contributions from three eddy processes, namely, the transient eddies, the standing eddies (zonal), and the standing eddies (meridional).

Vertical motion and temperature results. The January latitudinal averages of the vertical motion, in centimeters per second, for each of the pressure layers are listed in Table 1, and the corresponding values for April are listed in Table 2.

The January and April mean values of temperature in degrees Kelvin as a function of pressure layer and latitude are listed in Tables 3 and 4.

Figure 1 shows the January zonal averages of the atmospheric vertical motion as a function

TABLE 2. Mean Vertical Motion in Centimeters per Second by Pressure Layer and by Latitude for April 1958

Pressure Layer, mb	Latitude						
	20°	30°	40°	50°	60°	70°	80°
1000–850	0.152	0.225	0.614	0.861	1.133	0.639	0.060
850–700	−0.063	0.175	0.517	0.678	0.697	0.428	0.128
700–500	0.192	0.117	0.042	0.131	0.161	−0.022	−0.011
500–300	0.003	−0.311	−0.722	−0.320	0.631	−0.064	−0.281
300–200	0.222	0.028	−0.256	0.372	0.375	0.475	0.511
200–100	−0.065	−0.083	−0.072	0.015	−0.010	0.014	−0.103
100–50	−0.041	0.026	0.094	0.066	0.024	0.007	0.012

TABLE 3. Mean Temperature in Degrees Kelvin by Pressure Layer and Latitude for January 1958

Pressure Layer, mb	Latitude						
	20°	30°	40°	50°	60°	70°	80°
1000–850	288.8	282.2	272.6	264.7	258.4	251.9	247.6
850–700	283.0	276.1	268.6	262.4	256.8	251.0	247.9
700–500	273.0	265.6	257.8	251.6	246.6	242.6	240.3
500–300	250.9	244.7	238.3	232.7	228.3	224.8	223.8
300–200	228.3	224.6	221.3	218.0	216.3	212.8	210.9
200–100	207.8	212.0	216.8	216.9	214.6	212.9	212.0
100–50	204.0	209.7	215.8	218.6	217.5	215.0	213.4

of latitude and as a function of the following layers: 1000–50, 850–50, 700–50, and 500–50 mb. Figure 2 is the comparable presentation for April. Both figures also have the zonal average of temperature for the layer from 1000 to 50 mb plotted as a function of latitude. In general, the combined effects of the vertical motion and temperature curves in Figures 1 and 2 tend to support the historical feature of a strong indirect meridional cell in mid-latitudes with the suggestion of less intense direct cells to the north and to the south. The indirect cell, or one with rising cold air and sinking warm air, appears to be quite striking and gives credence to ideas presented by *Ferrel* [1856] and subsequently modified by *Rossby* [1941]. The peak value of the cellular motion of approximately 0.3 cm/sec for the layer from 700 to 50 mb compares reasonably well with the peak value of about 0.1 cm/sec found for the month of February 1959 by *Saltzman and Fleisher* [1959] who used vertical-motion data obtained by the numerical weather prediction method. *Phillips* [1954] found that the simple baroclinic waves in his two-level, quasi-geostrophic model are accompanied by a weak meridional circulation pattern, similar to the observational evidence of Figures 1 and 2 but with a somewhat reduced maximum vertical motion of about 0.05 cm/sec.

In the long-time mean, the area under each of the vertical-motion curves in Figures 1 and 2 should be zero when closed pressure surfaces over the whole hemisphere are considered. The actual values of the integrals are listed in Table 5.

The integrated values of the vertical motion given in Table 5 decrease in algebraic magnitude as the lower layers of the atmosphere are consecutively eliminated from the computations. In fact, the values for April pass through the null point. A zero value for the integral appears to occur for a lower boundary of about 700 mb when the combined January and April data (Table 5) are considered. This suggests the influence in the lower atmosphere of two effects which were neglected in the computations. First, the nonadiabatic effects would have to act in

TABLE 4. Mean Temperature in Degrees Kelvin by Pressure Layer and by Latitude for April 1958

Pressure Layer, mb	Latitude						
	20°	30°	40°	50°	60°	70°	80°
1000–850	291.5	285.9	279.3	273.3	267.5	257.1	248.5
850–700	285.7	281.4	274.3	267.3	261.4	254.2	248.8
700–500	274.5	269.3	262.5	255.6	250.0	243.9	238.9
500–300	252.6	247.3	241.4	235.5	231.5	228.5	226.5
300–200	228.8	224.7	222.3	220.7	219.7	222.3	225.3
200–100	208.4	210.6	214.8	219.1	221.7	223.6	224.8
100–50	205.2	210.7	216.2	220.0	222.0	223.0	221.6

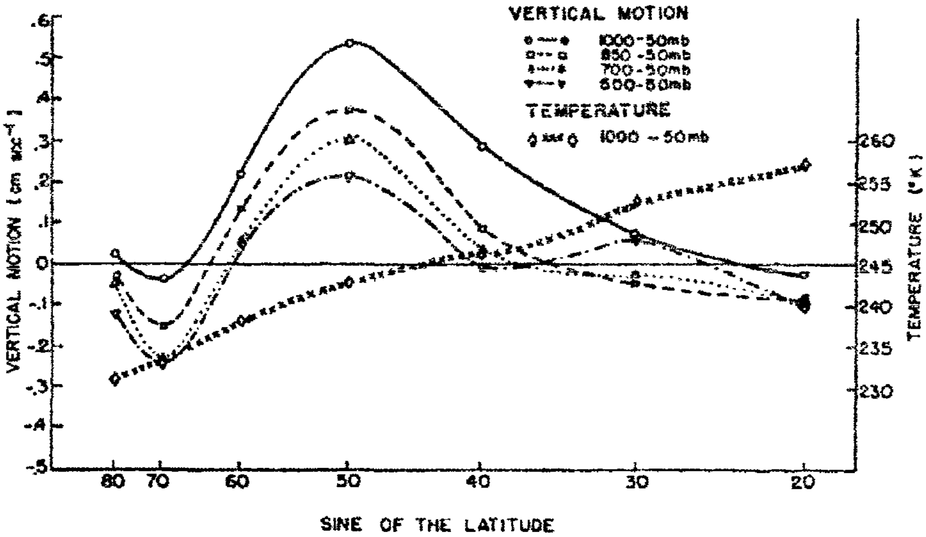

Fig. 1. Mean vertical motion and mean temperature as a function of latitude and pressure layer for January 1958.

the sense of a net cooling in order to influence the computed vertical velocities in the upward or positive sense. For the lower layers of the atmosphere, at least, this would seem to rule out the neglect of the nonadiabatic effects as being the dominant cause of the seemingly large positive values of vertical motion. The second influence involves the nongeostrophic effects within the friction layer which may be defined as the portion of the atmosphere below 800 meters. Within this layer, the wind veers with height approximately in accordance with the *Ekman* [1902] spiral, where the angle between the actual wind at anemometer level and the geostrophic wind at the top of the friction layer is about 20° [*Taylor*, 1915]. The veering of the

TABLE 5. Integrated Values of the Vertical Motion in Centimeters per Second by Pressure Layer between 20°N and 80°N for January and April 1958

Pressure Layer, mb	January	April
1000–50	+0.1702	+0.1330
850–50	+0.0501	+0.0626
700–50	+0.0149	−0.0016
500–50	+0.0045	−0.0518

wind with height implies warm-air advection from the thermal wind relation. This latter relation was applied in evaluating the temperature advection term in (9), but actual rather than geostrophic winds were used; thus there is a strong possibility that nongeostrophic effects produced a systematic positive error in the computations of vertical motion for the layer from 1000 to 850 mb. Some of this effect undoubtedly extends into the next higher layer (850 to 700 mb) since there is an indication in Tables 1 and 2 that the vertical motion for this layer is perhaps too great on the positive side. In general, Table 5 shows that the layer from 700 to 50 mb is fairly representative of quasi-geostrophic conditions in the free atmosphere when the integrated values of vertical motion for both January and April are considered.

Energy transformation due to vertical motion and temperature covariance. The energy transformation rates due to the transient eddies and to the standing eddies (zonal) shown in (10) have been added, and the results appear in tabular form in Table 6 and as a function of pressure-height in Figure 3. The curves for January and April show the same two positive modes, the lower one in the boundary region from 1000 to 850 mb and the other in the mid-tropospheric region from

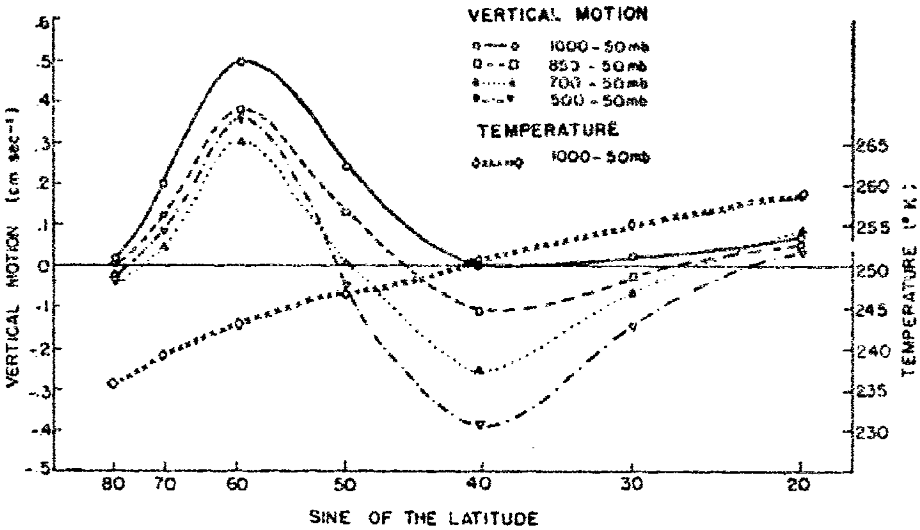

Fig 2. Mean vertical motion and mean temperature as a function of latitude and pressure layer for April 1958.

700 to 500 mb. The lower and more predominant mode in the boundary region is questionable because of previously discussed nongeostrophic and nonadiabatic effects. On the other hand, this could be a realistic phenomenon and a reflection of observed low-level convergence and divergence. The prominent positive mode in the mid-tropospheric region centered at about 600 mb is quite significant and not wholly unexpected. There seems to be little doubt that this is a region in the atmosphere where energy transformation processes take place in a very effective, organized manner. The curve for April gradually turns negative at about 200 mb and remains so above that surface. The curve for January oscillates between negative and positive values above 400 mb and finally turns to negative values above 100 mb.

Saltzman and Fleisher [1959] obtained a value of 2.682 ergs/cm² sec mb for the total or net rate of energy transformation for the month of February 1959. They used thickness data for

TABLE 6. Energy Transformation Rates Due to the Transient Eddies and to the Standing Eddies (Zonal) Listed as a Function of Pressure Layer for January and April 1958
(Units are ergs/cm² sec mb. Positive values signify a transformation from potential to kinetic energy.)

Pressure Layer, mb	January			April		
	Transient Eddies	Standing Eddies (zonal)	Total	Transient Eddies	Standing Eddies (zonal)	Total
1000–850	3.068	5.905	8.973	0.616	5.845	6.461
850–700	2.338	1.739	4.077	2.410	1.202	3.612
700–500	3.495	3.531	7.026	1.953	2.238	4.191
500–300	1.057	3.233	4.290	2.146	0.219	2.365
300–200	−1.392	1.057	−0.335	1.450	−0.950	0.500
200–100	−0.246	1.624	1.378	−0.594	−0.273	−0.867
100–50	−0.422	−0.405	−0.827	−0.144	−0.625	−0.769

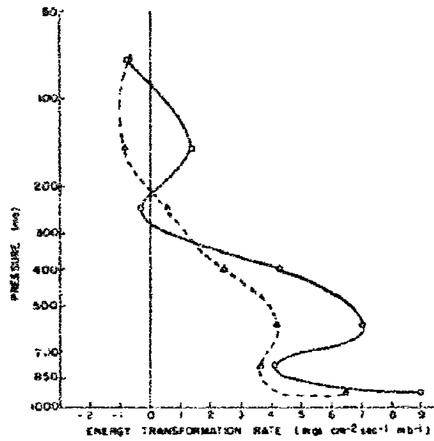

Fig. 3. Energy transformation rates due to the combined effect of the transient eddies and the standing eddies (zonal) as a function of pressure-height for the layer from 1000 to 50 mb for January 1958 (solid curve) and April 1958 (dashed curve).

the layer from 850 to 500 mb to obtain the field of temperature with values of ω computed at 600 mb obtained from the Joint Numerical Weather Prediction Unit two-parameter, quasi-geostrophic, adiabatic, frictionless model. Values of the transformation rates between 850 and 500 mb for January (Table 6), integrated and combined with the tropospheric contribution of -1.488 ergs/cm² sec mb for the remaining standing eddies (meridional), would yield a net transformation rate of 4.274 ergs/cm² sec mb, which is somewhat larger than the corresponding one obtained by Saltzman and Fleisher.

Energy transformation due to vertical motion and potential energy covariance. From (6) and (7) it is shown that the integral of $\omega\alpha$ plus the integral of $\omega\Phi$ jointly represent the energy transformation process. The integral of $\omega\Phi$ is evaluated, however, as a boundary value of the flux of potential energy through some arbitrary pressure surface with units of ergs/cm² sec.

Since the integrals of $\omega\Phi$ and $\omega\kappa$ in (6) both entail boundary values, it would appear reasonable to dispense with both integrals by specifying boundary conditions such that both integrals vanish. This seems to be not too unrealistic, since $\omega \to 0$ at the outer limits of the atmosphere and since it is difficult to visualize a flux of kinetic or potential energy through the surface of the earth. If this is the case, the roles played by the integrals of $\omega\Phi$ and $\omega\kappa$ in (6) are probably nothing more than those involving some internal adjustments which are required in response to the energy transformation function represented by the integral of $\omega\alpha$.

It appears worthwhile, however, in consideration of the energy balance equation, to investigate the magnitude of the flux of kinetic energy as well as that of the flux of potential energy. The former is treated in detail in a later section, and we are indebted to *Roberts* [1960] for the values of the flux of potential energy which are listed in Table 7 as a function of pressure layer and eddy process for January 1958.

The contribution from the transient eddies in Table 7 is negative up to 300 mb and becomes positive within the layers from 300 to 200 mb and 100 to 50 mb. This suggests that rising motion is associated with troughs and sinking motion with ridges in the troposphere, whereas the reversal of the sign of the flux above 300 mb suggests an opposite-phase relationship in certain layers of the stratosphere, with ascending motion in ridges and descending motion in troughs. This phase shift has been previously noted for the upper stratosphere by *Kochanski* [1954] and by *Austin and Krawitz* [1956].

Energy transport due to vertical motion and kinetic energy covariance. This transport process is represented by (13). A cursory examination of the hemispheric distribution of the covariance between vertical motion and horizontal kinetic energy shows that, in general, and for both months, there is a net upward flux of kinetic

TABLE 7. Vertical Flux of Potential Energy as a Function of Pressure Layer and Eddy Process for January 1958
(Units are ergs/cm² sec. Positive values signify an upward flux of potential energy.)

Pressure Layer, mb	Transient Eddies	Standing Eddies (zonal)	Standing Eddies (meridional)
1000-850	−322.5	−2749.5	−137.4
850-700	−1263.2	−207.3	−2158.0
700-500	−241.3	+1049.4	−1673.1
500-300	−465.2	+747.2	−576.9
300-200	+756.2	+270.9	+205.6
200-100	−102.8	+338.3	+108.0
100-50	+107.9	−48.0	−78.2

TABLE 8. Vertical Transport of Kinetic Energy Due to the Three Eddy Processes Integrated with Respect to Longitude and Listed as a Function of Pressure Layer for January and April 1958 (Units are ergs/cm² sec. Positive values signify an upward transport of horizontal kinetic energy.)

	January			April		
Pressure Layer, mb	Transient Eddies	Standing Eddies (zonal)	Standing Eddies (meridional)	Transient Eddies	Standing Eddies (zonal)	Standing Eddies (meridional)
1000–850	306.43	80.37	70.79	219.79	156.18	23.26
850–700	159.35	−22.83	34.40	220.99	91.92	45.67
700–500	18.40	−158.08	5.12	71.84	9.20	−6.72
1000–500	147.06	−37.37	46.71	.59.95	78.15	19.71

energy due to the transient eddies for the layers 1000–850, 850–700, and 700–500 mb. This seems contradictory to intuitive reasoning, which would lead one to suspect that the flux of kinetic energy should be directed downward in order to balance the dissipation of kinetic energy in the boundary region. By the same token, however, one might also reason that there should be an upward flux of kinetic energy in order to help maintain the strong westerlies against dissipative action. It has been shown by *Starr and White* [1951], for example, that angular momentum is transported poleward mainly by the transient eddies and that this flux of momentum also helps to maintain the westerlies. Both the vertical transportation of kinetic energy and the poleward transportation of angular momentum are directed against the gradient of a wind velocity. The significance of a net positive value of the covariance between vertical motion and horizontal kinetic energy is that rising motion must occur predominantly in regions of maximum wind velocity. In the synoptic sense, this means that on the average there must be stronger winds to the lee of troughs where rising motion is normally evident. The vertical transport of kinetic energy due to the three eddy processes is shown in Table 8.

Energy balance results. From a scale factor analysis the values obtained for the integral of $\omega\Phi$ should be about an order of magnitude larger than the values for $\omega\kappa$, or approximately equal to the ratio of the Coriolis parameter to the relative vorticity. This appears to be borne out fairly well, as can readily be verified by comparing Tables 7 and 8.

The integrals in the mechanical energy equation (6) representing energy transformation and vertical flux processes have been individually evaluated for various pressure layers. The sum of these integral values should balance, ideally at least, the dissipation of horizontal kinetic energy within the region of the atmosphere being sampled. In this analysis, the contributions due to the standing eddies (meridional) will be omitted, since these values are not considered to be as stable as the contributions computed for the two other eddy processes. Using Tables 6, 7, and 8 for data for the integral values of $\omega\alpha$, $\omega\Phi$, and $\omega\kappa$, respectively, we have computed energy dissipation rates for selected layers of the atmosphere for January 1958 (Table 9). For the layers which extend to 50 mb the flux of energy across that pressure surface is neglected. This appears to be justified by the small net flux of potential energy shown in Table 7 for the layer from 100 to 50 mb. In addition, the flux of kinetic energy is probably negligible at high levels; for example, the kinetic energy flux has been computed by *Hanrote and Lambert* [1960] to be approximately 10 ergs/cm² sec for the layer from 200 to 100 mb for April 1958. It is also assumed (Table 9) that the transports of potential energy and kinetic energy vanish at 1000 mb, which is taken to be the surface value of pressure at $Z_s = 0$. Furthermore, the flux of kinetic energy is neglected above 500 mb, and this seems justified since it is an order of magnitude smaller than the flux of potential energy.

A separation of data in Table 9 is made in order to distinguish between the dissipation rates computed, in general, for rather deep layers, each extending to 50 mb, versus the rates computed for rather shallow layers with depths of from 50 to 75 mb.

Brunt [1941] estimated the energy dissipation rate to be approximately −3000 ergs/cm² sec in

TABLE 9. Energy Dissipation Rates within Certain Atmospheric Layers for January 1958 (in ergs/cm² sec)

Pressure Layer, mb	$\omega\alpha$	$(\omega\Phi)_s - (\omega\Phi)_t$	$(\omega K)_s - (\omega K)_t$	Energy Dissipation Rates
1000–50	+4283.6	0	0	−4283.6
925–50	+3610.7	−3072.0	+386.8	−925.5
850–50	+2937.7	−2271.3	+261.7	−928.1
775–50	+2631.9	−1470.5	+136.5	−1297.9
700–50	+2326.1	−662.4	−1.6	−1662.1
600–50	+1623.5	+808.1	−139.7	−2291.9
500–50	+920.9	+595.1		−1516.0
400–50	+491.9	+282.0		−773.9
300–50	+62.9	+654.6		−717.5
250–50	+79.6	+1027.1		−1106.7
200–50	+96.4	+631.3		−727.7
150–50	+27.5	+235.5		−263.0
100–50	−41.4	+147.7		−106.3
1000–925	+672.9	+3072.0	−386.8	−3358.1
925–850	+672.9	−800.7	+125.1	+2.7
850–775	+305.8	−800.8	+125.2	+398.8
775–700	+305.8	−808.1	+134.9	+367.4
700–600	+702.6	−1470.5	+138.1	+629.8
600–500	+702.6	+213.0	−139.7	−775.9
500–400	+429.0	+313.1		−742.1
400–300	+429.0	−372.6		−56.4
300–250	−16.8	−372.5		+389.3
250–200	−16.8	+395.8		−379.0
200–150	+78.9	+395.8		−474.7
150–100	+68.9	+87.8		−156.7
100–50	−41.4	+147.7		−106.3

the layer from the ground up to about 1 km, and this compares favorably with the value of −3358.1 ergs/cm² sec in Table 9 for the layer from 1000 to 925 mb. Brunt further estimated the energy dissipation rate to be approximately −2000 ergs/cm² sec in the atmosphere above 1 km, with a total dissipation rate of about −5000 ergs/cm² sec. This latter estimate would correspond to the value of −4283.6 ergs/cm² sec (Table 9) computed for the layer from 1000 to 50 mb.

The curious feature in Table 9 is the appearance of a group of positive values of the dissipation rate within the four shallow pressure layers located between 925 and 600 mb. Also, there is one additional positive value in the layer from 300 to 250 mb. The locations of these positive dissipation rates suggest that on the average they occur just above the friction layer and near the tropopause, respectively. Both these regions are characterized by wind maxima, although the lower-level jet is certainly not as pronounced as the more familiar upper-level jet. Further, both these regions contain turbulent eddies of the type often referred to as clear air turbulence. It is entirely possible that these eddies are of such scale that they were not detected in the data used in this study and further that these eddies act to produce an amount of kinetic energy sufficient to change the sign of the dissipation rate in the energy balance equation.

Conclusions. The energy-transformation process represented by the integral of $\omega\alpha$ in the mechanical energy equation (6) exhibits two significant positive (potential to kinetic energy) modes, one in the boundary region from 1000 to 850 mb and one in the middle troposphere from 700 to 500 mb. It is probably not too surprising to find that one of the regions of more intense energy-transformation activity is close to the commonly observed region of nondivergence at 600 mb. The occurrence of very intense energy transformations within the boundary region is questionable in view of certain nongeostrophic and nonadiabatic effects which probably influenced the computations of vertical motion in this region. It is conceivable, however, that the computed rates of energy transformation are not spurious for this boundary region, since the rates of energy dissipation increase rather sharply in layers close to the ground. Reference is made to the work of *Davidson and Lettau* [1957], for example.

The evidence for meridional cell activity appears quite striking and conforms in general with the usual arrangement of an intense mid-latitude indirect cell with direct cells to the north and to the south. The northward phase shift of the cells from January to April seems to be consistent with synoptic evidence of the northward displacement of the baroclinic storm tracks and the semipermanent pressure systems from winter to summer. Granting the existence of the indirect mid-latitude cell, the inclusion of the implied nonadiabatic effects would have resulted in an intensification of the cellular motion.

The flux of potential energy has been determined to be about an order of magnitude larger than the transport of kinetic energy, and this is in agreement with an analysis of these two

processes in which appropriate scale factors were used.

As there is no net change in the kinetic energy of the atmosphere over a long period of time, values corresponding to the integrals of $\omega\alpha$, $\omega\Phi$, and $\omega\kappa$ were introduced into the mechanical energy equation (6), and the residual was taken to represent the dissipation rate for the horizontal kinetic energy. This was done for various regions of the atmosphere. The resulting values agree very well with other independent estimates, being about -3000 ergs/cm^3 sec in the friction layer and about -1000 ergs/cm^3 sec above the friction layer. There are certain shallow layers in the atmosphere, located just above the friction layer and also near the tropopause, where the dissipation rates are computed to be positive. This indicates that the energy-transformation process, as measured, is not producing the required amount of kinetic energy for a proper atmospheric energy balance, and thus the dissipative action must unrealistically produce rather than destroy kinetic energy. It is more likely, of course, that the dissipative action is small in these regions and that there are other eddies, not discerned by the methods of this study, which tend to make up the deficit of kinetic energy in the energy balance. There is the suggestion that eddies representing clear air turbulence, for example, are of the type of process which has not been detected in the present analysis.

This study has been based upon data from two specific months (January and April 1958), and it is not necessarily a representative sample in time. Further, the portion of the atmosphere that was sampled covered only the northern hemisphere horizontally between 20° and 80° and vertically between 1000 to 50 mb, which is not necessarily a representative sample in space. The only data that were tested for significance were the set of station values of $\bar{\omega}T$ for a few pressure layers. The 95 per cent confidence factor of $2\sigma/\sqrt{N}$ was used. The sets of data examined produced confidence limits which indicate a reasonable degree of significance, e.g., a mean value of 207 ± 75. A more rigorous significance test could be made only with considerable effort; daily maps of vertical motion, temperature, kinetic energy, and potential energy would be needed in order to get daily values of the various covariances which could then be tested in sets. On the other hand, one could perform the same type of study as the present one, but for several additional months, and then test the significance of the acquired set of monthly mean values of the covariances.

Acknowledgments. I am grateful to Dr. Robert M. White for his suggestion of the topic of this report and for his initial guidance. Much appreciation is due Professor Victor P. Starr for his never-ending encouragement and indispensable advice throughout the 14 months devoted to this study.

Basic meteorological data were machine-processed in an expert manner by the staff of the Air Weather Service Climatic Center, and these processed data formed the foundation for this study. In particular, I wish to acknowledge the cooperation of Lt. Col. George W. Moxon, the Director of the Center.

I am also indebted to the staff of the M.I.T. Computation Center for providing IBM 704 computer time and other assistance in the initial phase of this study.

Considerable support in the form of card punching, machine programming and IBM 650 computer time were provided by the computer branch of the Geophysics Research Directorate at L. G. Hanscom Field. I appreciate the efforts of Mrs. Janet Dwyer, Miss Dottie Quinn, Mr. Dan La Chance, Mr. Milton Jones, and Airman Carl Collins.

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Further Statistics on the Modes of Release of Available Potential Energy

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The measurements of the hemispheric spectra of individual pressure change, temperature, available potential energy, and rate of conversion between available potential energy and kinetic energy made for each day of February 1959, and reported previously in this Journal [*Saltzman and Fleisher*, 1960], have been extended to cover the winter half-year period, January through March and October through December, 1959. We present these new results in Tables 1 to 3 and Figures 1 to 7, using the same notation as was employed in the previous article.

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TABLE 1. Mean Values and Standard Deviations of $[\omega]$, $[T]$, $[\omega'^2]$, $[T'^2]$ and $[\omega'T']$ as a Function of Latitude ϕ, for the Winter Half-Year, January through March and October through December, 1959

ϕ	80	75	70	65	60	55	50	45	40	35	30	25	20
$\overline{[\omega]}$*	−9	+312	−9	−441	−618	−504	−48	+638	+743	+768	+349	−231	−491
σ	(±)3353	3517	2825	3274	3648	3668	3059	3663	3232	3416	2203	1494	1928
$\overline{[T]}$†	246.84	248.27	249.70	251.13	252.83	254.81	257.54	261.11	265.28	269.31	272.92	275.82	278.30
σ	(±)4.22	3.63	3.19	2.89	2.94	3.12	3.39	3.69	3.77	3.51	3.02	2.24	1.34
$\overline{[\omega'^2]}$‡	122	155	189	244	292	428	569	667	760	606	466	279	163
σ	(±)114	94	117	138	155	220	249	255	316	259	245	156	106
$\overline{[T'^2]}$§	20.38	28.24	34.02	37.68	39.66	42.85	43.08	37.88	23.29	18.32	11.43	6.35	3.06
σ	(±)13.01	13.65	12.19	13.17	13.21	15.19	17.43	16.46	12.85	8.86	5.68	3.27	1.37
$\overline{[\omega'T']}$‖	−422	−414	−541	−698	−696	−945	−1256	−1453	−1261	−722	−209	+60	+69
σ	(±)604	557	558	569	573	725	764	715	678	581	366	244	149

* g cm^{-1} sec^{-3} × 10^{-4}.
† degrees absolute.
‡ g^{-2} cm^{-2} sec^{-6} × 10^{-3}.
§ (degrees absolute)^2.
‖ g cm^{-1} sec^{-3} deg abs × 10^{-3}.

TABLE 2. Mean Values and Standard Deviations of $2\,|\Omega(n)|$, $2\,|B(n)|$, $\mathcal{P}(n)$, and $\mathcal{C}(n)$ as a Function of Wave Number n, for the Winter Half-Year, January through March and October through December, 1959

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$2\,\lvert\overline{\Omega(n)}\rvert^*_{\phi=45°N}$	1764	1992	2017	2115	2660	2957	3360	3460	3727	2845	2798	2514	2243	2060	1985
σ	(±)1041	1028	1185	1174	1458	1696	1641	1700	1832	1502	1304	1479	1144	1139	1160
$2\,\lvert\overline{B(n)}\rvert^†_{\phi=45°N}$	3.84	3.49	3.19	2.49	2.42	2.18	1.65	1.24	1.05	0.73	0.63	0.52	0.44	0.37	0.31
σ	(±)1.71	1.76	1.42	1.38	1.27	1.18	0.87	0.63	0.48	0.43	0.36	0.27	0.23	0.20	0.17
$\overline{\mathcal{P}}(n)$‡	379	382	206	121	97	79	45	31	18	11	8	5	4	3	2
σ	(±)175	190	129	74	66	52	32	18	10	7	5	3	2	2	1
$\overline{\mathcal{C}}(n)$§	+137	+336	+267	+273	+330	+473	+382	+308	+227	+120	+73	+42	+25	+17	+9
σ	(±)221	269	273	275	342	402	376	287	215	143	104	81	60	49	39

* g cm^{-1} sec^{-3} × 10^{-4}.
† degrees absolute.
‡ ergs cm^{-2} mb^{-1} × 10^{3}.
§ ergs sec^{-1} cm^{-2} mb^{-1} × 10^{-3}.

TABLE 3. Mean Values and Standard Deviations of the Available Potential Energy **P**, and the Rate of Conversion of Available Potential Energy into Kinetic Energy **C** for the Region Between 20°N and 80°N

$\bar{P}^{*} = 6687$	$C_{EST}\dagger = +2671$	
$\sigma \quad \pm 657$	$\sigma \quad \pm 792$	
$\bar{P}_Z = 5296$	$\bar{C}_Z = -344$	$\bar{C}_Z(\text{extrap}) = +352$
$\sigma \quad \pm 555$	$\sigma \quad \pm 578$	
$\bar{P}_E = \sum_{n=1}^{15} \mathcal{P}(n) = 1391$	$\bar{C}_E = \sum_{n=1}^{15} \mathcal{C}(n) = +3016$	
$\sigma \quad \pm 361$	$\sigma \quad \pm 884$	

* ergs cm^{-2} mb^{-1} $\times 10^3$.
† ergs cm^{-2} mb^{-1} sec^{-1} $\times 10^{-3}$.

As would be expected, the results based on the longer record display smoother variations with latitude and wave number than those for the single month. In almost all respects the essential features of the winter average conditions were already quite well represented in the February results—hence, most of the comments made in the previous article apply to those shown here. There are, however, two differences from the February values worth noting. These are (*i*) the

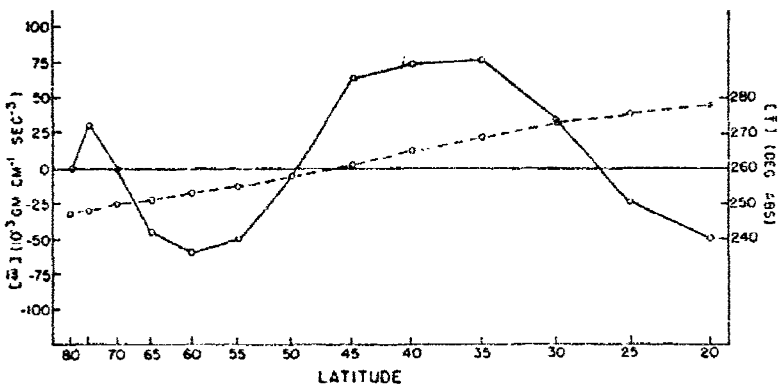

Fig. 1. Mean values of $[\omega]$ (solid curve) and of $[T]$ (dashed curve) for the winter half-year, January through March and October through December, 1959. See Table 1. For the area between 20°N and 80°N $\{[\bar{\omega}]\}$ is $+.013$ g cm^{-1} sec^{-3}, which corresponds to a net descending motion of roughly .02 cm sec^{-1}.

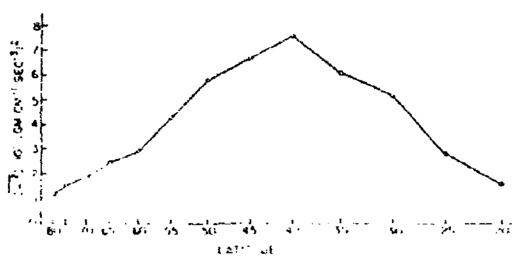

Fig. 2. Mean variance of ω around latitude circles, $\overline{[\omega'^2]}$, for the winter half-year, 1959. See Table 1.

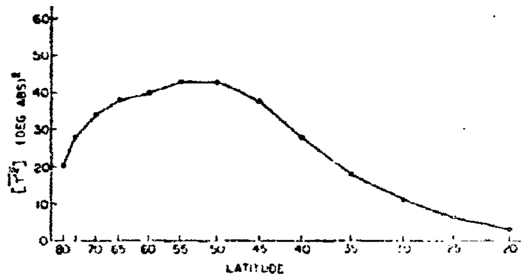

Fig. 3. Mean variance of T around latitude circles, $\overline{[T'^2]}$, for the winter half-year, 1959. See Table 1.

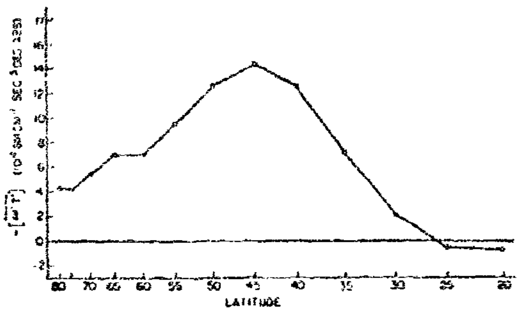

Fig. 4. Mean covariance between ω and T around latitude circles, $-[\overline{\omega' T'}]$, for the winter half-year, 1959. See Table 1.

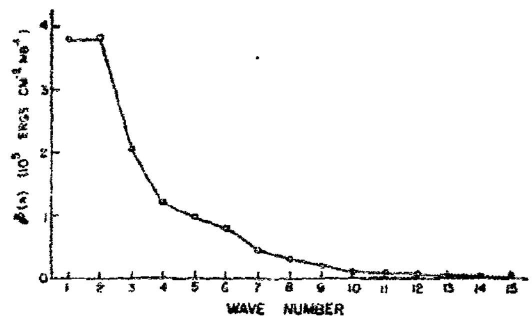

Fig. 6. Mean spectral function for eddy available potential energy, $\mathcal{P}(n)$, for the winter half-year, 1959. See Tables 2 and 3.

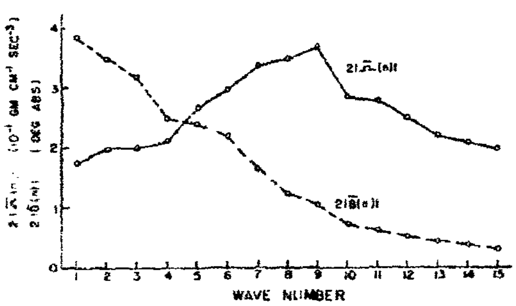

Fig. 5. Mean amplitude spectrum of ω, $2\,|\overline{\Omega(n)}|$ (solid line), and of T, $2\,|\overline{B(n)}|$ (dashed line), both at 45°N for the winter half-year, 1959. See Table 2.

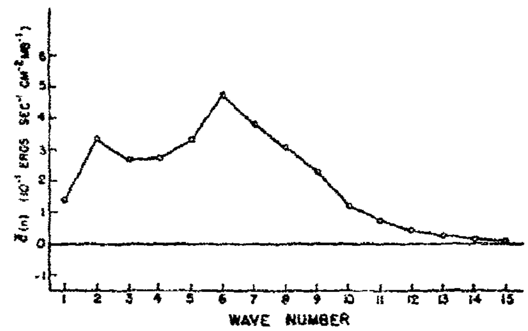

Fig. 7. Mean spectral function for the rate of conversion from eddy available potential energy to eddy kinetic energy, $\mathcal{C}(n)$, for the winter half-year, 1959. See Tables 2 and 3.

southward shift of the distribution of $[\omega]$ (Fig. 1) such that maximum ascending motion is located at 60°N and maximum descending motion is located at 35°N, in closer agreement with the classical concept of the mean meridional cell pattern, and (*ii*) the absence of the pronounced peak of $\mathcal{P}(n)$ at $n = 2$, which evidently was an anomalous condition for February (see Fig. 6).

If we apply the same assumption as was used in the previous article (p. 1220) to obtain an estimate of C_Z for the entire hemisphere, we obtain the value C_Z (extrap) $= +0.352$ ergs sec^{-1} cm^{-2} mb^{-1}, which is to be compared with the value of $+0.294$ ergs sec^{-1} cm^{-2} mb^{-1} for February.

Acknowledgments. We are indebted to the Joint Numerical Weather Prediction Unit for supplying the basic data and to the M.I.T. Computation Center for providing the IBM 704 computer time and running our program.

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Spectral Statistics of the Wind at 500 mb

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1. Introduction

We present a summary of statistics describing the kinetic energy and momentum transport distributions as a function of wave number for the year 1951. The energy transfer computations based on the same raw data as these statistics have already been reported (Saltzman and Fleisher, 1960).

Let us first set down definitions of symbols.

$\lambda \equiv$ longitude
$\phi \equiv$ latitude
$p \equiv$ pressure
$t \equiv$ time
$n \equiv$ wave number around a latitude circle
$a \equiv$ radius of the earth
$g \equiv$ acceleration of gravity
$u \equiv a \cos\phi d\lambda/dt \equiv$ zonal wind speed
$v \equiv ad\phi/dt \equiv$ meridional wind speed

$F(x) \equiv 1/2\pi \int_0^{2\pi} x(\lambda,\phi,p,t) \exp(-in\lambda)d\lambda$
$\equiv$ Fourier transform of x, representing a zonal harmonic analysis
$\equiv X(n,\phi,p,t) \equiv X_1(n,\phi,p,t) - iX_2(n,\phi,p,t)$

$U(n,\phi,p,t) \equiv F(u) \equiv U_1(n,\phi,p,t) - iU_2(n,\phi,p,t)$
$V(n,\phi,p,t) \equiv F(v) \equiv V_1(n,\phi,p,t) - iV_2(n,\phi,p,t)$.

2. Modes of averaging

$$[x] = \frac{1}{2\pi}\int_0^{2\pi} xd\lambda \equiv X(0,\phi,p,t)$$
$\equiv$ zonal average of x

$x' \equiv x - [x]$

$$\{x\} \equiv (\sin\phi_2 - \sin\phi_1)^{-1}\int_{\phi_1}^{\phi_2} x \cos\phi d\phi$$
$\equiv$ meridional average of x between ϕ_1 and ϕ_2

$x'' \equiv x - \{x\}$

$$\bar{x} \equiv \frac{1}{\tau}\int_0^{\tau} xdt$$
$\equiv$ time average of x over the interval τ

$x^* \equiv x - \bar{x}$

$\overline{xy} = \overline{xy}^{(S)} + \overline{xy}^{(T)}$

$\overline{xy}^{(S)} \equiv \bar{x}\bar{y} \equiv$ component of $\overline{xy}$ representing the "stationary" variability of x and y

$\overline{xy}^{(T)} = \overline{x^*y^*} \equiv$ component of $\overline{xy}$ representing the transient variability of x and y

$\sigma(x) \equiv (\overline{x^{*2}})^{\frac{1}{2}} \equiv$ standard deviation of x.

3. Quadratic forms

(1) $E(n,\phi,p,t) \equiv E_u(n,\phi,p,t) + E_v(n,\phi,p,t)$
$\equiv$ spectral function of the zonally averaged kinetic energy per unit mass

(2) $E_u \equiv |U|^2 \equiv U_1^2 + U_2^2 \equiv$ component of E from the zonal motions

(3) $E_v \equiv |V|^2 \equiv V_1^2 + V_2^2 \equiv$ component of E from the meridional motions

(4) $\bar{E}(n,\phi,p)\equiv\bar{E}^{(S)}(n,\phi,p)+\bar{E}^{(T)}(n,\phi,p)$

(5) $\bar{E}^{(S)}\equiv\bar{E}_u^{(S)}+\bar{E}_v^{(S)}\equiv|\bar{U}|^2+|\bar{V}|^2\equiv$ component of $\bar{E}$ due to stationary motions

(6) $\bar{E}^{(T)}\equiv\bar{E}_u^{(T)}+\bar{E}_v^{(T)}\equiv\overline{|U^*|^2}+\overline{|V^*|^2}\equiv$ component of $\bar{E}$ due to transient motions

(7) $K(n,p,t)\equiv\{E(n,\phi,p,t)\}\equiv K_u(n,p,t)+K_v(n,p,t)$

(8) $K_u\equiv\{E_u\}$

(9) $K_v\equiv\{E_v\}$

(10) $k(\lambda,\phi,p,t)\equiv\frac{1}{2}(u^2+v^2)\equiv$ kinetic energy per unit mass

(11) $[k]\equiv k_Z+k_E\equiv$ zonally averaged kinetic energy per unit mass

(12) $k_Z\equiv\frac{1}{2}([u]^2+[v]^2)\equiv\frac{1}{2}E(0,\phi,p,t)$
$\equiv$ component of $[k]$ representing kinetic energy per unit mass of the zonally averaged wind

(13) $k_E\equiv k_{Eu}+k_{Ev}\equiv\sum_{n=1}^{\infty}E(n,\phi,p,t)$

$\equiv$ component of $[k]$ representing the kinetic energy per unit mass of the departure from the zonally averaged wind ("eddy kinetic energy")

(14) $k_{Eu}\equiv\frac{1}{2}[u'^2]$

(15) $k_{Ev}\equiv\frac{1}{2}[v'^2]$

(16) $\bar{k}_Z\equiv\bar{k}_Z^{(S)}+\bar{k}_Z^{(T)}$

(17) $\bar{k}_Z^{(S)}\equiv\frac{1}{2}(\overline{[u]}^2+\overline{[v]}^2)$

(18) $\bar{k}_Z^{(T)}\equiv\frac{1}{2}(\overline{[u]^{*2}}+\overline{[v]^{*2}})$

(19) $\mathbf{k}(p,t)\equiv\{[k]\}\equiv\frac{1}{2}K(0,p,t)+\sum_{n=1}^{\infty}K(n,p,t)$

$\equiv$ total kinetic energy per unit mass averaged over a pressure surface

(20) $$\tau_M(\phi,p,t)\equiv\frac{2\pi a^2\cos^2\phi}{g}[u'v']\equiv\sum_{n=1}^{\infty}J(n,\phi,p,t)$$

$\equiv$ northward eddy transport of zonal angular momentum across latitude ϕ, per unit pressure difference and per unit time

(21) $$J(n,\phi,p,t)\equiv\frac{4\pi a^2\cos^2\phi}{g}(U_1V_1+U_2V_2)$$

$\equiv$ spectral function for the meridional transport of zonal angular momentum across latitude ϕ, per unit pressure difference and per unit time

(22) $J(n,\phi,p)\equiv J^{(S)}(n,\phi,p)+J^{(T)}(n,\phi,p)$

(23) $$J^{(S)}\equiv\frac{4\pi a^2\cos^2\phi}{g}(\bar{U}_1\bar{V}_1+\bar{U}_2\bar{V}_2)\equiv\text{component of } J \text{ due to stationary motions}$$

(24) $$J^{(T)}\equiv\frac{4\pi a^2\cos^2\phi}{g}\overline{(U_1^*V_1^*+U_2^*V_2^*)}\equiv\text{component of } J \text{ due to transient motions.}$$

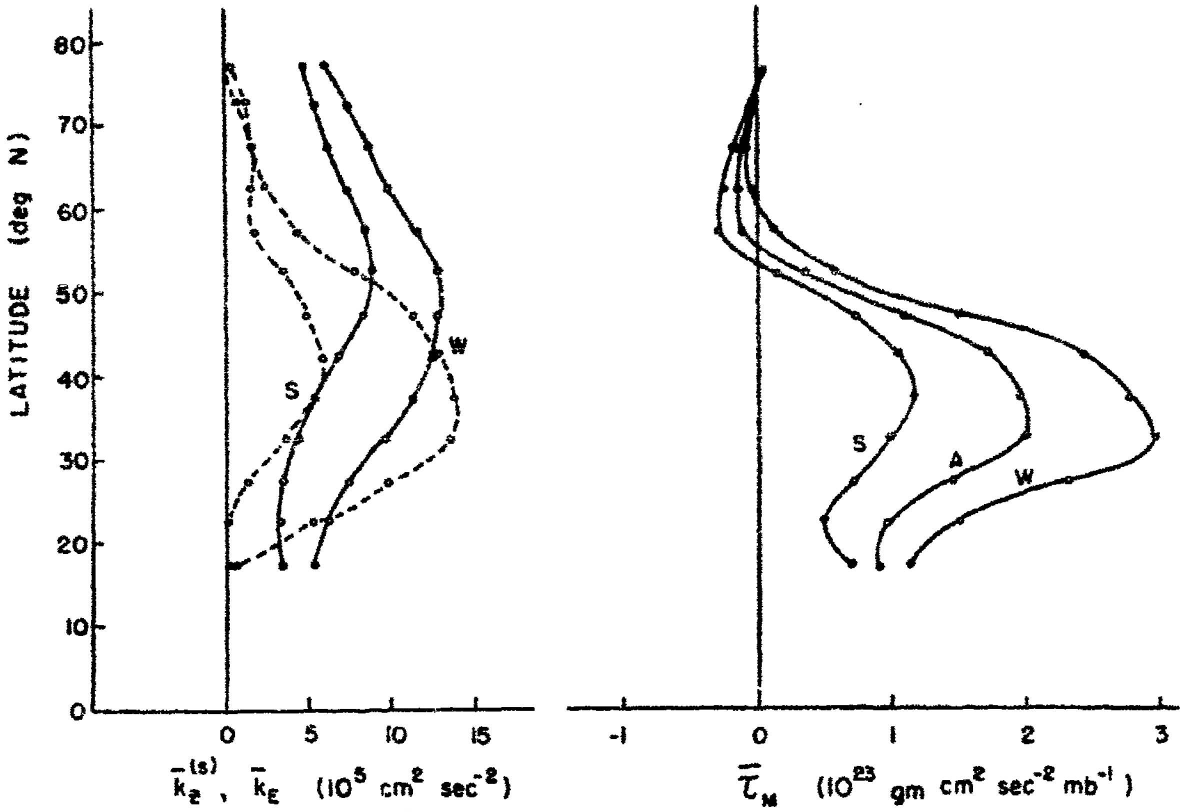

FIG. 1. Latitude-profiles of $\bar{k}_z^{(S)}$ (dashed), $\bar{k}_E$ (solid), and τ_M for winter and summer of 1951, at 500 mb.

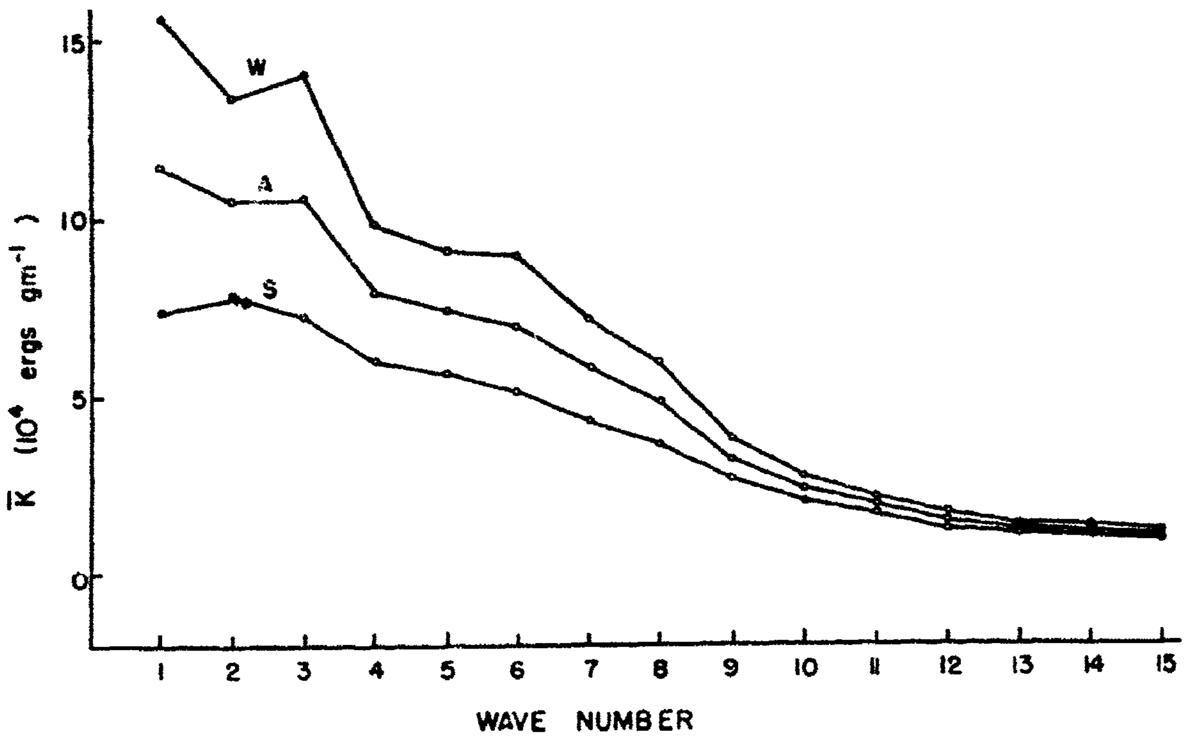

FIG. 2. Spectrum of total kinetic energy per unit mass at 500 mb, averaged over latitude band 17.5N to 77.5N, for winter, summer and year of 1951.

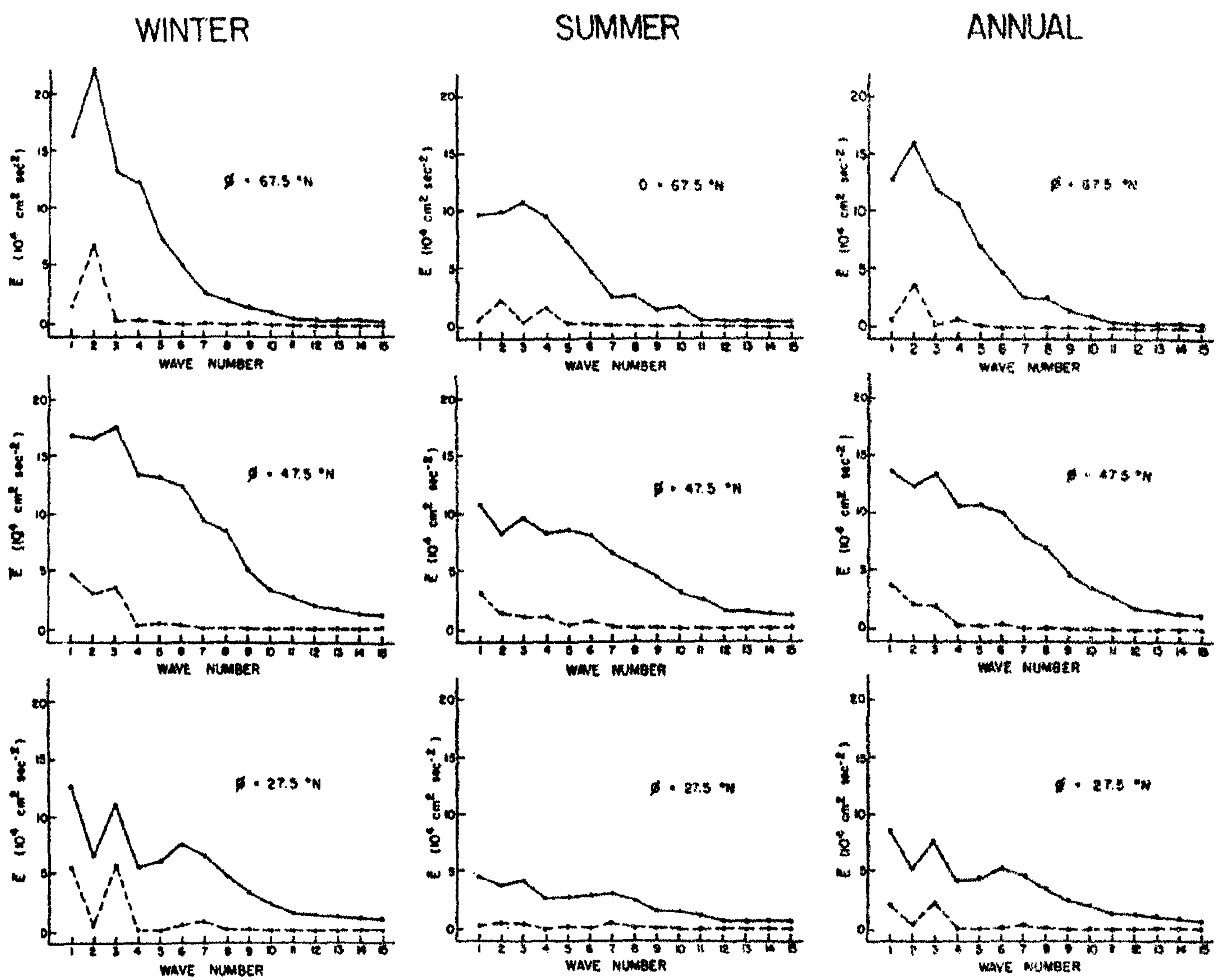

FIG. 3. Spectra of eddy kinetic energy per unit mass at 500 mb, for winter, summer and year of 1951, at latitudes 27.5N, 47.5N and 67.5N. Solid line denotes the total $\bar{E}$, and dashed line denotes the component of $\bar{E}$ due to stationary motions ($\bar{E}^{(S)}$), the difference being the component due to transient motions $\bar{E}^{(T)}$.

TABLE 1. 500-mb wind statistics as a function of latitude for winter, summer and year of 1951. See Table 2 for a resolution of k_E and τ_M into stationary and transient components at 27.5N, 47.5N and 67.5N. Units are 10^{-8} sec^{-1} for $d\lambda/dt$, cm sec^{-1} for $[u]$, 10^{9} ergs gm^{-1} for k, and 10^{20} gm cm^{2} sec^{-2} mb^{-1} for τ_M. Numbers in parentheses to the left of each quantity refer to the list of definitions.

	Statistics	Latitude (ϕ) 17.5°	22.5°	27.5°	32.5°	37.5°	42.5°	47.5°	52.5°	57.5°	62.5°	67.5°	72.5°	77.5°
								Winter						
	$[\overline{d\lambda/dt}]$	642	1744	2484	3059	3276	3410	3505	3242	2677	2254	2344	2549	2161
	$(\sigma)\pm$	(699)	(757)	(691)	(542)	(481)	(551)	(694)	(874)	(931)	(1105)	(1436)	(2288)	(3651)
	$[\bar{u}]=a\cos\phi[d\lambda/dt]$	390	1027	1404	1644	1656	1602	1509	1258	916	663	571	488	298
(17)	$k_Z^{(S)}$	761	5274	9856	13514	13712	12832	11385	7913	4195	2198	1630	1191	444
(18)	$k_Z^{(T)}$	899	986	724	426	298	338	445	567	515	532	620	969	1276
(16)	k_Z	1660	6260	10580	13940	14010	13170	11830	8480	4710	2730	2250	2160	1720
	$(\sigma)\pm$	(1546)	(4077)	(5110)	(4746)	(4221)	(4323)	(4584)	(4333)	(2879)	(2408)	(2287)	(2451)	(1927)
(14)	k_{E_u}	3524	4278	4741	5692	6178	6571	6173	5904	4980	4179	3930	3123	2338
	$(\sigma)\pm$	(1320)	(1982)	(2308)	(2569)	(2907)	(2653)	(1567)	(2550)	(2205)	(2087)	(1809)	(1517)	(1272)
(15)	k_{E_v}	1630	1807	2597	3895	5072	5853	6545	6864	6539	5606	4724	4142	3742
	$(\sigma)\pm$	(550)	(675)	(985)	(1424)	(1858)	(2252)	(2496)	(2510)	(2229)	(2158)	(1923)	(1849)	(1671)
(13)	k_E	5154	6085	7338	9587	11250	12424	12718	12768	11519	9785	8654	7265	6080
(11)	$[k]$	6814	12345	17918	23527	25260	25594	24548	21248	16229	12515	10904	9425	7800
(20)	$\bar{\tau}_M$	1092	1481	2250	2968	2819	2375	1494	475	63	−7	−95	−37	38
	$(\sigma)\pm$	(1653)	(1824)	(2276)	(2625)	(2886)	(2755)	(2666)	(2340)	(1896)	(1300)	(846)	(465)	(196)
								Summer						
	$[\overline{d\lambda/dt}]$	−187	380	928	1623	2097	2316	2330	2145	1760	1850	2097	2180	1602
	$\sigma\pm$	(695)	(880)	(970)	(918)	(625)	(480)	(539)	(1148)	(867)	(902)	(976)	(1358)	(2415)
	$[\bar{u}]$	−276	224	524	872	1060	1088	1003	832	603	544	511	418	221
(17)	$k_Z^{(S)}$	381	251	1373	3802	5618	5919	5030	3461	1818	1480	1306	874	244
(18)	$k_Z^{(T)}$	579	1339	1517	1208	492	241	270	139	432	350	284	336	556
(16)	k_Z	960	1590	2890	5010	6110	6160	5300	3600	2250	1830	1590	1210	800
	$(\sigma)\pm$	(926)	(2407)	(3898)	(4550)	(3447)	(2514)	(2325)	(2155)	(2005)	(1567)	(1353)	(1101)	(1335)
(14)	k_{E_u}	2504	2100	2328	2888	3223	3895	4302	4225	3896	3385	2641	2325	1924
	$(\sigma)\pm$	(1129)	(953)	(1400)	(1393)	(1650)	(1893)	(1761)	(1689)	(1661)	(1382)	(989)	(1299)	(1225)
(15)	k_{E_v}	1120	1023	1166	1629	2283	2995	3908	4606	4519	4056	3586	3264	3049
	$(\sigma)\pm$	(443)	(462)	(610)	(1048)	(1228)	(1340)	(1563)	(1608)	(1492)	(1374)	(1292)	(1263)	(1288)
(13)	k_E	3624	3123	3494	4517	5506	6890	8210	8831	8415	7441	6227	5589	4973
(11)	$[k]$	4584	4713	6384	9527	11616	13050	13510	12431	10665	9271	7817	6799	5773
(20)	τ_M	660	495	679	1044	1140	1057	704	158	−286	−221	−162	−20	10
	$(\sigma)\pm$	(1355)	(1160)	(995)	(1245)	(1442)	(1378)	(1648)	(1685)	(1302)	(908)	(559)	(351)	(153)
								Annual						
	$[\overline{d\lambda/dt}]$	227	1060	1704	2339	2685	2861	2916	2692	2217	2051	2221	2364	1881
	$(\sigma)\pm$	(812)	(1069)	(1152)	(1046)	(818)	(760)	(862)	(1163)	(1013)	(1032)	(1236)	(1892)	(3109)
	$[\bar{u}]$	138	624	963	1257	1357	1344	1255	1044	759	603	541	453	259
(17)	$k_Z^{(S)}$	95	1947	4637	7900	9207	9032	7875	5450	2880	1818	1463	1026	335
(18)	$k_Z^{(T)}$	1215	1973	2093	1560	843	628	685	590	600	462	457	654	925
(16)	k_Z	1310	3920	6730	9460	10050	9660	8560	6040	3480	2280	1920	1680	1260
	$(\sigma)\pm$	(1321)	(4083)	(5950)	(6444)	(5516)	(4981)	(4889)	(4203)	(2932)	(2078)	(1906)	(1957)	(1721)
(14)	k_{E_u}	3013	3186	3531	4286	4697	5230	5236	5062	4437	3781	3284	2723	2130
	$(\sigma)\pm$	(1330)	(1897)	(2276)	(2496)	(2786)	(2664)	(1911)	(2318)	(2025)	(1813)	(1593)	(1467)	(1266)
(15)	k_{E_v}	1374	1414	1880	2759	3674	4421	5223	5732	5526	4830	4153	3702	3395
	$(\sigma)\pm$	(560)	(699)	(1087)	(1687)	(2103)	(2339)	(2464)	(2390)	(2148)	(1967)	(1733)	(1643)	(1531)
(13)	k_E	4387	4600	5411	7045	8371	9651	10459	10794	9963	8611	7437	6425	5525
(11)	$[k]$	5697	8520	12141	16505	18421	19311	19019	16834	13443	10891	9357	8105	6785
(20)	$\bar{\tau}_M$	876	986	1462	2003	1977	1714	1098	316	−112	−114	−129	−28	24
	$\sigma\pm$	(1526)	(1605)	(1923)	(2267)	(2429)	(2274)	(2250)	(2044)	(1635)	(1125)	(718)	(412)	(176)

TABLE 2. Average 500-mb kinetic energy per unit mass over 15N to 80N, as a function of wave-number. Units are 10^3 ergs gm^{-1}. Numbers in parentheses to the left of each quantity refer to the list of definitions.

Statistics		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	(13)$\{k_E\}$
									Winter									
(8)	$\bar{K}_u$	1780	146	96	89	48	37	30	22	17	10	8	6	5	4	4	3	524
	$(\sigma)\pm$	(419)	(73)	(48)	(45)	(19)	(20)	(14)	(12)	(8)	(5)	(3)	(2)	(2)	(2)	(2)	(2)	(120)
(9)	$\bar{K}_v$		10	37	50	49	53	60	51	43	27	20	15	12	10	9	8	457
	$(\sigma)\pm$		(5)	(27)	(30)	(31)	(36)	(42)	(31)	(30)	(16)	(12)	(9)	(6)	(5)	(5)	(5)	(109)
(7)	$\bar{K}$	1780	156	133	140	97	91	90	72	60	38	28	21	17	14	13	12	981
									Summer									
(8)	$\bar{K}_u$	736	68	62	49	31	26	20	14	12	8	6	5	4	4	3	3	316
	$(\sigma)\pm$	(487)	(38)	(35)	(27)	(14)	(15)	(8)	(6)	(5)	(3)	(2)	(2)	(2)	(2)	(1)	(1)	(92)
(9)	$\bar{K}_v$		6	16	23	29	31	31	29	26	19	15	13	9	8	7	6	268
	$(\sigma)\pm$		(5)	(10)	(13)	(17)	(27)	(16)	(18)	(17)	(12)	(10)	(9)	(5)	(5)	(4)	(3)	(75)
(7)	$\bar{K}$	736	74	78	73	60	57	52	43	37	27	21	18	13	12	10	9	584
									Annual									
(8)	$\bar{K}_u$	1256	107	78	69	40	31	24	18	15	9	7	6	5	4	3	3	420
	$(\sigma)\pm$	(692)	(70)	(45)	(42)	(19)	(18)	(12)	(10)	(7)	(4)	(3)	(2)	(2)	(2)	(2)	(1)	(149)
(9)	$\bar{K}_v$		8	27	37	40	42	46	40	34	23	17	14	10	9	8	7	362
	$(\sigma)\pm$		(5)	(23)	(27)	(27)	(34)	(35)	(27)	(26)	(15)	(12)	(9)	(6)	(5)	(5)	(4)	(133)
(7)	$\bar{K}$	1256	115	105	106	79	74	70	58	49	33	24	20	15	13	12	10	783

Our measurements are based on the 500-mb contour heights for each day of 1951, at every 5 deg of latitude from 15N to 80N and at every 10 deg of longitude. The geostrophic approximation is used, nd therefore $[v]=0$.

The results are given in Tables 1 to 3, and some of the more interesting distributions are also shown graphically in Figs. 1 to 3. The spectra are discrete, and connecting lines are drawn only as a visual aid. The *winter* average covers the six-month period October through March (182 days), and the *summer* average the six-month period April through September (183 days). The spectral resolutions are shown only for the three latitudes, 27.5N, 47.5N and 67.5N.

Other presentations of some of these statistics, generally based on time records shorter than a year, are given in papers listed in the References. In several instances values for pressure levels other than 500 mb are given (e.g., Benton and Kahn, 1958).

We note especially (Table 3 and Fig. 3) that the stationary kinetic energy amounts approximately to 10 per c of the total kinetic energy, and less than 20 per cent of the long wave energy, (wave numbers 1 to 5); and that the long stationary waves account for more than 85 per cent of all the stationary kinetic energy. Most of the stationary eddies, then, are large; but only a small part of the large eddies are stationary. Therefore statements of the average transfer of kinetic energy between transient and stationary eddies carry little implication of spatial scale, which should be remembered should one be tempted to collate such results as reported by Murakami (1960) with those of Saltzman and Fleisher (1960).

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TABLE 3. 500 mb wind statistics as a function of wave-number for winter, summer and year of 1951, at 27.5N, 47.5N and 67.5N. Units are 10 ergs gm^{-1} for E, and 10^{19} gm cm^3 sec^{-2} mb^{-1} for J.

	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15}$
	Winter, $\phi=27.5°$																
(5)	$\bar{E}_u^{(S)}$	5453	1281	5596	47	7	192	3	4	34	32	3	24	2	2	16	12,697
(6)	$\bar{E}_u^{(T)}$	6901	4812	4088	3957	3750	2973	2127	1733	1080	835	650	468	481	456	393	34,711
(2)	$\bar{E}_u$	12,354	6093	9684	4004	3757	3165	2130	1737	1114	867	653	492	483	458	409	47,408
	$(\sigma)\pm$	(12,023)	(5886)	(9667)	(3883)	(3302)	(3143)	(2116)	(1563)	(1054)	(992)	(640)	(421)	(483)	(450)	(416)	(23,083)
(5)	$\bar{E}_v^{(S)}$	88	20	243	4	27	257	802	4	50	64	21	32	5	5	33	1655
(6)	$\bar{E}_v^{(T)}$	160	410	1143	1525	2260	4018	3723	3204	2163	1619	1060	995	728	651	645	24,311
(2)	$\bar{E}_v$	248	430	1386	1529	2287	4275	4525	3208	2213	1683	1081	1027	733	656	678	25,966
	$(\sigma)\pm$	(249)	(507)	(1123)	(1585)	(2725)	(4231)	(4420)	(2783)	(2055)	(1841)	(1054)	(1140)	(851)	(743)	(703)	(9850)
(23)	$J^{(S)}$	1053	−68	4560	−32	37	645	113	1	−9	−150	20	4	8	−10	55	6228
(24)	$J^{(T)}$	1118	589	1605	909	1627	3257	2405	1561	1146	806	374	279	282	150	167	16,273
(22)	J	2171	521	6165	877	1664	3902	2518	1562	1137	656	394	283	290	140	222	22,501
	$(\sigma)\pm$	(3583)	(5151)	(8578)	(5451)	(7407)	(7739)	(7531)	(5476)	(3425)	(3175)	(1761)	(1804)	(1657)	(1539)	(1471)	(22,761)
	Winter, $\phi=47.5°$																
(5)	$\bar{E}_u^{(S)}$	4030	2361	1289	20	46	171	19	47	5	55	3	7	2	4	3	8071
(6)	$\bar{E}_u^{(T)}$	11,627	11,213	8336	6704	4145	3427	2341	1613	1080	882	633	523	412	389	339	53,661
(2)	$\bar{E}_u$	15,657	13,574	9625	6724	4191	3589	2360	1660	1085	937	636	530	414	393	342	61,732
	$(\sigma)\pm$	(11,189)	(14,386)	(9115	(5123)	(4155)	(3014)	(2833)	(1488)	(1115)	(807)	(509)	(489)	(403)	(404)	(419)	15,666
(5)	$\bar{E}_v^{(S)}$	754	686	2475	195	549	148	19	44	31	1	35	25	10	6	10	4992
(6)	$\bar{E}_v^{(T)}$	426	2507	5347	6599	8647	8643	7089	6973	3911	2943	2248	1620	1454	1056	991	60,457
(2)	$\bar{E}_v$	1180	3193	7822	6794	9196	8791	7108	7017	3942	2944	2283	1645	1464	1062	1001	65,449
	$(\sigma)\pm$	(979)	(3350)	(8421)	(6498)	(8889)	(9063)	(6370)	(7336)	(3837)	(2779)	(2223)	(1496)	(1359)	(1079)	(965)	(24,958)
(23)	$J^{(S)}$	3937	−1056	3113	−145	359	−363	−43	−58	21	7	24	−12	6	8	−2	5796
(24)	$J^{(T)}$	582	1379	14	553	1824	1903	1136	1119	−148	341	239	139	71	72	−83	9141
(22)	J	4519	323	3127	408	2183	1540	1093	1061	−127	348	263	127	77	80	−85	14,937
	$(\sigma)\pm$	(5464)	(10,631)	(12,079)	(9557)	(10,739)	(10,184)	(7144)	(5540)	(3354)	(2763)	(2088)	(1517)	(1297)	(1031)	(891)	(26,659)
	Winter, $\phi=67.5°$																
(5)	$\bar{E}_u^{(S)}$	1395	2996	187	14	92	62	4	8	6	5	4	1	0	4	0	4783
(6)	$\bar{E}_u^{(T)}$	12,537	6806	5399	3725	2022	1438	849	499	402	235	174	143	109	97	81	34,519
(2)	$\bar{E}_u$	13,932	9802	5586	3739	2114	1500	853	507	408	240	178	144	109	101	81	39,302
	$(\sigma)\pm$	(13,948)	(9484)	(4903)	(3791)	(2031)	(4662)	(872)	(553)	(403)	(263)	(204)	(163)	(134)	(154)	(158)	(18,089)
(5)	$\bar{E}_v^{(S)}$	260	3937	115	386	205	34	40	15	100	47	14	3	8	2	11	5182
(6)	$\bar{E}_v^{(T)}$	2027	8519	7399	8111	4899	3627	2085	1553	1074	745	612	435	350	318	296	42,054
(2)	$\bar{E}_v$	2287	12,456	7514	8497	5104	3661	2125	1568	1174	792	626	438	358	320	307	47,236
	$(\sigma)\pm$	(2054)	(12,244)	(7614)	(7971)	(4908)	(3765)	(2003)	(1569)	(1217)	(780)	(788)	(510)	(378)	(384)	(385)	(19,226)

TABLE 3 (*continued*).

	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15}$
	Winter, $\phi=67.5°$ (*continued*)																
(23)	$J^{(S)}$	126	−2064	−111	52	80	−25	3	8	11	10	2	1	1	1	0	−1904
(24)	$J^{(T)}$	251	−459	86	511	309	221	7	−5	−2	36	1	−5	2	−1	0	952
(22)	J	377	−2523	−25	563	389	196	10	3	9	46	3	−4	3	0	0	−952
	$(\sigma)\pm$	(3268)	(5413)	(3248)	(2509)	(1782)	(1256)	(719)	(473)	(358)	(225)	(166)	(125)	(92)	(85)	(69)	(8463)
	Summer, $\phi=27.5$																
(5)	$\bar{E}_u^{(S)}$	318	569	291	37	75	41	172	42	69	34	7	1	3	9	23	1693
(6)	$\bar{E}_u^{(T)}$	4123	3117	3285	2259	1974	1612	1048	993	736	591	437	383	349	384	288	21,584
(2)	$\bar{E}_u$	4441	3686	3576	2296	2049	1653	1220	1035	805	625	444	384	352	393	311	23,277
	$(\sigma)\pm$	(6993)	(4174)	(4780)	(2550)	(2583)	(1406)	(1238)	(996)	(739)	(648)	(441)	(402)	(385)	(409)	(306)	(14,600)
(5)	$\bar{E}_v^{(S)}$	51	37	353	5	40	7	502	78	32	16	10	14	6	16	17	1183
(6)	$\bar{E}_v^{(T)}$	95	217	321	503	785	1274	1308	1391	1010	945	735	507	483	477	424	10,481
(2)	$\bar{E}_v$	146	254	674	508	825	1281	1810	1469	1042	961	745	521	489	493	441	11,664
	$(\sigma)\pm$	(116)	(289)	(582)	(682)	(858)	(1710)	(1797)	(1830)	(1256)	(933)	(807)	(478)	(549)	(550)	(475)	(6097)
(23)	$J^{(S)}$	−316	438	767	24	223	66	−180	121	1	83	−9	14	3	−33	79	1283
(24)	$J^{(T)}$	−263	1027	490	−7	469	555	915	959	438	304	49	17	85	382	91	5509
(22)	J	−579	1465	1257	17	692	621	735	1080	439	387	40	31	88	349	170	6792
	$(\sigma)\pm$	(1446)	(3753)	(3714)	(2617)	(3042)	(3316)	(3903)	(3281)	(2503)	(2104)	(1466)	(1301)	(1148)	(1447)	(1134)	(9948)
	Summer, $\phi=47.5°$																
(5)	$\bar{E}_u^{(S)}$	2802	644	886	340	118	206	81	17	21	25	8	36	4	5	6	5199
(6)	$\bar{E}_u^{(T)}$	7384	5782	6703	4463	4032	2473	1647	1388	930	787	582	537	460	348	308	37,830
(2)	$\bar{E}_u$	10,186	6426	7589	4803	4150	2679	1728	1405	951	812	590	573	464	353	314	43,029
	$(\sigma)\pm$	(9789)	(7145)	(7369)	(4344)	(5178)	(2584)	(2082)	(1159)	(962)	(873)	(585)	(552)	(543)	(340)	(333)	(17,607)
(5)	$\bar{E}_v^{(S)}$	279	900	261	758	258	532	189	31	35	21	14	7	10	10	11	3317
(6)	$\bar{E}_v^{(T)}$	257	947	1755	2647	4163	4878	4712	4120	3605	2342	2112	1218	1169	1023	810	35,763
(2)	$\bar{E}_v$	536	1847	2016	3405	4421	5410	4901	4151	3640	2363	2126	1225	1179	1033	821	39,080
	$(\sigma)\pm$	(553)	(1898)	(2362)	(3503)	(5337)	(4343)	(4872)	(3963)	(3365)	(2783)	(2379)	(1248)	(1288)	(1127)	(918)	(15,632)
(23)	$J^{(S)}$	1819	−925	1130	509	−414	−386	286	22	−53	45	−5	−33	14	7	9	2025
(24)	$J^{(T)}$	−179	1144	−750	30	1483	1653	310	636	−98	354	−10	151	185	48	57	5016
(22)	J	1640	219	380	539	1069	1267	596	658	−151	399	−15	118	199	55	66	7041
	$(\sigma)\pm$	(4209)	(6150)	(6422)	(6194)	(7784)	(5669)	(4090)	(3800)	(2873)	(2351)	(1982)	(1603)	(1383)	(944)	(857)	(16,480)
	Summer, $\phi=67.5°$																
(5)	$\bar{E}_u^{(S)}$	457	1595	153	124	169	27	40	9	2	9	8	1	3	1	2	2601
(6)	$\bar{E}_u^{(T)}$	7765	3817	3956	2728	1948	1291	634	545	281	246	180	142	99	84	91	23,813
(2)	$\bar{E}_u$	8222	5412	4109	2852	2117	1318	674	554	283	255	188	143	102	85	93	26,414
	$(\sigma)\pm$	(7404)	(4794)	(4058)	(2407)	(2057)	(1383)	(739)	(487)	(288)	(264)	(267)	(164)	(135)	(129)	(132)	(9886)

TABLE 3 (*continued*).

	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15}$
	Summer, $\phi=67.5°$ (*continued*)																
(5)	$\bar{E}_v^{(S)}$	33	455	297	1724	194	103	46	58	107	58	16	8	10	0	25	3136
(6)	$\bar{E}_v^{(T)}$	1514	4045	6429	5038	4741	3295	1974	1671	1080	784	562	550	358	335	316	32,726
(2)	$\bar{E}_v$ $(\sigma)\pm$	1547 (2109)	4500 (4312)	6726 (5264)	6762 (6572)	4935 (5527)	3398 (3466)	2020 (1778)	1729 (1742)	1187 (1266)	842 (886)	578 (691)	558 (653)	398 (556)	335 (353)	341 (355)	35,862 (12,923)
(23)	$J^{(S)}$	51	−448	−161	−280	106	39	−4	17	11	12	−8	2	−1	0	−3	−667
(24)	$J^{(T)}$	−706	−115	24	−200	8	27	41	−68	18	−24	13	2	7	9	9	−956
(22)	J $(\sigma)\pm$	−655 (2192)	−563 (2438)	−137 (3259)	−480 (2195)	114 (1527)	66 (1043)	37 (548)	−51 (435)	29 (296)	−12 (227)	5 (162)	4 (134)	6 (106)	9 (82)	6 (85)	−1623 (5590)
	Annual, $\phi=27.5°$																
(5)	$\bar{E}_u^{(S)}$	1999	827	2099	2	9	27	41	14	50	32	0	9	2	4	17	5133
(6)	$\bar{E}_u^{(T)}$	6388	4059	4523	3145	2892	2380	1633	1371	909	714	548	429	415	422	343	30,176
(2)	$\bar{E}_u$ $(\sigma)\pm$	8387 (10,595)	4886 (5240)	6622 (8209)	3147 (3392)	2901 (3084)	2407 (2547)	1674 (1791)	1385 (1356)	959 (923)	746 (846)	548 (559)	438 (415)	417 (442)	426 (431)	360 (368)	35,309 (22,762)
(5)	$\bar{E}_v^{(S)}$	64	15	180	4	5	45	469	13	40	19	2	18	6	8	23	913
(6)	$\bar{E}_v^{(T)}$	133	327	849	1013	1549	2729	2694	2323	1586	1302	910	755	605	566	536	17,883
(2)	$\bar{E}_v$ $(\sigma)\pm$	197 (201)	342 (421)	1029 (962)	1017 (1322)	1554 (2146)	2774 (3554)	3163 (3633)	2336 (2510)	1626 (1800)	1321 (1502)	912 (954)	773 (910)	611 (726)	574 (659)	559 (611)	18,796 (10,870)
(23)	$J^{(S)}$	97	241	2274	8	16	41	−336	8	−11	−27	−2	21	5	−16	74	2396
(24)	$J^{(T)}$	695	753	1431	438	1161	2216	1960	1313	798	548	218	135	184	261	122	12,229
(22)	J $(\sigma)\pm$	792 (3056)	994 (4529)	3705 (7045)	446 (4294)	1177 (5677)	2257 (6170)	1624 (6059)	1321 (4518)	787 (3019)	521 (2695)	216 (1629)	156 (1577)	189 (1429)	245 (1497)	196 (1313)	14,625 (19,226)
	Annual, $\phi=47.5°$																
(5)	$\bar{E}_u^{(S)}$	3356	1309	1065	49	45	67	5	24	4	37	4	16	0	5	5	5992
(6)	$\bar{E}_u^{(T)}$	9558	8681	7539	5712	4126	3070	2038	1508	1014	837	609	535	439	368	323	46,363
(2)	$\bar{E}_u$ $(\sigma)\pm$	12,914 (10,861)	9990 (11,898)	8604 (8348)	5761 (4845)	4171 (4692)	3137 (2844)	2043 (2505)	1532 (1339)	1018 (1043)	874 (843)	613 (549)	551 (522)	439 (479)	373 (374)	328 (379)	52,355 (19,112)
(5)	$\bar{E}_v^{(S)}$	469	718	1002	328	243	291	83	6	32	5	21	8	0	3	10	3222
(6)	$\bar{E}_v^{(T)}$	388	1801	3909	4766	6559	6805	5918	5574	3759	2648	2183	1427	1321	1044	901	49,006
(2)	$\bar{E}_v$ $(\sigma)\pm$	857 (857)	2519 (2803)	4911 (6825)	5094 (5484)	6802 (7706)	7096 (7298)	6001 (5775)	5580 (6063)	3791 (3611)	2653 (2796)	2204 (2304)	1435 (1393)	1321 (1331)	1047 (1103)	911 (946)	52,228 (24,636)
(23)	$J^{(S)}$	2767	−1177	2161	224	−53	−308	48	20	−15	24	12	−7	0	9	3	3708
(24)	$J^{(T)}$	309	1448	−411	250	1677	1711	796	839	−124	350	112	130	138	58	−12	7270
(22)	J $(\sigma)\pm$	3076 (5083)	271 (8679)	1750 (9763)	474 (8049)	1624 (9391)	1403 (8237)	844 (5822)	859 (4752)	−139 (3122)	374 (2565)	124 (2041)	123 (1561)	138 (1343)	67 (988)	−9 (878)	10,978 (22,497)

TABLE 3 (*continued*).

Annual, $\phi = 67.5°$

	n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15}$
(5)	$\bar{E}_u^{(S)}$	783	2197	147	19	124	33	18	4	2	7	5	1	0	2	1	3345
(6)	$\bar{E}_u^{(T)}$	10,286	5404	4698	3275	1991	1376	745	527	343	240	178	143	106	91	86	29,495
(2)	$\bar{E}_u$ $(\sigma)\pm$	11,069 (11,517)	7601 (7823)	4845 (4559)	3294 (3204)	2115 (2044)	1409 (1432)	763 (813)	531 (521)	345 (356)	247 (263)	183 (238)	144 (164)	106 (135)	93 (143)	87 (146)	32,840 (15,928)
(5)	$\bar{E}_v^{(S)}$	101	1704	170	918	194	35	0	9	72	33	10	2	8	1	17	3276
(6)	$\bar{E}_v^{(T)}$	1815	6763	6949	6709	4825	3494	2073	1640	1108	784	592	496	370	327	307	38,257
(2)	$\bar{E}_v$ $(\sigma)\pm$	1916 (2114)	8467 (9995)	7119 (6554)	7627 (7355)	5019 (5228)	3529 (3621)	2073 (1894)	1649 (1660)	1180 (1242)	817 (835)	602 (741)	498 (589)	378 (476)	328 (369)	324 (371)	41,533 (17,331)
(23)	$J^{(S)}$	60	−1172	−117	−67	93	15	2	4	5	11	−3	1	−1	1	−1	−1170
(24)	$J^{(T)}$	−200	−368	36	107	158	116	21	−28	14	6	7	−1	6	4	4	−118
(22)	J $(\sigma)\pm$	−140 (2829)	−1540 (4307)	−81 (3254)	40 (2414)	251 (1664)	131 (1156)	23 (639)	−24 (455)	19 (329)	17 (228)	4 (164)	0 (130)	5 (99)	5 (84)	3 (78)	−1288 (7176)

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LARGE SCALE VERTICAL EDDIES IN THE ATMOSPHERE AND THE ENERGY OF THE MEAN ZONAL FLOW

by VICTOR P. STARR (*) & ROBERT E. DICKINSON (*)

Summary — The zonal eddy stress across horizontal surfaces due to large scale vertical motions was evaluated for two months from data for the northern hemisphere for a number of levels up to 50 mb. From this information and from the corresponding distributions for each of the two months of the mean zonal winds, the rate of transformation of kinetic energy from eddy to mean zonal form was calculated. The two sets of data gave rather small values for the hemisphere which were of opposite sign.

It is well known that large scale eddies in the atmosphere in the long period average lose kinetic energy through countergradient horizontal eddy momentum transport, to maintain the mean zonal flow against dissipative processes and the forcing of an indirect mean meridional circulation in middle latitudes. A recapitulation of results of this kind for the northern hemisphere, together with comparable new results for the southern hemisphere, is being currently published by OBASI (1963). It has usually been assumed in work of this kind that momentum transport upward or downward by large scale vertical eddies is of small importance for these energy considerations, although direct measurements were lacking. The validity of this assumption is, however, suggested by geostrophic scale theory. Its is the purpose of this discussion to present the results of some preliminary measurements of the large scale vertical eddy effects.

As a part of our larger program of studying the general circulation in its various aspects, JENSEN (1960, 1961) calculated with the aid of the adiabatic assumption the values of the instantaneous individual pressure change ω for all conveniently obtainable stations of the northern hemisphere for all standard pressure layers up to 50 mb, for each day of the month of January 1958 and also for each day of the month of April of the same year. In addition to the use which JENSEN himself made of these values, i.e., correlating them with the temperature, use was made of them by MOLLA & LOISEL (1962) who computed their covariance with the northward component of wind velocity v. The present writers have, pursuant to the purpose mentioned, made still further use of JENSEN's values of ω and calculated their covariance with u, the eastward component of the wind. As in the studies of MOLLA & LOISEL, the vertical transport of eastward momentum

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obtained may be separated into two parts, one due to the transient eddies and another due to standing eddies. For interest's sake the vertical momentum transports are listed in tables 1 through 4. In JENSEN's work the values of ω are given in units of velocity, so that in the tables the symbol w is to be interpreted as $-\omega/\rho g$ where ρ is the appropriate density and g is the acceleration of gravity. In what follows the bar is used to denote a time average and the primes denote deviations therefrom. The brackets signify a zonal average with longitude around the earth and the superscript asterisk signifies a departure therefrom. Lastly, p is pressure.

The transformation term under consideration, written in terms of ω in pressure coordinates, may be stated as follows:

$$-[\bar{u}]\frac{\partial}{\partial p}\{[\overset{*}{\bar{\omega}}\overset{*}{\bar{u}}]+[\overline{\omega' u'}]\} = \{[\overset{*}{\bar{\omega}}\overset{*}{\bar{u}}]+\overline{\omega' u'}]\}\frac{\partial[\bar{u}]}{\partial p} - \frac{\partial}{\partial p}([\bar{u}]\{[\overset{*}{\bar{\omega}}\overset{*}{\bar{u}}]+[\overline{\omega' u'}]\}) \,.$$

This is to be integrated over the vertical extent of the atmosphere and over the hemisphere, whereupon the last term essentially gives no contribution. The tabulated data together with corresponding values of $[\bar{u}]$ from JENSEN's material, were assumed to furnish an adequate sampling of the integrand for an initial trial evaluation. Actually the computation could be extended down to 15° N and not to the equator.

The finite difference integration gave a value of $-1.45 \cdot 10^{20}$ erg sec^{-1} for January and a value of $+1.84 \cdot 10^{20}$ erg sec^{-1} for April. If the figures are assumed correct, the action of the large vertical eddies was to decrease the mean zonal kinetic energy in January and to increase it in April. The corresponding action of the horizontal eddies on an annual mean basis ranges from about 5 to $10 \cdot 10^{20}$ erg sec^{-1} as summarized by OBASI.

As far as problems of the kind here considered are concerned, a sample of two individual months is hardly large enough for proper measurement of the effects. Save for the labor entailed it would be best to consider a season or a year. This has not as yet been done, however, and it is therefore felt that the figures above are of interest as indicating the smallness of the action in agreement with scale theory.

TABLE 1: *Zonally averaged standing eddy term* $[\overset{*}{\bar{w}}\overset{*}{\bar{u}}] = [\bar{w}\,\bar{u}] - [\bar{w}][\bar{u}]$ in cm^2 sec^{-2} $\cdot 10^2$ *by pressure layer and latitude for January* 1958.

Pressure layer in mb	Latitude						
	20	30	40	50	60	70	80
1000-850	− 0.11	+ 0.82	+ 0.93	− 0.52	+ 0.17	+ 0.47	+ 0.43
850-700	+ 0.06	− 0.24	− 0.63	− 0.49	− 0.81	+ 0.29	− 0.08
700-500	− 0.21	− 2.56	+ 1.47	− 1.25	− 0.39	− 0.12	− 0.21
500-300	− 0.01	− 2.15	0.00	+ 0.14	+ 0.13	+ 0.72	+ 0.56
300-200	− 0.36	+ 2.96	+ 0.10	− 0.05	+ 0.82	− 0.15	+ 0.09
200-100	+ 0.92	+ 1.03	+ 0.61	+ 0.06	+ 0.92	− 0.08	+ 0.06
100-50	+ 0.02	− 0.05	− 0.33	− 0.68	− 2.36	− 0.42	+ 1.21

Table 2: *Zonally averaged standing eddy term* $[\bar{w}^{*} \bar{u}^{*}] = [\bar{w}\ \bar{u}] - [\bar{w}][\bar{u}]$ in $cm^2\ sec^{-2} \cdot 10^2$ *by pressure layer and latitude for April* 1958.

Pressure layer in mb	Latitude						
	20	30	40	50	60	70	80
1000-850	— 1.59	+ 0.15	+ 0.36	+ 1.58	+ 1.24	+ 0.17	+ 0.58
850-700	+ 0.07	+ 0.46	+ 0.63	+ 0.17	— 1.06	— 1.95	— 1.51
700-500	+ 0.38	+ 0.07	— 0.53	+ 0.70	— 0.98	— 1.36	— 0.96
500-300	— 0.85	— 1.71	— 0.51	— 2.88	— 2.08	— 5.10	— 2.48
300-200	+ 0.12	+ 2.79	+ 0.54	+ 0.05	— 1.10	— 0.43	+ 0.30
200-100	+ 0.26	— 0.78	+ 0.08	+ 0.42	— 0.19	+ 0.31	— 0.08
100-50	+ 0.05	— 0.07	+ 0.21	— 0.12	— 0.31	— 0.22	— 0.20

Table 3: *Zonally averaged covariance of vertical motion and northward component of the horizontal wind (transient eddy term)* $[\overline{w' u'}]$, *in* $cm^2\ sec^{-2} \cdot 10^2$ *by pressure layer and latitude for January* 1958.

Pressure layer in mb	Latitude						
	20	30	40	50	60	70	80
1000-850	+ 0.14	+ 2.25	+ 1.16	+ 1.39	— 0.58	— 1.00	— 2.50
850-700	— 0.03	+ 0.28	+ 0.50	+ 0.62	+ 1.17	+ 2.50	+ 2.00
700-500	+ 0.72	+ 1.94	— 0.61	+ 1.42	+ 5.16	+ 3.33	+ 0.08
500-300	— 1.89	— 4.65	— 4.14	+ 0.44	— 1.25	+ 0.92	— 2.26
300-200	— 0.42	+ 1.67	— 3.22	— 2.39	+ 1.00	+ 1.17	— 0.33
200-100	0.00	+ 1.05	+ 1.47	+ 0.67	— 0.42	+ 0.42	+ 2.33
100-50	— 0.25	— 0.72	— 0.94	+ 0.17	+ 1.67	+ 1.83	+ 0.25

Table 4: *Zonally averaged covariance of vertical motion and northward component of the horizontal wind (transient eddy term)* $[\overline{w' u'}]$, *in* $cm^2\ sec^{-2} \cdot 10^2$ *by pressure layer and latitude for April* 1958.

Pressure layer in mb	Latitude						
	20	30	40	50	60	70	80
1000-850	+ 0.53	+ 1.08	+ 1.08	+ 2.94	0.00	+ 0.08	0.00
850-700	— 0.14	+ 1.42	+ 1.22	+ 0.06	— 1.83	— 1.00	— 2.17
700-500	+ 1.06	+ 1.31	— 1.78	— 0.08	+ 1.25	— 1.33	— 1.42
500-300	+ 0.97	+ 2.86	— 0.64	+ 0.36	— 0.67	— 1.33	+ 0.33
300-200	— 0.17	+ 0.89	+ 0.14	+ 2.67	— 0.42	0.00	0.00
200-100	+ 0.81	+ 0.17	— 0.14	— 0.25	— 0.17	— 0.17	0.00
100-50	— 0.22	— 0.36	— 0.14	+ 0.06	— 0.17	— 0.08	0.00

Systematic nonadiabatic effects could alter the results somewhat. Thus condensation processes occurring chiefly east of trough lines in the upper westerlies where u is larger could augment the upward flow of momentum in middle latitudes at intermediate levels. This would increase algebraically the above values for the transformation.

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Transfer through the tropopause and within the stratosphere*

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Summary

The evidence for tropospheric-stratospheric mass exchange is reviewed. A large fraction of the exchange appears to occur in the vicinity of the middle latitude tropopause discontinuity and the concomitant jet stream. Calculations of the flux of radioactive substances in the troposphere support this view. Computations of the horizontal flux of ozone in the lower stratosphere are presented for the I.G.Y. period divided into three-month seasons. Indications are that the large-scale quasi-horizontal transient eddies can transport ozone polewards in sufficient quantities to account for the spring build-up of ozone. Such large-scale mixing, as opposed to mean meridional motions, also allows explanation of the distribution of radioactive tungsten in the stratosphere. Transports appear to be polewards and downwards between latitudes 20° and 60°. Independent meteorological evidence, in the form of isentropic cross-sections and values of the covariance between the meridional and vertical components of the wind, support the mixing scheme. The observed counter-gradient eddy flux of heat can be explained from the model. Distributions of certain other trace substances are not at variance with the model. From a preliminary examination of the angular momentum transport processes up to 60 km it appears that transient eddies are also important to these levels but there is not yet sufficient global coverage to evaluate the contributions due to mean motions and standing eddies.

1. Introduction

Meteorologists presently claim an understanding of the behaviour of the atmosphere below the tropopause; but comparable claims cannot yet be made for the stratosphere, the mesosphere and the thermosphere. As with most physical systems our comprehension is limited by our ability to observe events properly. In the past few years this ability has increased by leaps and bounds. Reliable data to about 30 km (10 mb) are now gathered on a routine basis from the radiosonde network; with special attention, balloons may be used to altitudes of 35 or 40 km (Conover, Lowenthal and Taylor 1960; Hopper and Laby 1960). The Meteorological Rocket Network over North America provides synoptic wind data, and some temperature data, in four periods of each year at altitudes to 65 km (0·1 mb) (Webb, Hubert, Miller and Spurling 1961). A tantalizing glimpse of the wind and temperature structure to 85 km ($\sim$ 0·004 mb) has been provided by the rocket grenade experiments (Stroud, Nordberg, Bandeen, Bartman and Titus 1960; Groves 1960; Nordberg and Stroud 1961; Teweles 1961) which have shown variability in the mesophere corresponding to that found in tropospheric weather. Between 80 and 100 km meteor trail drift observations (Elford 1959; Greenhow and Neufeld 1955, 1956), taken over several years, have provided the wind velocities for one station in each hemisphere, including the tidal components whose origin is still not completely resolved. Just above this layer ionospheric discontinuities have been tracked and interpreted in terms of winds (Mitra 1949). Sodium vapour trails extending to altitudes of 200 km have also been tracked to obtain wind velocity in a few cases (Manring and Bedinger 1960). Certain rocket flights (Horowitz and LaGow 1957, 1958; Horowitz, LaGow and Giuliani 1959) have enabled temperatures to be calculated, with various assumptions, up to these levels. At higher altitudes, to about 1,000 km, our knowledge is restricted to that of density (King-Hele 1961) and composition (Istomin 1959). Again under certain assumptions temperatures can be calculated.

* This paper gained highly honourable mention in the 1962 Napier Shaw Essay Competition.

The atmospheric circulation patterns derived from the whole gamut of techniques used below 100 km have been presented by Murgatroyd (1957) and Batten (1961); wind data are more abundant than temperature data in the section above 30 km. The patterns above the tropopause suggest to the meteorologist very similar questions to those that were raised many years ago concerning the circulation of the troposphere. One may enquire, for example, about the source of the kinetic energy represented by these circulations. How much of the kinetic energy is advected across the lower and upper boundaries, either as kinetic or potential or internal energy, and how much is generated *in situ* from the effects of solar and infra-red radiation ? This particular topic was the subject of a recent paper by Starr (1959b) in which he emphasized the importance of a study of the energy flux across levels such as the tropopause. Another striking fact is the reversal of the wind direction that takes place between summer and winter in the region below about 70 km. Where are the sources and sinks of relative angular momentum for this region ? Does the summer easterly regime correspond to some systematic removal of angular momentum from the region into the troposphere ? Or are there some torques, not yet understood, that are operating *in situ* ? One may also enquire about the character of the mixing processes in the region. If a foreign trace substance is introduced into the atmosphere at say 60 km what happens to it ? Does it rapidly mix throughout the region 50-80 km, so that in the course of a month or so it is uniformly distributed ? Or is it confined to a thin layer in the vertical and limited in latitudinal extent ? In contrast to the opinions prevalent some years ago it seems that the atmosphere is reasonably well mixed up to about 100 km (Meadows and Townsend 1960) but the time scale associated with the mixing is not yet known.

Answers to the first two sets of questions can be obtained by application of the principles of conservation of energy and momentum. The third set indirectly can be attacked with the principle of the conservation of mass. One of the first steps should be the construction of a budget to keep track of the energy and angular momentum involved in the region from the tropopause to 100 km. When one constructs from the circulation patterns cross-sections of the kinetic energy density and angular momentum density of the region it immediately becomes obvious that from physical considerations the kinetic energy and momentum advected across the upper boundary makes a negligible contribution to the total budget. Such statements cannot at present be made about the higher layers, for example 900-1,000 km. It is equally clear that relatively small changes near the lower boundary can give rise to enormous changes in the upper half of the layer considered – if indeed there is any relationship between events. It therefore seems logical that we build from our knowledge of the troposphere to gain some understanding of the vertical fluxes of energy, angular momentum and mass at the tropopause, then investigate in detail the next layer above, say from the tropopause to 30 km, with the ultimate aim that we shall evaluate the vertical fluxes at 30 km as soon as events in the layer are understood and as soon as observations allow.

During the past three years the Planetary Circulations Project at the Massachusetts Institute of Technology, directed by Professor V. P. Starr, has been evaluating the horizontal and vertical fluxes of energy and momentum in the layer from 100 to 10 mb. Data collected during the International Geophysical Year have been used. As is well known, similar computations have been made over the past twelve years for the atmosphere between the surface and 100 mb. A picture of the workings of the stratospheric region is emerging that is not only of considerable interest in its own right but which will provide an indispensable springboard for the study of the layers above – say 30-50 km, 50-80 km and 80-100 km. Some of the findings will be quoted below.

There are several approaches to the study of transport processes within, and into, the stratosphere. If direct observations of temperature, pressure and wind velocity are available one can either attempt a comprehensive study of events over a short period by isentropic trajectory analysis or some similar technique, or one can collect the observations together over a long period of time and study the average values both of the elements themselves and of derived quantities such as the transport of momentum and energy. The

latter, the climatological approach, is the one adopted by the Planetary Circulations Project. Yet another approach is available if one has observations of some trace substance as a function of latitude, longitude, height and time. If the substance can be treated as conservative in its passage from its sources to its sinks then changes in concentration and gradients of concentration can be interpreted in terms of atmospheric transport processes. This technique is essentially an application of the principle of mass conservation.

Until quite recently much of the literature concerning the general circulation of the lower stratosphere has contained conclusions based on observations of the distribution of trace substances. Craig (1948), for example, interpreted ozone observations as implying a mean meridional motion from equatorial to polar regions in the lower stratosphere. Brewer (1949), from water vapour and ozone data, suggested that there is a direct mean meridional cell involved with rising motion through the tropical tropopause, northward motion (in the northern hemisphere) between low and middle or high latitudes and sinking motion to the north. Similar types of mean meridional circulations have been suggested by Goldie (1950) from meteorological observations, Dobson (1956) from ozone and water vapour data, Stewart, Osmond, Crooks and Fisher (1957), Machta (1957), Dyer and Yeo (1960) Libby and Palmer (1960) all from observations of fission products, and Burton and Stewart (1960) from observations of natural radioactive substances. Murgatroyd and Singleton (1961) have deduced the existence of such a circulation in meridional planes from calculations based on the radiation budget alone with consideration of possible eddy heat transport omitted. Brewer, Machta and Murgatroyd and Singleton have each pointed out that one of the major drawbacks of these schemes is that the angular momentum budget of the atmosphere is not balanced by the circulations postulated, at least in certain regions.

There is, of course, a parallel between the suggestions of a mean meridional circulation in the stratosphere and the similar suggestions made many years ago of a direct meridional circulation in the troposphere. Such a direct circulation was supposed to transfer heat energy from the source regions at low latitudes to the sink regions near the poles. Later it turned out that the heat transfer mechanism was somewhat different. Certainly the differential heating produces available potential energy, but instead of this being realised as kinetic energy of a mean meridional motion it appears that under the combined influence of the differential heating and rotation of the earth the potential energy is converted into available potential energy of the large scale quasi-horizontal eddies, thence into kinetic energy of the eddies and thence into kinetic energy of the mean zonal flow. It is these large-scale eddies which produce the heat flux poleward and not the mean motion. Indeed these eddies in the process also transport relative angular momentum northward and for a complete angular momentum balance an indirect mean meridional cell has been postulated (Eady 1950) and observed (Starr 1954). In view of the experience with the troposphere it is thus not necessarily logical to suggest that the differential heating in the stratosphere and mesosphere produces a direct mean meridional circulation. Murgatroyd and Singleton carefully pointed out that eddy transports would ultimately have to be included in their calculations.

Several authors have proposed that the distribution of tracers can best be explained by large-scale eddy-mixing processes with a flow down the concentration gradient for any particular tracer. Reed (1953) felt that vertical-eddy mixing was important but that horizontal-eddy mixing should also be considered. Martin (1956) was one of the first to investigate the latter suggestion with regard to ozone; his work was based on computations of the horizontal ozone flux using actual ozone and wind data. Godson (1960) and Ramanathan and Kulkarni (1960) also pointed out that baroclinic waves may govern the ozone flux. In a thesis written in 1959, the author (Newell 1960a; 1960b) pointed out that Martin's work could possibly explain the observed distributions of fission-product radioactivity and simultaneously account for the stratospheric countergradient flux of heat reported by White (1954). Evidence in favour of the idea came when observations of tungsten 185, collected by U-2 aircraft at altitudes up to 70,000 ft, were released (Feely and Spar 1960a). There is now evidence at hand from extensive meteorological data

that the eddy-mixing interpretation gives a stratospheric model which is not at variance with considerations of angular momentum and energy balance. Some of these data have been discussed elsewhere (Newell 1961). The main purpose of the present paper is to summarize the work on tracers that has led to the current picture of stratospheric-tropospheric exchange and to the current ideas concerning transport processes within the stratosphere. Calculations of the ozone flux during the International Geophysical Year will be presented in detail as the main observational basis for the model and an effort will be made to fit the observations of trace substances into a picture which is also consistent with the extensive meteorological observations processed by the members of the Planetary Circulations Project.

2. Tropospheric-stratospheric interchange from tracer studies

The exchange of air between the troposphere and stratosphere is a topic which is still not thoroughly understood but which has received considerable attention of late in attempts to account for observations of trace substances in the two regions. One of the first studies of the exchange which used methods of synoptic meteorology was that of Reed and Sanders (1953); in evaluating the mechanisms and motions that occurred in the formation of a baroclinic frontal zone they found evidence that stratospheric air was entrained into the zone. Sawyer (1955), in a study of detailed aircraft observations of a frontal zone, noticed that a tongue of very dry air was often present in association with the zone and on one occasion was able to trace it backwards in time along a quasi-horizontal path into the stratosphere. In similar vein, Ramanathan (1956) suggested that ozone may escape into the troposphere via the quasi-horizontal circulations associated with the jet stream.

Intense interest in the topic was aroused when it was found that many months after nuclear-weapon tests had ceased, considerable concentrations of radioactivity were still observed in the troposphere in spite of the fact that the debris had, supposedly, a mean life in the troposphere of only 30 days (the removal being principally by precipitation). The debris originated from those parts of the original nuclear clouds that had penetrated into the stratosphere; such high yield explosions occurred either at high or low latitudes. Machta (1957) suggested that the material entered the troposphere in the vicinity of the middle-latitude tropopause discontinuity. Some of the data on the radioactivity of the air is now examined to see if there is any support for the hypothesis.

One of the best sets of observations of the fission-product radioactive substances in the lower troposphere is that collected by the United States Naval Research Laboratory. There are 20 stations along 80°W. At each station air is blown through a circular piece of asbestos-based filter paper, 8 in. diameter, at a flow rate of about 30 cubic feet per minute. The paper has essentially 100 per cent retention for 0·3 μ particles and 90 per cent retention for particles as small as 0·02 μ. Filters are changed daily at 0800 hr local time, and forwarded to Washington where they are ashed at 650°C and then all counted for gross β-activity with one and the same end-window Geiger-Müller Tube. Each day the count of a standard sample and the background count is obtained. Most stations commenced daily sampling at the beginning of the I.G.Y. period and continued until November 1959, at which point the low levels of activity made necessary a change to three-day sampling periods. (Dr. L. B. Lockhart of the Naval Research Laboratory has kindly supplied the author with this valuable geophysical record). Monthly mean meridional profiles have been constructed and are shown in Fig. 1 (see also Lockhart and Patterson 1960; Lockhart, Patterson, Saunders and Black 1960). Tests ceased on 4 November 1958, and apart from the French tests in February, April and December 1960 and April 1961, which are reflected directly in the monthly data, most of the tropospheric radioactivity during 1959, 1960 and 1961 had come from the stratosphere. As previously mentioned, the mean residence time of radioactive particles in the troposphere is about 30 days (Stewart, Crooks and Fisher 1956). It is very difficult to make a detailed

interpretation of these profiles because they are essentially the end product of stratospheric-transport processes, stratospheric-tropospheric exchange, tropospheric-transport and tropospheric-removal processes. All of the factors may vary with latitude and season. Nevertheless three general points are outstanding; there are maxima in middle latitudes of both hemispheres : the maxima move north and south with the sun : the concentrations are highest in the spring.

The first point cannot be explained by the variation of rainfall with latitude. In the tropics there is an inverse relationship between the radioactivity of the air and the rainfall (Lockhart, Baus and Blifford 1959) but it cannot be extended to middle latitudes for in the periods examined in detail there was more rainfall at the stations with high activity (25-40°N) than in the region to the north with much lower activity. The strontium-90 content of soil samples taken from the entire globe shows a similar middle-latitude maximum, even when the samples are collected from a line of equal rainfall rate (Alexander 1959). In an effort to elucidate the second point, profiles of mean monthly zonal component of the 200 mb wind at stations along 80°W in the northern hemisphere were compared with the radioactivity profiles and it was noted that the respective maxima move north and south in phase with the radioactivity maxima displaced to the south by 5-10°. The displacement is in the direction that would be expected from physical considerations, as the potential temperature surfaces from the lower middle-latitude stratosphere that penetrate the tropical troposphere or baroclinic zone slope downwards towards the equator. It is not intended to assert that the transport is purely meridional. The secondary maximum in the northern hemisphere at Thule (74°N) that appears after tests at high latitudes by the U.S.S.R. represents either direct stratospheric-tropospheric exchange at high latitudes or a longer wash-out time.

There is clearly much more information to be gleaned from the concentration values and no doubt will be gained when the injection into, and removal from, the troposphere is more thoroughly understood. For the present the very simple assumption will be made that the daily concentration values along 80°W in the northern hemisphere are not directly related to the rainfall. The actual tropospheric wind data can then be used to estimate the flux of fission products in the lower troposphere. Surface concentrations were combined with wind velocities from 1·0 or 1·5 km and calculations were made of the north-south eddy flux of fission products by means of the technique which will be outlined later in the discussion of ozone. The average monthly fission-product concentration at 4,000 ft has a correlation coefficient with the concentration at the surface of + 0·85 (calculated from data tabulated by Pierson, Crooks and Fisher 1960) and thus it seemed reasonable to take the transports as representative of the lower troposphere. From the flux values at adjacent stations along 80°W, with the assumption that the zonal flux is divergence free, the divergence of radioactivity in a given volume of the lower troposphere was calculated. The equation of continuity was then used to estimate the vertical flux into or out of the volume. If convergence was indicated it could only come about by descent of material from above, whereas divergence could be interpreted either as removal by precipitation or settling on to the earth's surface or as upward transport. Four two-month periods free of tests in the winters of 1958 and 1959 were chosen for the calculations. The results showed a divergence of the meridional flux from a region of 30-35°N which suggests that downward transport occurred into that region. There was also an indication of a southward transport from high latitudes which may have corresponded to direct stratospheric-tropospheric exchange. The results have been discussed in detail elsewhere (Newell 1960b) and have since been extended to include the continent of North America. Both zonal and meridional fluxes have been calculated for the entire year 1959. Again in the region of 80°W there are indications of downward transport in middle latitudes with the region of downward flux moving northward in summer. A complete description of this more complex procedure will be published in a separate paper. The main object here is to point out that not only the daily values themselves but also the tropospheric-transport values derived from these values can be explained quite well with the assumption that the majority of the radioactivity enters the troposphere in the region of the baroclinic

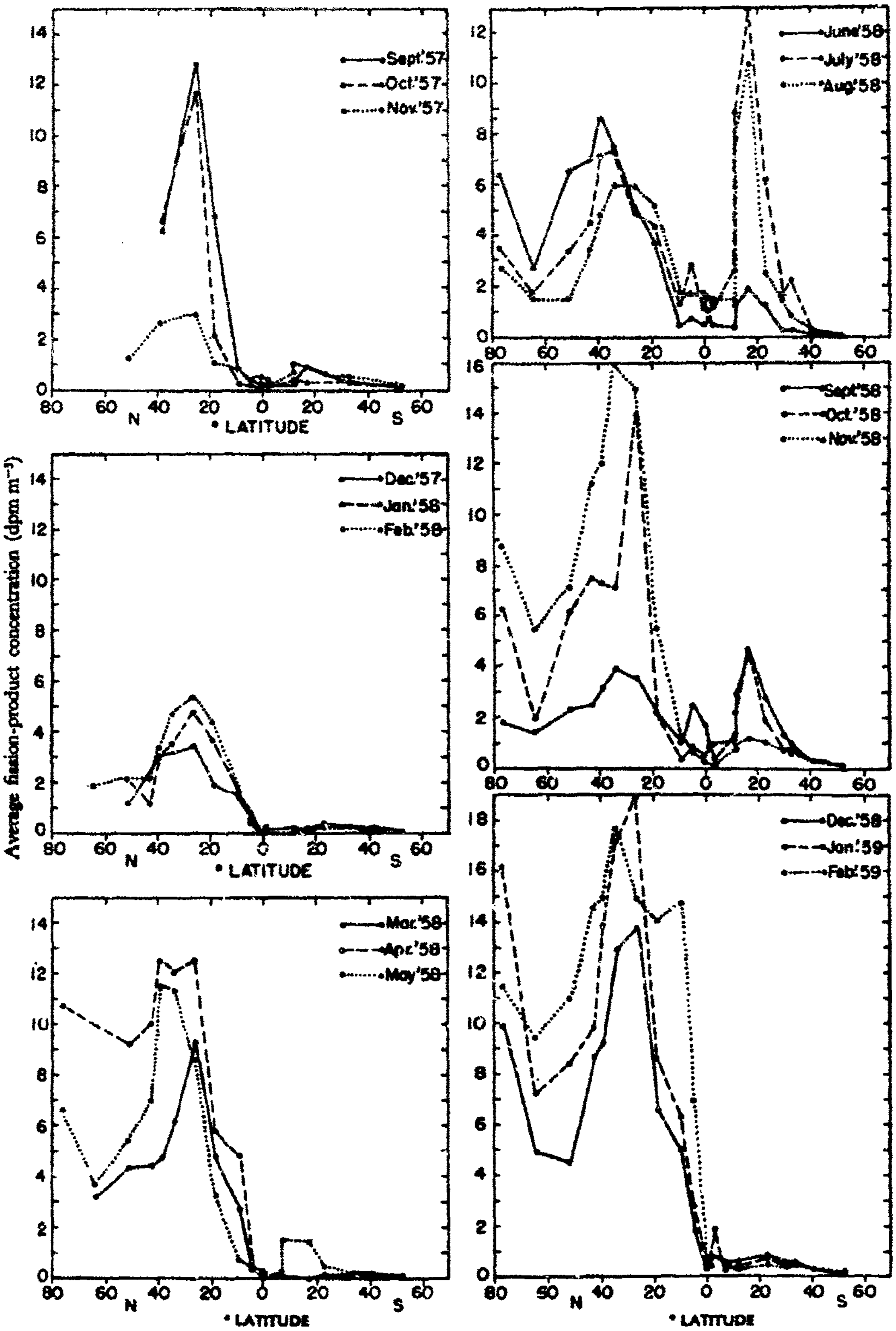

Figure 1 (a). Average fission-product concentration in surface air along 80°W. Units are in disintegrations per minute per cubic metre.

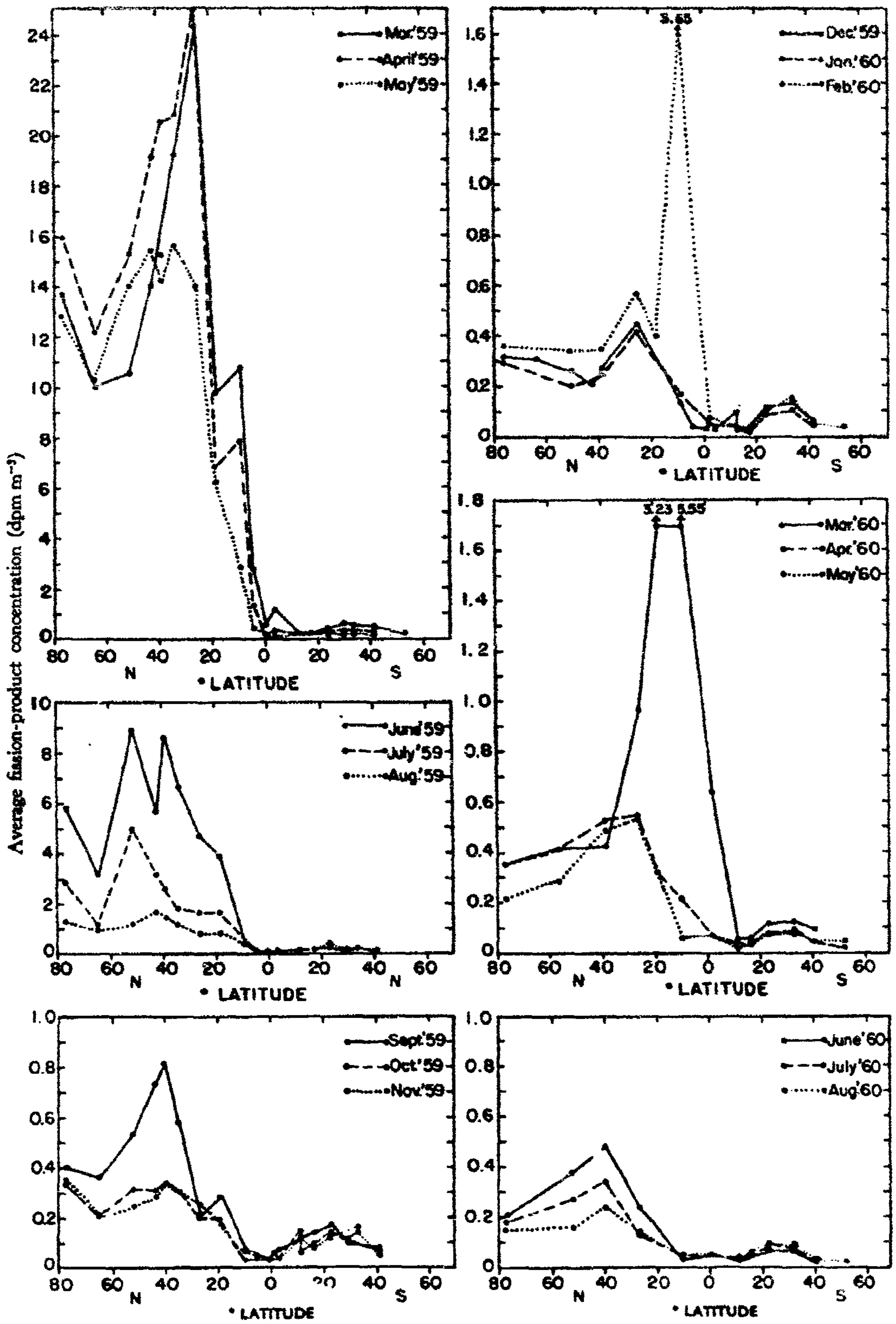

Figure 1 (b). Average fission-product concentration in surface air along 80°W. Units are in disintegrations per minute per cubic metre.

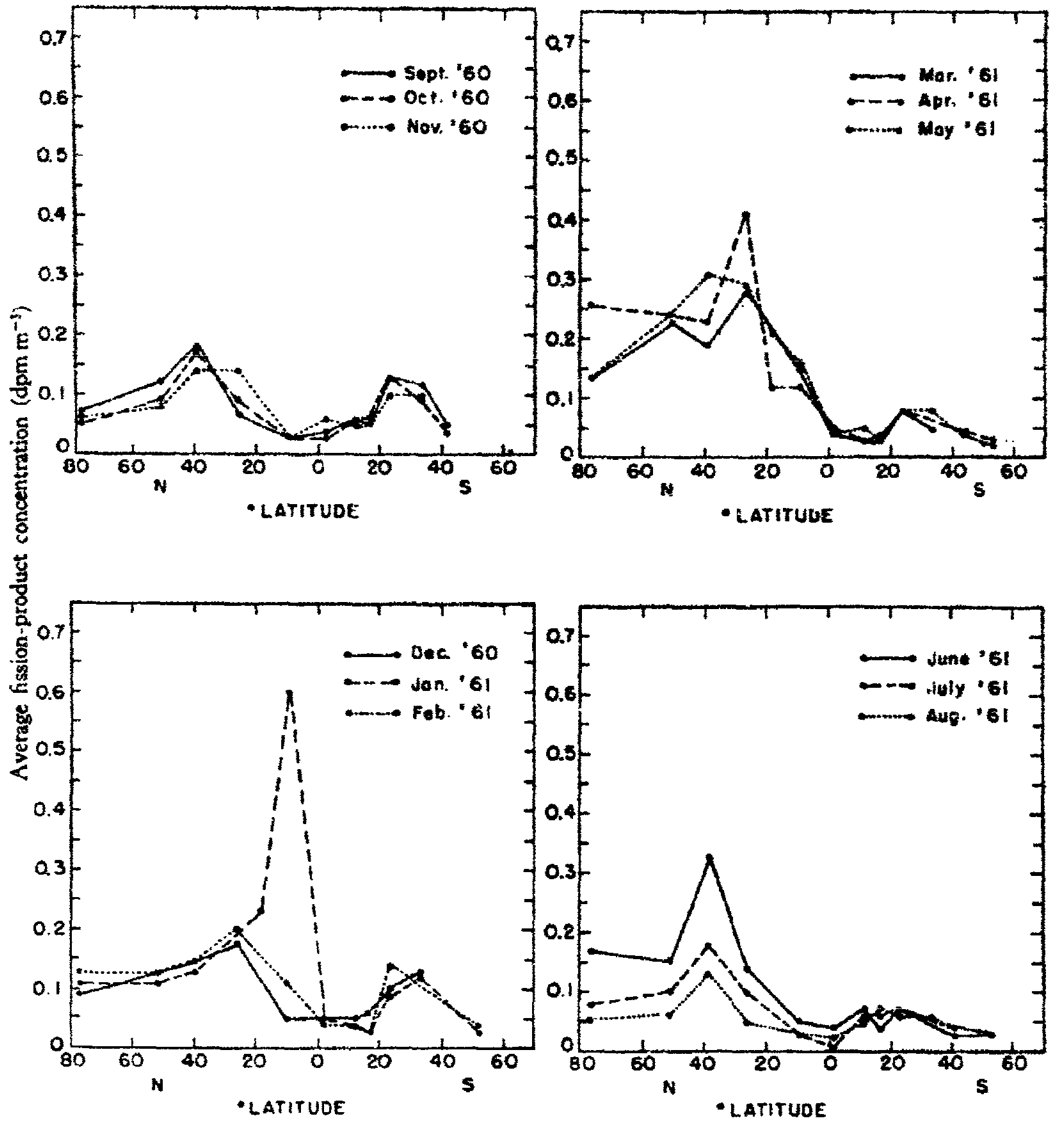

Figure 1 (c). Average fission-product concentration in surface air along 80°W. Units are in disintegrations per minute per cubic metre.

zones and tropopause discontinuities. The possibility of a secondary direct source at high latitudes from tests by the U.S.S.R. is quite reasonable on meteorological grounds because the vertical gradient of temperature in the lower stratosphere in winter is negative, as in the upper troposphere. Presumably small-scale vertical-eddy mixing can proceed more easily in these conditions than when there is an inversion at the tropopause (see, for example, the work of Ball (1960)).

There have recently been several detailed studies of the structure of the isentropic surfaces in the vicinity of frontal zones over North America. These studies (see Reed and Danielsen 1959; Danielsen 1959; Staley 1960) show several good examples where isentropic surfaces pass from troposphere to stratosphere. Consideration of constant potential vorticity trajectories on these surfaces shows the physical possibility of transfer

in both directions. The laminar structure revealed suggests that transfer is accomplished quasi-horizontally rather than by direct vertical circulations.

Having established that, according to the meteorological analysis, air can pass between the two regions in the vicinity of the jet stream and baroclinic zone, and having shown that such passage provides a reasonable explanation for the high concentrations of fission products observed in the middle-latitude troposphere, one must enquire if there is any *direct* evidence of the transfer of trace substances in the region of interest. Sawyer (1955) showed cases where the dry air in the frontal zone had probably come from the stratosphere. Helliwell's (1960) measurements suggest transfer in the opposite sense, for he reports several occasions when the air of the lower stratosphere was relatively rich in water vapour in the vicinity of frontal zones. Brewer (1960), in a discussion of some of the measurements of ozone concentration made with his chemical sonde, has pointed out several ways in which ozone may be transferred from the stratosphere into the troposphere, including direct transfer downwards through the tropopause and also the schemes presently under discussion. His measurements show evidence of ozone-poor layers in the lower temperate stratosphere which may have come from the troposphere and there are also indications of ozone-rich layers in the troposphere. Ney and Kroening (1961) using the chemiluminescent type of ozonesonde described by Regener (1960) have also detected ozone-rich layers, about 1 km thick, in the upper troposphere which may have entered from the stratosphere. It would seem that the quasi-horizontal motions indicated from the work of Danielsen and Staley could adequately account for these findings.

Roach (1961) has recently presented some meridional cross-sections of ozone and frost-point constructed from observations made by the Meteorological Research Flight. The isopleths of ozone concentration show a definite protrusion from the stratosphere into the troposphere in the region of the jet stream, the baroclinic zone, and the tropopause gap, which are all at the same general latitude. The effect is much more pronounced in winter. Roach also confirmed Murgatroyd's (1959) finding of a high negative correlation between water vapour and ozone in the vicinity of the jet stream. Ozone-poor, water-vapour-rich layers in the stratosphere had probably recently been in the troposphere.

Giles (1961) has summarized the data on the concentration of strontium-90 in the vicinity of jet streams obtained in the troposphere and stratosphere from air samples collected by aircraft. Again there is a tendency for the isolines to follow the tropopauses with higher concentrations above these surfaces and there is a protrusion into the troposphere in the vicinity of the jet stream. Paetzold and Piscaler (1961) have reported the protrusion evident from ozone soundings made during and after the International Geophysical Year. Their observations suggest protrusions at both the polar and the subtropical jets in winter.

In summarizing the status of the tropospheric-stratospheric exchange problem it can be said that there is now considerable experimental evidence from the observations of trace substances which indicates that much of the mass exchange occurs in the region of the jet stream and tropopause discontinuity. Meteorological evidence in the form of isentropic cross-sectional diagrams is not at variance with this view. Direct vertical exchange at the tropopause or exchange by virtue of the local change of height of the tropopause are factors whose contribution is not yet known. Since many of the isolines of the trace substances tend to parallel the tropopause away from the regions of the jets, it would appear that these factors are smaller than the mass exchange discussed above.

The first two facets of the surface-air radioactivity values have now been discussed at some length. The third aspect was the higher concentrations observed in the spring. Reference to Fig. 1 shows that the phenomenon is best marked in the northern hemisphere in the spring of 1959. In 1960, events were confused by the French tests of February and April but nevertheless a rise in levels appeared in December 1959 and January 1960. In 1961, there was again confusion from the French tests of December 1960 and April 1961 but yields were low and again the rise appeared at stations not influenced by these

tests. It is actually possible by using isotope ratio data or age determinations to eliminate the confusion entirely, but the pertinent data will not be discussed here. Suffice it to say that a seasonal effect of meteorological origin has been established. One may ask whether the effect is due to increased stratospheric-tropospheric exchange in the winter and spring or to increased stratospheric mixing that leads to higher concentrations in the middle latitude stratosphere or to both causes. The fact that ozone amounts are a maximum in the spring suggests that stratospheric transport is the principal variable. An attempt to examine the stratospheric mixing process is outlined in the following section.

3. Transport of ozone within the stratosphere

The problems relating to the distribution of ozone in the atmosphere have been discussed at length by many authors (see, for example, Dütsch 1946, 1956; Craig 1950; Normand 1953; Paetzold 1956; Godson 1960; Martin 1956; Martin and Brewer 1959) and as they have had considerable airing in the pages of this journal no attempt will be made to present a comprehensive review. The photochemical theory of ozone predicts maximum amounts in the layer between 20 and 30 km, with total amounts in a vertical column decreasing from the equator polewards and from summer to winter. Observations of the vertical distribution of ozone confirm the presence of the layer but observations of the horizontal distribution show an increase in the total amount of ozone with latitude in winter, and higher values in middle and high latitudes in the late winter and spring than in the late summer and autumn. The time necessary to reach 50 per cent of the photochemical equilibrium concentration, after a disturbance in which the ozone content of an air parcel is entirely removed, is 30 minutes, three days, and seven months, for heights of 50, 30 and 20 km respectively for the overhead sun, and 90 minutes, one month and 12 years for the sun at the horizon (Nicolet 1958). Hence it is obvious that the concentration of ozone in the layers below about 30 km is not influenced directly by the sun. The air in these regions is effectively shielded from the ultraviolet radiation and the ozone can thus be treated, in a sense, as a conservative tracer. Theory and observations can be reconciled if atmospheric circulations are postulated which transport ozone from low to middle and high latitudes or downwards out of the photo-chemical equilibrium regions in middle and high latitudes. Such atmospheric circulations have previously been referred to in the Introduction. In this section, calculations of the horizontal flux of ozone within the stratosphere will be presented and an effort made to assess the role of quasi-horizontal circulations in the ozone budget.

Supposing the ozone concentration at a particular point at a given time is O; we can write

$$O = \bar{O} + O',$$

where the bar represents the time mean and the prime the departure from the mean.

Likewise the northward component of the wind V may be written

$$V = \bar{V} + V'$$

The instantaneous northward transport of ozone will be given by

$$OV = \bar{O}\,\bar{V} + O'\,V' + \bar{O}\,V' + O'\,\bar{V},$$

and the time averaged transport by

$$\overline{OV} = \bar{O}\,\bar{V} + \overline{O'\,V'}.$$

The equation is an expression of the fact that the northward transport of ozone at a particular point is due to transport by mean meridional motions $\bar{O}\,\bar{V}$ and transport by

transient eddy processes $\overline{O'V'}$. For the latter term to be effective in the present problem it is necessary that northward-moving parcels of air contain more ozone than southward-moving parcels. The eastward transport of ozone could be written similarly as

$$\overline{OU} = \bar{O}\,\bar{U} + \overline{O'U'}.$$

If the discussion is extended from a point to the hemisphere and the northward transport at a particular level is considered, then a third possibility exists for meridional transport over extended time periods, namely, that in the pattern of the so-called standing eddy disturbances around the globe, the large-scale troughs and ridges, there exists some systematic relationship between the meridional components of the wind and the ozone concentration. For example, if there is more ozone in the region to the east of a trough line and to the west of a ridge line than elsewhere then a northward transport of ozone must exist. Formally such a transport can be expressed as $[\bar{O}\,\bar{V}] - [\bar{O}]\,[\bar{V}]$ where the square brackets represent average values around latitude circles. These procedures whereby a transport process is resolved into three components, the mean, the transient eddy and the standing eddy transports, have been used extensively in the study of planetary circulations in the past ten years (Priestley 1949; Starr 1951; Starr and White 1952; Starr 1954, 1957, 1959a). Originally they were applied to such intrinsic properties of the atmosphere as wind velocity and temperature to calculate momentum and heat transports, but in recent years they have been extended to the study of trace substances such as water vapour (Starr and White 1955; Hutchings 1957), ozone (Martin 1956) and radioactive substances (Newell 1960b).

In order to discuss with rigour the transport of ozone over the hemisphere it is necessary to know the concentration of ozone as a function of latitude, longitude, height and time as well as the concomitant wind distribution including both vertical and horizontal components. In practice the only synoptic observations of ozone on a global basis are those of the total amount of ozone measured by the Dobson spectro-photometer. This total amount refers to all the ozone in a vertical column above the station and a myriad of wind velocities are occurring at any one time in the column. It is difficult to see how to compute the ozone flux from these data. Fortunately it has been found that when the total amount of ozone changes, much of the change occurs, at least in middle latitudes, between 12 and 24 km. Mateer and Godson (1960) find the coefficient of correlation between the total amount of ozone and the ozone in the 12-24 km layer to be + 0·97 (for a Canadian station). They find that changes of ozone in this layer account for three-quarters of the total change, on a daily or seasonal basis. The amount of ozone at the Canadian stations on which the study was based ranged from about 0·29 to 0·50 cm at STP. Ramanathan (1956) found a similar result for the 18-27 km layer at an Indian station (the ozone centre of gravity in the vertical is higher at low latitudes). There, ozone amounts ranged from 0·15 to 0·21 cm and vertical distributions in each case were found by the *umkehr* method. The distributions and findings based on them suggest that a first approximation to the transport of ozone in the stratosphere can be obtained from wind velocities representative of the 12-24 km layer together with a measure of the total amount of ozone.

The Planetary Circulations Project of the Massachusetts Institute of Technology is engaged in an extensive study of the general circulation of the stratosphere, based on observations collected during the International Geophysical Year. Computations of the seasonal wind velocities and temperatures and fluxes of angular momentum and energy are performed by machine methods; a similar approach to that already extensively applied at tropospheric levels (see Starr 1954, 1957) is being used. Data from levels of 100, 50, 30 and 10 mb are used; in the standard atmosphere they correspond to heights of 16, 21, 24 and 30 km. Altogether some 220 stations in the northern hemisphere are used. Twenty-five stations which were either at, or close to, good wind-reporting sites, reported daily values of the total amount of ozone.

The amounts have been incorporated into the machine programme and three-month seasonal averages calculated for the six periods of the I.G.Y. In addition to average values $\overline{O}$, $\overline{V}$, $\overline{U}$, $\overline{T}$ and their standard deviations $\sigma(O)$, $\sigma(V)$, $\sigma(U)$, $\sigma(T)$, covariances $\overline{O'V'}$, $\overline{O'U'}$, $\overline{O'T'}$, $\overline{V'T'}$, $\overline{U'V'}$ and correlation coefficients $r(O, V)$, $r(O, U)$, $r(O, T)$, $r(V, T)$, $r(U, V)$ were calculated. The data which involve ozone values are presented here, although reference will be made to the other results which will be discussed in detail elsewhere by my colleagues. There are too few data to make an analysis of the 30 and 10 mb results. The fluxes reported refer only to levels of 100 and 50 mb (approximately 16 and 21 km respectively). These are the most appropriate levels according to the vertical distribution data examined above. At the lower latitudes it would perhaps be better to examine only the 50 mb results but there is always present some inter-level wind correlation (Charles 1959) and so both levels are presented. The ozone stations used are listed in Table 1

TABLE 1. Ozone stations

Station	International Index No.	Location	
Marcus Island, Pacific	91-131	24° 17'N	153° 58'E
Torishima, Japan	47-963	30° 29'N	140° 18'E
Tateno, Japan	47-646	36° 03'N	140° 08'E
Sapporo, Japan	47-412	43° 03'N	141° 20'E
Washington, D.C., U.S.A.	72-405	38° 51'N	77° 02'W
Abastumani, U.S.S.R.	37-506	41° 43'N	42° 50'E
(Tbilisi, U.S.S.R.)	37-549	41° 41'N	44° 57'E
Rome, Italy	16-239	41° 48'N	12° 36'E
Vladivostok, U.S.S.R.	31-960	43° 07'N	131° 54'E
Alma Ata, U.S.S.R.	36-870	43° 15'N	76° 56'E
Green Bay, Wisconsin, U.S.A.	72-645	44° 2[illegible]'N	88° 08'W
Bismarck, N. Dakota, U.S.A.	72-764	46° 46'N	100° 45'W
Arosa, Switzerland	—	46° 47'N	09° 41'E
(Milano, Italy)	16-080	45° 28'N	09° 17'E
Caribou, Maine, U.S.A.	72-712	46° 50'N	68° 00'W
Cambourne, England	03-808	50° 13'N	05° 19'W
Moosonee, Canada	72-836	51° 16'N	80° 39'W
Oxford, England	—	51° 46'N	01° 16'W
(Crawley, England)	03-774	51° 05'N	00° 13'W
Edmonton, Canada	72-879	53° 34'N	113° 31'W
Eskdalemuir, Scotland	03-162	55° 19'N	03° 12'W
(Leuchars, Scotland)	03-171	56° 23'N	02° 53'W
Aarhus, Denmark	06-070	56° 18'N	10° 37'E
(Copenhagen, Denmark)	06-180	55° 38'N	12° 40'E
Uppsala, Sweden	02-076	59° 52'N	17° 37'E
(Stockholm, Sweden)	02-077	59° 21'N	17° 57'E
Leningrad, U.S.S.R.	26-063	59° 57'N	30° 42'E
Lerwick, Scotland	03-005	60° 08'N	01° 11'W
Reykjavik, Iceland	04-030	64° 08'N	21° 54'W
(Keflavik, Iceland)	04-018	63° 57'N	23° 37'W
Resolute, Canada	72-924	74° 43'N	94° 59'W
Alert, Canada	74-082	82° 30'N	62° 20'W

( ) = Wind stations where wind and ozone were measured at different sites.

TABLE 2. Average ozone amounts

Units: cm of ozone at STP (Number of observations in parentheses)

Station	July-September 1957 $\bar{O}$	July-September 1957 $\sigma(O)$	October-December 1957 $\bar{O}$	October-December 1957 $\sigma(O)$	January-March 1958 $\bar{O}$	January-March 1958 $\sigma(O)$
Marcus Island	—	—	—	—	0·241 (86)	0·012
Torishima	—	—	0·262 (25)	0·012	0·310 (74)	0·030
Tateno	0·301 (82)	0·022	0·290 (85)	0·025	0·356 (77)	0·044
Sapporo	—	—	—	—	0·445 (29)	0·039
Washington D.C.	0·319 (32)	0·020	0·296 (31)	0·022	0·358 (40)	0·036
Abastumani	0·257 (46)	0·018	0·280 (37)	0·021	0·347 (29)	0·035
Rome	0·315 (81)	0·018	0·306 (81)	0·028	0·378 (90)	0·048
Vladivostok	0·268 (22)	0·015	0·259 (14)	0·026	—	—
Alma Ata	—	—	0·226 (11)	0·021	0·292 (13)	0·019
Green Bay	—	—	—	—	0·386 (12)	0·028
Bismarck	—	—	—	—	0·375 (52)	0·049
Milano	0·303 (74)	0·018	0·284 (70)	0·029	0·360 (64)	0·052
Caribou	—	—	—	—	—	—
Cambourne	0·301 (84)	0·029	0·277 (77)	0·030	0·352 (57)	0·035
Moosonee	0·325 (75)	0·024	0·319 (72)	0·031	0·397 (83)	0·046
Crawley	0·331 (91)	0·030	0·288 (86)	0·036	0·368 (78)	0·054
Edmonton	0·304 (76)	0·027	0·291 (74)	0·044	0·379 (35)	0·062
Leuchars	0·325 (5)	0·024	0·294 (64)	0·029	0·380 (62)	0·073
Copenhagen	0·352 (91)	0·026	0·306 (85)	0·033	0·428 (78)	0·073
Stockholm	0·318 (73)	0·017	0·275 (71)	0·030	0·404 (68)	0·074
Leningrad	0·347 (57)	0·025	0·303 (10)	0·019	0·460 (24)	0·023
Lerwick	0·330 (83)	0·032	0·280 (46)	0·026	0·405 (56)	0·049
Keflavik	0·307 (73)	0·022	0·291 (71)	0·032	0·401 (72)	0·075
Tromso	0·302 (80)	0·018	0·277 (86)	0·040	0·420 (86)	0·093
Resolute	0·310 (57)	0·040	0·281 (20)	0·035	0·442 (8)	0·049
Alert	0·301 (70)	0·020	—	—	0·433 (8)	0·116

Station	April-June 1958 $\bar{O}$	April-June 1958 $\sigma(O)$	July-September 1958 $\bar{O}$	July-September 1958 $\sigma(O)$	October-December 1958 $\bar{O}$	October-December 1958 $\sigma(O)$
Marcus Island	0·287 (80)	0·019	0·282 (91)	0·012	0·257 (90)	0·013
Torishima	0·341 (76)	0·021	0·300 (84)	0·024	0·256 (83)	0·016
Tateno	0·355 (81)	0·026	0·302 (84)	0·021	0·290 (70)	0·023
Sapporo	0·422 (69)	0·044	0·326 (82)	0·028	0·352 (70)	0·049
Washington D.C.	0·352 (77)	0·052	0·312 (75)	0·037	0·301 (86)	0·025
Abastumani	0·324 (24)	0·035	0·276 (40)	0·017	0·275 (44)	0·028
Rome	0·374 (90)	0·040	0·311 (91)	0·017	0·311 (79)	0·032
Vladivostok	0·322 (22)	0·038	0·273 (24)	0·014	0·272 (37)	0·045
Alma Ata	0·283 (22)	0·022	0·254 (36)	0·024	0·252 (15)	0·032
Green Bay	0·356 (66)	0·028	0·295 (64)	0·023	0·313 (37)	0·048
Bismarck	0·369 (81)	0·046	0·305 (87)	0·045	0·300 (71)	0·033
Milano	0·366 (66)	0·034	0·312 (83)	0·026	0·294 (66)	0·028
Caribou	0·376 (57)	0·031	0·321 (82)	0·035	0·318 (78)	0·046
Cambourne	0·368 (89)	0·032	0·320 (68)	0·031	0·296 (63)	0·050
Moosonee	0·413 (91)	0·050	0·339 (92)	0·028	0·359 (89)	0·050
Crawley	0·394 (87)	0·028	0·331 (90)	0·033	0·299 (71)	0·039
Edmonton	0·380 (90)	0·041	0·317 (91)	0·027	0·334 (91)	0·042
Leuchars	0·381 (68)	0·032	0·328 (85)	0·031	0·308 (78)	0·040
Copenhagen	0·414 (88)	0·038	0·337 (90)	0·033	0·308 (58)	0·032
Stockholm	0·374 (82)	0·050	0·309 (81)	0·033	0·283 (56)	0·042
Leningrad	0·397 (54)	0·039	0·337 (54)	0·031	0·272 (15)	0·018
Lerwick	0·389 (91)	0·037	0·319 (92)	0·032	0·277 (50)	0·034
Keflavik	0·385 (87)	0·036	0·316 (92)	0·038	0·319 (56)	0·047
Tromso	0·396 (85)	0·039	0·299 (80)	0·027	0·290 (76)	0·046
Resolute	0·406 (68)	0·047	0·333 (92)	0·024	0·383 (26)	0·079
Alert	0·416 (77)	0·065	0·310 (79)	0·033	0·313 (2)	0·013

together with the wind velocity stations in parenthesis where wind and ozone were measured at different sites. There are other ozone stations in the northern hemisphere but either ozone or wind values were not available here at the time the computations were made (during 1960 and early 1961). As soon as ozone amounts are made available we propose to extend the computations to 1959, 1960 and 1961.

Wind velocities were received for both 0001 GMT and 1200 GMT for all stations except Marcus Island, which relayed data only for 0001 GMT. All the calculations mentioned above were performed separately for these two times. Ozone stations in Canada, Japan, the United States and the U.S.S.R. reported values of the total amount of ozone applicable to several different times each day (on World Meteorological Organization form 0-1); in such cases the ozone value nearest in time to the wind observation was selected for the computations. Completed forms for Moosonee and Edmonton were not received until after the calculations were made and forms for the European stations still have not been received. In these cases one tentative value was available for each day and was used in the calculations with the two wind velocities nearest in time.

A summary of the average ozone amounts, by station and season, principally compiled from the results of the 1200 GMT calculations, appears in Table 2. Ozone amounts for Tromso (69° 40′N 18° 57′E) are also included for reference; this is, of course, one of the oldest ozone-reporting stations and of great interest, but winds were not available to the heights necessary for our transport calculations. The average amounts show the well-known seasonal variation, although it is not so well marked when three-month averages are considered. Most stations show higher amounts in the January-March 1958 period than the April-June period but the reverse is true for the two low-latitude stations and for those in the latitude belt from 45-55°N. There is a general tendency for ozone amounts to increase with latitude except in the October-December periods. The variation of ozone amount with longitude is quite striking. Sapporo, at 43°N 141°E, has very much higher values in the first half of 1958 than stations in comparable latitudes at other longitudes. It is difficult to decide on the basis of just one station whether the high values are meteorological in origin or are due to instrumental effects. A recent paper by Sekiguchi (1961) favours the former explanation.

The standard deviations, also shown in Table 2, are largest in middle and high latitudes and in the January-March 1958 period. In the July-September periods the variability exhibited is low at all latitudes.

Perusal of the covariance values shows considerable variation from one season to the next and even between the same season in successive years. Variations with longitude are also evident. These variations are not surprising to the meteorologist, particularly when the relatively small number of observations is taken into account. The principal interest in the present paper is in the general circulation of ozone. We have therefore collected together observations in the same latitude belt to form some estimates of the global flux of ozone. Calculations for both time sets have been combined and thus the number of cases quoted does not always represent the number of independent observations but may be up to a factor of two higher than this quantity. There was an exception to the grouping by latitude belts in the case of Japan. For reasons to be discussed presently it was decided to treat these three stations as a separate group rather than include them in the latitude belts with others. There were not sufficient stations at high latitudes to warrant combination into groups. Covariance values, henceforth referred to as transient eddy fluxes, are shown in Table 3.

The levels 50 mb and 100 mb are both presented for comparison. The resemblance between the results reflects the inter-level wind correlation. At low latitudes it is more pertinent to consider the 50 mb flux as the 100 mb level is often within the troposphere. The 50 mb level fluxes are generally positive in middle latitudes with largest values appearing in the spring. The Japanese stations show a fairly strong negative flux in the same period. Keflavik too, unfortunately the only good ozone and wind station north of 60°, shows a very strong negative flux in the spring. At 50 mb the one low-latitude station shows a northward flux throughout the year. When the average seasonal winds

TABLE 3. MERIDIONAL FLUX OF OZONE BY TRANSIENT EDDIES

Units: cm of ozone at STP m sec^{-1} (Number of cases in parentheses)

Latitude Belts	July-Sept. 1957	Oct.-Dec. 1957	Jan.-March 1958	April-June 1958	July-Sept. 1958	Oct.-Dec. 1958
100 mb						
Marcus Island 24°N	—	—	− 0·0111 (23)	− 0·0316 (68)	+ 0·0022 (81)	− 0·0442 (72)
Japan (3 stations)	− 0·0664 (172)	− 0·0050 (228)	− 0·0558 (377)	+ 0·0183 (404)	− 0·0621 (511)	− 0·0572 (459)
38°-45°N (6 stations)	+ 0·0099 (161)	+ 0·0005 (184)	+ 0·0820 (232)	+ 0·0156 (419)	+ 0·0048 (398)	− 0·0008 (332)
45°-55°N (7 stations)	+ 0·0519 (568)	+ 0·0510 (529)	+ 0·0725 (499)	+ 0·0043 (738)	+ 0·0340 (749)	+ 0·0461 (606)
55°-60°N (5 stations)	+ 0·0382 (310)	+ 0·0819 (261)	+ 0·0781 (380)	− 0·0096 (548)	+ 0·0488 (546)	+ 0·0464 (345)
Keflavik 64°N	+ 0·0381 (140)	+ 0·0240 (125)	− 0·1779 (136)	+ 0·0736 (167)	− 0·0081 (166)	− 0·1968 (105)
Resolute 75°N	+ 0·0647 (29)	+ 0·1022 (18)	− 0·0421 (6)	+ 0·0383 (99)	+ 0·0442 (121)	+ 0·1111 (18)
Alert 82·5°N	+ 0·0064 (120)	+ 0·0049 (13)	− 0·9798 (6)	+ 0·0183 (124)	+ 0·0220 (128)	+ 0·0752 (10)
50 mb						
Marcus Island 24°N	—	—	+ 0·0313 (12)	+ 0·0021 (55)	+ 0·0006 (75)	+ 0·0140 (60)
Japan (3 stations)	− 0·0075 (148)	− 0·0031 (211)	− 0·0416 (312)	+ 0·0183 (319)	− 0·0148 (461)	+ 0·0126 (393)
38°-45°N (6 stations)	+ 0·0034 (79)	+ 0·0276 (94)	+ 0·1175 (166)	+ 0·0083 (321)	+ 0·0049 (302)	− 0·0208 (220)
45°-55°N (7 stations)	+ 0·0319 (409)	+ 0·0106 (393)	+ 0·1151 (372)	+ 0·0099 (568)	+ 0·0222 (592)	+ 0·0162 (638)
55°-60°N (5 stations)	+ 0·0188 (180)	+ 0·0683 (151)	+ 0·0782 (192)	+ 0·0103 (359)	+ 0·0318 (335)	+ 0·0970 (207)
Keflavik 64°N	+ 0·0169 (141)	+ 0·0572 (117)	− 0·2680 (125)	+ 0·0390 (166)	+ 0·0030 (161)	− 0·2420 (96)
Resolute 75°N	+ 0·0406 (25)	− 0·1011 (13)	− 0·0578 (4)	+ 0·0077 (87)	+ 0·0341 (110)	+ 0·2015 (14)
Alert 82·5°N	+ 0·0022 (113)	+ 0·0675 (5)	− 0·4754 (7)	− 0·0186 (126)	+ 0·0087 (121)	+ 0·0833 (9)

over Japan were studied it was found that there is a strong jet over the area in the spring and autumn; it is possible that the high negative transports actually represent ozone amounts being transported downwards into the troposphere in the vicinity of the jet, as discussed in Section 2. The very high values of ozone amount in the area may also be due to these circumstances.

There is considerable interest attached to these two findings – that the transient eddy flux is generally northward and that the maximum is in the spring season – for these are precisely the nature of the transports required to reconcile theory and observations as pointed out earlier. But it is necessary to establish that these transports are of sufficient magnitude to be of importance in the global balance of ozone. For this purpose a crude ozone budget of the stratosphere has been constructed.

The contributions of mean motions and standing eddies to the budget are, with the present number of stations, very difficult to assess. One of my colleagues on the Planetary Circulations Project has recently completed a study of the first six months of stratospheric data from the I.G.Y. and has constructed charts of the seasonal averages of the meridional wind component which have been used to estimate the average values of the wind around latitude circles. For the six-month period July-December 1957 these values (see Barnes 1961) show a southward motion between 25° and 55°N with a maximum magnitude of about 15 cm sec^{-1}, and a northward motion with a maximum of 10 cm sec^{-1} between the equator and 25°N at 50 mb. The values are not strictly comparable with the ozone fluxes for they represent a six-month period. Even using all the available data from 220 stations in the northern hemisphere Barnes found that the hemispheric mean meridional motions were so small that they were almost lost in the meteorological 'noise' unless two three-month periods were combined. At some later date we shall have seasonal averages for several years available and then this problem will perhaps have been circumvented.

A purely objective appraisal of the contribution to the ozone flux by standing eddies appears in Table 4 as calculated directly from the formula shown. Stations in the 40-60°N latitude belt, widely spaced in longitude, were included. There is an indication of northward transport which again apparently has a maximum in winter and spring. Once more we must stress the crudeness of this approach – ideally we would like to use many more stations – but this seems to be the only step possible at the present time.

TABLE 4. MERIDIONAL FLUX OF OZONE BY STANDING EDDIES
(Units as in Table 3)

	July-Sept. 1957	Oct.-Dec. 1957	Jan.-March 1958	April-June 1958	July-Sept. 1958	Oct.-Dec. 1958
100 mb	− 0·0065	+ 0·0190	+ 0·0214	+ 0·0046	− 0·0059	+ 0·0236
50 mb	− 0·0008	+ 0·0123	+ 0·0422	+ 0·0078	− 0·0086	+ 0·0220

TABLE 5. OZONE BUDGET
(Units: cm^3 of ozone at STP sec^{-1} when multiplied by 10^9)

	July-Sept. 1957	Jan.-March 1958	July-Sept. 1958
Transport of ozone across 50°N			
(a) by transient eddies	+ 2·9	+ 10·4	+ 2·0
(b) by standing eddies	− 0·1	+ 3·8	− 0·8
(c) by mean meridional motions	− 2·9	?	?
Transport from content change north of 50°N	− 5·2	+ 9·0	− 5·2
Transport of ozone across 40°N			
(a) by transient eddies	+ 0·4	+ 12·6	+ 0·5
(b) by standing eddies	− 0·1	+ 4·5	− 0·9
(c) by mean meridional motions	− 4·9	?	?
Transport from content change north of 40°N	− 7·4	+ 11·6	− 7·4
Tropospheric downward flux	≃ 6		

To obtain an independent estimate of the seasonal changes of ozone amount, the curves presented by Godson (1960) of the average total ozone amount as a function of latitude and month of the year were used. The ozone amount from the pole to 50°N and 40°N was integrated on a monthly basis and three monthly changes were averaged to

give the average change in ozone content of the region during the season. For comparison with the January-March fluxes, the content change from December to January, January to February and February to March was computed. The budget calculations for the 50 mb level are shown in Table 5. In order to convert from the fluxes recorded in Tables 3 and 4 to actual ozone transports it was assumed that 35 per cent of the total amount of ozone was actually involved in the layer in which transport was occurring centred on 50 mb. This approximate figure was obtained from the summaries of *umkehr* observations presented by Ramanathan (1956), Dütsch (1959) and Mateer and Godson (1960). Barnes's (1961) values of the mean meridional motion were used to calculate the flux in the July-September period. The actual transports given by the direct calculation of the standing-eddy transport are also included. For reasons which have already been discussed, both of these transports should be viewed with caution. Perhaps the most important point to note is that the transient eddy flux, by itself, is of the same order of magnitude as the observed change in ozone content of the polar cap in the January-March period. Thus the large-scale quasi-horizontal transient eddies may be the prime factors in the movement of ozone northwards in the stratosphere.

From ozone-concentration measurements taken near the earth's surface, Regener (1957) has been able to estimate the vertical flux of ozone in the troposphere : his value is $1{\cdot}2 \times 10^{11}$ molecules cm^{-2} sec^{-1}. The measurements were in middle latitudes. Let us suppose that a flux one-half as great is representative of the entire hemisphere; the middle latitude values might be expected to be higher than elsewhere if most of the ozone enters the troposphere in the vicinity of the tropopause gap. Let us further suppose that all the ozone flowing downwards in the troposphere initially flowed northwards quasi-horizontally across an imaginary vertical boundary along 40°N; in this case the equivalent flux in the stratosphere corresponding to the ozone flux at the ozone sink is $\sim 6 \times 10^{9}$ cm^{3} sec^{-1} as shown in the Table. It is of the same order of magnitude as the calculated fluxes and provides an independent piece of evidence that these fluxes are important in the general circulation of ozone.

If we combine the results of the ozone budget estimate with those in Section 2 of this paper we can hypothesize the following transport path for ozone. Ozone moves northwards from low to middle and high latitudes in the lower stratosphere. There it is subjected to vertical mixing and can reach the isentropes which sometimes pass into the troposphere in the vicinity of the jet stream. Once there, it can be removed from the stratosphere. As mentioned earlier, the Japanese stations may reflect this exchange. The largest northward transport of ozone into the middle latitude stratosphere occurs in the period January-March and the well-known maximum in the total amount appears also at this time at the high-latitude stations. Afterwards much of this ozone is presumably lost to the troposphere. Junge (1961a) has recently collected values of the tropospheric concentration of ozone and shown that this, too, exhibits a seasonal variation with the maximum in the tropospheric concentration at the surface being 1-2 months later than the maximum in the total amount of ozone : his finding fits very nicely with the present model. It would thus seem that greater stratospheric mixing in winter is the reason for the observed spring maxima in both fission products and ozone.

The hypothetical path followed, as outlined above, does not seem to be subject to any long time delays. After the increased mixing in the stratosphere there soon appears a maximum value in the ozone amounts followed one or two months later by higher values at the surface as the ozone passes into the troposphere. The speed of these eddy processes can be gauged from the length of time necessary to replenish the polar cap. It has already been demonstrated that the increases in the polar-cap content can be accounted for on the basis of the calculated eddy flux values. If the ozone in the region from the pole to 50°N from the tropopause to 25 km were completely removed and then replenished by the transient eddy flux it could attain the maximum content (the value for March) in just under four months. Similarly if the maximum polar-cap content passed into the troposphere through the tropopause gaps at one-half the rate equivalent to the flux measured by Regener it would take about four months to remove all the ozone.

4. Transport of radioactive tungsten within the stratosphere

Several moderate-yield tests in the equatorial Pacific in the summer of 1958 produced radioactive tungsten-185 (whose half-life is 74 days) which was injected into the troposphere and lower stratosphere. Tungsten is not a fission product; it was produced in the bombardment of tungsten-184 by neutrons. It is believed that the sole source was the 1958 summer test series by the United States and the isotope therefore serves as a unique tracer for the debris from this series. It was first produced on 13 or 14 May 1958 and was detected in surface air by the Naval Research Laboratory stations along 80°W before the end of May (Lockhart, Baus, Patterson and Saunders 1960). By the end of July the profile along 80°W appeared very similar to that usually observed for gross-fission products; there were peaks in both hemispheres, as can be seen in Fig. 2, some 20-30° north and south of the latitude of injection at 11°N. Lockhart *et al.*, consider that the transports northward and southward to the maxima occurred in the upper troposphere. It is possible that the transport may have occurred in the lower stratosphere, in which case the two maxima represent regions of stratospheric-tropospheric exchange. Stebbins (1959) has pointed out that the first of the radioactive clouds which contained tungsten moved westwards around the globe in the stratosphere to reach eastern United States some 44 days after the explosion, during which time the debris spread in latitude by about 45°. The equivalent meridional velocity is about 1·2 m sec^{-1} which is large enough to support the argument that the two peaks came from stratospheric debris.

The data of Fig. 2 show an increase in the surface air concentration of tungsten in the spring of 1959. Similar spring maxima had been noted earlier in the concentration of gross-fission products and in the rainwater content of radioactivity but interpretation had been difficult because of the pattern of the tests. Martell (1959) considered that spring maxima were principally due to the removal from the polar stratosphere of debris introduced by Russian tests in the previous autumn. The tungsten measurements demonstrate that debris whose ultimate source is the equatorial stratosphere also enters the troposphere at a greater rate during the spring. Lockhart *et al.*, from considerations of the ratio of strontium-89 to tungsten-185 (Sr^{89} has a half-life of 50·5 days), show that debris younger than that from the 1958 summer series is also involved in the 1959 spring maximum. Walton (1960), who has measured the concentration of tungsten-185 and strontium-90 in rainfall, estimates that only 8 per cent of the total strontium-90 in rainfall collected in the spring of 1959 was from the equatorial tests – the remainder being from the October 1958 Soviet tests in the Arctic. Lockhart, Patterson, Saunders and Black (1960) estimate a value of 10 per cent for the same fraction. The principal point to note is that debris from both stratospheric sources arrives at middle latitudes and produces a maximum in the spring. A polewards mean meridional motion is difficult to reconcile with these facts.

The various arguments involving fission-product radioactivity have so far referred to measurements made at the earth's surface, except for the work reported by Stebbins (1959). In 1960, a considerable body of data was released on the concentration of radioactivity in the stratosphere (Stebbins 1960; Feely and Spar 1960b). Samples of stratospheric particulates were collected over the Americas to heights of 70,000 ft by Lockheed U-2 aircraft. Two examples of the stratospheric content of tungsten-185 are shown in Figs. 3 and 4. Concentrations represent average values over the two-month periods shown. The figures have recently been published by Stebbins (1961). In an earlier publication on this topic (Newell 1961) similar diagrams were reproduced with permission from Dr. Feely, which were kindly provided to us by Major Stebbins. All tungsten-185 disintegration rates are corrected for radioactive decay back to 15 August 1958. As a background we have used isentropic cross-sections for July 1957 and December 1957 taken from the work of Taylor (1960). The tropopause positions are from Taylor's work. The Planetary Circulations Project has data for the northern hemisphere only, at present. In the previous superposition, isentropes for the stratosphere of the northern hemisphere for longitude 80°W have been used (Newell 1961). Mean isentropes for the hemisphere give a very similar picture.

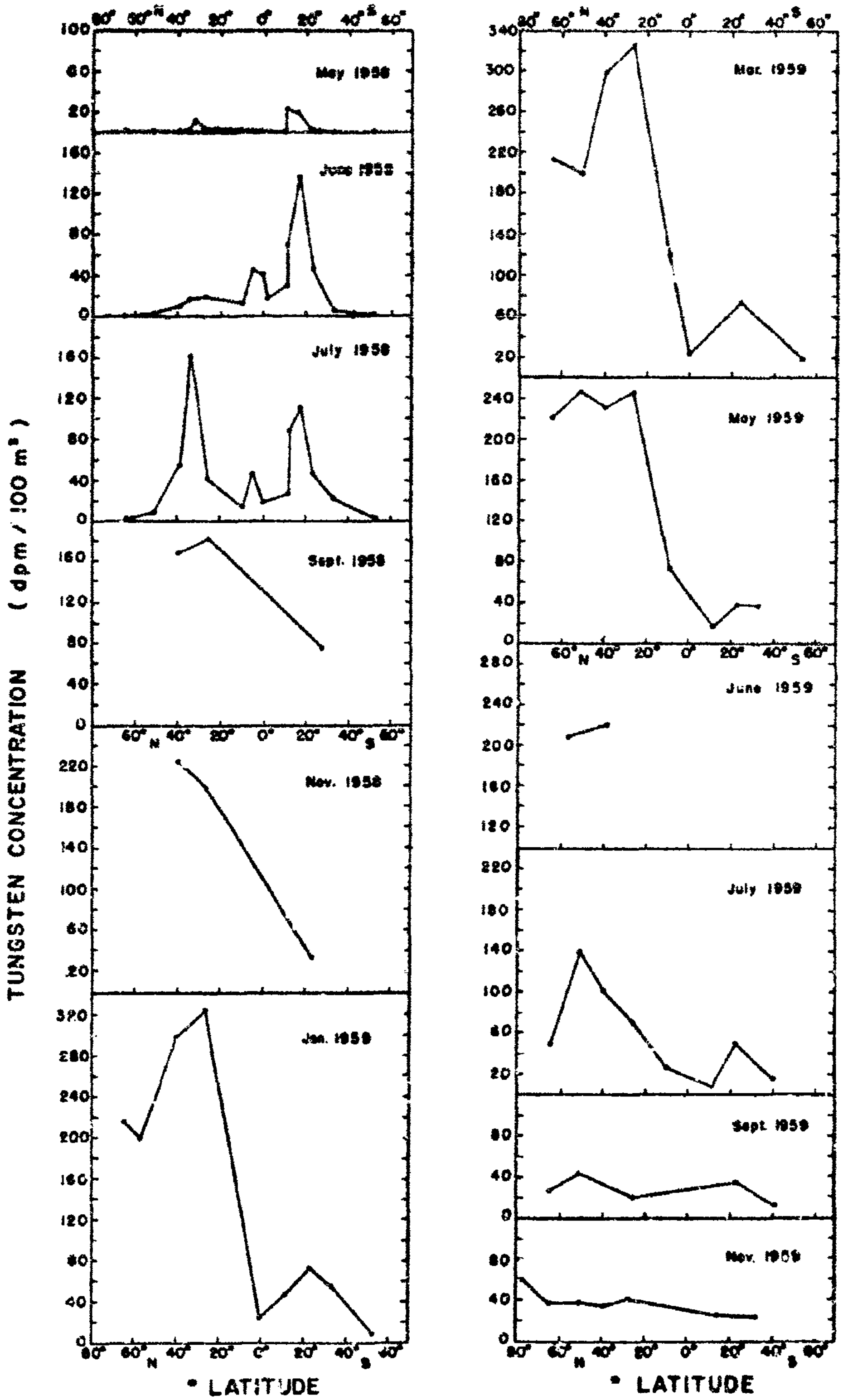

Figure 2. Average concentration of tungsten-185 in surface air along 80°W. Units are in disintegrations per minute per 100 cubic metres.

Ultimately our work will provide isentropes for 1958 but for the present paper the temperature cross-sections for 1957 will be compared with the tungsten concentrations for 1958. Both figures suggest that tungsten was transported polewards more or less along the isentropic surfaces with the greatest transport occurring in the winter hemisphere.

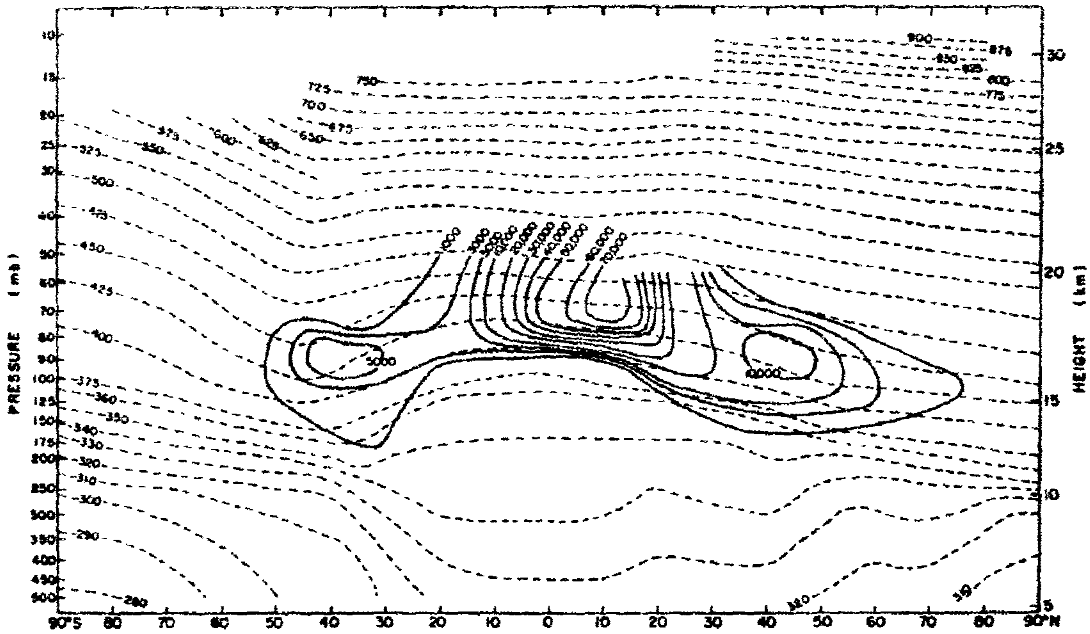

Figure 3. Distribution of tungsten-185 (solid lines) and potential temperature (dotted lines) in the stratosphere. Tungsten values for September-October 1958. Units are disintegrations per minute per 1,000 standard cubic feet of air. Potential temperatures for July 1957 (°K).

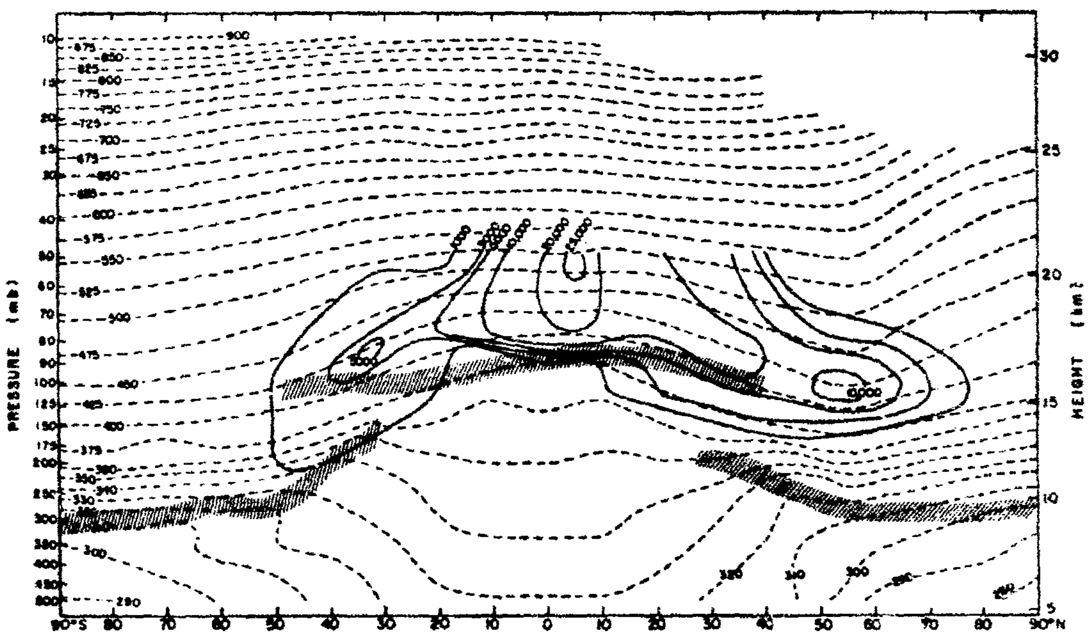

Figure 4. Similar to Fig. 3. Tungsten values for November-December 1958. Potential temperature for December 1957. Shading represents region of tropopause.

As Feely and Spar point out, indications are that the motion is some type of turbulent mixing rather than an organized mean meridional circulation. The zone of maximum concentration rather than moving northwards with time, as would be expected for a mean meridional motion, actually moved southwards. The two secondary maxima are possibly formed by the removal of tungsten into the troposphere in the region of the tropopause gaps. The secondary maxima have almost identical potential temperatures in both periods. Between the two periods the equivalent potential temperatures of the maxima decreased, in spite of the fact that the southern hemisphere maxima actually rose in altitude while that in the northern hemisphere sank. Because of the complexities involved in the interpretation it is not possible to argue unequivocally that the decrease is a diabatic cooling. Gravitational settling in conjunction with large-scale diabatic vertical motion is another possibility – as also is the chance that coincidence played a part. Table 6 shows the terminal velocities of small spherical particles with a density of 2 gm cm^{-3}. The values were obtained from a graph published by Junge, Chagnon and Manson (1961). The distance fallen by these particles in two months, the time between the tungsten cross-sections, is shown in Table 7. Although the actual distribution of radioactivity among the stratospheric particle size spectrum is not well known at present it appears that radioactive material is present on particles in the size range from 0·01 μ to 1·0 μ (Stebbins 1960). If it turns out that most of the activity is on the smallest particles in this range then essentially none of the vertical displacement of the tungsten is due to gravitational settling; but as the table illustrates it will be necessary to measure the distribution of the radioactivity as a function of particle size before a ruling can be made on the matter. In later periods the maxima depart quite chaotically from the mean isentropes given here (see Stebbins 1961). It is part of our plan to examine the actual isentropes along longitudes close to the collection sites as soon as the data are processed.

TABLE 6. Terminal velocity of spherical particles

(Units : cm sec^{-1})

Altitude (km)	Radius 0·01 μ	0·03 μ	0·1 μ	0·3 μ	1·0 μ
30	0·0022	0·0067	0·021	0·067	0·24
25	0·0010	0·0030	0·010	0·032	0·13
20	0·00048	0·0015	0·005	0·017	0·06
15	0·00022	0·00065	0·0023	0·008	0·048
10	0·00010	0·00031	0·0013	0·0048	0·038

TABLE 7. Distance fallen in two months at terminal velocity

(Units : metres)

Altitude (km)	Radius 0·01 μ	0·03 μ	0·1 μ	0·3 μ	1·0 μ
30	114	347	1083	3473	12442
25	52	156	518	1659	6739
20	25	78	259	881	3110
15	11	34	119	415	2488
10	5	16	67	249	1970

When the tungsten data were first made available in late 1960, it was very gratifying to us to see the general agreement between our indirect findings based on ozone calculations and the direct measurements in the stratosphere. The fact that eddy transports were predominant in both cases and that transport was polewards and downwards at least to about 60°N, gave reason to believe that a beginning had been made in the formulation of a model of stratospheric transport processes which was consistent with meteorological observations (to be mentioned later), as well as with measurements of trace substances. A more detailed study cannot be made until meteorological data for other seasons has been processed and until the distribution of ozone in three dimensions is at hand. Such steps may take several more years. A few generalized comparisons between meteorological evidence, tungsten-185 and ozone observations are presented below.

Let us suppose that the meridional displacement of the mid-latitude maximum in the northern hemisphere or the displacement of the 5,000 dpm/1,000 scr contour between the two profiles gives a representation of the meridional component of the quasi-horizontal eddy speed. For the region 40-60°N a speed of about 21 cm sec^{-1} is obtained. In like fashion the equivalent speed for the ozone flux may be obtained by dividing the calculated flux by the average ozone amount. Such a procedure yields a value of about 17 cm sec^{-1} from the transient eddy fluxes averaged over the two October-December periods in the 45-60°N latitude belts.

Spar, quoted by Stebbins (1960), has used the tungsten profiles and a quasi-Gaussian model of turbulent diffusion to estimate diffusion rates in both vertical and horizontal directions. He finds a value for the horizontal diffusion coefficient of 10^9 cm^2 sec^{-1}. A value for the same parameter in the troposphere is 10^8-10^{10} cm^2 sec^{-1} obtained by Grimminger (1941) from a study of isentropic charts. The difference between the two regions lies not so much in their horizontal motions, for it is well known that strong winds with a certain degree of variability exist in the stratosphere, as in their vertical structure. Measures of the variability will be quoted later.

In principle an equivalent eddy diffusion coefficient could be calculated from the ozone flux calculations in conjunction with the observed ozone distributions. Some of these distributions have been presented by Ramanathan and Kulkarni (1960) from *umkehr* measurements and by Brewer (1960) and Paetzold and Piscaler (1961) from measurements with ozone sondes. But there are not really enough soundings available yet to make quantitative estimates of diffusion from such profiles.

Spar's estimates of the vertical diffusion coefficient, which are somewhat more difficult as the source had a finite height, are 10^3 cm^2 sec^{-1} for the tropical stratosphere and about 4×10^4 cm^2 sec^{-1} for middle latitudes. The difference is in the direction that would be expected on meteorological grounds as the temperature increases with height in the tropical stratosphere, whereas in middle and high latitudes it increases by a much smaller amount and in the winter season at high latitudes it decreases with height.

The difference in the vertical diffusion coefficients in the two regions of the stratosphere may be an important factor in the physical explanation of how the ozone budget can be balanced by the quasi-horizontal motions discussed. Ozone can diffuse down the concentration gradient and therefore northwards and downwards as long as appropriate eddy shuffling occurs apparently following the general pattern of the isentropic surfaces. The process can only continue as long as ozone is removed from the region to which it is transported. The large vertical eddy diffusion allows ozone to be transferred vertically, also down the gradient, so that a particular column at middle latitudes can actually build up more ozone than a similar column at low latitudes where vertical diffusion is so much smaller. In this way ozone is apparently transferred northwards against the gradient but the gradient now is envisaged as that due to total amounts of ozone; actually quasi-horizontal eddies are transporting ozone down the concentration gradient in a particular isentropic layer. It is possible in this fashion to remove one of the major objections to the quasi-horizontal transfer approach which has hitherto been that ozone could not be transferred against the gradient of total ozone. Such thinking has led to the conception that ozone must come into the column from above. Of course in the simple picture outlined here

no attempt has been made to specify the actual processes which produce the vertical eddy diffusion – they may just be the vertical components of what have been referred to as large-scale quasi-horizontal eddies. Undoubtedly a particular isentrope has a considerable vertical displacement both with time at a given position and with latitude and longitude at a given time. But events cannot be completely isentropic, otherwise debris would remain in the stratosphere indefinitely. There is a general tendency for the debris to reach lower isentropes where it has access to the troposphere. Another point evident from the tungsten cross-sections is that inter-hemispheric mixing can occur fairly easily in the lower stratosphere. This is not the case in the lower troposphere as is evident from the radioactivity measurements shown in Fig. 1, although Lockhart *et al.* suggest that tropospheric inter-hemispheric exchange can occur at certain times. Consideration of the whole set of tungsten cross-sections shows that more poleward transfer occurs in the winter hemisphere.

TABLE 8. Latitude of maximum concentration of tungsten

	Latitude
September-October 1958	10°N
November-December 1958	4°N
January-February 1959	5°S
March-April 1959	7°N
May-June 1959	0°N
July-August 1959	0°N
September-October 1959	5°N
November-December 1959	5°N
January-February 1960	5°N
March-April 1960	?
May-June 1960	6°N

Although the tungsten data provide good evidence that mean meridional motions are not the major contributors to the poleward flux of material it is by no means ruled out that small meridional motions cannot exist at certain times. The latitude of the maximum concentration of tungsten as a function of time taken from the cross-sections published by Stebbins (1961) is shown in Table 8. During the first winter there was apparently a southward drift with a corresponding velocity of 12 cm sec^{-1} followed by a northward drift. But because the maximum was for the majority of the time just above the sampling altitudes it is very difficult to interpret this as a mean meridional motion. Another point is that the tungsten data were collected between longitudes of 70° and 130°W and do not therefore represent a global mean. It is quite probable that large-scale standing eddies exist in the tungsten concentration just as they do for ozone or wind velocity and in such a case a phase change of the standing eddy pattern could be interpreted as a mean meridional motion.

5. Meteorological evidence of meridional transport

In the January-March period it has been shown that both transient and standing eddies transport ozone northwards. Northward-moving parcels must therefore contain more ozone than southward-moving parcels. The implication from the ozone calculations taken alone was that the transport is horizontal. An equally fair interpretation would be that the transports are quasi-horizontal and that northward-moving parcels are descending so that they tend to be removed from the layer where photochemical equilibrium prevails. The tungsten data too have revealed polewards transports that are also downwards and the configuration of the isentropes leads to the suggestion that these motions are, to a certain extent, isentropic.

One might ask if there is any meteorological evidence of such an effect, namely that northward-moving parcels are sinking and southward-moving parcels are rising in the middle-latitude lower stratosphere. White (1954) reported a counter-gradient eddy flux of heat at 200 mb and 100 mb at latitudes 31°, 42·5°, 55° and 70°N. The exact mechanism whereby this flux came about was unknown at the time but in view of the ozone and tungsten data a possible interpretation is that northward-moving parcels are sinking and therefore warming adiabatically, and southward-moving parcels are rising and cooling. In Table 9 average values are shown of the covariance between the 50 mb meridional velocity and temperature at the ozone stations used. Time, seasonal and longitudinal averages are performed in exactly the same way as for the ozone flux data in Table 3. At 50 mb in the latitude range from 30° to 60°N the covariance is positive and this is just what would be expected in view of the configuration of the isentropes if the motions are largely adiabatic. The same configuration leads one to suspect that the particularly strong negative value at the Icelandic station in the spring and winter of 1958 corresponds to northward-moving parcels ascending. It will be recalled from Table 3 that in the same seasons the station exhibited a southward flux of ozone by transient eddies. As there is little information on the actual distribution of ozone north of this station it is not possible to ascribe the flux values to an increase of ozone concentration with latitude or altitude.

TABLE 9. COVARIANCE OF MERIDIONAL VELOCITY AND TEMPERATURE AT OZONE STATIONS
Units : $°C\ m\ sec^{-1}$ (Number of cases in parentheses)

Latitude Belts	July-Sept. 1957	Oct.-Dec. 1957	Jan.-March 1958	April-June 1958	July-Sept. 1958	Oct.-Dec. 1958
50 mb						
Marcus Island			(No 50 mb temperature data available)			
Japan 30°-43°N (3 stations)	+ 0·39 (404)	+ 2·84 (476)	+ 2·94 (427)	+ 2·98 (351)	+ 0·18 (480)	+ 2·18 (442)
38°-45°N (6 stations)	+ 0·87 (405)	+ 2·48 (320)	+ 5·88 (388)	+ 2·01 (459)	+ 2·02 (416)	+ 2·83 (279)
45°-55°N (7 stations)	+ 3·19 (583)	+ 1·88 (583)	+ 5·75 (640)	+ 2·64 (651)	+ 2·56 (639)	+ 5·88 (656)
55°-60°N (5 stations)	+ 4·19 (289)	+ 11·80 (209)	+ 9·47 (223)	+ 7·83 (384)	+ 4·83 (355)	+ 7·01 (331)
Keflavik 64°N	+ 0·70 (177)	+ 6·24 (149)	− 40·85 (149)	+ 0·62 (174)	− 1·23 (160)	− 20·82 (151)
Resolute 75°N	+ 3·42 (38)	− 11·69 (42)	+ 0·78 (42)	+ 6·87 (116)	− 1·23 (109)	+ 17·82 (68)
Alert 82·5°N	− 1·42 (142)	− 10·54 (106)	− 31·85 (124)	+ 0·11 (156)	− 2·46 (137)	− 18·89 (127)

TABLE 10. COVARIANCE OF MERIDIONAL AND VERTICAL VELOCITIES, NORTHERN HEMISPHERE, JANUARY [illegible]
TRANSIENT EDDY EFFECTS
Units : $cm^2\ sec^{-2}$

Pressure layer (mb)	Latitude 20°	30°	40°	50°	60°	70°	80°
1,000-850	+ 109	+ 366	+ 350	+ 197	+ 92	+ 150	− 83
850-700	+ 86	+ 324	+ 363	+ 247	+ 96	+ 196	+ 4
700-500	+ 74	+ 354	+ 463	+ 365	+ 154	+ 233	+ 133
500-300	+ 261	+ 1,192	+ 906	+ 343	+ 317	+ 725	+ 183
300-200	+ 246	+ 697	+ 844	+ 494	+ 475	+ 350	+ 58
200-100	+ 240	+ 232	+ 167	+ 258	+ 367	+ 167	+ 100
100- 50	− 35	− 97	− 103	− 11	+ 142	+ 108	+ 100

Further evidence has come from some recent work by Loisel and Molla (1961). They have evaluated the covariance between northward and vertical components of the motion in the northern hemisphere using the vertical velocities computed by Jensen (1960) by the adiabatic method together with the horizontal components of the wind. Their results for the transient eddies are quoted, with permission, in Table 10. In the troposphere the covariance values are positive indicating that, in general, northward-moving parcels are rising and southward-moving parcels are sinking. In the 50-100 mb layer, however, the sign reverses and it appears that northward-moving parcels are sinking south of 50°N and rising to the north of this latitude.

Individual trajectories of air parcels over Europe drawn by Martin and Brewer (1959) in their study of ozone changes showed, contrary to their expectations, that at the end points of northward trajectories local changes of total ozone and temperature were both positive. Their findings could be explained by the mechanisms discussed above in which northward moving ozone-rich parcels are subsiding and warming adiabatically. Although no attempt has been made here to apply the climatological findings to individual situations it seems that the relationships between temperature and ozone amount discussed by Godson (1960) for such situations are not at variance with the present interpretation. The covariance between 50 mb temperature and ozone amount is shown in Table 11 in the same form as the previous covariance summaries. Positive values are predominant, this circumstance being presumably due to the descent and adiabatic heating of ozone-rich parcels. Ultimately it will be desirable to examine the diabatic factors involved in the temperature changes. It should be noted from Table 11 that the largest covariance values occur in the spring. Calculations performed for Tromso (not shown) for 100 mb also showed the same sign as the Icelandic station with large values in the spring. It should be noted that Meetham's (1937) early work, in which a positive correlation between total amount of ozone and the potential temperature at 18 km was found, can also be interpreted by these same mechanisms.

TABLE 11. Covariance between ozone amount and temperature

Units : cm of ozone at STP °C (Number of cases in parentheses)

Latitude Belts	July-Sept. 1957	Oct.-Dec. 1957	Jan.-March 1958	April-June 1958	July-Sept. 1958	Oct.-Dec. 1958
50 mb						
Japan 30°-43°N	+ 0·0033	+ 0·0433	− 0·0091	+ 0·0358	+ 0·0172	+ 0·0296
(3 stations)	(148)	(218)	(324)	(373)	(462)	(408)
38°-45°N	+ 0·0044	+ 0·0128	+ 0·0858	+ 0·0344	+ 0·0273	+ 0·0081
(6 stations)	(299)	(192)	(243)	(472)	(479)	(331)
45°-55°N	+ 0·0408	+ 0·0286	+ 0·1182	+ 0·0043	+ 0·0333	+ 0·0605
(7 stations)	(596)	(577)	(542)	(843)	(913)	(851)
55°-60°N	+ 0·0320	+ 0·0382	+ 0·1535	− 0·0292	+ 0·0896	+ 0·0652
(5 stations)	(356)	(281)	(290)	(454)	(499)	(348)
Keflavik 64°N	+ 0·0663	+ 0·0764	+ 0·4840	+ 0·1034	+ 0·0373	+ 0·0703
	(144)	(119)	(129)	(168)	(174)	(100)
Resolute 75°N	+ 0·0852	− 0·0063	+ 0·2295	− 0·1036	+ 0·0215	+ 0·3065
	(93)	(21)	(10)	(115)	(171)	(42)
Alert 82·5°N	+ 0·0465	− 0·1245	+ 1·3418	− 0·2246	+ 0·0407	+ 0·1250
	(126)	(5)	(8)	(141)	(145)	(10)

Ozone and tungsten measurements have shown transports from which one could infer greater quasi-horizontal shuffling in the stratosphere in winter than in summer. A meteorological measure of the shuffling might be given by the variance of the meridional component of the wind. Dr. T. Murakami of the Planetary Circulations Project has recently completed a summary of the hemispherical averages of these variances as a function of latitude for the first two 3-month periods of the I.G.Y. and he was kind enough to allow me to peruse the values which he will be publishing shortly. At 50 mb, during July-September 1957, the standard deviations were about 3 m sec^{-1} and showed little dependence on latitude with a maximum close to 4 m sec^{-1} near 25°N. In the October-December 1957 period, standard deviations were again about 3 m sec^{-1} near the equator but increased to about 7 m sec^{-1} near 65°N with values of about 5 m sec^{-1} near 80°N. The main points to note are the increase of variance in the winter season and the increase from the equator polewards in this season. Both points substantiate the conclusions from the ozone and tungsten data. Such large values make it very difficult to detect a mean meridional motion of only 10 cm sec^{-1} or so, such as was discussed in relation to the ozone budget. Barnes (1961) discussed this point in his paper in which the mean meridional motions for the six-month period July-December 1957 were presented.

6. Zonal flux of stratospheric ozone

The results of the calculations concerning the zonal flux of ozone by transient eddies appear in Table 12 for the 50 mb level. While little is presently known about the balance requirements and circulation of ozone in the zonal direction apart from the fact that there are changes in ozone amount from one longitude to another as shown in Table 2, it is possible that the mean zonal motion and its standing eddies control the zonal flux. The flux ascribed to the transient eddies may be a reflection of the pattern of angular momentum transport in so far as the covariance $\overline{O'U'}$ will be predominantly positive in regions where the covariances $\overline{O'V'}$ and $\overline{U'V'}$ are *large* and positive. For example, the covariance $\overline{O'U'}$ in the October-December 1957 period is positive in middle latitudes as also is the covariance

TABLE 12. Zonal flux of ozone by transient eddies

Units: cm of ozone at STP m sec^{-1} (Number of cases in parentheses)

Latitude Belts	July-Sept. 1957	Oct.-Dec. 1957	Jan.-March 1958	April-June 1958	July-Sept. 1958	Oct.-Dec. 1958
50 mb						
Marcus Island	—	—	+ 0·0485 (12)	− 0·1135 (55)	− 0·0054 (75)	− 0·0357 (60)
Japan 30°-43°N (3 stations)	− 0·0095 (148)	+ 0·0501 (211)	+ 0·0066 (312)	+ 0·0898 (319)	− 0·0472 (461)	+ 0·0544 (393)
38°-45°N (6 stations)	− 0·0424 (79)	+ 0·0665 (94)	− 0·0480 (166)	+ 0·0925 (321)	− 0·0253 (300)	+ 0·0312 (220)
45°-55°N (7 stations)	− 0·0317 (409)	+ 0·0827 (388)	− 0·1429 (372)	+ 0·0825 (531)	− 0·0378 (592)	+ 0·0550 (518)
55°-60°N (5 stations)	− 0·0292 (180)	+ 0·0506 (151)	− 0·1739 (192)	+ 0·0870 (359)	− 0·0165 (335)	+ 0·0207 (207)
Keflavik 64°N	− 0·0328 (141)	− 0·0055 (117)	− 0·5801 (125)	+ 0·1550 (166)	− 0·0351 (161)	− 0·1132 (96)
Resolute 75°N	− 0·0582 (25)	− 0·0670 (13)	+ 0·0819 (4)	+ 0·1223 (87)	+ 0·0050 (110)	− 0·5867 (14)
Alert 82·5°N	− 0·0248 (113)	+ 0·1663 (5)	+ 0·3502 (7)	+ 0·1221 (126)	− 0·0270 (121)	+ 0·1174 (9)

$\overline{O'V'}$. The hemispheric averages show that $\overline{U'V'}$ too is positive in this season, that is, relative angular momentum is transported northwards by the transient eddies. The final interpretation of the zonal eddy flux values must therefore await completion of our studies now underway concerning the angular momentum budget of the stratosphere.

7. Application of findings to other trace substances

(a) *Strontium-90*

The distribution of strontium-90, whose half-life is about 28 years, shows middle latitude maxima in the soil, surface air and rainwater (Alexander 1959; Lockhart *et al.* 1960; Stewart *et al.* 1957). No attempt will be made to give a comprehensive bibliography of the results. Air samples to 70,000 ft in the stratosphere have been collected by the Lockheed U-2 aircraft (Stebbins 1960; 1961), and the Atomic Energy Commission has used balloons to raise sampling equipment (the Ashcan programme) to 100,000 ft (Holland 1959). Recently the United States Weather Bureau has also conducted sampling missions in the troposphere and lower stratosphere (Giles 1961). The pattern in the stratosphere, somewhat chaotic during the period of intensive weapons testing in 1958, has gradually evolved since that time as the relative contribution of the various groups of tests has altered. Figs. 5 and 6 show the stratospheric distribution during the first six months of 1959 and 1960. In 1959 when the Ashcan balloon programme enabled the isolines to be drawn to 100,000 ft, albeit from a somewhat inadequate sample-size as far as meteorologists are concerned, there is a broad zone of high concentration stretching from low to high latitudes in the northern hemisphere. Undoubtedly this is a result of the winter eddy mixing of debris from the high and low latitude tests of 1958. The maximum concentrations are at a higher elevation than those of tungsten-185 because the latter was injected by tests of only moderate yield whereas much of the strontium was injected by high-yield tests whose clouds penetrated higher into the stratosphere. The effect of the greater stability and consequently smaller mixing in the lower tropical stratosphere can be clearly seen. In 1960 (Fig. 6) the pattern had changed and there were maxima at middle and high latitudes and at the high altitudes sampled by the U-2 aircraft. Feely and Spar (1960b) and Stebbins (1961) have quoted some work by Kalkstein which shows that the pattern of rhodium-102 was similar to these strontium patterns. The majority of the rhodium, another unique radioactive tracer (half-life 210 days) that is not a fission product, was injected by two high-yield weapons fired from rockets above 100,000 ft in the equatorial mesosphere in late summer of 1958. The clouds from one of these tests apparently reached a height of 1,000,000 ft. While interpretation of this tracer is made difficult by its complex characteristics the general pattern of the results is thought to be reliable. The rhodium from the high-level explosion first appeared in the stratosphere in the summer of 1959 and by the summer of 1960 there were almost equal amounts in both hemispheres. The interpretation given by Stebbins to both the strontium-90 and rhodium-102 1960 patterns is that they are due to the entry into the sampling network at high latitudes and high altitudes of the debris from the rocket shots. The debris is then supposed to mix equatorwards down the concentration gradient in much the same manner as debris from the equatorial tests mixed polewards. The presence of the region of relatively low mixing in the equatorial stratosphere is again apparent from Fig. 6. It is presently too difficult to estimate how much of the strontium came from the rocket shot and how much is residual debris from the tests of 1958 and earlier. Further indirect evidence that the debris at high latitudes and altitudes originates from the rocket shots is provided by the age estimates derived from cerium-144 to strontium-90 ratios (cerium-144 has a half-life of 285 days). These show that debris in the northern hemisphere is younger over the pole and at high latitudes than over the equator; of course another possible interpretation is that the debris came from high-altitude Russian tests in the October 1958 series. The meteorological interpretation of the 1960 strontium and rhodium patterns is that the debris from the explosion in the mesosphere mixed laterally at high altitudes, possibly in the mesosphere, before mixing vertically down to the sampling regions, the vertical mixing being an

accompaniment of the winter polar vortices. It is important to notice here that a model based upon observations of a single element could be quite misleading. In the case of strontium-90 a natural interpretation of Figs. 5 and 6 would be that a mean meridional circulation of the Dobson-Brewer type was in operation. Yet the tungsten-185, ozone and rhodium-102 tracers suggest that this is not the case below 30 mb. It is not at present possible to present a concrete case for either type of circulation at higher levels.

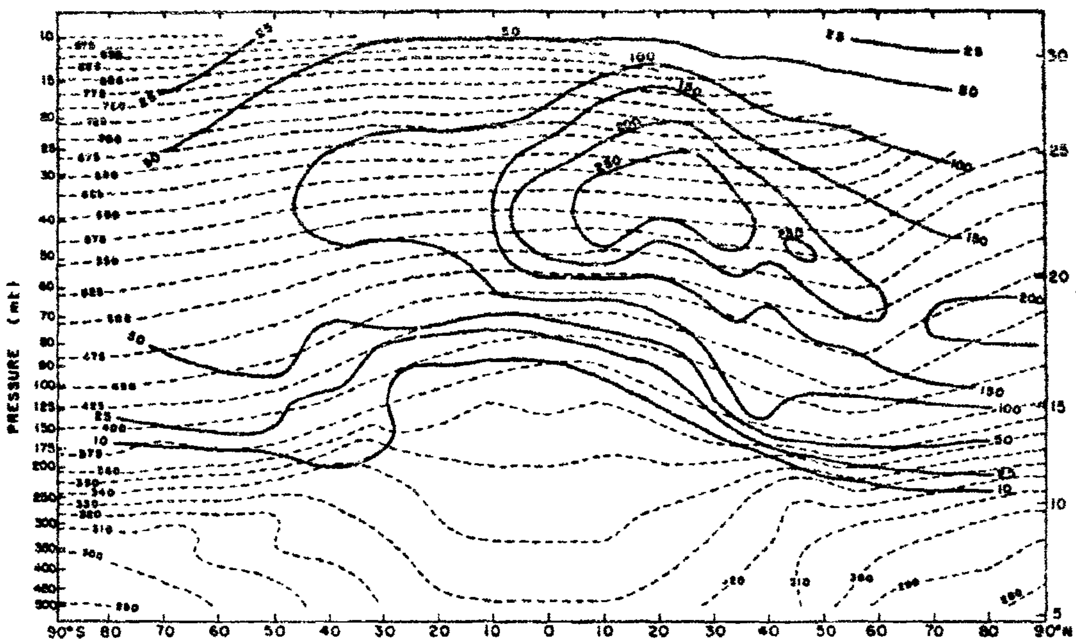

Figure 5. Distribution of strontium-90 in the stratosphere January-August 1959. Units are disintegrations per minute per 1,000 standard cubic feet of air. Potential temperatures for December 1957.

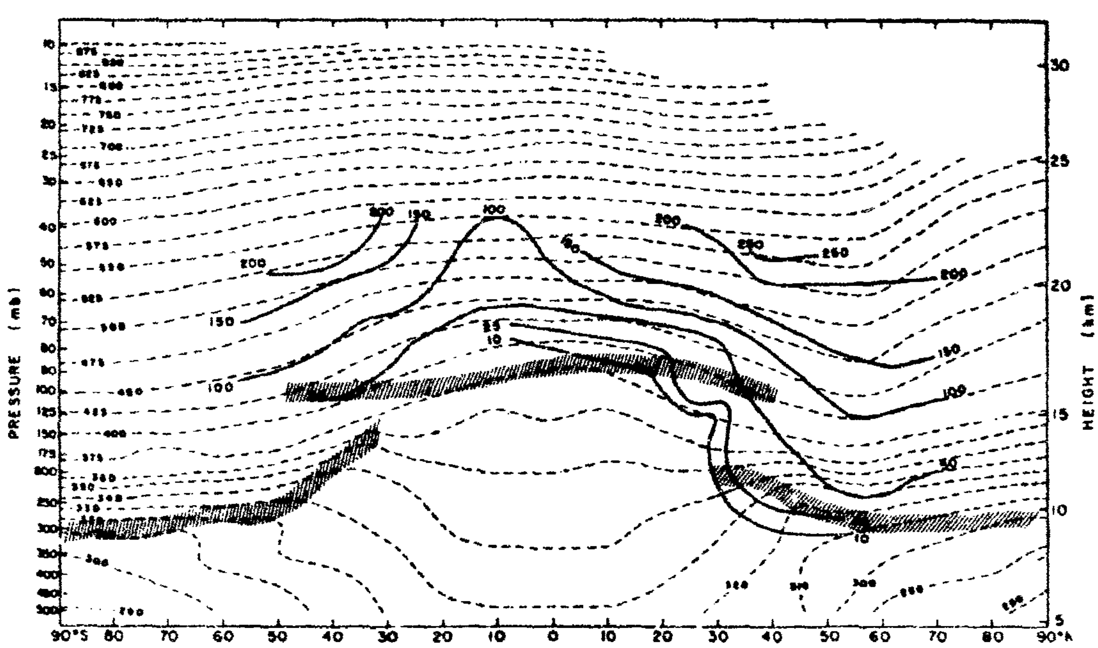

Figure 6. Similar to Fig. 5. Strontium values for January-June 1960.

(b) *Strontium-89*

The principal use of strontium-89 in tracer studies has been its application to provide a measure of the age of samples. Its half-life is 50·5 days and its physical similarity to strontium-90 makes it unlikely that atmospheric events differentiate between the two. It has therefore been the case that the ratio of strontium-89 to strontium-90, in air, water or soil, has been considered as an age parameter. Feely and Spar (1960b) presented values of the ratio in stratospheric air for the 1958-59 period. Values in the equatorial stratosphere, between 60,000 and 70,000 ft decreased with a half-time of 51 days between January and September 1959 and when the rate of decrease was extrapolated backwards to a value considered appropriate for the production ratio, the date obtained was July 1958, which was the time at which tests were carried out at low latitudes. After September 1958 the ratio no longer decreased. Feely and Spar consider that this may have been due to the difficulty of dealing with very low concentration of strontium-89 or to an influx of younger debris presumably from the October 1958 Russian tests. Samples from the southern hemisphere stratosphere gave lower values of the ratio between September 1958 and early 1959, indicating the presence of older debris. The ratio increased to reach approximately the same values as the equatorial region by the middle of the year. The increase was gradual which is in accordance with the view that large-scale eddy mixing was bringing debris from the equatorial stratosphere into the region.

(c) *Carbon-14*

In the stratosphere an excess of radioactive carbon-14, whose half-life is 5,760 years, over the natural background produced by cosmic rays, has been introduced by the high-yield weapons tests. Hagemann, Gray, Machta and Turkevich (1959) have reported measurements of the concentration of carbon-14 in samples to heights of 100,000 ft collected with the aid of high-altitude balloons. They have constructed a meridional cross-section illustrating the distribution in July 1955. The concentration lines in the stratosphere extend from the equatorial regions downwards and polewards and concentrations increase with altitude. At this time the majority of the bomb carbon had come from tests conducted at low latitudes. A mechanism which could bring about the observed distribution from the single source is the quasi-horizontal large-scale eddy mixing referred to above; in fact this was mentioned by the authors but at the time they favoured an interpretation in terms of the Brewer-Dobson direct mean meridional circulation.

(d) *Beryllium-7*

Cosmic rays are thought to be the sole source of berryllium-7 in the atmosphere. The half-life of beryllium-7 is 53 days. The theoretical concentrations have been compared with those observed by the U-2 sampling network by Stebbins (1961). Both vertical and horizontal gradients are less steep than those predicted and amounts are generally lower except in the equatorial stratosphere. Again eddy mixing provides a good explanation for the difference whereas, as Stebbins points out, a mean meridional motion model would predict higher concentrations in the polar regions rather than the observed lower amounts.

(e) *Radon and its daughter products*

A tracer offering the possibility of examining exchange from the troposphere to the stratosphere is radon-222, a gas with half-life of 3·8 days exuded from rocks in the earth's crust, and its daughter products, radium A, B and C, all particulates with very short half-lives, radium D (or lead-210 as it is more usually termed) with half-life of 19·4 years, radium E with half-life of 5 days and radium F with half-life of 138 days which decays to stable lead-206. Measurements of radium D and F in air in the troposphere and lower stratosphere have been reported by Burton and Stewart (1960). They find that specific

concentrations increase with height in the troposphere and increase more sharply with height just above the tropopause. Burton and Stewart interpret the higher values in the lower stratosphere as being due to transport of radon and its daughter products from the equatorial stratosphere northwards by mean meridional motions of the Dobson-Brewer type. The radon-rich air is supposed to enter the stratosphere by vertical motions in the equatorial regions. It is possible to account for the observations equally well with the assumptions that the radon and its daughters enter the stratosphere by quasi-horizontal mixing in the vicinity of the jet stream and tropopause gap and then are transported northwards by eddy mixing as discussed earlier. Under various sets of assumptions Burton and Stewart estimate the circulation time from equatorial to middle latitudes to be between 177 and 212 days. If we suppose that the meridional eddy speeds derived from the ozone and tungsten data are applicable to the radon and its daughters and suppose that about 30° of latitude has to be traversed northwards of the tropopause gap associated with the subtropical jet then we arrive at a transit time of between 173 and 197 days. It would seem that meridional eddy mixing provides an equally valid explanation of the observations. Telegadas and List (1961) have reported some lead-210 concentrations over North America in samples collected in the spring, which show much lower concentrations than those over England, and show maximum values at 20,000 and 30,000 ft with values at 50,000 ft some 10 times smaller than those over England at that level. In view of this situation and several other complicating factors, such as the possible production of lead-210 in tests by the neutron bombardment of bismuth-209 and lead-208, it is best to postpone further speculation about the role of the atmosphere until more data are forthcoming. One additional complication that is not usually mentioned in discussions concerning the daughter products of radon is that some become positively charged by recoil on decay and as has been well known for many years (see Rutherford 1904) they can be collected on a negatively charged wire. The author was fortunate to witness some experiments on the top of Mount Withington, New Mexico, by Professor M. H. Wilkening and Dr. A. W. Kawano in which they sampled radon and its daughter products separately at the same site. When thunderstorms were close their electric fields removed all the charged daughter products from the air; the fact that the radon content remained constant demonstrated that the effect was electrical in origin rather than being due directly to air motion.

(*f*) *Water vapour*

While ozone measurements provided the seeds for the Brewer-Dobson circulation model (Dobson, Harrison and Lawrence 1929) the strongest impetus came from the early measurements of the water-vapour content of the lower stratosphere. Brewer (1949) reported very dry air just above the troposphere over England with frost points close to $-80°C$ which correspond to mixing ratios of about $2 \times 10^{-3}\,g\,kg^{-1}$. Brewer suggested that the air had been dried by passage through the equatorial tropopause region where temperatures are about $-80°C$ and he theorized that the air then moved northwards in a mean meridional motion and subsided over middle and high latitudes. More recent measurements by the British Meteorological Research Flight (Murgatroyd, Goldsmith and Hollings 1955; Helliwell, Mackenzie and Kerley 1957; Helliwell 1960) have confirmed the presence of these low frost points. Roach (1961) reports a frost point of $-80°C$ above the polar troposphere in summer, and Roach (1961) and Kerley (1961) report similar low frost points in the upper equatorial troposphere but the latter region is somewhat isolated from the lower troposphere according to their cloud data and has a different lapse rate. These facts are difficult to reconcile with the idea that air in the tropical regions slowly rises through the tropical tropopause. On the basis of present knowledge it would probably be an equally fair interpretation to claim that the atmospheric circulation somehow acts to bring about very low concentrations of water vapour near the equatorial tropopause and that as a consequence very low temperatures appear because there is little water vapour to absorb the outgoing long-wave radiation.

Soon after Brewer's report, three high-altitude balloon flights over North America showed much higher values of the mixing ratio and a tendency for the ratio to increase with altitude above about 15 km to values close to 100×10^{-3} g kg^{-1} in two out of three cases (Barrett, Herndon and Carter 1950). Such high values were initially viewed with suspicion based mainly on the thought that water vapour may have been taken up by the balloon and apparatus and caused contamination of the high-level readings. But recently Mastenbrook and Dinger (1960) reported a similar increase in mixing ratio with height and a value of about 80×10^{-3} g kg^{-1} at 30 km. At the 100 mb level Mastenbrook and Dinger's data were comparable to the British data. Barclay, Elliott, Goldsmith and Jelley (1960) using a cooled vapour trap, measured the humidity directly at 27 km and found a value of 37×10^{-3} g kg^{-1}. Houghton and Seeley (1960) presented some spectroscopic evidence that is not inconsistent with such values at and above the 27 km level. Murcray, Murcray and Williams (1961) from infra-red absorption measurements made with a spectroscope carried aloft on a balloon also found an increase of mixing ratio above 17 km and for the path length above 30 km the absorption corresponded to mixing ratios of about 100×10^{-3} g kg^{-1} (if the mixing ratio is supposed independent of height above that level). These authors emphasize the patchiness of the moist layers.

The Japan Meteorological Agency (1961) has reported measurements from a series of special dew-point sondes, several of which have reached 10 mb, launched during the I.G.Y. and 1959-60. Considerable variability has been revealed; sometimes an increase of dew point does occur at the highest levels with mixing ratios exceeding 100×10^{-3} g kg^{-1}, but on other occasions a decrease of mixing ratio with height is noted. Independent evidence of the existence of considerable water vapour above the tropopause is provided by the mother-of-pearl and noctilucent clouds. Mother-of-pearl clouds occur at northern latitudes in winter at levels between 23 and 29 km (Störmer 1948). Present evidence suggests that they are composed of water even though temperatures are apparently about -75 to -80°C when they are reported. Saturation vapour densities over water are not normally tabulated at such low temperatures, as it is the general opinion that the water will be in the form of ice. Extrapolation from the values calculated for temperatures down to -50°C and quoted in the Smithsonian Meteorological Tables (List 1951) gives a vapour density of about 0·004 g m^{-3} at -75°C which corresponds to a mixing ratio at 25 km of about 100×10^{-3} g kg^{-1}. The value is the same order of magnitude as the mixing ratios already quoted for these levels. The noctilucent clouds are usually interpreted as either ice or dust (Ludlam 1957). They are seen at high latitudes in summer at levels of about 80 km. The rocket grenade experiments (Nordberg and Stroud 1961) indicate that at just this time and in this region occur the lowest temperatures observed anywhere in the upper atmosphere. Temperatures of 170°K have been reported. The saturation vapour density over ice at -103°C is about 0·000010 g m^{-3} (List 1951) and the density at 80 km is about 2×10^{-5} kg m^{-3}, hence a mixing ratio of 500×10^{-3} g kg^{-1} is obtained. Although this is higher than the reports at 30 km by a factor of five it is by no means unreasonable. In fact some of the Japanese results approach the value. Furthermore, in the presence of respectable vertical motion such as apparently exists with the clouds (Paton 1954) and which is to be expected in the mesosphere, it is not necessary to have such high humidity values to get cloud formation, as is well known for the case of the troposphere.

Thus there are several lines of evidence that point to the existence of higher water vapour concentrations above 25 km than are found in the lower stratosphere and one of the most intriguing present-day questions about the upper atmosphere concerns the origin of this water vapour. De Turville (1961) claims that sufficient hydrogen is received at the earth from space to account for all the water present in the oceans. He feels that water may be formed in the higher atmosphere essentially by the oxidation of incoming protons. But although it may be possible to argue that there is a down-gradient flux of water vapour between 30 km and the tropopause it was noted in Section 2 that in the vicinity of the tropopause gap water vapour seemed to be entering, rather than leaving, the stratosphere. It is difficult to explain the high values on the basis of large-scale eddy

diffusion working in the opposite direction to the ozone flux, as the present observations would suggest that such a flux would be counter-gradient. More extensive coverage of the globe geographically and in altitude is required before one can satisfactorily argue this point. Other possibilities are that the water vapour has been introduced to the higher levels by volcanic eruptions and nuclear explosions. Or it may be that we have not yet come to grips with the problem; perhaps the water vapour is bound to the stratospheric particulates such as those studied by Junge, Chagnon and Manson (1961). Junge (1961b) has reported some preliminary evidence that some of the particles are wet. Diffusion may then proceed down the gradient of particulates yet in the opposite direction to the gradient of water-vapour concentration. Finally, it should be emphasized that there is not a thorough understanding of the radiative effects of the water vapour. It is not meteorologically impossible for the layers or clouds of water vapour to be introduced into the stratosphere near the tropopause gap and to rise diabatically at certain times and in certain regions by the absorption of long-wave radiation. Indeed it is already clear that the radiation budgets constructed by Murgatroyd and Goody (1958) with the assumption of low water-vapour concentrations in the stratosphere will have to be revised to take account of the higher concentration of water vapour now thought to be present.

8. Conclusions

At the opening of this essay it was stressed that the logical approach to the study of the high atmosphere is to examine first of all the vertical fluxes of energy, angular momentum and mass at levels such as the tropopause and then to study transports within the layer immediately above, say to 30 km, and so forth. The greater part of the discussion has been concerned with the stratospheric transport and stratospheric-tropospheric exchange of mass. It has been established that a large fraction of the exchange occurs by what are essentially quasi-horizontal exchange processes in the vicinity of the baroclinic zones, the jet streams and the tropopause gaps. Within the lower stratosphere the ozone and tungsten data provide good evidence that large-scale quasi-horizontal eddy processes accomplish most of the mixing although the effects of mean meridional motions cannot be ignored entirely. Such processes satisfactorily account for the general distribution of other trace substances with the notable exception of water vapour. There is considerable meteorological evidence which also favours the model: the counter-gradient heat transport data and the covariance between the meridional and vertical components of the wind are two good examples.

No attempt has been made here to make use of the two other diagnostic tools, namely the principles of conservation of energy and angular momentum. My colleague Mr. A. A. Barnes, Jr. will shortly present a comprehensive study of those terms in the energy balance of the stratosphere that can be determined from meteorological observations and it would seem that there is nothing in the present work which conflicts with his findings. It will be recalled that a preliminary study of a selected situation by White and Nolan (1960) brought out the importance of eddy processes in the energy budget and demonstrated that they acted to convert kinetic to potential energy in the lower stratosphere. Other members of the Planetary Circulations Project are presently giving attention to the various components of the angular momentum budget of the stratosphere. Their work too has not so far produced any evidence of conflict with the circulation processes deduced from the distributions of trace substances, in fact the situation is quite to the contrary. It will be recalled that in the older pictures of the general circulation of the stratosphere a polewards mean motion imposed certain difficulties when the angular momentum balance was considered. The northwards motion actually produced a northwards transport of momentum that was too large on the basis of the observed westerly winds. In the present view there is a southwards mean motion in middle latitudes, rather like that in the upper troposphere. Preliminary evidence shows that in the October-December 1957 period the quasi-horizontal eddies transport angular momentum northwards south of 60°N and

TABLE 13. Mean zonal and meridional components of Rocket Network winds

Units: knots (N): the number of observations

Height		58·8°N, 94·3°W Ft. Churchill				38·0°N, 116·5°W Tonopah Range, Nev.				37·8°N, 75·5°W Wallops Is., Va.				34·1°N, 119·1°W Pt. Mugu., Cal.			
Km	Ft	$\bar{U}$	$\bar{V}$	(N)	σ(V)	$\bar{U}$	$\bar{V}$	(N)	σ(V)	$\bar{U}$	$\bar{V}$	(N)	σ(V)	$\bar{U}$	$\bar{V}$	(N)	σ(V)
61·0	200,000	+ 62·0	+ 12·0	(1)		+ 53·5	− 5·0	(2)		+ 100·2	+ 7·1	(14)	19·2	+ 105·9	+ 15·0	(9)	
54·9	180,000	+ 110·5	− 0·75	(4)		+ 48·5	+ 19·0	(2)		+ 82·2	+ 14·8	(29)	21·6	+ 90·3	+ 18·9	(26)	26·2
48·8	160,000	+ 94·6	− 1·7	(7)		+ 25·0	+ 5·0	(1)		+ 77·0	+ 15·5	(38)	22·0	+ 78·8	+ 10·6	(39)	17·1
42·7	140,000	+ 46·5	− 13·6	(15)	18·7	+ 68·5	+ 11·5	(15)	17·0	+ 61·4	+ 14·0	(47)	21·9	+ 65·7	+ 2·1	(44)	17·0
36·6	120,000	+ 34·8	− 13·5	(24)	28·1	+ 26·7	− 2·4	(36)	12·6	+ 43·8	+ 1·3	(51)	17·5	+ 44·0	+ 5·7	(55)	12·0
30·5	100,000	+ 17·2	− 9·3	(30)	23·9	+ 9·6	− 0·47	(36)	10·9	+ 23·6	+ 5·4	(57)	10·7	+ 22·2	+ 3·4	(63)	10·7
24·4	80,000	+ 16·9	− 10·1	(33)	17·0	− 0·23	− 2·3	(30)	6·8	+ 7·3	+ 2·5	(42)	7·6	+ 5·5	+ 1·2	(60)	7·9
18·3	60,000	+ 14·3	− 5·3	(24)	10·2	+ 10·7	− 2·4	(16)	9·3	+ 23·4	− 2·2	(31)	13·7	+ 19·4	− 0·39	(49)	9·0

Height		33·3°N, 106·5°W White Sands, New Mex.				32·9°N, 106·1°W Holloman, New Mex.				30·5°N, 86·5°W Eglin Field, Fla.				28·2°N, 80·6°W Cape Canaveral, Fla.			
Km	Ft	$\bar{U}$	$\bar{V}$	(N)	σ(V)	$\bar{U}$	$\bar{V}$	(N)	σ(V)	$\bar{U}$	$\bar{V}$	(N)	σ(V)	$\bar{U}$	$\bar{V}$	(N)	σ(V)
61·0	200,000	+ 107·2	+ 23·8	(32)	24·5	+ 113·3	+ 28·3	(7)		+ 84·0	− 0·27	(11)	16·0	+ 112·3	+ 7·5	(6)	
54·9	180,000	+ 103·6	+ 28·7	(40)	24·1	+ 60·6	+ 17·0	(8)		+ 99·8	+ 9·8	(12)	16·2	+ 114·7	+ 32·4	(11)	22·1
48·8	160,000	+ 96·0	+ 19·2	(40)	23·9	+ 62·0	+ 0·17	(12)	25·3	+ 109·4	+ 14·9	(14)	19·2	+ 83·4	+ 30·3	(17)	28·9
42·7	140,000	+ 79·6	+ 2·7	(43)	16·0	+ 54·0	− 8·1	(16)	28·3	+ 93·7	+ 8·5	(15)	23·3	+ 68·5	+ 1·8	(16)	20·4
36·6	120,000	+ 47·7	− 0·25	(56)	10·5	+ 33·0	− 2·4	(19)	15·3	+ 71·9	− 5·6	(14)	10·5	+ 52·4	+ 0·10	(21)	14·7
30·5	100,000	+ 16·6	+ 1·9	(59)	9·9	+ 8·1	− 4·6	(19)	8·4	+ 32·1	+ 7·0	(11)	6·0	+ 22·0	+ 2·3	(21)	8·5
24·4	80,000	+ 8·6	+ 0·02	(62)	7·4	+ 17·5	− 6·2	(4)		+ 5·0	− 8·0	(1)		+ 10·9	+ 1·6	(12)	5·2
18·3	60,000	+ 21·6	− 1·3	(45)	9·2	+ 20·2	+ 4·0	(4)						+ 37·2	+ 1·8	(6)	

southwards north of this latitude. Thus the eddies act in the right direction to bring about the formation of a polar jet. Although the detailed budget has not yet been drawn up it is clear that the eddy transport coupled with the southward transport by the mean motion does not violate angular momentum considerations.

At the outset it was pointed out that higher layers could be considered in turn. While there are no vertical velocities available for 30 km some wind data has been collected by the Meteorological Rocket Network over North America (Webb, Hubert, Miller and Spurling 1961) at heights to 60 km. Preliminary wind data have been supplied to us very soon after the firings and have been used so far principally in conjunction with class-room instruction. While it is appreciated that no firm conclusions can be based on these preliminary data it is of some interest to treat them rather as the stratospheric data have been treated, bearing in mind that it will be many years before good climatological values are available for these levels. The mean zonal and meridional wind velocities for the winter seasons 1959-60 and 1960-61 are shown in Table 13. All data have been assigned either

TABLE 14. Angular momentum transport from Rocket Network winds

Units: knots^2 (Number of cases in parentheses)

Height Km	Ft		58·8°N, 94·3°W *Ft. Churchill*	38·0°N, 116·5°W *Tonopah Range, Nev.*	37·8°N, 75·5°W *Wallops Is., Va.*	34·1°N, 119·1°W *Pt. Mugu., Cal.*
61·0	200,000	$\bar{U}\,\bar{V}$			+ 708·6 (14)	
		$\overline{U'V'}$			+ 257·4	
54·9	180,000	$\bar{U}\,\bar{V}$			+ 1212·8 (29)	+ 1708·9 (26)
		$\overline{U'V'}$			+ 362·2	+ 213·5
48·8	160,000	$\bar{U}\,\bar{V}$			+ 1193·9 (38)	+ 835·0 (39)
		$\overline{U'V'}$			+ 385·6	− 95·3
42·7	140,000	$\bar{U}\,\bar{V}$	− 632·9 (15)	+ 789·6 (15)	+ 858·9 (47)	+ 137·3 (44)
		$\overline{U'V'}$	+ 183·5	+ 176·4	+ 134·1	+ 70·5
36·6	120,000	$\bar{U}\,\bar{V}$	− 468·8 (24)	− 65·3 (36)	+ 55·8 (57)	+ 253·0 (55)
		$\overline{U'V'}$	+ 160·8	+ 264·8	+ 342·6	+ 24·0
30·5	100,000	$\bar{U}\,\bar{V}$	− 160·5 (30)	− 4·5 (36)	+ 128·2 (57)	+ 74·9 (63)
		$\overline{U'V'}$	+ 187·4	+ 133·6	+ 73·7	+ 4·7
24·4	80,000	$\bar{U}\,\bar{V}$	− 170·9 (33)	+ 0·54 (30)	+ 18·3 (42)	+ 6·3 (60)
		$\overline{U'V'}$	− 56·3	+ 42·5	+ 8·4	+ 68·3
18·3	60,000	$\bar{U}\,\bar{V}$	− 75·8 (24)	− 26·1 (16)	− 50·5 (60)	− 7·5 (49)
		$\overline{U'V'}$	− 41·2	+ 29·1	− 51·2	+ 7·3

Height Km	Ft		33·3°N, 106·5°W *White Sands, New Mex.*	32·9°N, 106·1°W *Holloman, New Mex.*	30·5°N, 86·5°W *Eglin Field, Fla.*	28·2°N, 80·6°W *Cape Canaveral, Fla.*
61·0	200,000	$\bar{U}\,\bar{V}$	+ 2555·0 (32)		− 22·9 (11)	
		$\overline{U'V'}$	+ 524·8		+ 48·5	
54·9	180,000	$\bar{U}\,\bar{V}$	+ 2974·8 (40)		+ 980·9 (12)	
		$\overline{U'V'}$	+ 401·0		− 179·4	
48·8	160,000	$\bar{U}\,\bar{V}$	+ 1846·1 (40)	+ 10·3 (12)	+ 1632·5 (14)	+ 2525·1 (17)
		$\overline{U'V'}$	+ 459·5	− 404·8	− 161·9	+ 177·8
42·7	140,000	$\bar{U}\,\bar{V}$	+ 218·4 (43)	− 435·4 (16)	+ 800·0 (15)	+ 124·2 (16)
		$\overline{U'V'}$	+ 115·2	+ 299·8	+ 354·3	+ 79·3
36·6	120,000	$\bar{U}\,\bar{V}$	− 11·9 (56)	− 79·9 (19)	− 405·5 (14)	+ 5·0 (21)
		$\overline{U'V'}$	+ 37·2	+ 408·7	+ 86·8	+ 16·7
30·5	100,000	$\bar{U}\,\bar{V}$	+ 32·4 (59)	− 37·1 (19)	+ 224·6 (11)	+ 50·2 (21)
		$\overline{U'V'}$	− 4·4	+ 38·9	+ 132·0	+ 62·5
24·4	80,000	$\bar{U}\,\bar{V}$	+ 0·14 (62)			
		$\overline{U'V'}$	+ 49·6			
18·3	60,000	$\bar{U}\,\bar{V}$	− 28·3 (45)			
		$\overline{U'V'}$	+ 19·4			

to winter or summer in the summaries. (N) represents the number of observations. Units are knots. It can be seen that there are indications of a mean meridional motion polewards at middle latitudes at about 50 km. A similar motion with smaller magnitude is evident in the summer summary. But caution should be exercised in the interpretation for this may just be a reflection of a standing-eddy pattern such as is observed at lower altitudes. Thus the existence of mean meridional motions at these levels will remain in doubt until the network is extended to Europe and Asia. The standard deviation of the meridional component $\sigma(V)$ is also shown for cases where 10 or more values were available. The meridional eddy shuffling apparently increases with altitude although part of this variance may be of instrumental origin. Table 14 illustrates the transport of angular momentum calculated from the observations. The important point to note here is that even at 50 km the transport by transient eddies is not negligible compared with the calculated total transport. It is not yet possible with so few observations to make a start on the construction of a budget to 60 km but this will be done in a few more years if observations are continued and extended. This concludes our general argument that eddy-mixing processes are important above the tropopause as well as below. The fact that the molecular weight of air remains constant with altitude up to about 100 km is good evidence that mixing occurs up to this level; within the next few years we should reach an understanding as to the amount of the mixing that can be ascribed to large-scale eddy motions such as are in evidence at lower levels.

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Further statistics on the exchange of kinetic energy between harmonic components of the atmospheric flow

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ABSTRACT

The transfers of kinetic energy between harmonic components of the 500 mb geostrophic flow over the Northern Hemisphere have been measured for an ensemble of daily maps covering a nine-year period, based on a truncation at zonal wave number, $n = 15$. The results show (1) that, in the mean, all waves ($n = 1$–15) transfer their energy to the zonally-averaged motion ($n = 0$) and, of more physical significance, the aggregate of all waves in the group $n = 2$–15 transfer energy to support the asymmetric polar vortex comprised of wave numbers zero and one, and (2) that, among the waves themselves, waves of $n = 2$ and 5–10 are sources of kinetic energy and all the rest are sinks. The energy source at $n = 2$ seems to be a significant new result indicating a strong forced conversion of energy on the scale of the major continents and oceans. Seasonal variations are discussed.

Daily measurements of the rates of transfer of kinetic energy to the zonal current from the eddies of different wave number, n (denoted by M) (SALTZMAN & FLEISHER, 1960*a*), and between the different eddy scales themselves (denoted by L) (SALTZMAN & FLEISHER, 1960*b*), have now been extended to cover a nine-year period within the years 1955 to 1964.

In view of the longer period of record, we feel enough confidence has been added to the averages to present a resolution into three-month and half-year "seasonal" averages in addition to the annual average, and also into individual wave numbers instead of the groups of wave numbers given previously. The measurements are still only for the 15° to 80° N zonal band at the 500-mb level and for $n = 1$–15. They are based on the same assumptions used in the previous studies (e.g., only the horizontal, geostrophic, components of the motion are considered).

The new nine-year averages are given in Table 1 along with probable errors (computed using half the total number of cases) for the ensemble of daily values over the entire year, the warmer and colder half years, and the three-month seasons. The six-month means are also shown graphically in Fig. 1, and the annual budget is represented schematically in Fig. 2.

From the M values, it can be seen that all waves tend to feed their energy into the zonal current, with maxima at $n = 2$ and 7 in the annual, colder six-month, and winter means. A minimum in M occurs at $n = 4$, as it did in the previous study covering the year 1951. For all wave numbers, the mean values of M appear to be significantly different from zero. The total gain of kinetic energy by the zonal current, measured by $\sum_{n=1}^{15} M(n)$, is given in the last column of Table 1.

The redistribution of kinetic energy among the individual waves, measured by L, shows, in general, a net gain by the long waves $n = 1$, 3, and 4, and the short waves $n = 11$–15. The large loss from $n = 2$ appears to be an important new finding that suggests a large forced conversion of potential energy on this scale associated with the continent–ocean structure. The loss from the cyclone band $n = 5$–10 is probably compensated by the normal free baroclinic development processes within the troposphere.

From a synoptic viewpoint, we note that the so-called "polar vortex" is no more than the sum of an axially-symmetric component of motion corresponding to $n = 0$ and an axially-asymmetric component corresponding to $n = 1$. Thus, the results here show that the whole polar vortex tends to derive an important

TABLE 1. *Mean values of M and L and their probable errors ε, in units of 10^{-3} ergs/cm² sec mb.*

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	$\sum_{n=1}^{15}$
Year																	
M		5	26	21	11	14	21	22	15	12	8	5	4	2	2	1	169
$\varepsilon(M)\pm$		*3*	*5*	*5*	*4*	*4*	*4*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*0*	*0*	*12*
L		87	−53	29	12	−3	−37	−39	−35	−14	−11	4	5	12	17	26	0
$\varepsilon(L)\pm$		*21*	*24*	*23*	*23*	*24*	*22*	*23*	*20*	*17*	*14*	*11*	*9*	*8*	*7*	*6*	—
Gain	169	82	−79	8	1	−17	−58	−61	−50	−26	−19	−1	1	10	15	25	
Colder months (Oct.–Mar.)																	
M		12	42	27	12	19	23	31	18	14	8	6	4	3	2	1	222
$\varepsilon(M)\pm$		*5*	*9*	*10*	*8*	*7*	*6*	*5*	*4*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*22*
L		148	−107	47	26	−4	−38	−57	−55	−21	−20	1	4	12	27	38	0
$\varepsilon(L)\pm$		*38*	*43*	*43*	*42*	*44*	*39*	*40*	*36*	*30*	*25*	*20*	*17*	*15*	*12*	*11*	—
Gain	222	136	−149	20	13	−23	−61	−88	−73	−35	−28	−5	0	9	25	37	
Warmer months (Apr.–Sept.)																	
M		−2	10	14	11	10	20	14	11	10	7	4	3	2	1	1	116
$\varepsilon(M)\pm$		*2*	*4*	*4*	*4*	*3*	*3*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*0*	*0*	*11*
L		26	1	13	−1	−2	−35	−22	−15	−8	−3	7	5	12	8	14	0
$\varepsilon(L)\pm$		*17*	*19*	*19*	*19*	*20*	*20*	*20*	*18*	*16*	*13*	*10*	*8*	*7*	*6*	*5*	—
Gain	116	28	−9	−1	−12	−12	−55	−36	−26	−18	−10	3	2	10	7	13	
Winter (Dec.–Feb.)																	
M		15	54	25	11	19	22	28	17	11	7	5	4	2	1	1	222
$\varepsilon(M)\pm$		*7*	*15*	*17*	*12*	*11*	*9*	*7*	*6*	*4*	*4*	*3*	*2*	*1*	*1*	*1*	*35*
L		210	−154	61	22	−18	−16	−73	−53	−15	−35	−10	1	9	26	45	0
$\varepsilon(L)\pm$		*63*	*71*	*68*	*69*	*70*	*61*	*65*	*59*	*49*	*40*	*32*	*27*	*24*	*19*	*18*	—
Gain	222	195	−208	36	11	−37	−38	−101	−70	−26	−42	−15	−3	7	25	44	
Spring (Mar.–May)																	
M		2	20	17	21	12	22	27	12	11	8	5	4	2	1	1	165
$\varepsilon(M)\pm$		*6*	*9*	*9*	*8*	*7*	*7*	*6*	*5*	*3*	*3*	*2*	*1*	*1*	*1*	*1*	*22*
L		73	−27	14	16	28	−67	−28	−53	−24	−5	9	9	10	13	32	0
$\varepsilon(L)\pm$		*36*	*45*	*45*	*42*	*46*	*46*	*46*	*41*	*33*	*32*	*23*	*20*	*16*	*15*	*12*	*32*
Gain	165	71	−47	−3	−5	16	−89	−55	−65	−35	−13	4	5	8	12	31	
Summer (Jun.–Aug.)																	
M		−6	9	7	3	4	13	8	10	10	6	3	3	2	1	1	74
$\varepsilon(M)\pm$		*2*	*4*	*4*	*4*	*4*	*4*	*3*	*2*	*2*	*1*	*1*	*1*	*1*	*1*	*0*	*11*
L		15	4	9	−3	−1	−12	−23	−8	−7	−1	6	3	7	6	5	0
$\varepsilon(L)\pm$		*17*	*17*	*16*	*18*	*18*	*19*	*19*	*18*	*15*	*12*	*10*	*8*	*7*	*6*	*5*	—
Gain	74	21	−5	2	−6	−5	−25	−31	−18	−17	−7	3	0	5	5	4	
Autumn (Sep.–Nov.)																	
M		8	23	35	10	22	28	27	20	16	9	7	4	4	3	1	217
$\varepsilon(M)\pm$		*4*	*8*	*9*	*8*	*8*	*8*	*6*	*5*	*4*	*3*	*2*	*2*	*2*	*1*	*1*	*23*
L		54	−37	36	13	−22	−51	−32	−26	−12	−3	10	5	20	24	21	0
$\varepsilon(L)\pm$		*37*	*39*	*43*	*44*	*43*	*40*	*40*	*33*	*28*	*23*	*20*	*16*	*15*	*11*	*10*	—
Gain	217	46	−60	1	3	−44	−79	−59	−46	−28	−12	3	1	16	21	30	

part of its energy from the higher wave numbers ($n \geq 2$) by the non-linear transfer processes measured by $\sum_{n=2}^{15} M(n)$ and $L(1)$.

As should be expected from these geostrophic calculations, the results over the spectral region studied are in accord with the theorems on energy transfer in two-dimensional, non-divergent flows presented by FJØRTOFT, 1953: e.g., the loss of energy from intermediate scales is accompanied by a gain of energy by larger

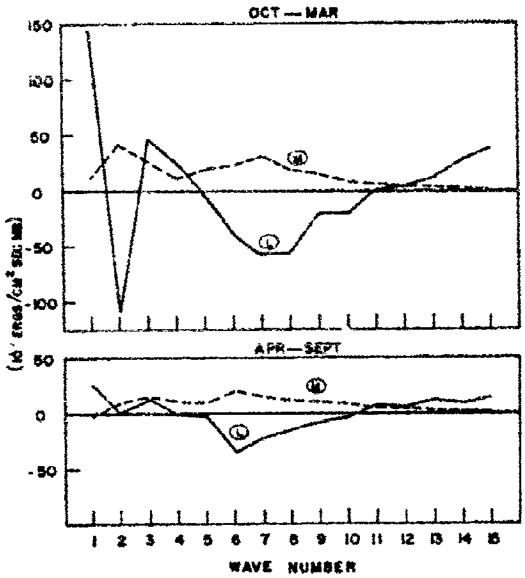

Fig. 1. Energy transfer spectral functions, L (solid line) and M (dashed line), for the cold and warm half years, based on a nine-year record. Lines are drawn between discrete values only for visual aid.

scales as well as by smaller scales. The consistent gain of energy by the higher wave numbers, $n = 11$–15, gives the appearance of a Kolmogoroff-type cascade constrained by the truncation at $n = 15$. In this connection, it is important to recognize that the transfer spectrum is a function of the particular truncation point chosen. For example, if we could extend the calculation to include infinitely high wave numbers (and, even more so, if we included *vertical* transfer processes) we would thereby encompass dissipative energy transfers associated with eddy viscosity, and, as a result, this would markedly affect the entire transfer spectrum. In our case, we have arbitrarily truncated at $n = 15$ in the belief that this represents a rough limit of the scales describable on hemispheric synoptic charts. Accordingly, we consider the aggregate of all scales $n \geqslant 16$ as a sort of viscous sink for the surplus energies acquired in the long-time average by eddies in the group $n = 1$–15, through the processes measured by L and M and by conversion from potential energy. In truth, however, the region $n \geqslant 16$ is itself rich in energetical detail involving all subsynoptic phenomena, and, in fact, certain portions of this region may even be significant *sources* of kinetic energy for the synoptic motions (e.g., organized cumulus convective motions).

Inspection of the error estimates ($\varepsilon = 2\sigma/\sqrt{N/2}$ where σ is the standard deviation and N is the number of days) shows much more variability in L than in M. In fact, for most wave numbers, L varies on a daily or longer period basis between positive and negative values so that, from time to time, the dominant kinetic energy source appears to shift from one wave number to another.

The month-to-month variation in kinetic energy transfer by those waves demonstrating a large and distinct annual cycle is shown in

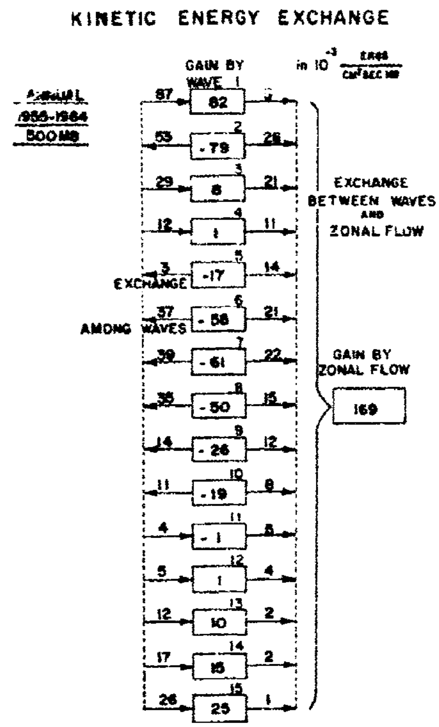

Fig. 2. Annual means of L and M represented in the form of a budget. L values are in the first open column and M values in the second open column with the plus sign shown by an arrow pointing toward the right. The net gain of kinetic energy by individual waves $[L(n) - M(n)]$ and by the zonal flow $\sum_{n=1}^{15} M(n)]$ is shown by the figures within boxes. A negative value within a box thus represents an exported quantity of kinetic energy which, it is assumed, was generated within the given wave number principally by conversion from available potential energy and which is in excess of any amount consumed by friction.

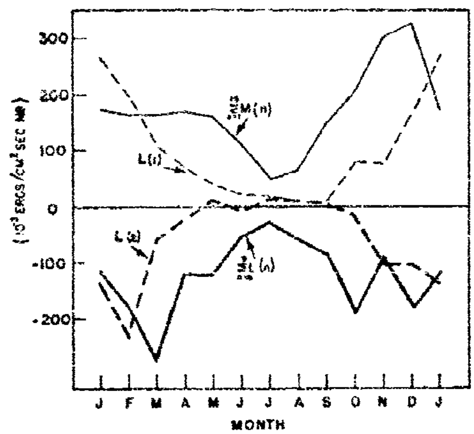

Fig. 3. Monthly means of $\sum_{n=1}^{15} M(n)$, $L(1)$, $L(2)$, $\sum_{n=6}^{9} L(n)$.

Fig. 3. It is interesting to note the degree to which wave numbers 1 and 2 appear to complement each other. The transfer *into* wave number 1 and the transfer *out* of wave number 2 both have large values in the colder months and near zero values in the warmer months, wave number 2 in particular having little net transport between April and October. Although there is a similar annual cycle in the transfer *into* the zonal flow, $\sum_{n=1}^{15} M(n)$, and the transfer *out* of the group of wave numbers 6–9, the rate of transfer remains substantial even in summer when a minimum rate of transfer appears in July. A unique characteristic of the individual month of February, included in the winter statistics, is the tendency for a large positive value of L in wave number 4.

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On the energetics of the mean and eddy circulations in the lower stratosphere[1]

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ABSTRACT

A hemispheric network of radiosonde stations is used in order to study the energetics of the lower stratosphere during the IGY period July 1957 through June 1958. For a hemispheric polar cap with 30 and 100 mb as top and bottom boundaries the balance equations of zonal and eddy kinetic energy, and zonal and eddy available potential energy are considered in detail. The eddies appear to build up the kinetic energy of the zonal flow at the expense of the eddy kinetic energy during all seasons. The eddies lose also eddy potential energy to the mean zonal distribution, in agreement with the abnormal upslope direction of the eddy heat transport. Thus, no source of energy for the eddy motions appears to be present *in situ* and the eddies must be forced by the circulation in the adjacent layers, probably by the tropospheric motions.

The energy-cycle in the lower stratosphere is in many respects different from the one usually found in the troposphere, where, as is well known, eddy kinetic energy is destroyed by friction and the energy source is found in the creation of zonal available potential energy by radiation and the subsequent baroclinic processes. For the 100–30 mb layer the necessary kinetic energy is supplied by interaction at the top and/or bottom boundaries, while ultimately the energy is destroyed in the form of zonal available potential energy by radiation.

1. Introduction

The research reported in this paper forms part of an extensive stratospheric study by the M.I.T. Planetary Circulations Project. The basic observations, namely the horizontal components of the wind, the temperature and the height of the pressure levels at 100, 50 and 30 mb, are taken from microcards which were issued by the World Meteorological Organization for the International Geophysical Year (IGY). A selected network of about 240 radiosonde stations, giving a good coverage over the northern hemisphere, is used. The selected time period is July 1957 through June 1958; this year of data was subdivided into four periods of three months each, in order to study possible seasonal variations. The data were reduced for every station separately. Then, after plotting the calculated three-monthly means on hemispheric maps, the latter were analyzed by hand. By this method the mean fields of the basic quantities and of their covariances in time were obtained. Gridpoint values which were read off the maps, form the basic material for calculating the terms in the energy cycle.

This paper deals with the energy budget for a stratospheric layer which is bounded in the vertical by the constant pressure levels 100 and 30 mb and in the horizontal direction by a "wall" at the equator. Our intention is to determine what processes might be responsible for maintaining the mean state of the general circulation in this polar cap. Masswise the layer between 100 and 30 mb is of little importance, since it contains only 1/14 of the total mass of

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the atmosphere, while about 8/10 is concentrated in the troposphere. As we shall see, the general circulation in the lower part of the stratosphere is very different from the one either below in the troposphere or above in the middle stratosphere. In those layers it is evident that the atmospheric disturbances tend to release kinetic energy at the cost of potential and internal energy.

The present work is one of the first attempts in studies of the general circulation to consider the complete energy cycle for a separate layer in the atmosphere. Much of the earlier research of hemispheric scope has been aimed exclusively at the behavior of the atmosphere integrated over all layers in the vertical. There is much to be gained from detailed information on the individual layers of the atmosphere. However, our approach has on this account certain limitations and additional problems, such as the following: (1) boundary effects in the vertical have to be taken into account, (2) the terms at the boundaries cause a certain arbitrariness in the definitions of energy conversions for the layer, and (3) the total mass transport in the layer across a latitude circle is not necessarily equal to zero.

2. Notation

(x, y, p) = coordinates with axes in eastward and northward directions and a pressure coordinate

ϕ = latitude

λ = longitude

H = height isobaric level

u = west-east component of the wind (positive if from the west)

v = south-north component of the wind (positive if from the south)

$c = \sqrt{u^2 + v^2}$

$\omega = dp/dt$ = "vertical velocity"

$\Phi = gH$ = geopotential

T = temperature

θ = potential temperature

α = specific volume

$\gamma = -\partial T/\partial z$ = lapse rate

γ_d = dry adiabatic lapse rate

$dm = a^2 \cos\phi \, d\lambda \, d\phi \, (dp/g)$ = increment of mass

a = radius of the earth

g = acceleration due to gravity

Ω = rotation rate of the earth

$f = 2\Omega \sin\phi$ = Coriolis parameter

R = gas constant

c_p = specific heat at constant pressure

Q = heating rate

F = frictional force

$\overline{(\quad)} = 1/(t_2 - t_1)\int_{t_1}^{t_2} (\quad)\, dt$ = time average

$(\quad)' = (\quad) - \overline{(\quad)}$ = deviation from time average

$[(\quad)] = 1/2\pi \int_0^{2\pi} (\quad)\, d\lambda$ = zonal average

$(\quad)^* = (\quad) - [(\quad)]$ = deviation from zonal average

$\widetilde{(\quad)} = 1/\pi^2 \int_0^{2\pi} \int_0^{\frac{1}{2}\pi} (\quad)\, d\lambda\, d\phi$ = hemispheric average

$(\quad)'' = (\quad) - \widetilde{(\quad)}$ = deviation from hemispheric average

$\overline{[ab]}_E = [\overline{a'b'}] + [\bar{a}^* \bar{b}^*]$

$[\overline{a'b'}]$ = transient eddy covariance of a and b

$[\bar{a}^* \bar{b}^*]$ = standing eddy covariance of a and b

$K_M = K_{M,x} + K_{M,y}$ = zonal kinetic energy

$K_{M,x} = \frac{1}{2} \int [\bar{u}]^2 dm$

$K_{M,y} = \frac{1}{2} \int [\bar{v}]^2 dm$

$K_E = K_{E,x} + K_{E,y}$ = eddy kinetic energy

$K_{E,x} = \frac{1}{2} \int \{[\overline{u'^2}] + [\bar{u}^{*2}]\} dm$

$K_{E,y} = \frac{1}{2} \int \{[\overline{v'^2}] + [\bar{v}^{*2}]\} dm$

$P_M = \frac{1}{2} g \int \frac{[\bar{T}]''^2}{\widetilde{T}(\gamma_d - \widetilde{\bar{\gamma}})} dm$ = zonal available potential energy

$P_E = \frac{1}{2} g \int \frac{[\overline{T'^2}] + [\bar{T}^{*2}]}{\widetilde{T}(\gamma_d - \widetilde{\bar{\gamma}})} dm$ = eddy available potential energy

$D(K)$ = rate of frictional dissipation of kinetic energy due to the effects of (1) below grid-size eddies inside the volume and (2) eddy stresses at the boundaries by a similar scale of motion

$G(P)$ = rate of generation of available potential energy by diabatic heating

$C(K_E, K_M)$ = rate of conversion from eddy into zonal kinetic energy by the eddy momentum transport

$C(K_{M,y}, K_{M,x})$ = rate of conversion from $K_{M,y}$ into $K_{M,x}$ by mean meridional circulations

$C(P_M, P_E)$ = rate of conversion from zonal into eddy available potential energy by the eddy heat transport

$C(P_M, K_M)$ = rate of conversion from zonal available potential energy into zonal kinetic energy by mean meridional circulations

$C(P_E, K_E)$ = rate of conversion from eddy available potential energy into eddy kinetic energy by large-scale eddy convection

$W_e(K)$ = rate at which work is done on the considered layer by the measured eddy stresses at the boundaries

$W_p(K)$ = rate at which work is done on the considered layer by the pressure forces at the boundaries.

3. Energy budget for 100–30 mb layer

We shall consider a stratospheric layer with the 100 and 30 mb levels (resp. at about 16 and 24 km height) as bottom and top boundaries and a "wall" at the equator as vertical boundary. It is of great interest to establish by what mechanism the time-mean and zonal-mean state of the circulation is maintained in this polar cap and, with this in mind, we shall compute most of the terms in the budgets of zonal and eddy kinetic energy, and zonal and eddy available potential energy.

No attempt will be made to compute the balance of total kinetic and total potential energy. Such calculations were performed by Craig & Lateef (1962) for the kinetic energy during one month in 1957 using only North American stations, and by Barnes (1963) for both forms of energy with a hemispheric network during the first six months of the IGY. It is evident from Barnes' work that with the present data coverage uncertain terms such as those involving transports by mean meridional overturnings, may spuriously dominate the equations. On the other hand, one generally finds that transient and standing eddy transports can be measured more accurately than transports by mean meridional circulations. In view of this, we have decided to consider the equations for the balance of the zonal and eddy forms of energy separately. These calculations may give us in addition valuable and quite reliable information on the redistribution of kinetic and available potential energy among the zonal-mean and eddy forms.

The interpretation of the calculated terms in both the kinetic and the available potential energy equation forms a difficult problem. There is a separation of the terms possible into groups with a distinct physical meaning, but this division is not unique. For example, one may combine some of the terms which indicate a transport, and the conversion term and call the sum "conversion". However, as a rule the name conversion should be reserved for the term which expresses clearly the physical mechanism by which one thinks the transformation takes place. Of course, this conversion term should appear with the opposite sign in the balance equation for the forms of energy from or into which the form of energy in question is converted. In the following discussions we shall adopt the nomenclature which in our opinion makes most physical sense (for the subject of energy transformations see: Miller, 1950; Lettau, 1954; Lorenz, 1955, and Pfeffer, 1957).

The levels of the basic observations are 100, 50 and 30 mb. The assumption is made in the calculations of the integrals that: (1) 100 mb is representative for the layer 100–75 mb, (2) 50 mb is representative for the layer 75–40 mb, and (3) 30 mb is representative for the layer 40–30 mb. Since values of adiabatic vertical motions were available only at 75 and 40 mb, it is further assumed that the calculated vertical transport at 75 mb represents the flux at the lower boundary, while the 40 mb transport is assumed to represent the flux at the upper boundary of our volume. It is possible that the measured

transport at 75 mb considerably underestimates the actual transport at 100 mb. In the vertical transports by mean meridional circulations, $\overline{\omega_{ad}}$ is corrected in such a way that the hemispheric average of $\overline{\omega_{ad}}$ vanishes. This is done in order to satisfy continuity of mass. However, it was found afterwards that this correction did not change the mean vertical transports appreciably. No correction has been applied to the eddy transports involving vertical motions, because they are probably only slightly affected by diabatic effects, such as (1) heating or cooling due to radiation and (2) heating due to the release of latent heat. The effects of radiation are slow; thus radiation will not change the computed transient eddy covariances, though it might be important for the standing eddy contributions. Since the amount of water vapor in the lower stratosphere is quite small, the effects of condensation may be neglected.

At this point it is of interest to add some remarks concerning the different scales of motion which we measure in our eddy transports. Let us first consider the contribution of the eddies in space, i.e., the standing eddies. The horizontal resolution in space is limited because of the station distribution and our method of analysis of hemispheric maps, to a scale of motions which is larger than about 500 km. There is practically no resolution in the vertical since we used only data at 100, 50 and 30 mb. The contributions of transient eddies are in general larger than those of the standing eddies; thus, it is of particular interest to look at the time scales involved. In the *vertical* transport by transient eddies (such as the transport of momentum $\overline{u'\omega'}$) we catch only eddies with time scales of more than one day. This is because the adiabatic vertical motions which are used here, were evaluated from two soundings 24 hours apart. They represent some kind of 24 hours smoothed values. On the other hand, the *horizontal* transport by transient eddies (such as the transport of momentum $\overline{u'v'}$) is measured almost instantly and contains, in principle, also the contribution from eddies with shorter time scales from a few hours down to a few minutes. Diurnal effects might be important, but they are not included in the transports, since the observations were taken at approximately the same time of the day. The frictional force is defined as a flux of momentum at the boundaries by eddies with a time or space scale below the scale of our grid. This implies that meso- and micro-scale eddies are included in the frictional force at the top and bottom boundaries, while the frictional force at the vertical wall contains mainly micro-scale, transient eddy contributions and is hence probably of minor importance.

3.1. Zonal kinetic energy

In this section we shall consider the problem of how the zonal kinetic energy is maintained against frictional dissipation inside the volume. The term "zonal kinetic energy" was defined in section 2 as the kinetic energy of the zonal mean and time mean horizontal circulation,[1] i.e., $K_M = K_{M,x} + K_{M,y} = \frac{1}{2}\int\{[\bar{u}]^2 + [\bar{v}]^2\}dm$. Since the mean meridional motions are two orders of magnitude smaller than the mean zonal motions, K_M is very nearly equal to $K_{M,x}$ and we are justified to restrict ourselves in the discussion of the mean motions mainly to the zonal component. Kuo (1951) derived a balance equation for the zonal kinetic energy in the (xyz) coordinate system. A similar equation, rewritten for the pressure coordinate system, was used by Starr (1953, 1959) and Barnes (1963) in their computations of the kinetic energy balance in the northern hemisphere and by Obasi (1963) in his study of the southern hemisphere circulation. In the derivation, the zonal equation of motion is first multiplied by $[\bar{u}]$, next averaged in time over the period considered, and finally integrated in space over the mass of a polar cap. The resulting expression may be written in a form where the effects of eddy and mean motions are separated. To derive equation (1) use is made of the Gauss' divergence theorem and the equation of continuity.

$$0 = C(K_{E,x}, K_{M,x}) + C(K_{M,y}, K_{M,x}) + D(K_{M,x}) + A(K_{M,x}) + W_i(K_{M,x}) + \text{Seasonal correction.} \quad (1)$$

[1] This definition of zonal kinetic energy is different from the one originally given by Lorenz (1955) who used $K_M = \int\{[\overline{u}]^2 + [\overline{v}]^2\}dm$. Also the conversion rates considered here are different from those derived by Lorenz. The present approach is more appropriate in this case because no daily gridpoint data were available.

Equation (1) states that there should be a balance between the following processes:[1]

(*a*) The conversion from eddy into zonal kinetic energy by horizontal and vertical eddies (only x-component of process considered):

$$C(K_{E,x}, K_{M,x}) = \int [\overline{uv}]_E \cos\phi \frac{\partial}{a\partial\phi}\left(\frac{[\bar{u}]}{\cos\phi}\right) dm + \int [\overline{u\omega}]_E \frac{\partial[\bar{u}]}{\partial p} dm.$$

The horizontal part of the integrand of $C(K_{E,x}, K_{M,x})$ consists of the product of the meridional transport of relative angular momentum by eddy motions and the gradient of the angular velocity. If an eddy transport of angular momentum takes place from latitudes where the air has a small relative angular rotation to latitudes where the air rotates more rapidly, the integral indicates a conversion from eddy into zonal kinetic energy.

(*b*) The conversion of energy from the mean meridional motions into the mean zonal motions due to essentially the Coriolis effect:

$$C(K_{M,y}, K_{M,x}) = \int f[\bar{u}][\bar{v}]dm + \int [\bar{v}][\bar{u}]^2 \frac{\tan\phi}{a} dm. \cong \int f[\bar{u}][\bar{v}]dm.$$

Its magnitude is extremely uncertain due to the great difficulty of estimating $[\bar{v}]$ accurately enough.

(*c*) The sum of two processes, which are included in the term "frictional dissipation", i.e., (1) the change of kinetic energy due to the effects of below grid-scale eddy stresses at the boundaries, and (2) the change of kinetic energy due to similar internal friction; this sum being:

$$D(K_{M,x}) = \int [\bar{u}][\bar{F}_x] dm.$$

(*d*) The advection of zonal kinetic energy through the boundaries by mean meridional overturnings:

$$A(K_{M,x}) = \iint_{\phi=0} [\bar{v}] \tfrac{1}{2} [\bar{u}]^2 dx \frac{dp}{g} - \iint_{100\,mb} [\bar{\omega}] \tfrac{1}{2} [\bar{u}]^2 \frac{dx\,dy}{g} + \iint_{30\,mb} [\bar{\omega}] \tfrac{1}{2} [\bar{u}]^2 \frac{dx\,dy}{g}.$$

(*e*) The work performed on our volume by the measured eddy stresses at the boundaries:

$$W_e(K_{M,x}) = \iint_{\phi=0} [\bar{u}][\overline{uv}]_E dx \frac{dp}{g} - \iint_{100\,mb} [\bar{u}][\overline{u\omega}]_E \frac{dx\,dy}{g} + \iint_{30\,mb} [\bar{u}][\overline{u\omega}]_E \frac{dx\,dy}{g}.$$

(*f*) The so-called "seasonal correction":

$$-\int [\bar{u}] \frac{\Delta[u]}{\Delta t} dm.$$

This term contains the considered time period Δt in the denominator and, therefore, decreases in importance with time. Aside from the seasonal trend, this correction contains random daily fluctuations superimposed upon the trend. The term thus arises because of our method of time averaging. In principle, one should average over a few cycles of the observed phenomenon in order to be able to define the time mean state.

An often used alternative form for the production of zonal kinetic energy at the cost of the eddy kinetic energy $C(K_{E,x}, K_{M,x})$ is the following:

$$C'(K_{E,x}, K_{M,x}) = C(K_{E,x}, K_{M,x}) + W_e(K_{M,x}) = -\int [\bar{u}] \frac{\partial[\overline{uv}]_E \cos^2\phi}{a\cos^2\phi\,\partial\phi} dm - \int [\bar{u}] \frac{\partial[\overline{u\omega}]_E}{\partial p} dm.$$

Since $W_e(K_{M,x})$ evidently represents an interaction term at the boundaries, we prefer the earlier mentioned form for the conversion where one splits off the contribution of $W_e(K_{M,x})$. Thus, we shall continue to use the expression $C(K_{E,x}, K_{M,x})$ for the conversion. For the entire atmosphere the boundary term vanishes and $C(K_{E,x}, K_{M,x}) = C'(K_{E,x}, K_{M,x})$.

In Table 1 one finds the estimated terms in equation (1). The values presented in this table permit us to make the following remarks:

(*a*) The action of horizontal eddies, both transient and standing, is throughout the year a significant source for the zonal kinetic energy at the expense of the eddy kinetic energy. In all

[1] In this symbolic form of the balance equation for the zonal kinetic energy no term indicating the rate of change of zonal kinetic energy appears on the left hand side of the equation. Because the balance of the time-mean state is considered, the term which contains the product of $[\bar{u}]$ and $\Delta u/\Delta t$ has a slightly different meaning and is written on the right hand side of the equation as "seasonal correction".

TABLE 1. *Zonal kinetic energy balance for 100–30 mb layer (northern hemisphere).*

Units: 10^{18} erg sec^{-1}, except for K_M and K_E. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(K_{E,z}, K_{M,z})$ horizontal part	t.e	9.0	29.8	6.8	14.7	16.1
	s.e	4.5	14.1	24.2	3.2	3.6
$C(K_{E,z}, K_{M,z})$ vertical part	t.e	−0.2	1.7	4.1	3.2	1.9
	s.e	0.5	0.9	2.4	0.0	0.4
$C(K_{M,y}, K_{M,z})$		(−3)[a]	(7)	(−298)	(−214)	(−94)
$W_e(K_{M,z})$ at equatorial wall		−0.4	−0.3	0.5	0.2	0.1
$W_e(K_{M,z})$ at top and bottom boundaries		−0.4	−1.2	−6.4	−1.1	−1.4
$A(K_{M,z})$		(−2)[a]	(−5)	(4)	(−4)	(−4)
"Seasonal" correction		9.2	−26.5	12.6	−0.4	−0.2
				(Units: 10^{24} erg)		
$K_{M,z} \simeq K_M$		6.45	8.44	12.65	3.74	4.10
$K_{E,z}$	t.e.	3.20	5.01	5.98	5.17	4.69
	s.e.	1.40	1.80	1.98	0.61	1.27

[a] Numbers in parentheses are considered to be uncertain since they involve mean meridional circulations, which are difficult to measure.

four periods the direction of this process is the same and extracts energy from the smaller-scale motions in favor of the zonal flow. This indicates a "negative" coefficient of eddy viscosity. The same more familiar process dominates in the troposphere and is one of the most interesting phenomena in meteorology.

(*b*) The large-scale vertical eddies tend to increase the energy of the zonal flow, but their action is of minor importance since their contribution only amounts to about 10 % of the contribution of the horizontal eddies (see also STARR & DICKINSON, 1963).

(*c*) On the other hand one notices that the mean meridional circulations give during three of the four seasons a negative contribution through the term $C(K_{M,y}, K_{M,z})$. This is probably due to the dominating indirect circulation in middle latitudes. STARR (1959) computed the same integral for the atmosphere of the entire northern hemisphere for the two years 1950 and 1951, and found a small negative value in both years. The present measurements give also a negative conversion in the 100–30 mb layer, but the magnitude appears to be too large, probably due to the difficult estimation of $[\bar{v}]$.

(*d*) The work done by the measured eddy stresses at the boundaries is negligible. This has the implication that $C(K_{E,z}, K_{M,z})$ is approximately equal to $C'(K_{E,z}, K_{M,z})$ for the considered volume. An independent calculation of the two expressions confirmed this fact.

(*e*) The seasonal correction can be neglected in the balance for the year. For a three-months period this term gives a definite contribution, but it does not dominate the equation. For the computation one needs the difference of the mean zonal wind at the first and at the last day of the considered period. This difference was computed from the U.S. Weather Bureau Stratospheric Daily Maps (1960).

If one assumes as the southern boundary a vertical wall not at the equator but one at a higher latitude, the work term $W_e(K_{M,z})$ at the vertical wall becomes important and the eddy conversion $C(K_{E,z}, K_{M,z})$ does not contribute significantly anymore. The Coriolis term $C(K_{M,y}, K_{M,z})$ gives a positive contribution for a polar cap north of 60° N, since the mean meridional motions are towards the north at high latitudes.

3.2. Eddy kinetic energy

In the preceding discussion we have looked at the energy in the zonal mean wind field. We

TABLE 2. *Eddy kinetic energy balance for 100–30 mb layer (northern hemisphere).*

Units: 10^{18} erg sec^{-1}. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(K_E, K_M)$[a]	t.e.	8.8	31.5	9.9	17.9	18.0
	s.e.	5.0	15.0	26.6	3.2	4.0
$C(P_E, K_E)$	t.e.	−13.7	−12.2	−13.1	−26.3	−16.3
	s.e.	6.0	−1.4	−31.8	−8.0	−9.2
				Units: 10^{25} erg		
K_M		6.45	8.44	12.66	3.74	4.10
K_E	t.e.	4.99	8.25	9.60	7.21	7.25
	s.e.	1.69	2.29	2.94	0.89	1.70

[a] In the computation $C(K_E, K_M)$ is approximated by $C(K_{E,x}, K_{M,x})$.

observed the existence of an important interaction with the eddy field in the sense of a buildup of the zonal motions at the cost of the eddying motions. In view of this, it is worthwhile to examine how the eddy field is maintained. In the case of the eddy kinetic energy, it is not sufficient to consider only the east–west motions. There is also a considerable amount of energy in the eddy components of the north–south motions: from the present data it appears that $K_{E,y} \cong \frac{1}{2} K_{E,x}$ in the lower stratosphere (compare estimates in tables 1 and 2). The balance equation for the eddy kinetic energy is derived in an analogous fashion as for the zonal kinetic energy. Thus, the zonal equation of motion is multiplied by $\bar{u}^* + u'$ and the meridional equation of motion with $\bar{v}^* + v'$ these two equations are added together, next averaged in time over the considered period and finally integrated in space over the mass of a polar cap. We will write the result in the form:

$$0 = -C(K_E, K_M) + C(P_E, K_E) + D(K_E) + A(K_E) + W_p(K_E) + W_e(K_E) + \text{Seasonal correction.} \quad (2)$$

where:

$$K_M = \frac{1}{2} \int \{[\bar{u}]^2 + [\bar{v}]^2\}\, dm$$

and
$$K_E = \frac{1}{2} \int \{\overline{[u^2]}_E + \overline{[v^2]}_E\}\, dm.$$

Similarly as in the balance equation for the zonal kinetic energy the "seasonal correction" which contains the time change of the horizontal wind components, is written on the right hand side of the equation. According to equation (2) there is a balance between the following processes:

(a) The conversion from eddy into zonal kinetic energy by horizontal and vertical eddies:

$$C(K_E, K_M) = \int \overline{[uv]}_E \cos\phi \frac{\partial}{a\partial\phi}\left(\frac{[\bar{u}]}{\cos\phi}\right) dm + \int \overline{[u\omega]}_E \frac{\partial[\bar{u}]}{\partial p} dm + \int \overline{[v^2]}_E \frac{\partial[\bar{v}]}{a\partial\phi} dm + \int \overline{[\omega v]}_E \frac{\partial[\bar{v}]}{\partial p} dm - \int [\bar{v}]\overline{[u^2]}_E \frac{\tan\phi}{a} dm.$$

The last three terms in this expression can be neglected with respect to the first two terms.

(b) The conversion from eddy available potential into eddy kinetic energy:

$$C(P_E, K_E) = -\int \overline{[\omega\alpha]}_E\, dm.$$

(c) The dissipation of eddy kinetic energy due to friction:

$$D(K_E) = \int \overline{[uF_x]}_E\, dm + \int \overline{[vF_y]}_E\, dm.$$

(d) The advection of eddy kinetic energy through the boundaries by mean meridional overturnings:

$$A(K_E) = \iint_{\phi=\phi_0} [\bar{v}]\, \frac{1}{2}\{\overline{[u^2]}_E + \overline{[v^2]}_E\}\, dx \frac{dp}{g}$$

$$-\iint_{100\,mb}[\bar{\omega}]\tfrac{1}{2}\{\overline{[u^2]_E}+\overline{[v^2]_E}\}\frac{dx\,dy}{g}$$

$$+\iint_{30\,mb}[\bar{\omega}]\tfrac{1}{2}\{\overline{[u^2]_E}+\overline{[v^2]_E}\}\frac{dx\,dy}{g}.$$

(e) The change of eddy kinetic energy due to the work perm:d on the layer by pressure forces at the boundaries:

$$W_p(K_E)=\iint_{\phi\ 0}\overline{[vH]_E}\,dx\,dp$$

$$-\iint_{100\,mb}\overline{[\omega H]_E}\,dx\,dy+\iint_{30\,mb}\overline{[\omega H]_E}\,dx\,dy.$$

(f) The change of eddy kinetic energy due to the work performed on the layer by the measured eddy stresses at the boundaries:

$$W_e(K_E)=\iint_{\phi\ 0}\{[\bar{u}^*\,\overline{u'v'}^*]+[\bar{v}^*\,\overline{v'^{2*}}]$$

$$+[\bar{v}^*\tfrac{1}{2}(\overline{u'^{2*}}+\overline{v'^{2*}})]\}\,dx\frac{dp}{g}$$

$$-\iint_{100\,mb}\{[\bar{u}^*\,\overline{u'\omega'}^*]+[\bar{v}^*\,\overline{v'\omega'}^*]$$

$$+\bar{\omega}^*\tfrac{1}{2}(\overline{u'^{2*}}+\overline{v'^{2*}})]\}\frac{dx\,dy}{g}$$

$$+\iint_{30\,mb}\{[\bar{u}^*\,\overline{u'\omega'}^*]+[\bar{v}^*\,\overline{v'\omega'}^*]$$

$$+[\bar{\omega}^*\tfrac{1}{2}(\overline{u'^{2*}}+\overline{v'^{2*}})]\}\frac{dx\,dy}{g}.$$

(g) The "seasonal correction":

$$-\int\overline{\left[u\frac{\partial u}{\partial t}\right]_E}dm-\int\overline{\left[v\frac{\partial v}{\partial t}\right]_E}dm.$$

The values of the measured terms in this equation are given in Table 2. The conversion from eddy to zonal kinetic energy $C(K_E, K_M)$ is for all practical purposes equal to the integral $C(K_{E,x}, K_{M,x})$, which was discussed in section 3.1. Thus, eddy kinetic energy is converted into zonal kinetic energy. The dissipation of eddy kinetic energy $D(K_E)$ is analogous to the dissipation of zonal kinetic energy by friction (section 3.1). It contains the effects of (1) below grid-size eddies inside the volume, which probably destroy kinetic energy, and (2) eddy stresses at the boundaries by a similar scale of motions, which could conceivably contribute positively to the eddy kinetic energy balance. The conclusion is that one cannot say beforehand what sign $D(K_E)$ should have. The interaction between the motions in the lower stratosphere and in the surrounding layers of the atmosphere gives rise to three forms of boundary terms: (1) the advection of K_E by mean motions: $A(K_E)$, (2) the work done by pressure forces at the boundaries: $W_p(K_E)$, and (3) the work done by the measured eddy stresses at the boundaries: $W_e(K_E)$. These interaction terms only redistribute the energy. Therefore, a gain of a certain amount of eddy kinetic energy by the layer must be accompanied by an equal loss of eddy kinetic energy by the surrounding atmosphere.

The term $C(P_E, K_E)$ apparently represents in the (x, y, p) coordinate system the important conversion process from eddy potential into eddy kinetic energy by convection. It forms the connecting link between on one side the eddy potential and internal energy and on the other side the eddy kinetic energy. In the troposphere this conversion is large and positive. It supplies the kinetic energy for the eddies and, through the interaction of the eddies with the mean flow, also the kinetic energy for the zonal currents. The transformation process is often visualized as the rising of warm air masses and the sinking of cold air masses, but this does not necessarily represent the complete picture. It is more correct, as was pointed out by Starr, to say that the warmer air has a systematic tendency to be correlated with motions towards lower pressure ($\omega<0$) in the troposphere and colder air with motions towards higher pressure ($\omega>0$). This description of the energy conversion leaves the possibility open that the process occurs for an important part horizontally and not predominantly in the vertical direction as is implied by the terms "rising" and "sinking". In the present study vertical motions computed according to the "adiabatic" method (i.e. from the first law of thermodynamics neglecting diabatic heating), are used in the evaluation of $C(P_E, K_E)$. It would be better to include here the effects of diabatic heating, but this is not yet possible since heating rates are not known on a day to day basis. WIIN-NIELSEN (1959) has estimated that the influence of a reasonable mean heating pattern could not be neglected in the standing

eddy contribution to $C(P_E, K_E)$. However, we expect that the inclusion of heating will not substantially alter the more important contribution of the transient eddies. Most other hemispheric computations of the conversion term (see e.g. JENSEN (1961) who used "adiabatic" vertical velocities, or WIIN-NIELSEN (1959) and SALTZMAN & FLEISHER (1960, 1961) who used vertical motions from the ω-equation) suffer also from the deficiency that diabatic heating is neglected. HOLOPAINEN (1963) has argued that especially "adiabatic" vertical velocities would lead to bad results for the conversion rate. This is only true if one keeps the static stability in the denominator of the expression for the vertical velocity constant. Therefore, great care has been taken to retain the variability of the static stability. The assumption of a constant static stability would change the meaning of the conversion integral. To avoid other difficulties, only actual wind data have been used in all computations. EDDY (1963) used a direct approach in the calculation of $C(P_E, K_E)$. He computed vertical velocities from the smoothed divergence field of the wind, and correlated them with simultaneous temperatures for a limited time period over North America. Eddy's results agree with those of Jensen and support the "adiabatic" method.

The computed values of $C(P_E, K_E)$ in our layer in the lower stratosphere are shown in Table 2. During all seasons a conversion from eddy kinetic into eddy potential energy takes place; this is in a direction opposite to the one usually found in the troposphere (see also WHITE & NOLAN, 1960).

For the January–March period the standing eddy contribution is larger than the transient eddy contribution both in $C(K_E, K_M)$ and in $C(P_E, K_E)$. In all other periods the transient eddies play a more important role.

Source of energy for the eddy motions in the lower stratosphere

The measurements show clearly that throughout the year: (1) the eddies lose kinetic energy to the mean flow, and (2) the eddies convert part of their kinetic energy into potential energy. Since the seasonal correction is not of much importance, unavoidably the conclusion must be drawn that the lower stratosphere receives its kinetic energy from the surrounding atmosphere and is consequently *forced*. The forcing presumably takes place at the bottom boundary, i.e. at 100 mb in this case, through the work done by pressure forces and/or eddy stresses.[1]

WHITE (1954) discovered that there occurs in the lower stratosphere a countergradient eddy flux of heat. This abnormal heat transport was found in our data during all seasons; only in winter at 30 mb a downgradient heat flux was observed, which probably indicates that the top of the abnormal layer is lower in the winter season (OORT, 1963). Also this process can only be explained if the eddies in the lower stratosphere are forced to act as a *refrigerating* mechanism causing heat to flow from a cold region at low latitudes to a warm one at high latitudes. Therefore, by two independent methods the phenomenon of the necessary forcing has been shown to be present.

From the available data the interaction in the vertical direction could be determined at the 75 and 40 mb levels. It enabled us to estimate the work done by eddy stresses and pressure forces at the top and bottom boundaries (see Table 3). This work is given by the terms:

$$W_e(K_M) + W_e(K_E) = -\tfrac{1}{2}\iint_{100\,mb} \overline{[\omega c^2]_E}\,\frac{dx\,dy}{g} + \tfrac{1}{2}\iint_{30\,mb} \overline{[\omega c^2]_E}\,dx\,dy$$

and

$$W_p(K_E) = -\iint_{100\,mb} \overline{[\omega H]_E}\,dx\,dy + \iint_{30\,mb} \overline{[\omega H]_E}\,dx\,dy.$$

The first term was not separated into the contribution from the balance of zonal kinetic energy, $W_e(K_M)$, and from the balance of eddy kinetic energy, $W_e(K_E)$. Still, the numbers will give us an idea of the magnitude of the measured interaction at the boundaries and may be compared with the deficit in the balance of the eddy kinetic energy. Table 3 shows, generally, an upward transport of energy at 75 mb. All eddy terms added together amount to about -3×10^{18} erg sec^{-1} for the year. Only during July–

[1] The advection by mean motions is probably small.

TABLE 3. *Estimates of interaction terms at the boundaries, computed with the aid of adiabatic vertical velocities.*

Units: 10^{18} erg sec^{-1}.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$\frac{1}{2}\iint [\overline{\omega c^2}]_E \frac{dx\,dy}{g}$	75 mb	0.7[a]	−1.8[a]	5.0	−0.0	−0.1[a]
	40 mb	—	—	−0.2	−0.3	—
$\iint [\overline{\omega H}]_E\, dx\, dy$	75 mb	1.0	−2.6	−11.1	−3.3	−2.7
	40 mb	—	—	−1.3	0.8	—

[a] Only contribution due to transient eddies included.

September is there a small downward transport. The transports around 40 mb appear to be small. If one compares the annual mean transport at 75 mb with the amount 48×10^{18} erg sec^{-1} which is needed in order to give a balance for the eddy kinetic energy for the year, a large discrepancy is in evidence. This discrepancy could arise due to the fact that the vertical eddy transport was measured at 75 mb and not at 100 mb. HANSROTE & LAMBERT (1960) found, for example, for one month of data at 150 mb a much larger value for the vertical eddy transport of energy than we measured at 75 mb. Another solution to the difference between the balance requirements and the values in table 3 could be that meso- and/or micro-scale eddies transport considerable amounts of eddy kinetic energy from the troposphere into the lower stratosphere. The eddies which we measure with the adiabatic method, have a time scale of over one day and it appears possible that eddies with shorter time scales accomplish an important part of the vertical transport of this energy.

3.3. Zonal available potential energy

In general circulation studies it is often convenient to deal with the sum of the potential and internal energy, since for a vertical column extending throughout the depth of the atmosphere the two forms of energy are proportional by the factor R/c_v. Only a small part of the potential and internal energy is really available for conversion into kinetic energy. One may consider the difference between the actual total potential plus internal energy and the potential plus internal energy if the mass were redistributed adiabatically in such a way that the isothermal surfaces would be parallel to the isobaric levels, as a measure of the "available potential energy" (LORENZ, 1955). To the approximation that the pressure surfaces are horizontal, this is a good definition for the energy available for conversion into kinetic energy.

In the case of the kinetic energy, we considered eddy and zonal kinetic energy separately. It will be of advantage to make a similar division for the available potential energy. In the case of a strictly zonal symmetric distribution of temperature, no eddy available potential energy is present and it is necessary to disturb first the zonal symmetry in order to make potential energy available for conversion into eddy kinetic energy. Once the zonal symmetry is distorted, the eddy heat fluxes can redistribute the available potential energy between the zonal and the eddy forms. The early studies of MARGULES (1903) concerning the potential energy of a fixed atmospheric volume were extended by LORENZ (1955). In his paper Lorenz gives an analysis of the concepts of zonal and eddy available potential energy. In Lorenz' formulation available potential energy is approximated by the variance of potential temperature on a constant pressure level, weighted by a factor containing the hemispheric average of the static stability. Difficulties come up if one tries to derive the balance equations for the zonal and eddy available potential energy. Aside from the terms which represent the expected energy transformations, extra terms enter into the balance equations. These

terms cannot be reduced to boundary integrals and therefore represent some kind of transformation of energy. They depend mainly upon the variation of the mean static stability with pressure and are probably caused by the approximations made in deriving the expressions for the available potential energy. It is not possible here to go into further detail since this would require an extensive theoretical study in itself. Despite the above mentioned difficulties, we shall use the formulae derived by Lorenz. Terms involving the vertical derivatives of measures of the mean static stability are left out and are thus explicitly *assumed* to be small. Neither shall we include boundary terms in the discussion of the balance equations.

One can derive the balance equation for the zonal available potential energy from the first law of thermodynamics in the form:

$$\frac{\partial\theta}{\partial t} = -u\frac{\partial\theta}{a\cos\phi\,\partial\lambda} - v\frac{\partial\theta}{a\partial\phi} - \omega\frac{\partial\theta}{\partial p} + \frac{1}{c_p}\frac{\theta}{T}Q. \quad (3)$$

In this equation Q is the diabatic heating rate, which is equal to the sum of the frictional heating and the radiational flux convergence. Effects of condensation and evaporation may be neglected in the stratosphere. Equation (3) is multiplied by $F(p)[\bar{\theta}]''$, where:

$$F(p) = -(T/\theta)(R/p)(\partial\underset{\sim}{\bar{\theta}}/\partial p)^{-1} = \left(\frac{T}{\theta}\right)^2 \frac{g}{\underset{\sim}{\bar{T}}(\gamma_d - \underset{\sim}{\gamma})},$$

a stability parameter. The resulting equation is averaged in time over the considered period and next integrated in space over a stratospheric polar cap. Triple correlations are neglected. By using the equation of continuity and the ideal gas law, the balance equation for the zonal available potential energy may be derived in the form:

$$0 = -C(P_M, P_E) - C(P_M, K_M) + G(P_M) + \text{Seasonal correction} \; (+\text{Boundary terms}). \quad (4)$$

Here: $$P_M = \tfrac{1}{2} g \int \frac{[\bar{T}]''^2}{\underset{\sim}{\bar{T}}(\gamma_d - \underset{\sim}{\bar{\gamma}})} dm \quad \text{and}$$

$$P_E = \tfrac{1}{2} g \int \frac{[\overline{T^2}]_E}{\underset{\sim}{\bar{T}}(\gamma_d - \underset{\sim}{\bar{\gamma}})} dm.$$

Equation (4) expresses that there should be an approximate balance between:

(*a*) The conversion from zonal into eddy available potential energy by horizontal or vertical eddy processes:

$$C(P_M, P_E) = -g\int \frac{[\overline{vT}]_E}{\underset{\sim}{\bar{T}}(\gamma_d - \underset{\sim}{\bar{\gamma}})} \frac{\partial[\bar{T}]''}{a\partial\phi} dm - g\int \frac{\theta}{T}[\overline{\omega T}]''_E \frac{\partial}{\partial p}\left(\frac{[\bar{T}]''}{\underset{\sim}{\bar{\theta}}(\gamma_d - \underset{\sim}{\bar{\gamma}})}\right) dm.$$

The conversion $C(P_M, P_E)$ depends upon the transport of sensible heat along the gradient of temperature. Thus $C(P_M, P_E)$ may be compared with the conversion from eddy to zonal kinetic energy $C(K_E, K_M)$, which depends upon the transport of angular momentum along the gradient of angular velocity. If the eddies transport heat down the temperature gradient, zonal available potential energy is converted into eddy available potential energy.

(*b*) The direct conversion from zonal potential energy into zonal kinetic energy by mean meridional circulations:

$$C(P_M, K_M) = -\int [\bar{\omega}]''[\bar{\alpha}]'' dm.$$

(*c*) The generation of zonal available potential energy by diabatic heating:

$$G(P_M) = \frac{g}{c_p}\int \frac{[\bar{T}]''[\bar{Q}]''}{\underset{\sim}{\bar{T}}(\gamma_d - \underset{\sim}{\bar{\gamma}})} dm.$$

(*d*) The "seasonal correction":

$$-g\int \frac{[\bar{T}]''\dfrac{\Delta[\bar{T}]''}{\Delta t}}{\underset{\sim}{\bar{T}}(\gamma_d - \underset{\sim}{\bar{\gamma}})} dm.$$

A similar term arising in the zonal kinetic energy balance was discussed before in section 3.1.

A qualitative argument can be given regarding the sign of the generation term $G(P_M)$. In the troposphere, low latitudes where the temperatures are high, are heated by the surplus of incoming, short wave over outgoing, long wave radiation and high latitudes where lower temperatures prevail, are cooled relatively. This action results in a large positive value of the hemispheric covariance $\underset{\sim}{[\bar{T}]''[\bar{Q}]''}$ and consequently in a large generation of zonal available potential energy. The presented picture applies well to the generation integrated over the entire

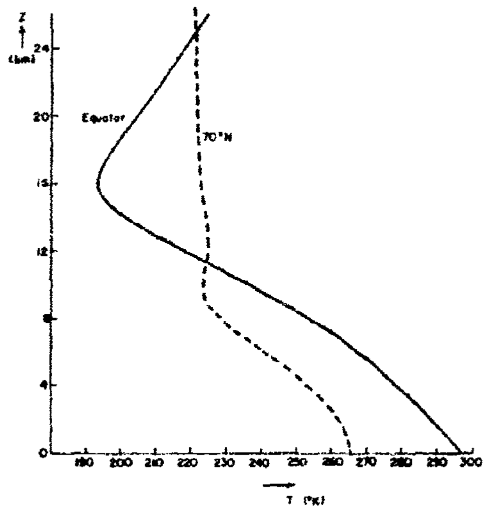


FIG. 1. Comparison of the temperature distribution with height at the equator and at 70° N. The tropospheric data are taken from PEIXOTO (1960) for the year 1950; the stratospheric data are for the year July 1957–June 1958.

atmosphere, but if one considers the lower stratosphere alone, the picture is not correct. Here, the temperature distribution has a maximum in high latitudes in winter and this maximum shifts even to polar latitudes during the summer season. In Fig. 1 curves are plotted of the temperature distribution with height at the equator and at 70° N. If one assumes that there is an excess of absorbed over emitted radiation at all levels at low latitudes, and a deficit at high latitudes, then it is clear that radiation tends to increase the meridional temperature gradient from the ground up to 12 km and above 24 km. However, radiation tends to decrease the meridional temperature gradient in a layer around 16 km height (see also BOVILLE, 1961). This layer contains our volume between 100 and 30 mb and it can be concluded that for the lower stratosphere $G(P_M) < 0$.

We shall compute $C(P_M, P_E)$ for the 100–30 mb layer and determine to what degree this conversion can balance the destruction of zonal available potential energy by radiation. The results are shown in Table 4. The seasonal correction was computed with the aid of temperatures which were read off the U.S. Weather Bureau Stratospheric Daily Maps (1960). Eddy available potential energy is converted into zonal available potential energy throughout the year, as might be anticipated from the counter-gradient heat transport (WHITE, 1954; OORT, 1963). The effect of the standing eddies is about 1/2 to 1/3 of the effect of the transient eddies. The vertical eddies, expressed in the vertical part of $C(P_M, P_E)$, give a slight negative contribution, but they are unimportant compared with the horizontal eddies.

The generation of zonal available potential energy due to the radiation flux convergence can be calculated directly from radiation estimates combined with our measured temperatures. We used the net cooling rates, obtained by DAVIS (1963), and computed for the rate of generation:

TABLE 4. *Zonal available potential energy balance for 100–30 mb layer (northern hemisphere).*
Units: 10^{18} erg sec^{-1}. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(P_M, P_E)$ horizontal part	t.e.	−26.6	−35.6	−42.0	−37.1	−35.6
	s.e.	−11.8	−14.1	−11.5	−25.5	−15.9
$C(P_M, P_E)$ vertical part	t.e. + s.e.	−1.9	0.4	−7.6	−5.2	−1.6
"Seasonal correction"		18.8	4.2	−21.3	−8.1	—
				Units 10^{25} erg		
P_M		26.4	18.2	25.7	30.2	23.6
P_E	t.e.	3.30	4.83	7.31	3.99	4.86
	s.e.	1.74	2.39	3.01	1.27	2.10

	Jul.–Sept. 1957	Oct.–Dec. 1957	Jan.–Mar. 1958	Apr.–Jun. 1958	Year
$G(P_M)$	−221	−127	−201	−230	−187

Units: 10^{18} erg sec^{-1}

The radiation estimates used by, for example, MURGATROYD & SINGLETON (1961), would give even larger destruction rates for the year: $G(P_M) = -4.8 \times 10^{20}$ erg sec^{-1}. The three-monthly mean vertical motions which are computed by the adiabatic method, are not appropriate to estimate the contribution of the term $C(P_M, K_M)$, since diabatic influences may alter these mean vertical motions significantly. Other investigators, as, e.g., REED *et al.* (1963), find evidence of a conversion from zonal kinetic to zonal potential energy in the lower stratosphere based upon data for a two week period. The discrepancy between the large destruction of P_M by radiation and the insufficient creation by the eddies, tends to support an additional conversion from K_M to P_M through the mean meridional circulations. On the other hand, the radiation estimates are still quite uncertain and this could also be the dominant factor in the discrepancy.

There is evidence of a seasonal variation of the zonal and eddy available potential energy in Table 4. The zonal available potential energy reaches a maximum in summer around April–June, and a minimum in the winter period October–December. The zonal kinetic energy (see Table 1) showed on the other hand a maximum value in winter and a minimum in summer. Both the eddy kinetic (Table 2) and the eddy available potential energy (Table 4) reach a maximum in the winter season.

3.4. Eddy available potential energy

With the same sort of approximations used in deriving the zonal balance equation, one can derive the balance for the eddy component. The first law of thermodynamics (3) is multiplied by $F(p)(\bar{\theta}^* + \theta')$, where $F(p)$ is the same stability parameter as the one used in the derivation of the balance of zonal available potential energy. Next the equation is averaged in time and then integrated in space, resulting in the balance equation for the eddy available potential energy for a polar cap:

$$0 = C(P_M, P_E) - C(P_E, K_E) + G(P_E) + \text{Seasonal correction } (+\text{Boundary terms}). \quad (5)$$

In sections 3.2 and 3.3 we considered $C(P_M, P_E)$ and $C(P_E, K_E)$ in detail. The estimated values are repeated in Table 5. It seems that the eddies lose more of their potential energy in building up the zonal energy than they gain by "negative" convection from their kinetic energy.

The rate of generation of eddy available potential energy by diabatic heating is given by

$$G(P_E) = \frac{g}{c_p} \int \frac{[\overline{TQ}]_E}{\tilde{T}(\gamma_d - \tilde{\gamma})} dm.$$

TABLE 5. *Eddy available potential energy balance for 100–30 mb layer (northern hemisphere).*

Units: 10^{18} erg sec^{-1}. t.e. = transient eddy contribution; s.e. = standing eddy contribution.

I = July–September 1957; II = October–December 1957; III = January–March 1958; IV = April–June 1958; V = July 1957–June 1958.

		I	II	III	IV	V (year)
$C(P_M, P_E)$ horizontal part	t.e.	−26.6	−35.6	−42.0	−37.1	−35.6
	s.e.	−11.8	−14.1	−11.5	−25.5	−15.9
$C(P_M, P_E)$ vertical part	t.e + s.e.	−1.9	0.4	−7.6	−5.2	−1.6
$C(P_E, K_E)$	t.e.	−13.7	−12.2	−13.1	−26.3	−16.3
	s.e.	−6.0	−1.4	−31.8	−8.0	−9.2

Very little is known about the magnitude of this generation. Even the sign of $G(P_E)$ is uncertain and cannot be inferred through a simple reasoning similar to the one used for $G(P_M)$. In the case of the troposphere, the studies of WIIN-NIELSEN & BROWN (1960), BROWN (1963) and KRUEGER, WINSTON & HAINES (1963) all indicate an eddy destruction of available potential energy probably due to the tendency of warmer air masses to be cooled and colder air masses to be heated.

GODSON (1960) and other investigators have found a high positive correlation between the temperature in the lower stratosphere and the amount of total ozone. Let us assume that under certain circumstances a high concentration of ozone is present in the 100–30 mb layer, then more short wave radiation will be absorbed in this layer. The effect will be equivalent to an extra heating of warm air and, consequently, a creation of eddy available potential energy $G(P_E)$, if in the layer the heating is determined mainly by the absorption of short wave radiation by ozone; this is certainly true at about 20 km and even more so at greater heights (see e.g. MANABE & MÖLLER, 1961). A creation of eddy available potential energy between 100 and 30 mb is supported by our measurements provided that the boundary terms in equation (5) are not very important.

4. The energy cycle

In the first part of this chapter we shall give a short description of the energy cycle as it is observed in the troposphere. Since the troposphere contains about 8/10 of the total atmospheric mass, this picture will be representative also for the cycle in the entire atmosphere. In section 4.2 we shall present the very different picture of the energy cycle of a layer in the lower stratosphere which was determined in the present study. Finally the contributions of different layers in troposphere and stratosphere will be compared in section 4.3.

4.1. ENERGY CYCLE FOR THE TROPOSPHERE

At low latitudes the earth-atmosphere system gains more energy per unit area by the absorption of short wave radiation from the sun than it looses to space by the emission of long wave radiation; the reverse is true at high latitudes. Due to the excess of incoming over outgoing radiation at low latitudes and the deficit at high latitudes, a meridional temperature gradient is created in the troposphere (this is equivalent to a creation of zonal available potential energy). The zonal currents resulting from the meridional temperature gradient are unstable for certain baroclinic wave disturbances. Maximum instability is found at about the wavelength of typical cyclones and anticyclones. These eddy circulations ("eddies") cause an east-west temperature gradient and thus convert zonal into eddy available potential energy. As the result of vertical motions the eddy potential energy is partly converted into eddy kinetic energy. The eddies appear to lose further potential energy through the combined action of radiation, condensation, evaporation and heat fluxes near the ground. The kinetic energy of these large-scale eddies is subject to the probable destructive influence of smaller-scale eddies which transfer the kinetic energy to still smaller eddies. This cascade of the kinetic energy to smaller and smaller scales, results finally, at the molecular scale in the transformation of the kinetic energy into heat. A significant percentage of the large-scale eddy kinetic energy is in this way destroyed. The remaining part of this energy is converted up the scale into kinetic energy for the zonal current and maintains thus the zonal current against the frictional dissipation of the smaller eddies. Finally, the conversion of zonal potential into zonal kinetic energy brings us back to the beginning of the cycle, i.e., the zonal available potential energy. This conversion has been proved to be small and has even probably a negative value, i.e. zonal kinetic energy is converted into zonal potential energy.

As was pointed out by SALTZMAN (1961), the zonal motions may be conceived of as strong free wheeling circulations with relatively small viscous damping or forcing.[1] The important process of the eddies supplying angular momentum for the zonal currents was first suggested by JEFFREYS in 1926. Jeffreys' ideas on the angular momentum balance have been extended to the eddy effects on the zonal kinetic energy, notably by KUO (1951), VAN MIEGHEM (1952), ARAKAWA (1953) and STARR (1953). Earlier concepts were based on the hypothesis of the existence of a direct mean meridional

[1] This result is based upon the large observed ratio of $C(P_E, K_E)$ and $C(K_E, K_M)$.

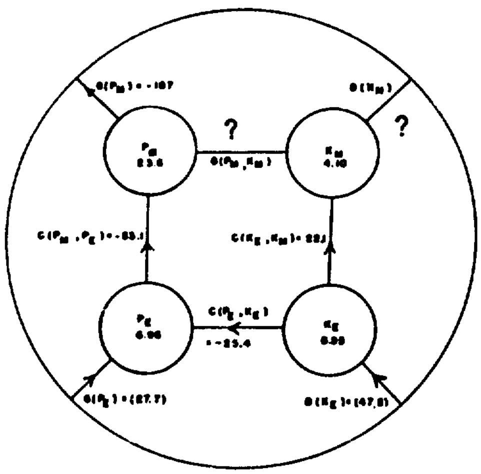


Fig. 2. Energy flow diagram during the year (July 1957–June 1958) for 100-30 mb layer. Energy transformation units: 10^{18} erg sec^{-1}. Energy units: 10^{25} erg. Numbers in parentheses are balance requirements as inferred from other observations. $G(P_M)$ is computed from radiation data by Davis (1963).

circulation. Starr realized as one of the first the importance of Jeffreys' suggestion. Through years of study of actual data on a hemispheric scale at the University of California (Los Angeles) under Jacob Bjerknes and principally at the Massachusetts Institute of Technology under Victor P. Starr, the important role of the eddy processes has been shown and thus the foundation has been laid for the modern concepts on the general circulation (see e.g. Starr, 1958).

4.2. The energy cycle in the lower stratosphere

A summary of the measured energy transformations and the total amount of each form of energy present is given in a diagram (Fig. 2), which was originally introduced by Lorenz (1954). Small circles represent the four forms of energy P_M, K_M, P_E and K_E. A line connecting two circles indicates a conversion between the two forms of energy. The interaction with the environment is given by a line connected with the large circle (environment) which contains the four smaller circles. The number in each circle represents the time-mean amount of the specific form of energy in units of 10^{25} erg. The conversions are labeled in units of 10^{18} erg sec^{-1}.

We should always keep in mind that we are dealing with a layer and not with the entire atmosphere. This implies that top and bottom boundary effects cannot be neglected without further consideration. In the diagram the generation $G(P)$ and dissipation $D(K)$ will thus contain these effects at the boundaries.

It is apparent from the energy diagram (Fig. 2) that pressure forces and/or eddy stresses at the top and bottom boundaries (presumably mainly at the bottom boundary with the troposphere) force the energy-cycle to be in a direction more or less opposite to the one in the troposphere.[1] This source of kinetic energy ultimately supplies the necessary balance for the loss of potential energy by radiation. The direct observations show the following special features of the circulation in the 100–30 mb layer:

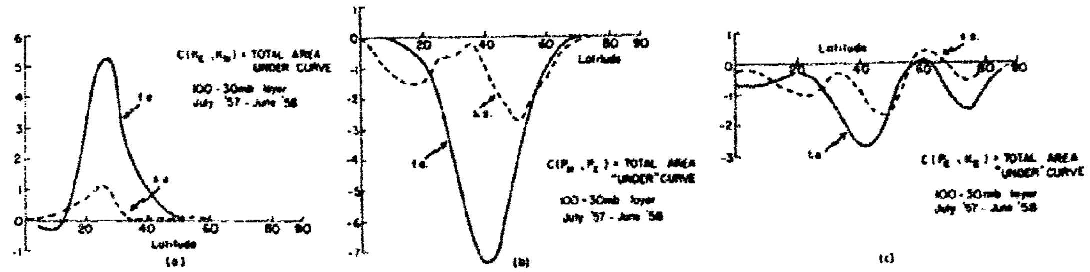
1. A "sinking" of warm air and a "rising" of cold air, which destroys eddy kinetic energy and converts it into eddy potential energy.
2. An eddy transport of heat against the mean temperature gradient, which destroys eddy available potential energy and converts it into mean zonal available potential energy.
3. A destruction of zonal available potential energy by radiation.

Indirectly, the observations imply or suggest by necessary balance requirements that:

1. Interaction with the surrounding atmosphere is a source rather than a sink of energy for the eddies and—through the eddies—also for the zonal flow.
2. Eddy available potential energy could be created by radiation.

The seasonal fluctuations of the three-monthly mean conversion and generation rates are small and no change in sign occurs. In this connection, it is of interest to look at the results of Reed, Wolfe & Nishimoto (1963) for a 14 day period during the stratospheric sudden warming of early 1957. They computed energy flow diagrams at 50 mb on a day to day

[1] In the troposphere, the energy cycle proceeds from P_M, via P_E and K_E, to K_M, i.e. in a counter-clockwise direction in the diagram; thus the tropospheric source of energy is found in the generation of zonal available potential energy by radiation and the sink in the destruction of kinetic energy by friction.



FIGS. 3a, b and c. The contributions at the different latitudes to (a) the conversion from eddy into zonal kinetic energy $C(K_E, K_M)$, (b) the conversion from zonal into eddy available potential energy $C(P_M, P_E)$ and (c) the conversion from eddy potential into eddy kinetic energy $C(P_E, K_E)$ by transient eddies (t.e.) and standing eddies (s.e.). All quantities are computed for the 100-30 mb layer and for the period July 1957–June 1958. The values of hemispheric integrals are as follows:

t.e. $C(K_E, K_M) = 16.1 \times 10^{18}$ erg sec^{-1}; $C(P_M, P_E) = -35.6 \times 10^{18}$ erg sec^{-1}; $C(P_E, K_E) = -16.3 \times 10^{18}$ erg sec^{-1}.

s.e. $C(K_E, K_M) = 3.6 \times 10^{18}$ erg sec^{-1}; $C(P_M, P_E) = -15.9 \times 10^{18}$ erg sec^{-1}; $C(P_E, K_E) = -9.2 \times 10^{18}$ erg sec^{-1}.

basis for the period January 25–February 9, 1957. A marked difference was found in the energy cycle between the first 10 days of the warming period and the next 5 days. The first 10 days showed a cycle similar to the one usually found in the troposphere, while the last period of 5 days might be compared with our results for the three-month period. Thus it appears that the energy fl w in the lower stratosphere is in an "abnormal" direction just after the sudden warming sets in, but returns later to its normal direction.

The contribution at the different latitudes to the conversion integrals $C(K_E, K_M)$, $C(P_M, P_E)$ and $C(P_E, K_E)$ is sketched in Figs. 3a, b and c. The transient eddies contribute most effectively in middle latitudes to the conversion from eddy potential and eddy kinetic energy into, respectively, zonal potential and zonal kinetic energy. The contribution from the standing eddies is of less importance. The conversion from eddy potential into eddy kinetic energy for the 100–30 mb layer has almost at every latitude a negative value.

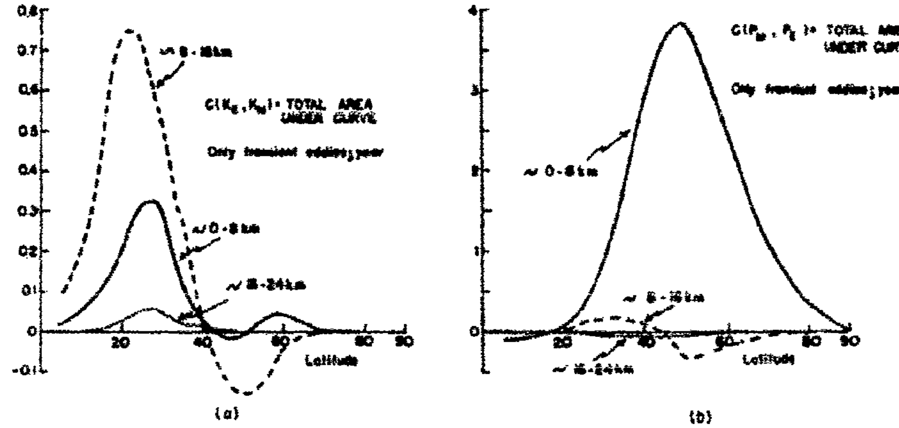
4.3. COMPARISON TROPOSPERIC AND STRATOSPHERIC CONVERSIONS

In order to avoid any distortion of the picture of energy conversions in the atmosphere by an over-emphasis on the stratospheric processes, graphs (Figs. 4a and b) have been prepared of the transient eddy contributions to the total integrals $C(K_E, K_M)$ and $C(P_M, P_E)$ of three atmospheric layers about 8 km thick, i.e., (1) lower troposphere 1000-400 mb (~0–8 km), (2) upper troposphere 400–100 mb (~8–16 km), and (3) lower stratosphere 100–30 mb (~16–24 km). The data for 1000–100 mb have been taken from the studies by BUCH (1954) of the wind and by PEIXOTO (1960) of the temperature conditions during the year 1950. Our data for the year July 1957–June 1958 are used for the 100–30 mb layer. Let us emphasize some important properties of the three layers:

1. The main contribution to the conversion from zonal to eddy available potential energy comes from the 0–8 km layer. From 8–16 km and 16–24 km the contribution is negative. The energy source for the eddies is therefore mainly situated in the lowest 8 km.
2. The main part of the conversion from eddy to zonal kinetic energy takes place in the upper troposphere. There is no reversal in the upper layers.
3. The contribution of the lower stratosphere to the hemispheric integrals is about 10 % in the kinetic energy and only 2–3 % in the (negative) potential energy conversion.

5. Some final comments

It is perhaps good to emphasize that in the present paper we have been interested in the maintenance of the energy of (a) the *time mean* and *zona l mean* state and (b) the transient and standing eddies; this approach was ap-



FIGS. 4*a* and *b*. The contributions at the different latitudes from 3 layers about 8 km thick to (*a*) the conversion from eddy into zonal kinetic energy $C(K_E, K_M)$ and (*b*) the conversion from zonal into eddy available potential energy $C(P_M, P_E)$, by the horizontal transient eddy processes. Data for 1000-100 mb are taken from the studies by Buch (1954) of the wind and by Peixoto (1960) of the temperature conditions during the year 1950. 100-30 mb data are for year July 1957–June 1958:

Layer 1000-400 mb ($\sim$0–8 km):
$C(K_E, K_M) = 1.27 \times 10^{20}$ erg sec^{-1};
$C(P_M, P_E) = 22.9 \times 10^{20}$ erg sec^{-1}.
Layer 400–100 mb ($\sim$8–16 km):
$C(K_E, K_M) = 2.53 \times 10^{20}$ erg sec^{-1};
$C(P_M, P_E) = -0.56 \times 10^{20}$ erg sec^{-1}.
Layer 100-30 mb ($\sim$16–24 km):
$C(K_E, K_M) = 0.20 \times 10^{20}$ erg sec^{-1};
$C(P_M, P_E) = -0.36 \times 10^{20}$ erg sec^{-1}.

propriate because of our method of analyzing maps of seasonal mean variables and of their covariances in time. If on the other hand we had computed the terms in the energy budget strictly on a *daily basis*, it would have been more appropriate to solve another problem, namely the question of how the energy of (*a*) the zonally averaged state and (*b*) the zonal inequalities (eddies) are maintained. This was the problem considered by LORENZ (1955) and in this case one should therefore use also the same expressions as in Lorenz' paper, and not the modified formulae as presented in this article. The numerical results for the comparable transformations would presumably be somewhat different.

The concept that there are no energy sources in the lower stratosphere itself, has been confirmed by this study and has been put on a firm basis. The conclusion must be drawn that interaction at the top and/or bottom boundaries supplies the necessary kinetic energy for maintaining the eddy circulations and indirectly also the zonal flow. It is to be expected that the influx of energy for the lower stratosphere occurs at the bottom boundary, i.e., from the underlying troposphere; this was already suggested by STARR (1960). The upward transport does not need to be large compared with the average energy in the troposphere since the contribution of the lower stratosphere to the energy transformations for the entire atmosphere has been shown to be less than 10 %.

From the data at the individual levels it appears that the forced layer generally extends from about 200 to 30 mb; the top boundary sinks below 30 mb during the winter season.

It is further of interest to look at an example of a probable forced regime of motion in the ocean. It appears from a fairly large sample of data on the Gulf Stream that the meanders in the surface layers of this current draw their energy neither from the kinetic energy of the main stream (WEBSTER, 1961) nor from the available potential energy connected with the temperature gradient between the slope water and the water of the Sargasso Sea (OORT, 1964). This would imply again an outside source of energy for these meanders. It is possible that also the eddies in the surface layers of the Gulf Stream are forced by the circulation in the lower atmosphere.

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ON ESTIMATES OF THE ATMOSPHERIC ENERGY CYCLE

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ABSTRACT

Several estimates of the rate at which the mean and eddy forms of both kinetic energy and available potential energy are generated, converted, and dissipated in the atmosphere are compared in tabular form. From these tables a selection is made of those values which are, in the author's opinion, representative for the yearly energy cycle in the Northern Hemisphere.

1. INTRODUCTION

In this article a critical survey will be given of statistics obtained by several investigators for the large-scale generation, dissipation, and conversion of energy in the atmosphere. Kinetic and available potential energy, both subdivided into their mean and eddy (deviations from the mean) contributions, will be considered as four separate forms of energy. This formulation was introduced by Lorenz [8].

The discussion will be restricted to the Northern Hemisphere. Only long time averages (i.e., averages over a period of a year, a winter or a summer half-year) will be considered; this is often a necessary condition for obtaining statistically significant values.

Only rough agreement is found among the estimates of different investigators regarding the fundamental characteristics of the general circulation. Therefore, it is of interest to examine the available material for the energy budget, and to attempt to construct from this material a quantitative picture of the energy cycle.

2. POSSIBLE DEFINITIONS OF MEAN AND EDDY FIELDS

If one considers the atmosphere, one can distinguish among three classes of eddies, namely eddies in the zonal, the meridional, and the vertical direction. We shall consider only the eddies in the zonal direction and also the eddies in time.[1] Thus, the eastward and northward components of the wind u and v, and the temperature T may be written as the sum of four components

$$u=[\bar{u}]+\bar{u}^*+[u]'+u'^*$$

$$v=[\bar{v}]+\bar{v}^*+[v]'+v'^*$$

$$T=[\bar{T}]+\bar{T}^*+[T]'+T'^*$$

where the brackets represent a zonal average, the star a deviation from the zonal average, the bar a time average, and the prime a deviation from the time average. The kinetic and potential energy connected with the four components will be denoted by, respectively, the subscripts 0, 1, 2, and 3. We shall be interested mainly in three methods of separating the total kinetic and available potential energy into their mean and eddy parts. Let us define these methods as follows:

(1) Space domain:

$$K_M=K_0+K_2=\frac{1}{2}\int\overline{([u]^2+[v]^2)}\,dm$$

= "zonal kinetic energy" in Lorenz [8] paper.

$$K_E=K_1+K_3=\frac{1}{2}\int\overline{([u^{*2}]+[v^{*2}])}\,dm$$

= "eddy kinetic energy" in Lorenz [8] paper.

[1] For a discussion of the exchanges between the vertical mean field and the vertical eddy field see Wiin-Nielsen [29] and Smagorinsky [20].

$$P_M=P_0+P_1=\frac{1}{2}c_p\int\overline{\gamma[T]''^2}\,dm$$

="zonal available potential energy" in Lorenz [8] paper. (A double prime represents a deviation from the area average over a closed pressure surface)

$$P_E=P_2+P_3=\frac{1}{2}c_p\int\overline{\gamma[T^{*2}]}\,dm$$

="eddy available potential energy" in Lorenz [8] paper.

(2) Time domain:

$$\mathsf{K}_M=K_0+K_1=\frac{1}{2}\int[\bar{u}^2+\bar{v}^2]\,dm$$

$$\mathsf{K}_E=K_2+K_3=\frac{1}{2}\int[\overline{u'^2}+\overline{v'^2}]\,dm$$

$$\mathsf{P}_M=P_0+P_1=\frac{1}{2}c_p\int\gamma[\bar{T}''^2]\,dm$$

$$\mathsf{P}_E=P_2+P_3=\frac{1}{2}c_p\int\gamma[\overline{T'^2}]\,dm$$

(3) Mixed space-time domain:

$$\mathcal{K}_M=K_0=\frac{1}{2}\int([\bar{u}]^2+[\bar{v}]^2)\,dm$$

$$\mathcal{K}_E=K_1+K_2+K_3=\frac{1}{2}\int[\overline{u'^2}+\overline{v'^2}+\bar{u}^{*2}+\bar{v}^{*2}]\,dm$$

$$\mathcal{P}_M=P_0=\frac{1}{2}c_p\int\gamma[\bar{T}]''^2\,dm$$

$$\mathcal{P}_E=P_1+P_2+P_3=\frac{1}{2}c_p\int\gamma[\overline{T'^2}+\bar{T}^{*2}]\,dm$$

All integrals are taken over the mass of the entire atmosphere. In these expressions

$(\tilde{\ })$=area average over a closed pressure surface,

$(\ )''$=deviation from this area average,

c_p=specific heat at constant pressure, and

$$\gamma=-\left(\frac{\Theta}{T}\right)^2\frac{R}{c_p p_0}\int_0^{p_0}\left(\frac{T}{\Theta}\right)\frac{1}{p}\left(\frac{\partial\tilde{\Theta}}{\partial p}\right)^{-1}dp,$$

where

Θ=potential temperature, R=gas constant, and p_0=1000 mb. (Lorenz used in this case $\gamma=-\frac{\Theta}{T}\frac{R}{c_p p}\left(\frac{\partial\tilde{\Theta}}{\partial p}\right)^{-1}$. However, if one uses this last expression for γ, extra terms will enter the equations for the balance of available potential energy aside from the terms which represent the expected energy transformations. These extra terms depend mainly upon the variation of the *mean* (i.e., averaged over a closed pressure level) static stability with pressure. In order to avoid these extra (and probably meaningless) terms, it appeared necessary to use from the beginning $\gamma=-k\left(\frac{\Theta}{T}\right)^2$ in the definition of available potential energy, where k is independent of pressure and time; see e.g., the expression given above. The numerical differences with Lorenz's formulae are quite small.)

In the space domain mean kinetic energy is defined as the kinetic energy of the zonally averaged motion, in the time domain it is defined as the kinetic energy of the time-mean motion, and in the mixed space-time domain as the kinetic energy of the time-mean and zonal-mean motion. What is left of the total kinetic energy is called in each case "eddy" kinetic energy. Analogous definitions are used in the case of mean and eddy available potential energy. One should clearly distinguish between these three methods, since it is obvious that the values of the mean kinetic and mean available potential energy computed in the space and time domain are both larger than those computed in the mixed space-time domain. On the other hand, eddy kinetic and eddy potential energy computed in the space and time domain are smaller than if they were calculated in the mixed space-time domain. Also, the numerical values of the rate at which mean or eddy energy is generated, dissipated, and transformed, are quite different in the three systems. Knowledge of the energy cycle in the space domain answers questions concerning the maintenance of the zonal-mean state and the zonal eddies. On the other hand, a similar knowledge in the time domain enables us to consider the maintenance of the time-mean state and the transient (in time) eddies. Finally, the energy cycle in the mixed domain gives information on how the time- and zonal-mean state and the sum of transient and standing eddies are maintained. The energy integrals for the three systems will be compared in more detail in the appendix of this paper. In general, it is difficult to decide which one of the three approaches is preferable. The choice must depend more or less on the information which one desires to obtain.

3. DATA AND METHODS OF COMPUTATION

A separation into three categories will be made according to the kind of data and the methods of reduction used in calculating the energy integrals in tables 1, 2, 3, and 4.

(A) Basic information consists of the observed values of the horizontal wind components and the temperature. Vertical motions are computed with the so-called adiabatic method. Especially over the ocean areas this method is hampered by inadequate data coverage. Principal investigators: V. P. Starr, H. S. Buch, E. Holopainen, C. E. Jensen, J. P. Peixoto, and R. M. White.

(B) Basic information consists of height data of the isobaric levels, which are obtained from the objective, daily analyses by the National Meteorological Center (NMC) of the U.S. Weather Bureau. Vertical motions are computed from a two- or three-level model using again only height data; in general, effects of heating and friction are neglected. Method B' allows for daily estimates of winds over data-sparse regions; further, the objective analyses are aided by extrapolation from sea

level observations and also by reports from reconnaissance aircraft. A drawback of method B is the geostrophic assumption which plays an essential role in this approach. On the other hand, method B can supply valuable indirect (i.e., through a model) information on, for example, the heating rates in the atmosphere, and thereby also on the generation of available potential energy, where direct information is not yet available. Principal investigators: A. Wiin-Nielsen, J. E. Brown, B. Saltzman, A. Fleisher, A. F. Krueger, J. S. Winston, and D. Haines.

(C) Basic input consists of the hydrodynamical equations with some necessary simplifications and suitable boundary conditions. No actual data are used at all. In such numerical experiments one attempts among other things to approximate the essential features of the general circulation as they are determined under A and B. Principal investigators: N. A. Phillips, J. Smagorinsky, and Y. Mintz.

Estimates of the terms in the energy cycle are available only in the "space" and in the "mixed space-time" domain. Most computations have been carried out in what was called above the space domain. In this method all integrals are computed on a daily basis, while the averaging in time occurs only as a last step. As far as is known to the author, no extensive calculations have been carried out in the time domain. On the other hand, some work has been done in the mixed space-time domain. In this method the evaluation of the energy transformations is less time-consuming than in the space or time domain since it is not necessary to calculate these integrals for each day separately. In the next section we shall present some tables which give the estimates of several investigators of the energy integrals in the space and mixed space-time domain.

The following symbols will be used:

K_M, K_E = mean, eddy kinetic energy in the space domain

P_M, P_E = mean, eddy available potential energy in the space domain

$\mathbf{K}_M, \mathbf{K}_E$ = mean, eddy kinetic energy in the time domain

$\mathbf{P}_M, \mathbf{P}_E$ = mean, eddy available potential energy in the time domain

$\mathcal{K}_M, \mathcal{K}_E$ = mean, eddy kinetic energy in the mixed space-time domain

$\mathcal{P}_M, \mathcal{P}_E$ = mean, eddy available potential energy in the mixed space-time domain

$G(P)$ = rate of generation of available potential energy by diabatic heating

$D(K)$ = rate of frictional dissipation of kinetic energy

$C(P_M, K_M)$ = rate of conversion from mean available potential energy into mean kinetic energy by mean meridional circulations

$C(P_E, K_E)$ = rate of conversion from eddy available potential energy into eddy kinetic energy by large-scale eddy convection

$C(K_E, K_M)$ = rate of conversion from eddy into mean kinetic energy by eddy momentum transport

$C(P_M, P_E)$ = rate of conversion from mean into eddy available potential energy by eddy heat transport.

4. DISCUSSION OF ENERGY INTEGRALS

In tables 1 and 2 some estimates are given of the amount of mean and eddy kinetic and available potential energy present in the atmosphere of the Northern Hemisphere. The estimates in the space domain are presented in table 1 and those in the mixed space-time domain in table 2. The tables contain, besides the estimates, some relevant information concerning the representativeness of the data.

Estimates of the energy generation, dissipation, and conversion in the space and mixed space-time domain may be found in tables 3 and 4. Some comments will be made concerning the energy integrals in these tables.

(1) Generation of eddy available potential energy takes place if at a certain pressure level relatively warm air masses are heated and cold air masses are cooled as a result of the combined effects of radiation, condensation or evaporation, and turbulent exchanges with the earth's surface. This eddy generation was computed in the space domain by Wiin-Nielsen and Brown [30] and Brown [1] with the aid of daily hemispheric heating fields which were calculated from the thermodynamic equation. According to their calculations diabatic heating destroys eddy available potential energy. This could be expected if the exchange of heat between the atmosphere and the earth's surface were the determining factor; in general this exchange will have the tendency to cool warmer air masses and to warm colder air masses. The 2-parameter baroclinic model used by Wiin-Nielsen and Brown for evaluating vertical motions may underestimate the creation of eddy available potential energy due to condensation heating. This would make their eddy generation too negative. Suomi and Shen [26] found a *buildup* of eddy potential energy which was due only to the influence of infrared cooling. This was derived from radiation measurements made by Explorer VII. The sample was limited to a period of 13 days and to a small horizontal area. Therefore, it is probably not representative and further measurements are needed.

From balance considerations it follows that in the mixed space-time domain not a destruction but a creation of eddy available potential energy may occur (see next section).

(2) The dissipation of kinetic energy by friction cannot be measured directly. The best method is to compute the dissipation as the residual term in the balance of kinetic energy. This was done for the sum of mean and eddy kinetic energy by Holopainen [5] for each day of a period of three winter months over England. Although the daily values cannot be trusted, the order of magnitude might be correct.

(3) Overturnings of the atmosphere on the largest scale (i.e., rings of air moving across latitude circles) give a con-

TABLE 1.—*Estimates of the average amount of energy (10^5 joule $m.^{-2}$)* per unit area in the Northern Hemisphere.** Computed in the "space domain" from height data only*

Investigators	Saltzman [18, 19], Fleisher [18, 19]		Tewels [27]	
Representative for area covering	20-80° N. for P_M, P_E; 15-80° N. for K_M, K_E		17.5°-77.5° N.	
P_M	53.0	6 winter mo. 1950; 850-500-mb. thickness [18]		
P_E	13.9	6 winter mo. 1950; 850-500-mb thickness [18]		
K_M	6.3	year 1951; 500-mb. height data [19]	14.4	3 winter mo. 1957-1958; 500-, 100-, 50-mb. height data [27]
	8.9	winter 1951; 500-mb. height data [19]		
	3.7	summer 1951; 500-mb. height data [19]		
K_E	7.8	year 1951; 500-mb. height data [19]	8.1	3 winter mo. 1957-1958; 500-, 100-, 50-mb. height data; wave numbers 1-8 [27]
	9.8	winter 1951; 500-mb. height data [19]		
	5.8	summer 1951; 500-mb. height data [19]		

*10^5 joule $m.^{-2}$ = 10^8 erg $cm.^{-2}$ is equivalent with 2.56×10^{26} erg for the atmosphere of the entire Northern Hemisphere.

**Values are integrated in vertical direction throughout the depth of the atmosphere. October, November, December, January, February, and March are considered as "winter" months. April, May, June, July, August, and September are considered as "summer" months.

TABLE 2.—*Estimates of the average amount of energy (10^5 joule $m.^{-2}$)* per unit area in the Northern Hemisphere.** Computed in the "mixed space-time domain"*

	A. From actual wind and temperature data		B. From height data only	
Investigators	Buch [3], Crutcher [4], Murakami [9], Peixoto [12]		Saltzman [19], Fleisher [19]	
Representative for area covering	Northern Hemisphere (10-70° N. for [9])		15-80° N.	
$\mathcal{P}_M$	26.4	year 1950: temperature data at 7 levels (computed from [12])		
$\mathcal{P}_E$	14.7	year 1950; temperature data at 7 levels (computed from [12])		
$\mathcal{K}_M$	3.5	5 years; winds at 6 levels (computed from [4])	6.2	year 1951; 500-mb. height data (computed from [19])
	3.2	year 1950; winds at 6 levels (computed from [3] by [9])	8.3	winter 1951; 500-mb. height data (computed from [19])
			2.9	summer 1951; 500-mb. height data (computed from [19])
$\mathcal{K}_E$	9.5	year 1950; winds at 6 levels (computed from [3] by [9])	8.9	year 1951; 500-mb. height data (computed from [19])
			10.4	winter 1951; 500-mb. height data (computed from [19])
			6.6	summer 1951; 500-mb. height data (computed from [19])

* ** See footnotes table 1.

version between mean potential and mean kinetic energy. This term is not important in the hemispheric energy balance; see in this connection Starr [22, 23], Wiin-Nielsen [28], Saltzman and Fleisher [17, 18]. Starr used the expression $\int f[\bar{u}][\bar{v}]\,dm$, where u, v=west-east, south-north component of the actual wind and f=Coriolis parameter, to compute the conversion from mean potential into mean kinetic energy in the mixed domain. This conversion is usually given by the expression $-\int[\bar{\omega}]\,[\bar{\alpha}]\,dm$, where ω= dp/dt="vertical velocity" and α specific volume. The two expressions are identical if u is replaced by u_g, the geostrophic component of u. Krueger, Winston, and Haines [7] using the new NMC 3-parameter model for computing vertical velocities, obtained large negative values for this conversion in the space domain, the reason being that they did not include the area south of 20° N. in their integration. If one integrates over the entire hemisphere, the influence of the direct Hadley circulation at low latitudes should bring these estimates closer to zero. The values obtained by Wiin-Nielsen [28] and Saltzman and Fleisher [17, 18] also for the area north of 20° N. are not so large negatively probably because of the use of the NMC 2-parameter vertical velocities which appear to give smaller conversions between potential and kinetic energy. Summarizing, our best estimate is that the conversion between mean potential and mean kinetic energy is small compared to, for example, the conversion from eddy potential into eddy kinetic energy.

(4) The eddy conversion from potential into kinetic energy in the space domain is given essentially by the covariance of ω and T within latitude circles. In all estimates of ω which were used for the calculation of $C(P_E,K_E)$ in table 3, the effects of diabatic heating have been neglected. Unfortunately, it is not known how reliable this approximation is. The same difficulty holds in the computations of the conversion in the mixed space-time domain.

(5) The horizontal area of integration does not always cover the entire hemisphere. For example, the objective NMC analyses which were used by the investigators of group B do not extend to latitudes lower than 17.5° N. For this reason the conversion and generation terms are integrated only over an area from the pole to about 20° N. It would, indeed, be incorrect to extrapolate these conversion rates to the equator. Recently indications have been found of a negative value of the rates of conversion from mean potential into eddy potential and from eddy potential into eddy kinetic energy at low latitudes (in middle latitudes these integrals are large and positive). In other words, there is evidence of an eddy heat transport against the mean meridional temperature gradient and also of a conversion from kinetic into potential energy by the large-scale eddy processes in tropical latitudes (see Peixoto [11], Starr and Wallace [24]). This indirect action of the eddies in the Tropics may be compared with the quite similar operation of the disturbances in the lower stratosphere (see Oort [10]).

(6) The rate of transformation between mean and eddy kinetic energy is given essentially by the product of the eddy transport of momentum and the gradient of mean angular rotation both taken in the north-south direction. Similarly, the rate of transformation between mean and

TABLE 3.—*Estimates of the energy integrals (watt m.⁻²)* in the "space domain" for the Northern Hemisphere***

Investigators	A. From actual wind and temperature data	B. From height data only (geostrophic approach): vertical motions are computed from a frictionless, adiabatic model using only height data			C. From numerical solution hydrodynamical equations	
	Starr [21, 23, 25], Brunt [2], Holopainen [5], Suomi [26], Shen [26], White [25]	Saltzman [15, 16, 18], Fleisher [16, 18]	Wiin-Nielsen [28, 31], Brown [1, 31], Drake [31]	Krueger [7], Winston [7], Haines [7], Teweles [27]	Phillips [14]; 2-level quasi-geostrophic model	Smagorinsky [20]; 2-level model using primitive equations
Representative for area covering	Northern Hemisphere	Northern Hemisphere	20-90° N.	20-90° N.	Rectangular region: -5000 km. $\le y \le 5000$ km, $0 \le x \le 6000$ km.	0-64.4° N
$G(P_M)$			1.94 year (9 mo. 1959-63); 850, 500-mb. height data and model for heating [1] 2.81 7 winter mo. 1959-63; 850, 500-mb. height data and model for heating [1] 1.07 6 summer mo. 1961-62; 850, 500-mb. height data and model for heating [1]	2.32 year; computed as residual term [7] 3.30 winter; computed as residual term [7] 1.34 summer; computed as residual term [7]	2.13 heating minus lateral heat diffusion	2.21 heating minus lateral heat diffusion
$G(P_E)$	0.58 13 days 1959-1960; infrared cooling measured from Explorer VII; 30-50° N.; sample is probably not representative for atmosphere [26]		−0.94 year (9 mo. 1959-63); 850, 500-mb. height data and model for heating [1] −1.57 7 winter mo. 1959-63; 850, 500-mb. height data and model for heating [1] −0.32 6 summer mo. 1961-62; 850, 500-mb. height data and model for heating [1]	−0.77 year; computed as residual term [7] −1.10 winter; computed as residual term [7] −0.44 summer; computed as residual term [7]	−0.10 lateral heat diffusion	−0.28 heating minus lateral heat diffusion.
$D(K_M)$	−5.0 mean + eddy dissipation based on mean wind profile; extremely uncertain [2]	−0.23 winter; computed as residual term [15]			−0.95 skin friction plus effects of lateral eddy viscosity taken into account	−1.25 skin and internal friction plus effects of lateral eddy viscosity taken into account
$D(K_E)$	−1.9 mean + eddy dissipation; Sept., Oct., Nov. 1954; area England; computed as residual term on a daily basis [5]	−2.37 winter; computed as residual term [15]			−0.89 skin friction plus effects of lateral eddy viscosity taken into account	−1.50 skin and internal friction plus effects of lateral eddy viscosity taken into account
$C(P_M, K_M)$	0.25 year 1950; winds at 7 levels; analysis by strings of stations (computed from [25])	0.35 6 winter mo. 1959; 850-500-mb. thickness and ω at 600 mb.; includes by extrapolation contribution of equatorial Hadley cell [18]	0.10 Jan. 1959; 850-500-mb. thickness and ω at 600 mb. [28] −0.11 Apr. 1959; 850-500-mb. thickness and ω at 600 mb. [28]	−0.66 year 1962-63; 3-parameter NMC model for ω [7] −0.79 6 winter mo. 1962-63; 3-parameter NMC model for ω [7] −0.53 6 summer mo 1962-63; 3-parameter NMC model for ω [7]	−0.30	−0.10
$C(P_E, K_E)$		3.92 6 winter mo. 1959; area 20-80° N; 850-500-mb. thickness and ω at 600 mb. [18]	1.46 Jan. 1959; 850-500-mb. thickness and ω at 600 mb. [28] 1.10 Apr. 1959; 850-500-mb. thickness and ω at 600 mb. [28]	2.21 year 1962-63, 3 parameter NMC model for ω [7] 2.98 6 winter mo. 1962-63; 3-parameter NMC model for ω [7] 1.44 6 summer mo. 1962-63; 3-parameter NMC model for ω [7]	3.47	2.44
$C(K_E, K_M)$	0.38 first 6 mo. 1950; winds at 5 levels; analysis by strings of stations in a latitude belt [21] 0.23 year 1951, winds at 7 levels; analysis by strings of stations in a latitude belt [23] 0.25 year 1951, only 500-mb. winds; analysis by strings of stations in a latitude belt [23]	0.15 year 1951; only 500-mb height data [16] 0.23 6 winter mo. 1951; only 500-mb. height data [16] 0.07 6 summer mo. 1951; only 500-mb. height data [16]	0.16 year (5 mo. 1962-63); height data at 5 levels [31] −0.06 4 winter mo. 1962-63; height data at 5 levels [31] 0.40 3 summer mo. 1962; height data at 5 levels [31]	0.14 IGY (5 mo.); area 17.5-77.5° N; 500, 100, 50-mb. height data [27]	[illegible].48	1.20
$C(P_M, P_E)$			3.25 year (5 mo. 1962-63), height data at 5 levels [31] 4.54 4 winter mo. 1962-63; height data at 5 levels [31] 1.96 3 summer mo. 1962; height data at 5 levels [31]	2.98 year (60 mo. 1958-63); 850, 500-mb. height data [7] 4.08 30 winter mo. 1958-63; 850, 500-mb. height data [7] 1.88 30 summer mo. 1958-63; 850, 500-mb. height data [7]	3.47	2.87

*1 watt m.⁻² = 10³ erg cm.⁻² sec.⁻¹ is equivalent with 2.56×10²¹ erg sec.⁻¹ for the atmosphere of the entire Northern Hemisphere.

**Values are integrated in vertical direction throughout the depth of the atmosphere. October, November, December, January, February and March are considered as "winter" months. April, May, June, July, August and September are considered as "summer" months.

TABLE 4.—*Estimates of the energy integrals (watt m.$^{-2}$)* in the "mixed space-time domain" for the Northern Hemisphere***

FROM ACTUAL WIND AND TEMPERATURE DATA		
Investigators	Starr [21, 23, 25], White [25]	Brunt [2], Buch [3], Holopainen [5], Jensen [6], Peixoto [12, 13]
Representative for area covering	Northern Hemisphere	Northern Hemisphere
$G(\mathcal{P}_M)$	1.12 year 1950 (computed as a residual term from [23, 25])	
$G(\mathcal{P}_E)$		
$D(\mathcal{K}_M)$	−0.10 year 1950 (computed as a residual term from [21, 23])	−5.0 mean + eddy dissipation based on mean wind profile; extremely uncertain [2]
$D(\mathcal{K}_E)$		−1.9 mean + eddy dissipation; Sept., Oct., Nov. 1954; area England; computed as residual term on a daily basis [5]
$C(\mathcal{P}_M, \mathcal{K}_M)$	−0.08 year 1950; winds at 7 levels; analysis by strings of stations [23] −0.12 year 1951; winds at 7 levels; analysis by strings of stations [23]	
$C(\mathcal{P}_E, \mathcal{K}_E)$		4.28 Jan. 1958; 7 layers; analysis of maps; adiabatic method for ω (computed from [6]) 2.74 Apr. 1958; 7 layers; analysis of maps; adiabatic method for ω (computed from [6])
$C(\mathcal{K}_E, \mathcal{K}_M)$	0.15 first 6 mo. 1950; winds at 5 levels; analysis by strings of stations in a latitude belt [21] 0.21 second 6 mo. 1950; winds at 5 levels; analysis by strings of stations in a latitude belt [21]	0.22 year 1950; winds at 6 levels; analysis of maps; transient and standing eddies included (computed from [3])
$C(\mathcal{P}_M, \mathcal{P}_E)$	1.20 year 1950; winds and temperatures at 7 levels; analysis by strings of stations in a latitude belt (computed from [25]).	0.98 year 1950; winds and temperatures at 7 levels; analysis of maps; only transient eddies included (computed from [12]) 2.02 6 winter mo. 1950; winds and temperatures at 7 levels; analysis of maps; transient and standing eddies included [13] 0.71 6 summer mo. 1950; winds and temperatures at 7 levels; analysis of maps; transient eddies only (computed from [12])

* **See footnotes table 3.

eddy potential energy is given by the product of the eddy transport of heat and the gradient of mean temperature, also both in the north-south direction. For both terms there is a considerable numerical difference between the conversion rates computed in the space and in the mixed space-time domain. As an illustration, we may quote Starr's [21] measurements of the conversion from eddy into mean kinetic energy for the first six months of 1950. Starr computes 0.38 watt m.$^{-2}$ in the space domain and with the same data 0.15 watt m.$^{-2}$ in the mixed space-time domain. There is evidence of a similar difference in the rate of conversion from mean into eddy potential energy. The investigators of group B, using Lorenz's approach, estimate a value of about 3.0 watt m.$^{-2}$ for the year, but the data of the investigators of group A, using the mixed space-time approach, give the much smaller value of about 1.2 watt m.$^{-2}$. The rate of conversion from mean into eddy potential energy is stronger in the space than in the mixed domain, since in the first case an extra term is included which depends on the covariance in time of the down-gradient eddy transport of heat and the meridional temperature gradient itself (see appendix). As will be seen in the next section, this extra term may significantly alter the value of the generation of eddy available potential energy as determined from balance conditions.

(7) Starr [23] noticed that the integral $C(K_E, K_M)$ had a positive value for each month of the year 1951; i.e., during each month the eddies gave kinetic energy to the zonal current. The same was true for four of the five months considered by Wiin-Nielsen, Brown, and Drake [31]; only January 1963 formed an exception. During this month the large-scale disturbances drew kinetic energy from the zonal flow. In this respect January 1963 must have been an exceptional month.

(8) A comparison of the energy integrals calculated in the numerical experiments (Phillips [14], Smagorinsky [20]) with those calculated from actual data shows that the dissipation of eddy kinetic energy in the experiments is too small, while the dissipation of mean kinetic energy is much too large. Presumably this is related to difficulties in formulating the dissipation mechanism. A further consequence of the incorrect dissipation rates is that too much eddy kinetic energy is converted into mean kinetic energy.

5. DIAGRAM FOR THE ATMOSPHERIC ENERGY CYCLE

From the generation, dissipation, and conversion rates in tables 3 and 4 a careful selection was made by considering the representativeness of the data. The author's best *estimates* of the order of magnitude of the hemispheric energy processes for the period of a year are presented in two diagrams, one for the space domain (fig. 1) and the other for the mixed space-time domain (fig. 2). No special averaging process was used in determining the numbers from the data given in tables 1 through 4. The necessary balance conditions for each energy component have been taken into account in the construction of these flow diagrams. In the diagrams the small circles indicate the four forms of energy. The large circle which encloses the four smaller ones represents the "environment". The numbers on the connecting lines indicate the rate of conversion, generation, or dissipation for the different forms of energy. The numbers within the small circles give the amount of energy present. The dissipation rates are derived indirectly from the other estimates using balance requirements; the same holds to a certain extent for the generation rates.

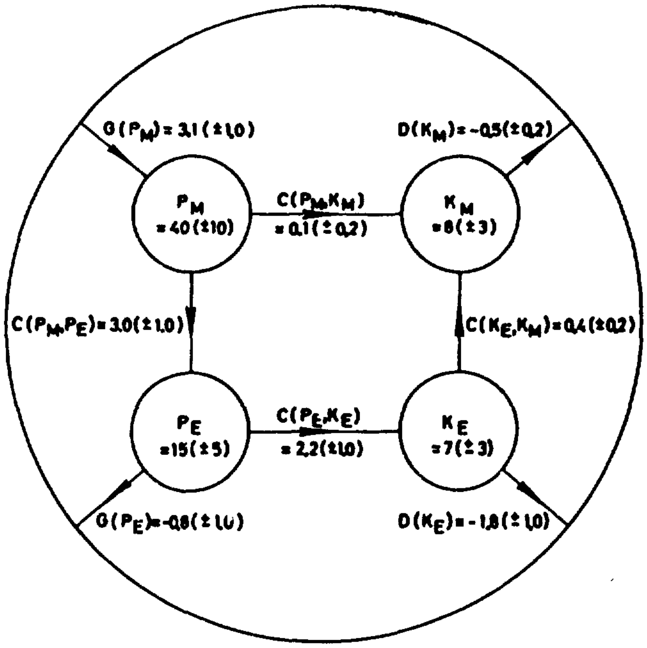

Figure 1.—Tentative flow diagram of the atmospheric energy in the space domain. Values are averages over a year for the Northern Hemisphere. Energy units are in 10^5 joule $m.^{-2}$ ($=10^8$ erg $cm.^{-2}$); energy transformation units are in watt $m.^{-2}$ ($=10^3$ erg $cm.^{-2}$ $sec.^{-1}$).

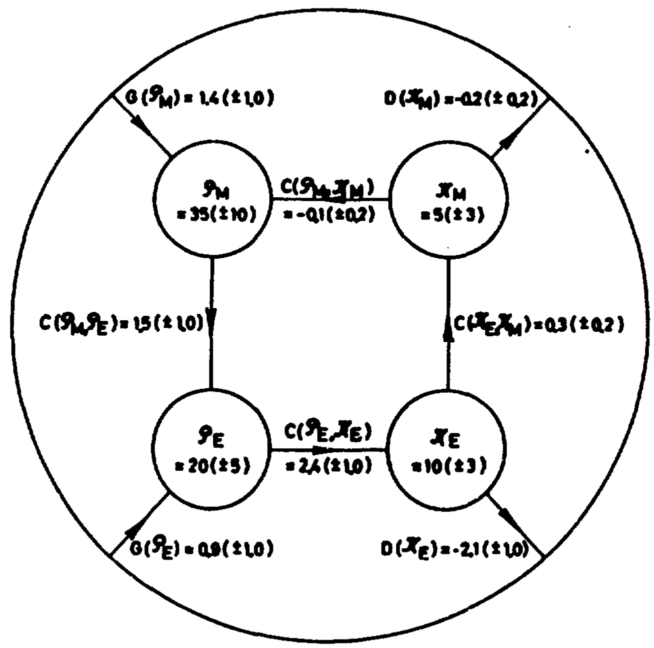

Figure 2.—Tentative flow diagram of the atmospheric energy in the mixed space-time domain. Values are averages over a year for the Northern Hemisphere. Energy units are in 10^5 joule $m.^{-2}$; energy transformation units are in watt m^{-2}

As is well known the energy cycle proceeds in the average from mean available potential energy via eddy available potential energy and eddy kinetic energy, to, finally, the mean kinetic energy, i.e., in a counterclockwise direction in figures 1 and 2. The important steps in this cycle will be discussed below in some detail. The numbers for the mixed space-time domain will be placed in parentheses after the numbers for the space domain.

At low latitudes the earth-atmosphere system gains more energy per unit area by the absorption of short-wave radiation from the sun than it loses to space by the emission of long-wave radiation; the reverse is true at high latitudes. Since in the troposphere, which contains the bulk of the atmosphere, the higher temperatures are found at low latitudes and the lower temperatures at high latitudes, mean available potential energy is created as a result of radiation. The creation due to all diabatic sources amounts to 3.1 (1.4) watt $m.^{-2}$. Without rotation the release of this energy would take place through an axially symmetric, meridional Hadley circulation. However, because of the actual conditions of rotation and heating of the earth and its atmosphere, asymmetric, eddy circulations take over the task of the mean meridional circulation as convective units. The zonal currents connected with the meridional temperature gradient have a maximum instability at the wavelength of the large-scale eddies. The conversion from mean available potential energy into its eddy counterpart occurs at the estimated rate of 3.0 (1.5) watt $m.^{-2}$. In the space domain heating processes appear to destroy (in the mixed domain they appear to create) eddy potential energy at the rate of -0.8 ($+0.9$) watt $m.^{-2}$. The remaining energy, i.e., 2.2 (2.4) watt $m.^{-2}$, is converted into eddy kinetic energy. The conversion from eddy kinetic into mean kinetic energy, which maintains the zonal currents against the frictional dissipation by small-scale eddies, is only about 0.4 (0.3) watt $m.^{-2}$. To maintain the balance of the eddy kinetic energy it is necessary that about 80 percent of the eddy kinetic energy which is released by convection, i.e., -1.8 (-2.1) watt $m.^{-2}$, be destroyed by friction. The dissipation of mean kinetic energy is comparatively small, namely -0.5 (-0.2) watt $m.^{-2}$, since the exchange between mean kinetic and mean available potential energy is only 0.1 (-0.1) watt $m.^{-2}$.

APPENDIX

Expressions for the energy integrals in the space (a), time (b), and mixed space-time (c) domain will be presented. Further, in order to obtain a better understanding of the differences in the flow diagrams (figs. 1 and 2) for the energy cycle, the integrals for the space and mixed domains will be compared (d).

NOTATION

λ, ϕ = longitude, latitude
p = pressure
u, v = eastward, northward components of the wind
$\omega = dp/dt$ = "vertical velocity"
Φ = geopotential "height"
T = temperature
Θ = potential temperature
dm = increment of mass
a = radius of the earth
f = Coriolis parameter
R = gas constant
c_p = specific heat at constant pressure
Q = rate of heat addition per unit mass
F = frictional force per unit mass

$$\gamma = -\left(\frac{\Theta}{T}\right)^2 \frac{R}{c_p p_0} \int_0^{p_0} \left(\frac{T}{\Theta}\right) \frac{1}{p} \left(\frac{\partial \tilde{\overline{\Theta}}}{\partial p}\right)^{-1} dp$$

(see discussion section

p_0 = 1000 mb.
$\overline{b}$ = time average of b
b' = deviation from time average of b
$[b]$ = zonal average of b
b^* = deviation from zonal average of b
$\tilde{b}$ = hemispheric average of b over isobaric surface
b'' = deviation from hemispheric average of b.

The basic equations used in deriving the formulae for the generation, destruction, and conversion rates of kinetic and available potential energy are the first law of thermodynamics

$$\frac{\partial \Theta}{\partial t} = -u \frac{\partial \Theta}{a \cos \phi \partial \lambda} - v \frac{\partial \Theta}{a \partial \phi} - \omega \frac{\partial \Theta}{\partial p} + \frac{1}{c_p}\left(\frac{\Theta}{T}\right) Q \qquad (1)$$

the zonal equation of motion

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{a \cos \phi \partial \lambda} - v \frac{\partial u}{a \partial \phi} - \omega \frac{\partial u}{\partial p} - \frac{\partial \Phi}{a \cos \phi \partial \lambda} + v\left(f + \frac{u}{a} \tan \phi\right) + F_x \qquad (2)$$

and the meridional equation of motion

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{a \cos \phi \partial \lambda} - v \frac{\partial v}{a \partial \phi} - \omega \frac{\partial v}{\partial p} - \frac{\partial \Phi}{a \partial \phi} - u\left(f + \frac{u}{a} \tan \phi\right) + F_y \qquad (3)$$

The balance equations may be derived as follows:

in the *space domain*

- multiply (1) by $(T/\Theta)^2 c_p \gamma [\Theta]''$ for the balance equation of P_M;
- multiply (1) by $(T/\Theta)^2 c_p \gamma \Theta^*$ for the balance equation of P_E;
- multiply (2) and (3) by $[u]$ and $[v]$, and add for the balance equation of K_M;
- multiply (2) and (3) by u^* and v^*, and add for the balance equation of K_E;

in the *time domain*

- multiply (1) by $(T/\Theta)^2 c_p \gamma \overline{\Theta}''$ for the balance equation of $\mathbf{P}_M$;
- multiply (1) by $(T/\Theta)^2 c_p \gamma \Theta'$ for the balance equation of $\mathbf{P}_E$;
- multiply (2) and (3) by $\overline{u}$ and $\overline{v}$, and add for the balance equation of $\mathbf{K}_M$;
- multiply (2) and (3) by u' and v', and add for the balance equation of $\mathbf{K}_E$;

in the *mixed space-time domain*

- multiply (1) by $(T/\Theta)^2 c_p \gamma [\overline{\Theta}]''$ for the balance equation of $\mathcal{P}_M$;
- multiply (1) by $(T/\Theta)^2 c_p \gamma (\overline{\Theta}^* + \Theta')$ for the balance equation of $\mathcal{P}_E$;
- multiply (2) and (3) by $[\overline{u}]$ and $[\overline{v}]$, and add for the balance equation of $\mathcal{K}_M$;
- multiply (2) and (3) by $(u' + \overline{u}^*)$ and $(v' + \overline{v}^*)$, and add for the balance equation of $\mathcal{K}_E$.

Finally, all equations are averaged in time over the period considered and integrated over the total mass of the atmosphere.

A. ENERGY INTEGRALS IN THE SPACE DOMAIN

$$P_M = \frac{1}{2} c_p \int \overline{\gamma [T]''^2} dm$$

$$P_E = \frac{1}{2} c_p \int \overline{\gamma [T^{*2}]} dm$$

$$K_M = \frac{1}{2} \int \overline{([u]^2 + [v]^2)} dm$$

$$K_E = \frac{1}{2} \int \overline{([u^{*2}] + [v^{*2}])} dm$$

$$G(P_M) = \int \overline{\gamma [T]'' [Q]''} dm$$

$$G(P_E) = \int \overline{\gamma [T^* Q^*]} dm$$

$$D(K_M) = \int \overline{([u][F_x] + [v][F_y])} dm$$

$$D(K_E) = \int \overline{([u^* F_x^*] + [v^* F_y^*])} dm$$

$$C(P_M, K_M) = -\int \overline{[\omega]''[\alpha]''} dm = \int f \overline{[u_g][v]} dm$$

$$C(P_E, K_E) = -\int \overline{[\omega^* \alpha^*]} dm$$

$$C(K_E, K_M)=\int\overline{[u^*v^*]\cos\phi\,\frac{\partial}{a\partial\phi}([u]\cos^{-1}\phi)}dm$$

$$+\int\overline{[u^*\omega^*]\frac{\partial[u]}{\partial p}}\,dm+\int\overline{[v^{*2}]\frac{\partial[v]}{a\partial\phi}}\,dm$$

$$+\int\overline{[v^*\omega^*]\frac{\partial[v]}{\partial p}}\,dm-\int\overline{[v][u^{*2}]\frac{\tan\phi}{a}}\,dm$$

$$C(P_M, P_E)=-c_p\int\overline{\gamma[v^*T^*]\frac{\partial[T]}{a\partial\phi}}\,dm$$

$$-c_p\int\overline{\gamma\left(\frac{T}{\Theta}\right)[\omega^*T^*]''\frac{\partial[\Theta]''}{\partial p}}\,dm$$

The corrections for the change in time in the balance equations of P_M, P_E, K_M, K_E are, respectively:

$$c_p\int\overline{\gamma\frac{\partial}{\partial t}\frac{1}{2}[T]''^2}dm,$$

$$c_p\int\overline{\gamma\frac{\partial}{\partial t}\frac{1}{2}[T^{*2}]}dm,$$

$$\int\overline{\frac{\partial}{\partial t}\frac{1}{2}([u]^2+[v]^2)}dm,$$

$$\int\overline{\frac{\partial}{\partial t}\frac{1}{2}([u^{*2}]+[v^{*2}])}dm.$$

B. ENERGY INTEGRALS IN THE TIME DOMAIN

$$\mathbf{P}_M=\frac{1}{2}c_p\int\gamma\,[\overline{T}''^2]dm$$

$$\mathbf{P}_E=\frac{1}{2}c_p\int\gamma\,[\overline{T'^2}]dm$$

$$\mathbf{K}_M=\frac{1}{2}\int[\overline{u}^2+\overline{v}^2]dm$$

$$\mathbf{K}_E=\frac{1}{2}\int[\overline{u'^2}+\overline{v'^2}]dm$$

$$G(\mathbf{P}_M)=\int\gamma\,[\overline{T}''\overline{Q}'']dm$$

$$G(\mathbf{P}_E)=\int\gamma\,[\overline{T'Q'}]dm$$

$$D(\mathbf{K}_M)=\int[\overline{u}\overline{F}_x+\overline{v}\overline{F}_y]dm$$

$$D(\mathbf{K}_E)=\int[\overline{u'F'_x+v'F'_y}]dm$$

$$C(\mathbf{P}_M, \mathbf{K}_M)=-\int[\overline{\omega}''\overline{\alpha}'']dm$$

$$C(\mathbf{P}_E, \mathbf{K}_E)=-\int[\overline{\omega'\alpha'}]dm$$

$$C(\mathbf{K}_E, \mathbf{K}_M)=\int\left[\overline{u'^2}\frac{\partial\overline{u}}{a\cos\phi\partial\lambda}\right]dm$$

$$+\int\left[\overline{u'v'}\cos\phi\,\frac{\partial}{a\partial\phi}(\overline{u}\cos^{-1}\phi)\right]dm$$

$$+\int\left[\overline{u'\omega'}\frac{\partial\overline{u}}{\partial p}\right]dm$$

$$+\int\left[\overline{v'u'}\frac{\partial\overline{v}}{a\cos\phi\partial\lambda}\right]dm$$

$$+\int\left[\overline{v'^2}\frac{\partial\overline{v}}{a\partial\phi}\right]dm+\int\left[\overline{v'\omega'}\frac{\partial\overline{v}}{\partial p}\right]dm$$

$$-\int\left[\overline{u'^2}\,\overline{v}\,\frac{\tan\phi}{a}\right]dm$$

$$C(\mathbf{P}_M, \mathbf{P}_E)=-c_p\int\gamma\left[\overline{u'T'}\frac{\partial\overline{T}}{a\cos\phi\partial\lambda}\right]dm$$

$$-c_p\int\gamma\left[\overline{v'T'}\frac{\partial\overline{T}}{a\partial\phi}\right]dm$$

$$-c_p\int\gamma\left[\left(\frac{T}{\Theta}\right)\overline{\omega'T'}\frac{\partial\overline{\Theta}''}{\partial p}\right]dm$$

The corrections for the change in time in the balance equations of $\mathbf{P}_M$, $\mathbf{P}_E$, $\mathbf{K}_M$, $\mathbf{K}_E$, are, respectively:

$$c_p\int\gamma\left[\overline{T}''\,\overline{\frac{\partial T''}{\partial t}}\right]dm,$$

$$c_p\int\gamma\left[\overline{T'\frac{\partial T}{\partial t}}\right]dm,$$

$$\int\left[\overline{u}\,\overline{\frac{\partial u}{\partial t}}+\overline{v}\,\overline{\frac{\partial v}{\partial t}}\right]dm,$$

$$\int\left[\overline{u'\frac{\partial u}{\partial t}}+\overline{v'\frac{\partial v}{\partial t}}\right]dm.$$

C. ENERGY INTEGRALS IN THE MIXED SPACE-TIME DOMAIN

$$\mathcal{P}_M=\frac{1}{2}c_p\int\gamma[\overline{T}]''^2dm$$

$$\mathcal{P}_E=\frac{1}{2}c_p\int\gamma[\overline{T'^2}+\overline{T}^{*2}]dm$$

$$\mathcal{K}_M=\frac{1}{2}\int([\overline{u}]^2+[\overline{v}]^2)dm$$

$$\mathcal{K}_E=\frac{1}{2}\int[\overline{u'^2}+\overline{v'^2}+\overline{u}^{*2}+\overline{v}^{*2}]dm$$

$$G(\mathcal{P}_M)=\int\gamma[\overline{T}]''[\overline{Q}]''dm$$

$$G(\mathcal{P}_E)=\int\gamma[\overline{T'Q'}+\overline{T}^*\overline{Q}^*]dm$$

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$$D(\mathcal{K}_M)=\int ([\bar{u}][\bar{F}_x]+[\bar{v}][\bar{F}_y])dm$$

$$D(\mathcal{K}_E)=\int [\overline{u'F_x'}+\overline{v'F_y'}+\bar{u}^*\bar{F}_x^*+v^*\bar{F}_y^*]dm$$

$$C(\mathcal{P}_M, \mathcal{K}_M)=-\int [\bar{\omega}]''[\bar{\alpha}]''dm-\int f[\bar{u}_g][\bar{v}]dm$$

$$C(\mathcal{P}_E, \mathcal{K}_E)=-\int [\overline{\omega'\alpha'}+\bar{\omega}^*\bar{\alpha}^*]dm$$

$$C(\mathcal{K}_E, \mathcal{K}_M)=\int ([\overline{u'v'}]+[\bar{u}^*\bar{v}^*]) \cos\phi \frac{\partial}{a\partial\phi} ([\bar{u}]\cos^{-1}\phi)\, dm$$

$$+\int ([\overline{u'\omega'}]+[\bar{u}^*\bar{\omega}^*]) \frac{\partial[\bar{u}]}{\partial p} dm$$

$$+\int ([\overline{v'^2}]+[\bar{v}^{*2}]) \frac{\partial[\bar{v}]}{a\partial\phi} dm$$

$$+\int ([\overline{\omega'v'}]+[\bar{\omega}^*\bar{v}^*]) \frac{\partial[\bar{v}]}{\partial p} dm$$

$$-\int [\bar{v}]([\overline{u'^2}]+[\bar{u}^{*2}]) \frac{\tan\phi}{a} dm$$

$$C(\mathcal{P}_M, \mathcal{P}_E)=-c_p\int \gamma ([\overline{v'T'}]+[\bar{v}^*\bar{T}^*]) \frac{\partial[\bar{T}]}{a\partial\phi} dm$$

$$-c_p\int \gamma \left(\frac{T}{\Theta}\right) ([\overline{\omega'T'}]+[\bar{\omega}^*\bar{T}^*])'' \frac{\partial[\bar{\Theta}]''}{\partial p} dm$$

The corrections for the change in time in the balance equations of $\mathcal{P}_M$, $\mathcal{P}_E$, $\mathcal{K}_M$, $\mathcal{K}_E$ are, respectively:

$$c_p\int \gamma[\bar{T}]'' \frac{\overline{\partial[T]''}}{\partial t} dm,$$

$$c_p\int \gamma \left[\overline{T' \frac{\partial T'}{\partial t}}+\bar{T}^* \frac{\overline{\partial T^*}}{\partial t}\right] dm,$$

$$\int \left([\bar{u}] \frac{\overline{\partial[u]}}{\partial t}+[\bar{v}] \frac{\overline{\partial[v]}}{\partial t}\right) dm,$$

$$\int \left[\overline{u' \frac{\partial u}{\partial t}}+\overline{v' \frac{\partial v}{\partial t}}+\bar{u}^* \frac{\overline{\partial u^*}}{\partial t}+\bar{v}^* \frac{\overline{\partial v^*}}{\partial t}\right] dm.$$

D. RELATION BETWEEN QUANTITIES IN SPACE DOMAIN (ITALIC CAPITALS) AND QUANTITIES IN MIXED SPACE-TIME DOMAIN (SCRIPT CAPITALS)

$$P_M=\mathcal{P}_M+\frac{1}{2}c_p\int \gamma\overline{[T]'^2}dm$$

$$P_E=\mathcal{P}_E-\frac{1}{2}c_p\int \overline{\gamma[T]'^2}dm$$

$$K_M=\mathcal{K}_M+\frac{1}{2}\int (\overline{[u]'^2}+\overline{[v]'^2})dm$$

$$K_E=\mathcal{K}_E-\frac{1}{2}\int (\overline{[u]'^2}+\overline{[v]'^2})dm$$

$$G(P_M)=G(\mathcal{P}_M)+\int \gamma\overline{[T]'[Q]'}dm$$

$$G(P_E)=G(\mathcal{P}_E)-\int \gamma\overline{[T]'[Q]'}dm$$

$$D(K_M)=D(\mathcal{K}_M)+\int (\overline{[u]'[F_x]'}+\overline{[v]'[F_y]'})dm$$

$$D(K_E)=D(\mathcal{K}_E)-\int (\overline{[u]'[F_x]'}+\overline{[v]'[F_y]'})dm$$

$$C(P_M,K_M)=C(\mathcal{P}_M,\mathcal{K}_M)-\int \overline{[\omega]'[\alpha]'}dm$$

$$C(P_E, K_E)=C(\mathcal{P}_E,\mathcal{K}_E)+\int \overline{[\omega]'[\alpha]'}dm$$

$$C(K_E, K_M)=C(\mathcal{K}_E, \mathcal{K}_M)+\int \overline{[u^*v^*]' \cos\phi\frac{\partial}{a\partial\phi}([u]'\cos^{-1}\phi)}dm$$

$$+\int \overline{[u^*\omega^*]' \frac{\partial[u]'}{\partial p}} dm+\int \overline{[v^{*2}]' \frac{\partial[v]'}{a\partial\phi}} dm$$

$$+\int \overline{[\omega^*v^*]' \frac{\partial[v]'}{\partial p}} dm-\int \overline{[u^{*2}]'[v]'} \frac{\tan\phi}{a} dm$$

$$-\int \overline{[u]'[v]'} \cos\phi \frac{\partial}{a\partial\phi} ([\bar{u}] \cos^{-1}\phi)dm$$

$$-\int \overline{[u]'[\omega]'} \frac{\partial[\bar{u}]}{\partial p} dm-\int \overline{[v]'^2} \frac{\partial[\bar{v}]}{a\partial\phi} dm$$

$$-\int \overline{[\omega]'[v]'} \frac{\partial[\bar{v}]}{\partial p} dm+\int [\bar{v}]\overline{[u]'^2} \frac{\tan\phi}{a} dm$$

$$C(P_M,P_E)=C(\mathcal{P}_M,\mathcal{P}_E)-c_p\int \overline{\gamma[v^*T^*]' \frac{\partial[T]'}{a\partial\phi}} dm$$

$$-c_p\int \gamma\left(\frac{T}{\Theta}\right) \overline{[\omega^*T^*]' \frac{\partial[\Theta]'}{\partial p}} dm$$

$$+c_p\int \gamma\overline{[v]'[T]'} \frac{\partial[\bar{T}]}{a\partial\phi} dm$$

$$+c_p\int \gamma\left(\frac{T}{\Theta}\right) \overline{[\omega]' [T]'} \frac{\partial[\bar{\Theta}]''}{\partial p} dm$$

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Mechanics of Eddy Processes in the Tropical Troposphere[1])

By VICTOR P. STARR[2]) and JOHN M. WALLACE[2])

Summary – It is suggested on the basis of the study of observational data gathered by J. P. PEIXOTO that the eddy processes in the tropical regions of the middle troposphere operate in an inverse manner as compared with a normal heat engine. Thus, in common with the situation in the lower stratosphere, the region is characterized by a countergradient horizontal eddy heat transport and a rising of colder air and sinking of warmer air on the scale of the large eddy processes.

1. *Introductory Remarks Concerning the Lower Stratosphere*

It is perhaps fair to say that during the past several years there has come to light for the first time a correct picture of the gross workings of the dynamical processes in the lower stratosphere. This is true, albeit that some large questions still remain. Of course residual problems will, however, never be exhausted. It is to be expected that the important new findings will presently appear in numerical and theoretical models of the atmosphere, as no doubt also will the features of the tropical troposphere to be presented below.

One method of describing briefly the actions which predominate in this new picture is to enumerate the following points:

a) The horizontal poleward eddy heat transport is by and large countergradient, since the temperature increases toward the pole.

b) In order that this should take place it is required that poleward moving parcels should be heated adiabatically, while equatorward moving parcels should be similarly cooled.

c) This action leads to a negative correlation between upward and poleward velocity components.

d) It also leads to a negative correlation between the upward velocity component and the temperature.

e) This rising of cold air and sinking of warm acts in the sense of a conversion from kinetic into internal and potential energy in eddy form.

f) The eddy potential and internal energy so manufactured is converted into mean zonal form by the countergradient heat flux.

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g) The angular momentum flux due to the eddies is against the gradient of angular velocity, thus giving a further drain of eddy kinetic energy through a conversion into kinetic energy of the mean zonal flow.

h) It must be concluded that the eddy kinetic energy is resupplied by vertical transport from the troposphere.

i) The general conclusion is that the lower stratosphere acts in the manner of a refrigerating machine, supplied with kinetic energy from the direct acting regions. This produces low temperatures near the equator and higher temperatures toward the pole, contrary to the tendency of the radiative processes, which it overcomes.

j) The selective action already mentioned, of the vertical and meridional eddy components of motion fits in with the fact that they are able to produce a poleward transport of ozone. This eddy flux of ozone leads to an accumulation of this material at more northerly latitudes in a manner contrary to what would be expected from the photochemical processes alone.

We can neither stop in our present exposition to descant further, in more technical detail, upon the manner in which the ten items enumerated unite to form a consistent system, nor can we, except as indicated below, render a full account of the rather interesting sequence of events through which the outlines of the material content of these items became apparent from global observations interpreted in the light of physical principles. Much less can we enter upon a consideration of the remaining problems. Since all these things, moreover, are not now our principal immediate concern, it will suffice simply to say that the work stemmed mainly from the following discussions, available in the literature: WHITE [17][1]); STARR and WHITE [15]; MARTIN [5]; STARR [16]; PEIXOTO [13]; JENSON [3]; NEWELL [7]; MOLLA and LOISEL [6]; NEWELL [8, 9]; OORT [11, 12]; SHEPPARD [14]. These the interested may read for a better background in the subject which is, of course, of considerable consequence for what follows, and constitutes, in its own right, no trifling chapter in the saga of man's effort to visualize correctly the operation of the atmospheric circulation.

2. Countergradient Heat Flux in The troposphere

In the development of our picture of stratospheric mechanics a key role is to be ascribed to the recognition of the existence of a horizontal countergradient eddy transport of heat as stated in item a). This fact together with the poleward eddy transport of ozone noted by MARTIN suggested the need for the coexistent scheme of vertical motions, corroborated more directly later. The other features then followed rather naturally.

We may now in similar fashion study the horizontal eddy heat transport in the troposphere more extensively than was done by STARR and WHITE [15]. Fortunately an extensive analysis of such heat flow in the northern hemisphere, which includes the region in the tropics — a deficiency of the previous work — was presented by PEIXOTO [13]. This material also was part of the subject matter of an essay for which a prize was awarded to Prof. PEIXOTO by the Academy of Science of Portugal. The chief results which concern us here are reproduced in Tables 1, 2 and 3.

[1]) Numbers in brackets refer to References, pages 143 and 144.

Expression should be given at the outset to the fact that the measurements tax to the utmost the resources of available observational material thus far analysed. Much more should by now be available, and one can only lament the listless attitude in meteorology which would assign considerably less than first priority to what is in fact the growing edge of our basic conceptions. No doubt many factors enter into this state of affairs, but it would seem on the face of it unbelievable that after the expenditure of the prodigious sums of money needed for making observations, so little value should be placed upon the discovery through their further elaboration, of permanent general information. This the more so, since the added cost is paltry in comparison with the main investment.

Upon examination of Table 1, it is evident that there is indicated a southward transport of sensible heat at low latitudes by the transient eddies. Although the action is weak at the lowest level, during both summer and winter taken individually the negative transport is found as far as 30°N latitude at 500 mb, and negative signs seem to predominate at and south of 20°N up to the stratosphere. The values no doubt cannot be relied upon to be accurate in detail because of various shortcomings of the original data, but probably the indication of a southward transient eddy heat transport in the middle and upper troposphere is correct.

A question at once arises as to the further contribution by standing eddies to the heat transport. Because of the nature of this action, it depends heavily upon a proper station sampling at all longitudes, and is more difficult to measure on this account. It was decided by PEIXOTO to give in his publication merely a sample calculation of this quantity for the winter season only. The values which are shown in Table 2 display an array of negative signs similar to those in Table 1, although there are minor differences. The magnitudes are also similar, so far as one may judge in spite of probable inaccuracies of detail.

In order to judge whether or not the transports are countergradient it is necessary to have information relating to the mean zonal temperature distribution. The compilation of these data prepared by PEIXOTO is reproduced in Table 3. For the inspection and study of this material, it should be noted that mean temperature gradients in the tropics are small and hence are probably as difficult to measure as the transports. It is a mitigating circumstance, however, that the standard deviations of temperature are smaller in the tropics, as shown by the results of PEIXOTO as well as those of other studies. Except for the well known reverse gradient essentially at 100 and 200 mb at all latitudes and for the reverse gradient in the tropics at low levels in summer, there is shown the normal horizontal poleward decrease of temperature in the tropics as well as elsewhere.

It now remains to be noted that whereas there exist some downgradient transports in the tropics at upper levels especially in summer and at low levels near the equator also in summer, by and large the transports due to the transient eddies in the middle troposphere are countergradient. This is most prominent at 500 mb, the effect being present as far north as 30° both in summer and in winter. The same general action is shown by the data for the standing eddies in winter.

3. *Concluding Discussion*

The outlines of the picture implied by the uphill eddy diffusion may be sketched tentatively as follows.

1. The countergradient heat flow suggests that in those levels and latitudes where it exists, namely in the vicinity of 500 mb, the southward moving parcels are warmer than average, while northward moving ones are colder—in spite of the mean gradient of temperature and contrary to simple mixing length notions.

2. This can happen if the former ones subside so as to increase their temperature sufficiently by adiabatic heating, and conversely if the northward moving ones rise enough to make them enough colder than normal.

3. This correlation between northward and upward velocity components is in the same sense as found in the troposphere farther north, but now the difference is in the increased vigour of the adiabatic effects *vis-à-vis* the magnitude of the temperature gradients. Some aid to understanding this is contained in the simple graphical aids due to GREEN [2] and KUO [4].

4. As in the lower stratosphere, we are led to the concept of a negative covariance or correlation between upward motion and temperature—a process acting to convert eddy kinetic to eddy potential and internal energy. This once more is characteristic of a refrigerating machine instead of a heat engine.

5. The countergradient heat transport has the added significance, as in the stratosphere, of converting eddy available potential and internal energy into mean zonal form, thus tending to build up the existing horizontal temperature gradient.

6. Due to the smallness of the temperature gradients and the weakness of the transports, the amount of energy involved in the reverse transformation is very small compared even with the magnitude of such processes in the stratosphere, not to say the main direct transformations in the other parts of the troposphere.

7. The supply of eddy kinetic energy to the region in question could easily be supplied by transport mechanisms from the ambient direct acting regions.

8. Since a small poleward transport of total energy should take place on the average across low latitudes, judging from our admittedly crude knowledge of radiational exchanges with space, additional processes must be at work in the atmosphere or the oceans.

9. It might appear that the proposals made in this paper are based upon insufficiently demonstrated data. There is of course the possibility that more evidence might nullify our expectations of corroboration. However, we have here an illustration of an exceedingly important point in the practical approach to research—one that is all too often forgotten or erroneously denied even. It is this—that many new conceptions concerning the operation of the atmosphere are to be looked for at the borderline of measurability. And here (to make a pun) wise indeed is the meteorologist who knows which way the wind is blowing.

10. Our colleague, Mr. P. A. GILMAN [1] has made the suggestion independently and on other grounds that there may be a zone of equatorward eddy heat transport not only in the tropics but also close to the pole. Perhaps better evidence may be forthcoming in regard to this.

11. In the measurement of the correlation between upward motion and temperature, use is often made of vertical motion computed by the so-called adiabatic method, as was done, for example, by JENSEN [3]. No doubt inaccuracies are inherent in the eddy correlations so obtained. However, much unjustified criticism has been leveled against this method. It is, in this technique, of prime importance to carry along the static stability as a variable quantity. If this is not done, the

elimination of the variability leads to a false physical identification of the main quantities computed. This point will be discussed elsewhere in order to clarify the not inconsiderable amount of confusion which seems to exist.

12. According to indications obtained by NEWELL [8] from rocket soundings, there may exist a layer in the atmosphere in the vicinity of 55 to 80 km elevation, in which the energy conversions proceed in the reverse sense, as in the lower stratosphere.

Table 1

Zonally averaged values of the mean meridional transient eddy transport of heat in units of degrees absolute meters per second for yearly and seasonal data at specified latitudes during 1950. The levels are given in millibars

Level	70°	60°	50°	45°	40°	30°	20°	10°	0°
				Yearly Data					
100	+ 6.08	+ 8.82	+ 6.93	+ 4.23	+ 1.89	+ 1.89	+ 1.39	− 0.87	− 0.67
200	+ 2.49	+ 4.49	+ 5.99	+ 6.25	+ 5.65	+ 3.37	− 0.62	− 1.11	− 0.25
300	+ 0.96	− 0.47	− 0.36	+ 1.79	+ 2.05	+ 1.75	+ 0.72	− 0.27	− 1.04
500	+ 7.33	+ 7.95	+ 6.59	+ 4.59	+ 2.54	+ 0.16	− 0.59	− 0.74	− 0.76
700	+ 9.71	+ 11.34	+ 10.48	+ 7.79	+ 6.69	+ 2.39	+ 0.48	− 0.46	− 0.46
850	+ 9.99	+ 15.60	+ 15.94	+ 13.57	+ 10.79	+ 4.00	+ 1.09	− 0.01	− 0.49
1000	+ 2.20	+ 4.58	+ 6.82	+ 7.83	+ 7.26	+ 4.29	+ 1.79	+ 0.46	− 0.35
				Summer Data					
100	+ 0.71	+ 1.64	+ 2.52	+ 1.95	+ 0.39	− 0.20	+ 0.69	− 0.18	− 0.11
200	+ 4.91	+ 5.56	+ 7.17	+ 7.74	+ 7.52	+ 3.15	− 1.34	− 0.60	− 0.88
300	+ 5.55	+ 3.95	+ 1.78	+ 2.35	+ 2.87	+ 2.29	− 0.26	− 0.54	− 0.62
500	+ 6.19	+ 5.88	+ 5.28	+ 3.86	+ 2.54	− 0.73	− 0.52	− 0.22	− 0.34
700	+ 5.45	+ 8.84	+ 7.61	+ 5.48	+ 3.73	+ 0.49	− 0.68	− 0.52	− 0.19
850	+ 9.11	+ 12.76	+ 11.69	+ 9.02	+ 6.16	+ 1.80	− 0.29	− 0.90	− 0.63
1000	+ 2.56	+ 4.93	+ 4.78	+ 4.21	+ 3.70	+ 1.66	+ 0.56	− 0.12	− 0.39
				Winter Data					
100	+ 2.90	+ 8.27	+ 9.52	+ 7.25	+ 6.32	+ 2.89	− 0.89	− 2.49	− 0.87
200	− 0.39	+ 2.67	+ 4.53	+ 6.07	+ 6.32	+ 2.46	− 1.21	− 1.89	− 1.11
300	− 1.35	+ 0.22	+ 2.29	+ 3.32	+ 4.14	+ 2.44	+ 1.08	− 0.56	− 1.58
500	+ 5.85	+ 6.35	+ 6.18	+ 5.06	+ 3.39	− 0.25	− 1.18	− 1.28	− 0.75
700	+ 9.28	+ 14.44	+ 13.56	+ 10.62	+ 7.03	+ 3.76	+ 0.32	− 0.50	− 1.12
850	+ 8.03	+ 18.00	+ 18.50	+ 15.93	+ 10.86	+ 5.50	+ 1.56	− 0.12	− 0.95
1000	+ 4.85	+ 4.90	+ 6.94	+ 7.47	+ 6.40	+ 3.18	+ 1.50	+ 0.32	− 0.46

Table 2

Zonally averaged values of the mean meridional standing eddy transport of heat in units of absolute degrees meter per second for the winter at specified latitudes. The levels are given in millibars

Level	70°	60°	50°	45°	40°	30°	20°	10°	0°
				Winter Data					
100	− 0.74	+ 0.40	+ 0.58	+ 0.60	+ 0.61	+ 0.77	− 0.30	− 0.20	—
200	− 1.48	+ 0.79	+ 1.17	+ 1.20	+ 1.21	+ 1.54	− 0.61	− 0.37	—
300	− 0.13	+ 5.53	+ 3.35	+ 0.20	+ 0.14	+ 0.71	− 0.65	− 1.12	—
500	− 0.53	+ 6.94	+ 5.71	+ 3.37	+ 1.56	− 0.44	− 0.62	− 0.47	—
700	+ 4.16	+ 5.87	+ 4.01	+ 2.24	+ 1.46	− 0.23	+ 0.05	− 0.22	—
850	+ 1.68	+ 0.62	+ 3.81	+ 2.05	+ 0.95	+ 0.24	− 0.72	+ 0.50	—
1000	+ 5.76	+ 6.49	+ 2.31	+ 1.43	+ 0.15	− 0.35	+ 0.43	+ 6.43	—

Table 3

Zonally averaged values of the mean temperature in absolute degrees for yearly and seasonal data at specified latitudes. The levels are given in millibars

Level	70°	60°	50°	45°	40°	30°	20°	10°	0°
				Yearly Data					
100	226.5	222.6	218.6	215.4	211.7	205.1	200.3	196.3	194.0
200	225.3	222.7	220.6	219.3	218.0	216.8	218.9	219.6	220.6
300	223.6	223.8	227.1	229.9	232.4	236.5	239.2	241.3	242.0
500	243.5	247.3	252.2	255.0	257.7	262.3	265.5	267.3	269.5
700	257.8	261.9	266.8	270 0	273.1	277.9	280.6	281.6	282.6
850	263.7	268.2	273.8	277.1	280.4	285.9	289.3	290.8	291.1
1000	265.5	272.0	279.0	282.1	285.9	293.3	298.2	299.2	296.1
				Summer Data					
100	228.9	224.7	218.8	215.3	212.2	206.0	201.6	197.4	194.5
200	226.7	224.4	222.1	220.3	219.5	219.0	218.9	218.8	213.3
300	226.0	227.5	231.1	233.4	235.7	238.9	240.8	242.0	241.4
500	247.9	252.3	256.6	259.2	261.6	264.7	266.5	267.6	269.0
700	262.1	266.9	272.0	274.4	277.1	280.4	282.3	282.0	281.3
850	269.2	274.3	279.5	282.3	285.3	289.7	291.8	291.3	289.4
1000	272.2	278.9	285.1	288.2	291.1	296.3	300.4	299.8	295.7
				Winter Data					
100	220.5	220.9	218.5	215.6	211.5	203.8	198.7	194.9	192.7
200	220.4	220.0	218.0	217.0	215.8	216.6	218.9	220.9	221.4
300	216.7	219.6	223.6	226.1	228.7	234.2	238.1	240.8	242.7
500	237.9	241.4	246.8	250.1	253.3	260.1	264.4	267.1	268.8
700	251.5	255.8	261.5	265.2	268.8	275.1	279.3	281.3	282.3
850	256.9	261.2	268.2	271.9	275.9	282.9	287.1	290.3	291.7
1000	254.2	261.2	271.4	276.4	281.5	290.5	295.8	299.6	300.0

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On the maintenance of the kinetic energy of mean zonal flow in the southern hemisphere

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ABSTRACT

Wind data reported at the eight pressure levels (850, 700, 500, 400, 300, 200, 100 and 50 mb) for 121 Southern Hemisphere plus 22 Northern Hemisphere equatorial stations during the calendar year 1958 has enabled us to study the rate of generation of the kinetic energy of the mean zonal flow. The results indicate that the kinetic energy of the mean zonal motion is maintained against frictional dissipation to a large extent through a conversion of transient eddy kinetic energy through the action of the horizontal wind. The generation of zonal kinetic energy by mean meridional motion through the action of the coriolis force cannot be measured well enough, but is probably small as in the Northern Hemisphere. The standing eddy transformation integral appears to be unimportant—a result which is not true for the Northern Hemisphere. If the conversion of the kinetic energy of the transient eddies into the kinetic energy of the mean zonal flow were to cease, the atmosphere of the Southern Hemisphere would be in solid rotation in about 2½ weeks. This assumes a continuation of a normal rate of dissipation during this period.

1. Introduction

In a previous article of this journal Kuo (1951) derived the equation of the balance of zonal kinetic energy. Because of the fragmentary observational data available, his computations of the conversion of transient eddy kinetic energy to the kinetic energy of the zonal flow were restricted to the North American continent. Starr (1953) made similar computations and in addition included the term involving the conversion of the kinetic energy of the mean meridional motion to the kinetic energy of mean zonal flow through the coriolis transformation. His computations were for the entire Northern Hemisphere.

In this paper some of the terms involved in the balance equation will be evaluated for the entire Southern Hemisphere and comparison will be made for analogous studies for the Northern Hemisphere. The data employed in this study has been fully discussed in another paper Obasi (1963). To avoid repetition the interested reader is directed to that paper.

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The following notation will be employed:

$$\bar{x} = \frac{1}{t_1 - t_0}\int_{t_0}^{t_1} x\,dt = \text{time average of } x.$$

$$[x] = \frac{1}{2\pi}\int_0^{2\pi} x\,d\lambda = \text{zonal average of } x.$$

$$x' = x - \bar{x} = \text{departure from time average.}$$

$$x^* = x - [x] = \text{departure from zonal average.}$$

Using the above notations, the equation of balance of zonal kinetic energy for the hemisphere of a spherical earth is given by

$$\frac{a^2}{g}\int_{p_G}^{0}\int_0^{\pi/2}\int_0^{2\pi}\frac{\partial}{\partial t}\left(\frac{[\bar{u}]^2}{2}\right)\cos\varphi\,d\lambda\,d\varphi\,dp \quad (1)$$

$$= -\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\phi}[\overline{u'v'}]\cos^2\varphi + \frac{\partial}{\partial P}[\overline{u'\omega'}]\right)\cos\varphi\,d\lambda\,d\varphi\,dp \quad (2)$$

$$-\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi + \frac{\partial}{\partial P}[\bar{u}^*\bar{\omega}^*]\right)\cos\varphi\,d\lambda\,d\varphi\,dp \quad (3)$$

$$+\frac{a}{g}\iiint[\bar{u}]^2[\bar{v}]\sin\varphi\, d\lambda\, d\varphi\, dp \quad (4)$$

$$-\frac{a}{2g}\iint[\bar{u}]^2[\bar{v}]\cos\varphi\, d\lambda\, dp\Big| \quad (5)$$

evaluated at the equator

$$+\frac{a^2}{g}\iiint f[\bar{u}]\,[\bar{v}]\cos\varphi\, d\lambda\, d\varphi\, dp \quad (6)$$

$$-\frac{a^2}{g}\iiint[\bar{u}]\,[T_\lambda]\cos\varphi\, d\lambda\, d\varphi\, dp. \quad (7)$$

In the above equation

a = radius of the earth
g = acceleration due to gravity
λ = longitude
φ = latitude
P = pressure
P_G = pressure at the ground
t = time
u = $a\cos\varphi(d\lambda/dt)$ = eastward component of the wind
v = $a(d\varphi/dt)$ = northward component of the wind
ω = dp/dt = individual pressure change
Ω = angular velocity of the earth
f = $2\Omega\sin\varphi$ = Coriolis parameter
T_λ = eastward component of viscous force per unit mass

In the long term average the left-hand side of the equation vanishes since there is no progressive increase or decrease in the kinetic energy of the mean zonal flow for the hemisphere. We now evaluate each of the terms involved in the right-hand side of the equation.

2. Evaluation of the integral

$$\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\overline{u'v'}]\cos^2\varphi + \frac{\partial[\overline{u'\omega'}]}{\partial p}\right)\cos\varphi\, d\lambda\, d\varphi\, dp.$$

This term represents the conversion of transient eddy kinetic energy to the kinetic energy of the zonal flow. At present there is no available data of vertical motion in the Southern Hemisphere. This data may be indirectly obtained from enthalpy study of the Southern Hemisphere (see for example JENSEN, 1961). However preliminary studies in the Northern Hemisphere, as well as theoretical considerations, indicate that

$$\frac{a^2}{g}\iiint[\bar{u}]\frac{\partial}{\partial p}[\overline{u'\omega'}]\cos\varphi\, d\lambda\, d\varphi\, dp \quad (2.1)$$

is much smaller than

$$\frac{a^2}{g}\iiint[\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\overline{u'v'}]\cos^2\varphi\right)\cos\varphi\, d\lambda\, d\varphi\, dp. \quad (2.2)$$

Because of this reason and since vertical motions are unavailable, we shall evaluate only the latter term. The integral

$$\frac{a^2}{g}\iiint\left(\frac{[\bar{u}]}{a\cos\varphi}\frac{\partial}{\partial\varphi}[\overline{u'v'}]\cos^2\varphi\right)d\lambda\, d\varphi\, dp \quad (2.3)$$

$$= -\frac{a^2}{g}\iiint[\overline{u'v'}]\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)d\lambda\, d\varphi\, dp \quad (2.4)$$

$$+\frac{a^2}{g}\iiint\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}[\overline{u'v'}]\cos^2\varphi\right)d\lambda\, d\varphi\, dp. \quad (2.5)$$

The integral (2.5) is

$$\int\frac{2\pi a}{g}[\bar{u}]\,[\overline{u'v'}]\cos\varphi\, dp \quad (2.6)$$

evaluated at the equator. All the three integrals, namely (2.3), (2.4) and (2.6) were evaluated by finite difference approximation.

A. Winter

Table 1 shows the integrand of (2.3). When the computations are performed by levels and then integrated over the entire hemisphere, we obtain a conversion of transient eddy kinetic energy to zonal kinetic energy. This value is 9.63×10^{20} ergs/sec.

When use is made of vertically-averaged relative angular velocity and vertically averaged gradient of transient eddy momentum flux, the integral (2.3) gives 9.89×10^{20} ergs/sec.

Use of the 500-mb data alone for the relative

TABLE 1. *Values of the term* $\frac{[\bar{u}]}{\cos\varphi}\frac{\partial}{\partial\varphi}[\overline{u'v'}]\cos^2\varphi$ *in winter 1958.*

Multiply by 4.09×10^{17} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.58	2.27	20.95	41.25	29.91	11.46	1.30	—	78.74
75–70	17.98	26.63	32.80	59.46	33.20	13.60	9.79	—	168.28
70–65	−3.74	40.46	21.88	16.55	33.67	10.69	4.92	−0.94	121.00
65–60	−2.33	−9.59	−39.25	−69.92	−29.91	−23.33	−10.66	−0.81	−201.68
60–55	15.56	−6.97	−153.49	−151.89	−95.33	−26.66	−21.53	−4.81	−482.75
55–50	23.29	−58.38	−247.29	−219.62	−164.84	−52.98	−41.86	−12.24	−838.32
50–45	3.22	−67.53	−169.50	−203.22	−175.60	−74.49	−49.77	−19.97	−840.32
45–40	0.22	−61.36	−194.30	−232.10	−143.02	−66.12	−22.64	−23.10	−806.05
40–35	−6.24	−19.54	−191.20	−267.50	−129.08	−49.72	−3.20	2.69	−681.24
35–30	−7.60	−5.66	−24.31	−12.29	4.11	−18.61	−3.28	−2.04	−80.67
30–25	−0.39	31.72	108.81	175.45	37.27	10.74	−1.06	−0.02	359.25
25–20	0	49.37	194.81	147.76	97.88	17.13	1.82	0.34	506.76
20–15	2.52	25.54	124.64	94.23	37.52	13.85	0.31	0.63	300.16
15–10	−1.14	10.16	45.87	32.93	4.74	1.04	−4.43	−0.41	83.25
10–5	0.16	0.51	5.69	−3.12	−8.12	−5.48	−5.02	−5.80	−35.10
5–0	−2.06	0.55	−13.45	−14.37	−13.61	−7.19	−1.68	−6.14	−70.79

Hemisphere 9.63×10^{20} ergs sec^{-1}

angular velocity and transient eddy momentum shear, assuming these to be representative of the average for the atmosphere resulted in the quantity 10.02×10^{20} ergs sec^{-1}.

Table 2 shows the integral of (2.4). When the computation is performed by levels and then integrated throughout the mass of the hemisphere we obtain 9.41×10^{20} ergs sec^{-1}.

The use of vertically-averaged momentum transport and a vertically-averaged shear in

TABLE 2. *Values of the term* $[\overline{u'v'}]\cos^2\varphi\,\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)$ *in winter 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0	0	0.35	−0.31	−1.20	−0.39	0.07	—	−1.22
75–70	0.21	0.29	1.39	2.30	0.69	0.74	0.94	—	8.88
70–65	0.25	0.98	3.06	4.67	3.77	2.35	1.75	1.70	20.75
65–60	−0.03	0.42	1.73	3.51	3.34	1.49	1.66	1.40	17.15
60–55	−0.57	−0.89	0.02	0.10	0.78	−0.01	0.56	0.76	2.48
55–50	−1.96	−0.96	6.25	4.06	0.60	0.07	−0.03	0.11	9.02
50–45	−2.34	2.11	13.68	12.33	3.09	1.16	1.03	0.09	32.67
45–40	−1.68	2.17	−1.30	−3.87	7.52	3.70	2.92	1.29	16.28
40–35	−0.72	−1.90	−18.47	−14.21	4.45	4.57	3.57	2.12	−12.32
35–30	0.07	−1.04	−12.15	−7.43	5.70	4.88	3.34	1.83	2.67
30–25	0.34	3.63	22.67	19.45	11.97	5.06	2.53	1.56	72.59
25–20	0.25	3.69	31.06	28.83	12.16	4.97	2.20	1.40	89.46
20–15	0.34	0.55	17.11	21.64	11.93	5.04	2.09	1.77	66.54
15–10	0.28	−1.17	5.28	9.46	6.61	2.67	0.90	0.04	26.34
10–5	−0.62	−1.21	−0.95	1.88	2.95	0.58	0.17	−0.55	2.44
5–0	−0.80	−0.97	−3.97	−0.70	0.40	−0.03	0.02	−0.18	6.01

Hemisphere 9.41×10^{20} ergs sec^{-1}

TABLE 3. *Values of the terms shown below at the equator in winter 1958.*

Units are in $m^3 sec^{-3}$.

Pressure in mb	$[\bar{u}][\overline{u'v'}]$	$[\bar{u}][\bar{u}^*\bar{v}^*]$
50	−8.78	−0.44
100	−5.56	−3.00
200	−44.09	−8.13
300	−18.24	−0.32
400	5.19	0.47
500	−4.94	−1.10
700	0.36	−1.54
850	4.26	−0.07
Vertical integral 10^{20} ergs sec^{-1}	−0.27	−0.06

evaluating the same integral gives 9.57×10^{20} ergs sec^{-1}.

If one uses the 500-mb data alone for the momentum flux and shear, assuming them to be representative of the average for the hemisphere one obtains 9.61×10^{20} ergs sec^{-1}.

Table 3 shows the values of

$$\frac{2\pi a}{g}[\bar{u}][\overline{u'v'}]\cos\varphi$$

at the equator. Computations by levels of this expression gives a vertical integral of 0.27×10^{20} ergs sec^{-1}. If one uses the vertically-averaged transient eddy momentum flux and the vertically-averaged zonal motion at the equator, one obtains 0.13×10^{20} ergs sec^{-1}. If one assumes the 500-mb level data to be representative of the entire column of the atmosphere above the equator, one obtains 0.20×10^{20} ergs sec^{-1}.

Assuming that the integration by levels is much more accurate one obtains 9.63×10^{20} ergs sec^{-1} for integral (2.3) and 9.68×10^{20} erg sec^{-1} for the sum of integrals (2.4) and (2.5). A difference of 0.08×10^{20} erg sec^{-1} is the difference in the truncation error.

B. Summer

Table 4 shows the integrand of (2.3). When one integrates over the entire mass of the hemisphere one obtains 9.72×10^{20} ergs sec^{-1}.

If use is made of the vertically-averaged relative angular velocity and vertically-averaged shear of transient eddy momentum flux, the integral (2.3) becomes 9.04×10^{20} ergs sec^{-1}.

If one assumes that the 500-mb relative velocity and momentum flux shear are representative of the average for the entire atmosphere, the integral results in 8.53×10^{20} ergs sec^{-1}.

Table 5 shows the integrand of (2.4). Using this table which gives the integrand by level

TABLE 4. *Values of the term* $\frac{[\bar{u}]}{a\cos\varphi}\frac{\partial}{\partial\varphi}[\overline{u'v'}]\cos^2\varphi$ *in summer 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	1.02	0.46	1.26	1.72	0.90	0.26	−0.22	—	5.15
75–70	1.86	1.52	1.08	1.64	1.36	0.81	−0.10	—	7.72
70–65	1.38	0.64	−4.77	−5.72	−0.69	−0.61	−0.08	−0.19	−11.15
65–60	0.75	−4.16	−15.60	−15.33	−5.10	−4.02	−1.85	−0.07	−48.02
60–55	−3.03	−8.27	−29.53	−24.49	−11.51	−7.82	−5.17	−2.10	−99.50
55–50	3.15	−9.20	−42.13	−33.34	−15.98	−8.34	−7.51	−6.41	−141.08
50–45	−0.71	−5.69	−39.16	−34.42	−17.07	−4.85	−1.92	−5.35	−118.12
45–40	−0.13	−0.54	−30.30	−23.46	−9.12	−2.45	−0.73	0.31	−67.63
40–35	0.06	−5.01	−22.34	−1.62	−1.96	−2.94	−1.18	2.37	−30.78
35–30	−0.52	−8.62	6.22	13.48	1.71	−1.60	−0.40	0.73	13.10
30–25	−0.48	−4.27	26.23	14.52	1.82	−0.91	0.47	0	38.47
25–20	−0.37	2.09	26.47	11.42	6.66	−0.11	−0.28	0.19	44.76
20–15	0.03	3.19	15.54	7.07	2.85	0.58	0	0.27	29.35
15–10	0.88	1.11	5.70	2.72	1.23	0.58	−0.27	0.06	11.52
10–5	2.39	0.11	−0.54	0.25	0.19	−0.38	−0.45	−0.89	−1.59
5–0	−0.26	0.08	−0.63	−0.02	−0.14	−0.81	−0.44	−0.84	−4.79

Hemisphere 9.72×10^{20} ergs sec^{-1}

TABLE 5. *Values of the term* $\overline{[u'v']}\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)$ *in summer 1958.*

Multiply by 2.61×10^{19} to obtain zonal kinetic energy generation in ergs sec^{-1}.

	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.07	0.03	0.58	1.02	0.27	0.26	0.08	—	2.25
75–70	0.20	0.23	1.09	2.29	0.99	0.79	0.23	—	6.10
70–65	−0.07	0.48	0.56	1.64	1.66	0.93	1.03	0.61	8.73
65–60	−0.61	0.06	−0.98	0.55	0.66	0.36	0.48	1.19	2.79
60–55	0.76	0.22	−1.19	−0.89	−0.51	−0.99	−1.10	0.07	6.24
55–50	0.15	1.18	4.08	2.61	−0.87	−0.67	−0.55	−1.07	2.44
50–45	0.94	2.76	9.73	6.22	0.68	0.95	2.74	0.80	27.41
45–40	1.12	1.64	9.74	16.23	5.87	0.46	5.07	3.58	57.65
40–35	0.96	1.61	10.14	15.00	8.20	5.32	4.81	3.70	59.99
35–30	0.63	3.51	12.90	13.74	8.47	5.50	4.01	1.97	57.91
30–25	0.18	7.58	24.30	15.03	8.74	5.25	3.17	1.12	69.82
25–20	−0.01	8.50	18.48	9.27	5.83	4.43	1.74	0.59	50.95
20–15	0.01	5.61	9.16	4.12	2.93	2.99	0.93	0.13	26.83
15–10	−0.07	2.33	2.35	0.57	1.49	2.05	0.32	−0.05	9.63
10–5	0.95	0.50	0.45	−0.35	0.43	0.94	0.18	0.10	2.13
5–0	−1.70	−0.07	0.55	−0.16	0.02	0.25	0.01	0.13	0.23

Hemisphere 9.86×10^{20} ergs sec^{-1}

one obtains a value of 9.86×10^{20} erg sec^{-1}. If use is made of the vertically-averaged transient eddy flux of momentum and vertically-averaged shear of relative angular velocity, the integral (2.4) becomes 8.99×10^{20} ergs sec^{-1}.

Use of the 500-mb data alone for the transient eddy flux of momentum and relative velocity shear gives a value of 8.58×10^{20} ergs sec^{-1}.

Table 6 shows the values of

$$\frac{2\pi a}{g}[\bar{u}]\overline{[u'v']}\cos\varphi$$

at the equator. Computation by level of this expression gives a vertical integral of 0.03×10^{20} ergs/sec. If one uses a vertically-averaged transient eddy flux of momentum and a vertically-averaged zonal motion at the equator, one obtains -0.03×10^{20} ergs sec^{-1}. The use of the 500-mb data alone, assuming this to be representative of the average of the atmospheric column over the equator, gives a value of 0.10×10^{20} ergs sec^{-1}.

Assuming the computation by levels to be more representative, one obtains a value of 9.72×10^{20} ergs sec^{-1} for the left hand side of (2) and a value of 9.89×10^{20} ergs sec^{-1} for the right hand side. The difference -0.17×10^{20} ergs sec^{-1} is the difference in the truncation errors.

C. Approximation of the term for the Northern Hemisphere

$$\frac{a^2}{g}\iiint\overline{[u'v']}\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)d\lambda d\varphi dp.$$

In order to have better appreciation for the generation of zonal kinetic energy by the transient eddies, we include in this section the results obtained in the Northern Hemisphere.

This integral has been evaluated by using Buch's yearly values of $[\bar{u}]$ and $\overline{[u'v']}$ for the Northern Hemisphere.

TABLE 6. *Values of the terms shown below at the equator in summer 1958.*

Units are in m^2 sec^{-2}.

Pressure in mb	$[\bar{u}]\overline{[u'v']}$	$[\bar{u}][\bar{u}^*\bar{v}^*]$
50	0.39	−0.05
100	−2.13	−0.87
200	−5.12	−12.08
300	0.70	−0.71
400	−0.71	−1.75
500	0	−0.36
700	−1.59	0.69
850	0	6.03
Vertical integral 10^{20} ergs sec^{-1}	−0.03	−0.00

When the integration was performed by levels a value of 4.84×10^{20} ergs sec^{-1} was obtained. This value is to be compared with the mean of summer and winter for the Southern Hemisphere, namely, 9.63×10^{20} ergs sec^{-1}.

If use is made of the vertically-averaged transient eddy momentum flux and vertically-averaged shear of the relative angular velocity, one obtains 4.55×10^{20} ergs sec^{-1}, the corresponding value for the Southern Hemisphere is 9.28×10^{20} ergs sec^{-1}.

If one uses the 500-mb data alone, and assumes this to be representative of the mean for the atmosphere, then one obtains a value of 4.95×10^{20} ergs sec^{-1} for the Northern Hemisphere, as compared to 9.09×10^{20} ergs sec^{-1} for the Southern Hemisphere.

Using the following sources of data:

(*a*) the first six months of the year 1950,

(*b*) the second six months of the year 1950,

(*c*) One month of data (January 1949) published by MINTZ, STARR (1953) evaluated an integral approximately similar to (2.4) for the Northern Hemisphere. Using vertically averaged winds and transports, the data (*a*) gives 4.2×10^{20} ergs sec^{-1} the data (*b*) gives 4.6×10^{20} ergs sec^{-1} and (*c*) gives 10.5×10^{20} ergs sec^{-1}.

Evaluation of the integrand at individual levels before integration yields for data (*a*) 3.9×10^{20} data (*b*) 5.5×10^{20} and data (*c*) 9.8×10^{20} ergs sec^{-1}.

Starr also used more elaborate methods in evaluation of the integral, namely by days and levels, but the results were similar to the above values.

3. Evaluation of the integral

$$\frac{a^2}{g}\iiint [\bar{u}]\left(\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi + \frac{\partial}{\partial P}[\bar{u}^*\overline{\omega}^*]\right)\cos\varphi\, d\lambda\, d\varphi\, dp.$$

This term represents the conversion of standing eddy kinetic energy to the kinetic energy of zonal motion. From BURGER's (1958) scale analysis we conclude that

$$\frac{1}{a\cos^2\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi \quad \text{and} \quad \frac{\partial}{\partial p}[\bar{u}^*\overline{\omega}^*],$$

are of the same order of magnitude. Although it is realised that by using the continuity equation one can obtain the $\overline{\omega}$ and $\overline{\omega}^*$ and so compute the vertical transport of zonal momentum by the standing eddies, because of the labour

TABLE 7. *Values of the term* $\frac{[\bar{u}]}{\alpha\cos\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi$ *in winter 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °*S*	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	1.74	1.42	0.01	−0.47	−0.52	−0.56	−0.11		0.43
75–70	4.11	1.70	0.78	−0.09	−0.42	−0.31	−0.11		4.05
70–65	2.00	0.46	1.70	0.23	0.01	0.64	0.44	−0.15	5.42
65–60	−5.38	−0.40	−0.28	−0.95	0.20	1.35	0.27	0.41	−2.57
60–55	−5.76	−5.34	−6.04	1.02	0.62	0.03	−0.37	0.21	−12.87
55–50	6.29	1.46	4.94	3.55	3.12	2.31	1.24	0.34	23.83
50–45	5.11	0.87	0.63	−1.93	3.30	0.10	0.4[illegible]	−0.13	7.07
45–40	−0.24	−0.31	−6.66	−1.80	−4.24	−0.33	−0.6[illegible]	−0.93	−16.87
40–35	−1.96	0.23	−4.18	−1.53	−4.05	−1.44	0.15	−0.06	−13.10
35–30	−0.55	1.94	10.03	0.23	−0.01	0.05	1.66	0.28	14.91
30–25	−0.07	3.12	7.01	2.80	2.40	−0.29	−0.35	0.00	13.43
25–20	0.00	0.63	−6.96	0.23	0.31	−0.66	−0.68	0.12	−7.87
20–15	−0.10	−0.35	0.21	−0.19	−0.26	0.06	−0.01	0.05	−0.39
15–10	0.07	−0.49	2.76	0.08	0.10	0.03	−0.41	1.47	5.28
10–5	−0.01	−0.13	−0.08	0.01	0.00	−0.15	−0.43	−2.00	−5.66
5–0	−0.06	−0.14	0.07	0.01	0.14	0.18	0.54	0.22	1.80

Hemisphere -0.44×10^{20} ergs sec^{-1}

TABLE 8. *Values of the term* $[\bar{u}^* \bar{v}^*] \cos^2 \varphi \dfrac{\partial}{\partial \varphi}\left(\dfrac{[\bar{u}]}{a \cos \varphi}\right)$ *in winter 1958.*

Multiply by 2.61×10^{18} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.16	0.02	0.00	0.00	0.12	0.11	−0.06		0.35
75–70	0.72	0.30	0.04	−0.06	−0.07	−0.20	−0.42		−0.04
70–65	0.79	0.50	0.33	−0.08	−0.31	−0.43	−0.44	−0.78	−1.09
65–70	0.00	0.23	0.35	−0.11	−0.26	−0.10	−0.26	−0.15	−0.58
60–55	−0.35	−0.19	0.00	0.00	−0.07	0.00	−0.14	0.20	−0.25
55–50	−0.64	0.10	0.15	−0.26	−0.04	−0.08	−0.02	0.16	−0.34
50–45	−2.74	−0.26	−0.53	−0.47	−0.34	−0.27	−0.03	−0.05	−4.15
45–40	−2.71	−0.13	−0.02	0.01	−0.41	−0.38	−0.02	−0.12	−3.43
40–35	−1.05	0.07	−1.19	−0.25	0.09	−0.16	0.05	0.04	−2.16
35–30	−0.18	0.09	−0.50	−0.15	0.27	−0.02	−0.32	−0.06	−1.18
30–25	0.05	−0.78	−0.50	0.21	0.34	0.01	−0.42	−0.13	−1.52
25–20	0.09	−1.97	−0.65	−0.08	0.07	0.18	−0.02	0.03	−1.76
20–15	0.04	−1.52	0.67	−0.07	0.13	0.39	0.35	0.22	1.31
15–10	−0.01	−1.07	−0.89	−0.06	0.06	0.13	0.08	0.01	−1.34
10–5	−0.03	−0.37	−1.40	−0.05	0.06	−0.11	−0.11	−0.16	−2.54
5–0	−0.02	−0.34	−1.01	−0.02	0.00	−0.05	−0.05	0.02	−1.43

Hemisphere -0.53×10^{20} ergs—sec^{-1}

involved and the probable insignificance of the standing eddies in the Southern Hemisphere, no effort has been made to compute

$$\frac{\partial}{\partial p}[\bar{u}^* \bar{\omega}^*].$$

The integral

$$\frac{a^2}{g}\iiint\left(\frac{[\bar{u}]}{a \cos \varphi}\frac{\partial}{\partial \varphi}[\bar{u}^* \bar{v}^*] \cos^2 \varphi\right) d\lambda d\varphi dp \quad (3.1)$$

$$= -\frac{a^2}{g}\iiint [\bar{u}^* \bar{v}^*] \cos^2 \varphi \frac{\partial}{\partial \varphi}\left(\frac{[\bar{u}]}{a \cos \varphi}\right) d\lambda\, d\varphi\, dp \quad (3.2)$$

$$+\frac{a^2}{g}\iiint \frac{\partial}{\partial \varphi}\left(\frac{[\bar{u}]}{a \cos \varphi}[\bar{u}^* \bar{v}^*] \cos^2 \varphi\right) d\lambda\, d\varphi\, dp. \quad (3.3)$$

Integral (3.3) is equivalent to the value of

$$\int \frac{2\pi a}{g}[\bar{u}][\bar{u}^* \bar{v}^*] \cos \varphi\, dp$$

at the equator.

Three different methods were again used in evaluating the above integrals.

(*a*) Computing the integrand at each level and finally integrating throughout the entire mass of the hemisphere.

(*b*) Using the vertically-averaged standing eddy transport of momentum and the vertically-averaged relative angular velocity.

(*c*) Using the 500-mb data alone and assuming this to be representative of the mean for the atmosphere of the hemisphere.

A. WINTER

Table 7 shows the integrand of (3.1). When use is made of method (*a*) one obtains -0.44×10^{20} ergs sec^{-1}. Method (*b*) gives -0.13×10^{20} ergs sec^{-1}, while (*c*) gives -0.27×10^{20} ergs sec^{-1}.

Using the same methods for integral (3.2) (see Table 8) method (*a*) gives -0.53×10^{20} ergs sec^{-1}; (*b*) gives -0.16×10^{20} ergs sec^{-1} and (*c*) gives -0.23×10^{20} ergs sec^{-1}.

The integral (3.3) gives for these three methods the values 0.06, 0.05 and 0.04×10^{20} ergs sec^{-1} respectively.

B. SUMMER

Table 9 shows the integrand of (3.1). When use is made of method (*a*) one obtains -0.05×10^{20} ergs sec^{-1}. Method (*b*) gives -0.31×10^{20} ergs sec^{-1}, while (*c*) gives -1.51×10^{20} ergs sec^{-1}.

TABLE 9. *Values of the term* $\frac{[\bar{u}]}{a\cos\varphi}\frac{\partial}{\partial\varphi}[\bar{u}^*\bar{v}^*]\cos^2\varphi$ *in summer 1958.*

Multiply by 2.61×10^{16} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	−0.09	−0.06	−0.40	−0.27	−0.29	−0.11	0.09		−1.14
75–70	0.69	0.22	−0.23	0.05	−0.33	−0.43	0.07		−0.41
70–65	1.93	0.46	1.21	1.34	0.22	−0.78	−0.07	0.17	3.40
65–60	−0.80	−0.83	0.56	−0.45	1.55	−1.15	0.67	0.06	−0.09
60–55	−2.53	0.57	0.40	1.49	2.08	2.27	0.68	−0.09	6.89
55–50	−1.53	1.28	2.50	5.72	3.00	4.26	0.44	−0.37	17.36
50–45	0.43	1.74	9.43	9.97	1.21	5.83	1.21	−0.34	32.34
45–40	0.15	0.60	4.26	0.81	−1.75	0.01	0.58	0.21	5.39
40–35	0.00	−1.52	−9.36	−6.73	−2.73	−1.14	0.18	−0.10	−21.60
35–30	−0.14	0.88	−9.34	−10.50	−2.85	−1.48	0.54	0.19	−22.99
30–25	−0.49	−0.24	−10.20	−3.53	−2.69	−1.29	−0.53	0.01	−19.81
25–20	0.48	−2.20	3.87	0.66	−0.54	−0.18	−0.18	−0.01	2.11
20–15	1.08	−0.44	0.56	0.07	0.35	−0.02	0.00	0.07	1.58
15–10	0.40	0.17	−1.11	0.55	0.80	0.07	−0.05	0.12	0.96
10–5	−0.65	0.17	−1.91	−0.09	0.20	0.00	−0.16	0.21	−1.97
5–0	−0.24	−0.17	−0.76	−0.01	−0.05	−0.09	0.03	0.53	−0.01
Hemisphere	-0.52×10^{19} ergs—sec^{-1}								

Using the same methods for integral (3.2) (see Table 10), method (*a*) gives -0.07×10^{19} ergs sec^{-1}; (*b*) gives -0.16×10^{19} ergs sec^{-1} and (*c*) gives -1.61×10^{19} ergs sec^{-1}.

The integral (3.3) gives for these three methods the values 0.00, −0.10 and 0.02×10^{19} ergs sec^{-1} respectively.

The yearly (summer plus winter) means of the interaction between the standing eddies and the mean zonal flow are respectively −0.25, −0.22 and -0.89×10^{19} ergs sec^{-1} with methods (*a*), (*b*) and (*c*). These results indicate that the standing eddies do not play a significant role in the maintenance of the mean zonal flow.

TABLE 10. *Values of the term* $[\bar{u}^*\bar{v}^*]\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)$ *in summer 1958.*

Multiply by 2.61×10^{16} to obtain zonal kinetic energy generation in ergs sec^{-1}.

Lat. °S	50 mb	100	200	300	400	500	700	850	Vertical integral
80–75	0.03	0.01	−0.19	−0.37	−0.17	−0.20	0.04		−0.97
75–70	0.04	0.02	−0.34	−0.53	−0.40	−0.45	−0.16		−2.06
70–65	−0.03	0.09	−0.18	−0.27	−0.61	−0.68	−0.59	−0.08	−3.14
65–60	−0.28	0.02	0.00	−0.07	−0.30	−1.16	−0.85	−0.10	−3.88
60–55	−0.12	−0.02	0.01	0.00	0.04	−0.86	−0.19	0.05	−1.56
55–50	0.47	−0.11	−0.11	−0.16	0.14	−0.09	0.00	−0.03	−0.06
50–45	0.70	−0.37	−0.72	−0.81	−0.11	−0.15	−0.13	0.04	−1.76
45–40	0.46	−0.28	−1.04	−2.35	−0.60	−1.20	−0.46	0.16	−6.08
40–35	0.33	−0.21	−0.71	−1.51	−0.37	−1.09	−0.53	0.17	−4.67
35–30	0.26	−0.28	−0.03	−0.05	0.20	−0.61	−0.57	0.07	−1.64
30–25	0.05	−0.56	2.09	1.80	1.01	−0.05	−0.38	−0.04	3.69
25–20	0.00	−0.07	3.08	2.07	1.36	0.23	−0.01	−0.03	6.72
20–15	−0.01	0.53	1.97	1.59	1.08	0.21	0.14	0.00	5.59
15–10	−0.42	0.36	1.76	0.96	0.59	−0.12	0.09	0.00	3.22
10–5	−0.41	0.02	1.64	0.43	−0.04	−0.04	0.05	0.03	1.95
5–0	−0.02	−0.01	1.51	0.18	−0.22	0.00	0.03	0.20	1.96
Hemisphere	-0.70×10^{19} ergs—sec^{-1}								

Buch's yearly data of $[\bar{u}^*\bar{v}^*]$ and $[\bar{u}]$ for the Northern Hemisphere give for the integral (3.1) the value 0.88×10^{20} ergs sec^{-1} for method (a), 0.80×10^{20} ergs sec^{-1} for method (b) and 1.02×10^{20} by method (c).

We therefore arrive at the important conclusion that while the standing eddies play an insignificant role in the maintenance of the kinetic energy of zonal motion in the Southern Hemisphere, the generation of the zonal kinetic energy by these eddies in the Northern Hemisphere is of significance.

The intergrals 4 to 6 involve the mean meridional motion. Owing to the difficulty in the measurement of $[\bar{v}]$ because of the presence of spurious effects, it was decided that the values could not be used in further computations.

4. Evaluation at the equator of the integrals

$$\frac{a}{g}\iint [\bar{u}]\,[\overline{u'v'}]\cos\varphi\, d\lambda\, dp \qquad (4.1)$$

and

$$\frac{a}{g}\iint [\bar{u}]\,[\bar{u}^*\bar{v}^*]\cos\varphi\, d\lambda\, dp. \qquad (4.2)$$

These two integrals evaluated at the equator are a measure of interhemispheric exchanges of zonal kinetic energy due respectively to transient and standing eddies and are therefore a measure of a certain interaction between the two hemispheres.

A. Winter

Integral (4.1) gives 0.27×10^{20} ergs sec^{-1} while integral (4.2) gives 0.06×10^{20} ergs sec^{-1}. For further details about the stress integrals, see Table 3.

B. Summer

Integral (4.1) gives -0.03×10^{20} ergs sec^{-1} while integral (4.2) gives 0.00×10^{20} ergs sec^{-1}. For further details about the stress integrals, see Table 6.

5. Nature and importance of transient eddies

The total relative mean zonal kinetic energy of the Southern Hemisphere is 18.93×10^{24} ergs during the winter. The corresponding summer value is 9.87×10^{24} ergs.

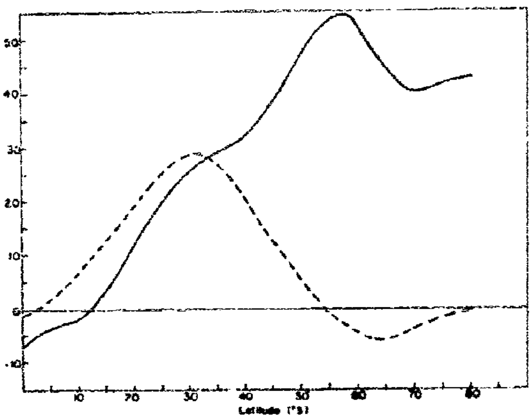

Fig. 1. The full curve gives the relative angular velocity in 10^{-7} sec^{-1} as function of latitude. The dashed curve gives the transient eddy flux of relative angular momentum in 10^{25} g cm^2 sec^{-2}. The curves are for winter 1958.

Discussions of the previous sections have shown that the transient eddies are the major producers of zonal kinetic energy. If these eddies cease to produce zonal kinetic energy, the atmosphere of the Southern Hemisphere will be in solid rotation with the earth, in about 2½ weeks, assuming normal rates of dissipation. Similar computations by Starr (1953) for the Northern Hemisphere show that it will take about 2 weeks for the atmosphere of that hemisphere to be in solid rotation with the earth. Because of the obvious importance of these eddies, it is of interest to examine further some of their properties.

The solid curves of Figs. 1 and 2 give the distribution with latitude of the angular velocity relative to the earth, averaged with respect to pressure and time. Fig. 1 denotes the situation in winter while Fig. 2 denotes the condition in summer.

From these profiles the effect of true lateral friction would be to retard the zones of most rapid rotation and to increase the angular velocity of the less rapidly rotating ones, so as to cause the whole to assume a more nearly uniform angular velocity. This means that lateral friction would then cause a flow of angular momentum southward and also northward away from the zone of most rapid rotation.

The dashed curves in the two figures show the transient eddy angular momentum transports in summer and winter. These curves show

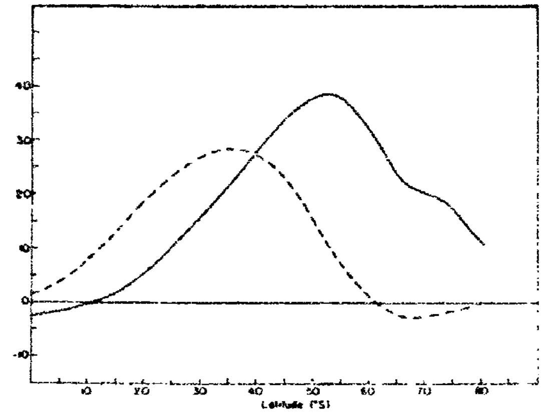

FIG. 2. The curves show the situation for summer 1958. The full curve gives the relative angular velocity in 10^{-7} sec^{-1} as function of latitude. The dashed curve gives the transient eddy flux of relative angular momentum in 10^{25} g cm^2 sec^{-2}.

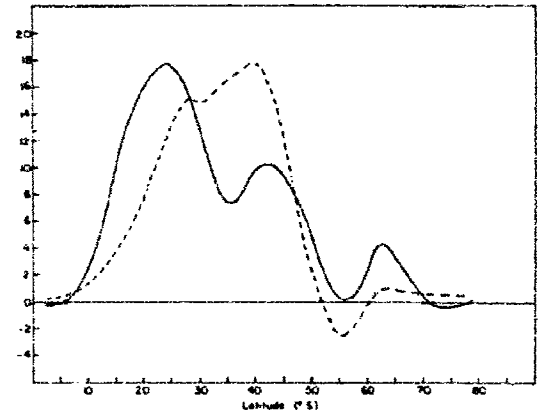

FIG. 3. Production of zonal kinetic energy using mean wind and transient eddy transport of relative angular momentum. The full curve represents the situation in winter while the dashed curve represents the summer condition. The units are in 10^{19} ergs sec^{-1}.

TABLE II

$$I \textit{ denotes } \frac{a^2}{g}\iiint \frac{[\bar{u}]}{a\cos\varphi}\frac{\partial}{\partial\varphi}([\text{eddy mom.}]\cos^2\varphi)\,d\lambda\,d\varphi\,dp.$$

$$II \textit{ denotes } \frac{a^2}{g}\iiint [\text{eddy mom.}]\cos^2\varphi\,\frac{\partial}{\partial\varphi}\left(\frac{[\bar{u}]}{a\cos\varphi}\right)d\lambda\,d\varphi\,dp.$$

$$III \textit{ denotes } \int\frac{2\pi a}{g}[\bar{u}]\,[\text{eddy mom.}]\cos\varphi\,dp \textit{ at the equator.}$$

W = Winter
S = Summer
M = Winter and Summer mean
} 1958 Southern Hemisphere.
N = Yearly mean using Buch's data 1950 for the Northern Hemisphere.

Units are in 10^{20} ergs sec^{-1}.

Method		Transient eddies I	Transient eddies II	Transient eddies III	Standing eddies I	Standing eddies II	Standing eddies III	All eddies I	All eddies II	All eddies III
Mean profiles	W	9.89	9.57	0.13	−0.13	−0.16	0.05	9.76	9.41	0.18
of zonal	S	9.04	8.99	−0.03	−0.31	−0.16	−0.10	8.73	8.83	−0.13
wind and momentum	M	9.46	9.28	0.05	−0.22	−0.16	−0.02	9.12	9.12	0.03
transport	N	4.35	4.55	—	0.80	0.80	—	5.15	5.35	—
Integration	W	9.63	9.41	0.27	−0.44	−0.53	0.06	9.19	8.88	0.33
by levels	S	9.72	9.86	0.03	−0.05	−0.07	0.00	9.67	9.79	0.03
	M	9.67	9.63	0.15	−0.25	−0.30	0.03	9.42	9.33	0.18
	N	5.84	4.84	—	0.88	0.87	—	6.72	5.71	—
Using 500 mb data	W	10.02	9.61	0.20	−0.27	−0.23	0.04	9.75	9.38	0.24
only	S	8.53	8.58	0.00	−1.51	−1.61	0.02	7.02	6.97	0.02
	M	9.27	9.09	0.10	−0.89	−0.92	0.03	8.38	8.17	0.13
	N	6.10	4.95	—	1.02	0.85	—	7.12	5.80	—

that there exists a strong *poleward* eddy transport of angular momentum towards the regions of maximum rotation. This state of affairs is contrary to classic eddy viscosity concepts and is compatible with them only if one assumes negative virtual viscosity coefficients.

The net influence of this observed property of the transient eddies is to increase the kinetic energy of zonal motions. Except for the equatorial boundary term, the rate of generation of the kinetic energy of the zonal motion is the mass integral of the product of eddy flux of momentum into the shear of relative angular velocity. This quantity can then easily be measured from Figs. 1 and 2. The areas under the curves of Fig. 3 measure the production of zonal kinetic energy. The full curve is for winter and the dashed curve for summer. In each case the total area for the hemisphere is positive by a wide margin.

9. Summary of results

Table 11 shows certain terms in the balance equation of the rate of generation of the kinetic energy of mean zonal flow. The preeminent importance of the transient eddies as major sources of the kinetic energy of the mean zonal flow is apparent.

In comparing the results in Table 11 with previous computations given by Starr and others, it appears that the rate of conversion of eddy kinetic energy to the kinetic energy of mean zonal flow is about twice as much in the Southern Hemisphere as in the Northern Hemisphere.

This result suggests then that the conversion rate of eddy available potential energy to eddy kinetic energy will be twice as much in the Southern Hemisphere as in the Northern Hemisphere. The verification of this has to await the study of the enthalpy budget.

Acknowledgement

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The mean meridional circulation of the southern hemisphere inferred from momentum and mass balance

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ABSTRACT

The mean meridional circulation for the southern hemisphere is inferred from momentum and mass balance considerations. Recent calculations by Obasi of the transient eddy transports of momentum and the mean zonal flow are the only data used. The vertical eddy transports are taken into account only in the surface layers, where the stress is assumed to decrease linearly with pressure from its surface value. The surface stresses themselves are calculated from the pressure integral of the momentum equation, and by a short iteration process the effect of horizontal transport of momentum by the mean meridional circulation is taken into account.

The resulting circulation is self consistent, needing only very small corrections to eliminate mass "drifts" across latitude circles, contrary to most direct measurements. The characteristic three cell pattern is very evident. The circulation is compared to that found in a somewhat similar manner by Mintz and Lang for the northern hemisphere. The winter circulations are found to be rather similar, but the polar direct cell is much stronger in the southern hemisphere. In the summer season the southern hemisphere circulation is much stronger.

The kinetic energy balance for the southern hemisphere is briefly considered. Given our assumptions about the vertical eddy fluxes of momentum, about half the kinetic energy fed into the mean flow by the horizontal transient eddies is extracted by the mean meridional motion. The other half is dissipated near the surface. In each season, energy balance is achieved to within 3 %, verifying the consistency of the calculations.

1. Introduction

Direct measurements of the mean meridional circulation in the atmosphere usually suffer from large uncertainties since they are a relatively small residual (of the order 1 m/sec) left from the averaging around latitude circles of time mean meridional winds of the order of 10 m/sec. For this reason, it is desirable to try to infer the mean meridional circulation from more accurately measured quantities. Since the mean meridional circulation is forced (Kuo, 1956; Gilman, 1964*b*), we may infer its strength in the steady state from the requirements that there be a momentum and a heat balance at every point in the meridional cross-section. General theoretical aspects of this problem have been discussed recently by Gilman (1964*b*). In any case, "indirect measurement" of the mean meridional circulation can be made only with sufficient knowledge of the other components (the eddy fluxes and diabatic heating) in the momentum and thermodynamic energy equations. As was pointed out by Gilman, 1964*b*, we have a choice of inferring the mean meridional circulation from the thermal forcing alone, or the mechanical forcing alone. (Ideally, of course, both should give the same result.) The choice would depend on which type of forcing we can measure more accurately.

For the southern hemisphere, Obasi (1963*a*, *b*) has calculated the mean zonal flow and the eddy transports of momentum for the IGY.[1] The eddy heat transports have not yet been computed. Therefore, we must use the mechanical forcing to infer the mean meridional circulation for the southern hemisphere.

Obasi (1963*a*) has also attempted to evaluate the mean meridional circulation directly, but his results suffered greatly from the uncertainty mentioned earlier. This uncertainty manifested itself in the very large meridional "drift" velocities (often as large as 40 cm/sec) which

[1] Certain corrections have been made to Obasi's (1963*a*, *b*) data.

had to be subtracted from the observed values to give zero mass transport across all latitude circles.

Having chosen to infer the mean meridional circulation from the momentum forcing rather than the thermal forcing, we must examine this momentum forcing more carefully. It consists of eddy convergences of momentum, both horizontal and vertical, and "friction". This "friction", for our purposes, is really just vertical eddy stresses on a scale smaller than measurable on a synoptic map. However, Obasi did not attempt to evaluate vertical eddy fluxes of momentum on *any* scale so really we can not separate the scales in our calculations.

Of the various forcing terms, the *transient* eddy convergences of momentum are the most accurately measurable. We may obtain the value of the time correlation of the zonal and meridional components of the wind at each *station*, and rely on map analysis and grid point readings only for obtaining the longitudinal averages of the momentum flux. This is not the case with the horizontal *standing* eddies, since there we must draw maps of the time averaged zonal and meridional components of the wind separately and evaluate the longitudinal correlation by the product of values at grid points. Furthermore, the fields of the meridional component of the wind have a much more frequent reversal in sign along a given latitude circle than will the time correlations of the meridional and zonal wind. For both these reasons, the standing eddy transports are generally less accurate quantities than are the transient eddies. In addition to this difference, OBASI (1963*b*) found that for the southern hemisphere, the standing eddies played a much smaller role in the horizontal transport of momentum than did the transient eddies.

The vertical eddy transports are much harder to evaluate than even the horizontal standing eddies, since the vertical motion is almost impossible to evaluate. Attempts have been made for the northern hemisphere by JENSEN (1961) in which he calculated "adiabatic" vertical motions, but there is some question as to the validity of this method (WIIN-NIELSEN, 1964). Furthermore, as stated earlier, Obasi did not even attempt to calculate the vertical eddy fluxes for the southern hemisphere.

In light of the above, the author decided to attempt to construct a mean meridional circulation for the southern hemisphere using *only* the horizontal transient eddy transports of momentum and the mean zonal wind measured by Obasi, feeling that from these most accurately measurable quantities it was likely that a more self consistent and accurate picture of the mean meridional circulation might be obtained.

2. Notation

The following notation will be adopted (generally the same notation as in GILMAN, 1964 *a*, *b*).

t = time
p = pressure
ϕ = latitude
λ = longitude
u = zonal wind (positive toward east)
v = meridional wind (positive toward north)
ω = dp/dt
Ω = angular velocity of the earth
f = Coriolis parameter = $2\Omega \sin \phi$
a = radius of the earth
g = acceleration of gravity

$\overline{(\quad)} = \dfrac{1}{t_2 - t_1}\displaystyle\int_{t_1}^{t_2} (\quad)\, dt$ = time average

$(\quad)' = (\quad) - \overline{(\quad)}$ = deviation from time average

$[(\quad)] = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} (\quad)\, d\lambda$ = zonal average

$(\quad)^* = (\quad) - [(\quad)]$ = deviation from zonal average

Z = mean absolute vorticity = $f - (a \cos\phi)^{-1}\partial/\partial\phi[\bar{u}] \cos\phi$
τ = stress in the zonal direction
χ = frictional force/unit mass

3. Equations and the method of calculation

The method of calculation of the mean meridional circulation is that given in GILMAN (1963 and 1964 *b*). In brief, the momentum balance equation for the zonal component and the continuity equation (both are time and zonally averaged and are written in the pressure coordinate system) are solved for the vertical motion $[\bar{\omega}]$. The horizontal component $[\bar{v}]$ is then obtained directly from the momentum balance.

Thus we have, for $[\bar{v}]$

$$[\bar{v}] = \frac{[\bar{\omega}]}{Z}\frac{\partial[\bar{u}]}{\partial p} - \frac{G}{Z}, \qquad (1)$$

where G is the momentum forcing function, i.e.,

$$G = -(a\cos^2\phi)^{-1}\partial/\partial\phi[\overline{u'v'}]\cos^2\phi + [\bar{\chi}]. \quad (2)$$

Here we have omitted the standing eddy convergences ($[\bar{u}^*\bar{v}^*]$ and $[\bar{u}^*\bar{\omega}^*]$) and grouped the vertical eddy fluxes of all scales in the friction term $[\bar{\chi}]$, in accordance with the remarks in section 1.

For $[\bar{\omega}]$ we have

$$[\bar{\omega}] = \left(\exp - \int_0^p J dp'\right)\int_0^p \left(\exp\int_0^{p'} J dp''\right) K dp', \qquad (3)$$

where the integration is along the absolute angular momentum isolines, which are the characteristics of the problem. Here

$$K = -(\cos\phi)^{-1}\partial/\partial\phi\, G\cos\phi, \qquad (4)$$

and $$J = (\cos\phi)^{-1}\partial/\partial\phi\, s\cos\phi, \qquad (5)$$

where s is the slope of the angular momentum characteristics, given by

$$s = \frac{1}{aZ}\frac{\partial[\bar{u}]}{\partial p}. \qquad (6)$$

We have no direct measurements of the vertical eddy fluxes of momentum for the southern hemisphere. However, their effect cannot be neglected at least near the surface because the horizontal transient eddy forcing alone will not give the return flow required to balance the mass transport across latitude circles at higher levels. Therefore we must make some assumption about the vertical eddy transports in the lowest layers. The simplest assumption to make would be that the stress due to these fluxes decreases from its surface value to a negligible value a certain pressure height above the surface. We chose a decrease linear in pressure, reaching zero at 850 mb (700 mb over the Antarctic continent). That is, for

$$[\bar{\chi}] = -g\frac{\partial[\bar{\tau}]}{\partial p} \qquad (7)$$

TABLE 1. *Surface stress and drift velocity for the* WINTER *season.*

Corrected surface stress includes the effect of horizontal convergence of momentum due to the mean meridional motions.

Latitude (°S)	Surface stress (dyne/cm²) Uncorrected	Corrected	Drift (cm/sec)
77.5	−2.00	−1.89	0
72.5	−1.91	−1.88	4
67.5	−0.75	−0.83	2
62.5	0.48	0.36	0
57.5	0.83	0.70	0
52.5	1.14	1.07	0
47.5	1.12	0.99	1
42.5	1.07	1.03	0
37.5	0.70	0.82	−1
32.5	0.12	0.27	0
27.5	−0.09	0.14	−3
22.5	−0.37	−0.52	3
17.5	−0.46	−0.49	1
12.5	−0.43	−0.63	2
7.5	−0.36	−0.65	4
2.5	−0.28		

we approximate

$$[\bar{\chi}] = -g\frac{[\bar{\tau}]}{\Delta p}, \qquad (8)$$

where Δp is the pressure depth of the surface layer. Now the surface stress can be inferred from the horizontal transient eddy transports of momentum themselves, simply by integrating the momentum equation in the vertical to the "top" of the atmosphere. (This method was used by Obasi in his estimation of the surface stress.) Furthermore, to include the horizontal transports of momentum by the mean meridional motion (the term $[\bar{u}][\bar{v}]$) in the calculation, a short iteration procedure was used. A first estimate of the surface stress was obtained from the vertical integral of the horizontal transient eddy convergences of momentum alone. These stress values were then used, together with the assumption about the rate of decrease of stress with height and the unintegrated distribution of horizontal transient eddy convergences of momentum, to calculate a first mean meridional circulation. The mean meridional circulation was then used to correct the surface stress values from which a second mean meridional circulation was then obtained. That this correction (see Tables 1 and 2) was not

TABLE 2. *Surface stress and drift velocity for the* SUMMER *season.*

Corrected surface stress includes the effect of horizontal convergence of momentum due to the mean meridional motion.

Latitude (°S)	Surface stress (dyne/cm²) Uncorrected	Corrected	Drift (cm/sec)
77.5	−1.39	−1.35	2
72.5	−0.93	−0.97	4
67.5	0.23	0.10	1
62.5	1.13	1.00	0
57.5	1.45	1.36	1
52.5	1.62	1.57	−1
47.5	1.16	1.17	0
42.5	0.46	0.50	0
37.5	0.10	0.17	0
32.5	−0.12	−0.02	0
27.5	−0.30	−0.20	0
22.5	−0.42	−0.38	0
17.5	−0.43	−0.46	0
12.5	−0.37	−0.45	−1
7.5	−0.25	−0.35	1
2.5	−0.22		

too large indicates the relatively small contribution of the mean meridional circulation in determining the *horizontal* transport of momentum. By the nature of our assumptions, however, we have forced it to be the mode of *vertical* transport between the upper levels and the surface layer; but this does not seem too unreasonable, as discussed by GILMAN (1964*a*, PHILLIPS (1954)) and others.

To actually carry out the integration for $[\bar{\omega}]$ indicated in (3), we must interpolate the integrand onto the characteristic curves, whose profiles must first themselves be calculated. The equation defining the characteristic in p, ϕ space is just the definition for absolute angular momentum/unit mass m, i.e.,

$$m = \Omega a^2 \cos^2\phi + a[\bar{u}]\cos\phi. \qquad (9)$$

To find ϕ as a function of p for $m = m_0$, the angular momentum at evenly spaced latitudes, we solve for $\cos\phi$, using the fact that

$[\bar{u}]/4\sqrt{\Omega m_0} < 1$, to obtain

$$\cos\phi = \frac{2\sqrt{\Omega m_0} - [\bar{u}]}{2\Omega a}.$$

Strictly speaking, we must use $[\bar{u}]$ at ϕ to determine $\cos\phi$, but we may to first order use $[\bar{u}]$ at ϕ_0, since for a given characteristic $|\phi - \phi_0| < 5°$ generally. Once the position of the characteristics was known, we interpolated the integrand values linearly onto them. The m_0 values (and therefore the ϕ_0 values) were chosen to coincide with the original latitude grid for the integrand, making the interpolation particularly simple. The results of the integration were then interpolated back to the ϕ_0 latitudes, from which the figures were made.

On a meridional cross-section, the characteristics appear as nearly vertical lines, each one not deviating from its reference latitude ϕ_0 by more than 5°. Since we may say that at the top of the atmosphere $[\bar{\omega}] = 0$, the solution (3) was chosen so that the integration along the characteristic begins at $p = 0$, and proceeds to the surface. Had we found the mean meridional circulation in the manner of KUO (1956) solving a second order equation for the meridional stream function, we would have imposed boundary conditions such that $[\bar{\omega}] = 0$ would hold both at $p = 0$, and at the surface. But here we are allowed to specify $[\bar{\omega}]$ at only one point on each characteristic. Nevertheless, $[\bar{\omega}]$ will still approach zero near the surface, as can be seen by a closer examination of (3). If the factors $\exp \pm \int J\,dp$ were deleted, and the characteristics straightened to become lines of constant latitude, (3) would just be the integrated continuity equation if we assumed that $[\bar{v}] = -G/Z$. In this simpler calculation we would expect $[\bar{\omega}] \to 0$ at the surface. Since, (*a*) $0.5 \leq \exp \pm \int J\,dp \leq 2.0$ generally, (*b*) the factors inside and outside the integral in (3) tend to cancel, and (*c*) the characteristics are nearly vertical, we should therefore expect $[\bar{\omega}]$ to be not radically different from the less refined calculation (10–30% was the result found in GILMAN 1963).

It is evident then that the above method of calculation of the mean meridional circulation will give a set of values to which no large "drift" correction will have to be applied to ensure vanishing meridional mass transport. We may therefore say that we have at least arrived at a self consistent mean meridional circulation. This does not say we have arrived at the correct one, but since the assumptions made in our calculation do not seem to be too unreasonable, it is probable that our results are more accurate than directly measured values would be.

4. Results and comparisons

The calculated mean meridional cross-sections for the winter (April–September 1958) and summer (January–March; October–December 1958) are presented in Figs. 1 through 4. The surface stress values used, both before and after correction for the horizontal convergence of momentum due to the mean meridional circulation, are presented in Tables 1 and 2, along with the meridional drift velocities which resulted from our method of calculation.

From examination of the results, the following facts are of note:

(*a*) Both seasons show very distinctly the characteristic three cell pattern, though the equatorward boundary of the direct cell in low latitudes is not in evidence. There is also evident a shift of the cell pattern equatorward in the winter season, as would be expected from the shifting of the heat equator. The amount of this shift is generally 10–20° latitude, and is more strongly evident in the vertical motion sections (Figs. 2 and 4), indicating a change in the shape of the cells with season.

(*b*) A calculation somewhat similar to ours was performed for the northern hemisphere by Mintz & Lang (1955). For the *winter* season in each hemisphere the boundaries between cells are found to be between 30° and 35°, and between 60 and 65°. The indirect cells in the two hemispheres have about the same mass circulation, as do the equatorial direct cells. In both cases, the equatorial direct cell has about three times the mass circulation of the middle latitude indirect cell. However, the southern hemisphere results show a very much stronger polar direct cell than has the northern hemisphere. Furthermore, the winter circulation in the southern hemisphere appears to overlap more across the equator than does the northern hemisphere circulation in its winter. This last result seems to be in agreement with the general conclusion of Obasi (1963*a*, *b*) that the southern hemisphere circulation is generally stronger. This conclusion is reinforced by the fact that Mintz and Lang's values are averages only over the most winterlike months, January and February.

In the *summer* season, the southern hemisphere mean meridional circulation is seen to be very much stronger. The polar cell is still very strong, while in the northern hemisphere it is practically non-existent. Undoubtedly the weaker northern hemisphere circulation is due partly to the choice by Mintz and Lang of two month averages over July and August, but it would appear to be so weak that even a 6 month average would be weaker than the southern hemisphere result.

The much stronger polar direct cell in the southern hemisphere during both seasons may be a manifestation of the katabatic wind descending off the antarctic continent.

(*c*) The surface stress values presented in Tables 1 and 2 indicate that the inclusion of the horizontal momentum convergences due to the mean meridional circulation does make some difference, but not a large amount, being generally on the order of 10–20%. The uncorrected values are slightly different from those calculated by Obasi (1963*b*), as the numerical integration procedures were slightly different.

(*d*) The drift velocities resulting from our method of calculation (which were subtracted out before the sections were drawn) are never larger than 4 cm/sec and average about 1 cm/sec. These drifts are probably a result of the finite difference computation methods. It should be noted that had the stress been assumed to decay more slowly with the decrease in pressure, the drifts would have been about the same, since the total meridional mass flow in the lower layers from the vertical convergence part of the forcing depends only on the size of the surface stress.

5. Energy computations

As a further check on the self consistency of our computations, the kinetic energy balance for the southern hemisphere is briefly considered. Obasi (1965) calculated the conversion of transient eddy kinetic energy to mean zonal kinetic energy for the hemisphere. The calculation of the conversion of mean meridional kinetic to zonal kinetic energy via the Coriolis parameter, however, was considered to be unreliable due to the uncertainty in the directly measured $[\bar{v}]$ values. Using our indirectly measured mean meridional circulation and mean zonal winds of Obasi (1963*a*) (extrapolating for the layer 850–1000 mb), we may make a first estimate of this conversion integral. Furthermore, we may estimate the dissipation of mean zonal kinetic energy near the ground in our model by using the calculated surface stress values, and the extrapolated zonal winds.

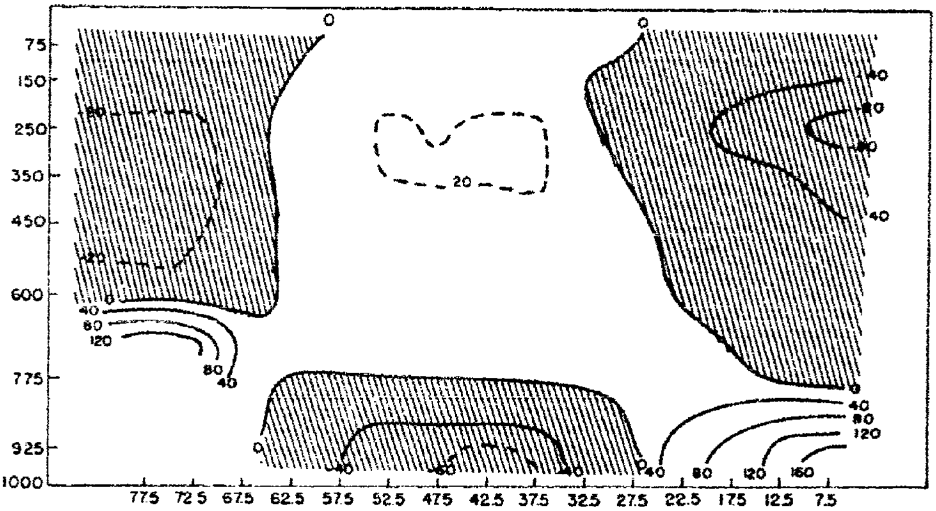

FIG. 1. Mean meridional motion $[\bar{v}]$ (cm/sec) for the southern hemisphere *winter* season, calculated from momentum and mass balance. Shaded areas indicate motion toward the south pole.

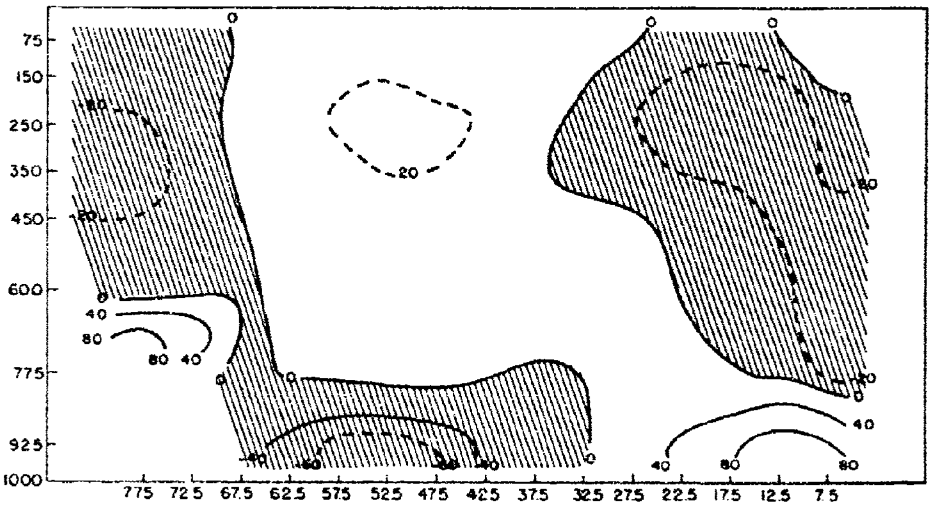

FIG. 3. Mean meridional motion $[\bar{v}]$ (cm/sec) for the southern hemisphere *summer* season, calculated from momentum and mass balance. Shaded areas indicate motion toward the south pole.

Mathematically, the conversion via the Coriolis parameter is given by the integral

$$\frac{2\pi a^2}{g}\int_0^{p_g}\int_0^{\pi/2} f[\bar{u}][\bar{v}]\cos\phi\, d\phi\, dp$$

while the dissipation in the surface layers is obtained from the integral

$$\frac{2\pi a^2}{g}\int_0^{p_g}\int_0^{\pi/2} [\bar{u}][\bar{\chi}]\cos\phi\, d\phi\, dp$$

which becomes, under our assumptions,

$$2\pi a^2\int_0^{\pi/2} [\bar{u}]/\,[\bar{\tau}]_{p_g-\Delta p/2}\cos\phi\, d\phi.$$

Here the subscript g refers to the ground level. The results of these integrations are presented in Table 3. As can be seen from the last column, which gives the amount of imbalance, our method of calculation arrives at energy conversion rates which balance to within 3%. In

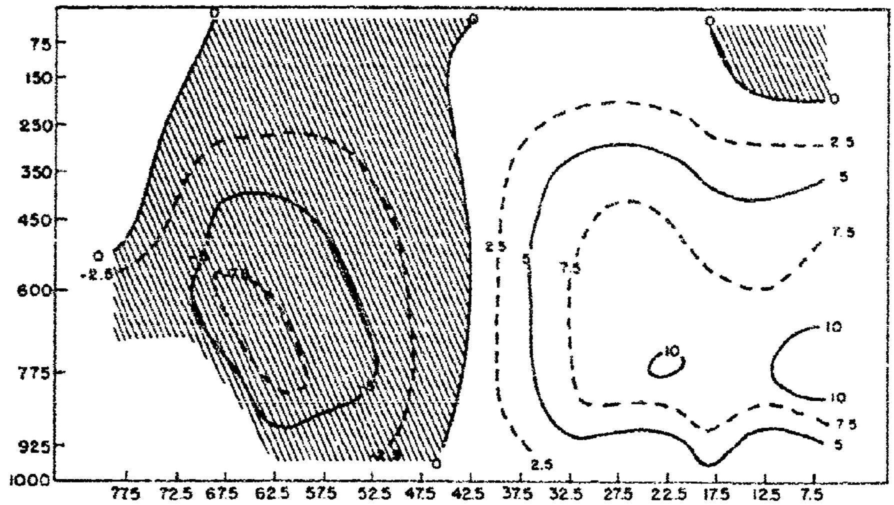

FIG. 2. **Mean vertical motion** $[\overline{\omega}]$ (10^{-5} mb/sec) **for the southern hemisphere** *winter* **season, calculated from momentum and mass balance.** Shaded areas indicate "upward" motion.

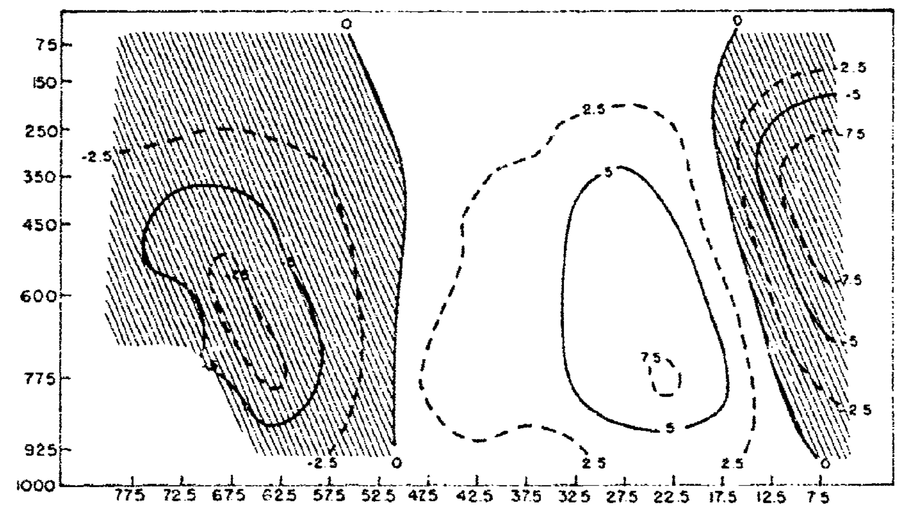

FIG. 4. **Mean vertical motion** $[\overline{\omega}]$ (10^{-5} mb/sec) **for the southern hemisphere** *summer* **season, calculated from momentum and mass balance.** Shaded areas indicate "upward" motion.

this balance the mean meridional circulation acts to remove about half the energy fed into the mean flow, the other half being dissipated. The degree of consistency will not be affected appreciably by a change in the extrapolated zonal wind, since a decrease in dissipation due to such a change will, by the nature of our method of calculation, be accompanied automatically by an increase in the conversion from mean zonal to mean meridional kinetic energy through the Coriolis parameter.

TABLE 3. *Comparison of the various energy conversions in the mean zonal kinetic energy balance.* A positive value indicates a conversion to mean zonal kinetic energy.

	Energy conversions (10^{20} ergs/sec)			
	Eddy to zonal	Meridional to zonal	Dissipation	Residual
Winter	+ 9.63	− 3.85	− 6.17	− 0.39
Summer	+ 9.72	− 5.40	− 4.02	+ 0.30

The values of the conversion from mean zonal to mean meridional kinetic energy via the Coriolis parameter are on the average about 75% greater than those obtained by direct measurement by STARR, 1959 for the northern hemisphere. This is a reasonable result since Obasi showed the rate of conversion of eddy kinetic to zonal kinetic energy to be almost twice as large in the southern hemisphere as in the northern.

6. Concluding remarks

Recapitulating, we have calculated a reasonable mean meridional circulation, surface stress distribution, and kinetic energy balance entirely from the observed horizontal transient eddy convergences of momentum and mean zonal wind. Since all the balance requirements are met, these statistics form a self consistent set. They are undoubtedly not the correct set. However, since the approximations made seem physically reasonable, this mean meridional circulation is probably more accurate than can be obtained at present by direct measurements. Even so, improvements in this method could be made. For example, more attention could be paid to the surface zonal wind values (though this would require a great deal more work). Even with better surface wind values, however, it will probably still be profitable to estimate the surface stress from the momentum convergences, rather than using drag coefficient assumptions, if, again, only for self consistency.

Acknowledgements

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III. THE BUDGET OF WATER VAPOR

DIRECT MEASUREMENT OF THE HEMISPHERIC POLEWARD FLUX OF WATER VAPOR

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ABSTRACT

The average meridional flux of water vapor in the atmosphere is evaluated for the northern hemisphere from suitably distributed daily observations of wind and moisture during the year 1950. Zonal averages for five latitude circles are presented, and these are compared with the corresponding requirements calculated from estimates of zonally averaged precipitation and evaporation given in climatological literature. Considering the nature and scope of the work, the extent of the agreement obtained is considered noteworthy.

INTRODUCTION

For several years the writers have been engaged in a long-term program of systematic compilation and analysis of hemispherically distributed observations of wind, temperature and moisture in order to elucidate the manner in which certain basic integral requirements of the general circulation are fulfilled in the atmosphere. As the studies proceeded, many of the results were published elsewhere (e.g., Starr and White 1951, 1952 a, b, c, 1954). However, the measurements relating to the poleward flux of water vapor have been completed only recently and have not been reported in their entirety save for some partial results contained in Starr and White (1954). The final outcome of the computations for the one complete year used (1950) is therefore presented herewith.

Since the divergence or convergence of the zonally averaged meridional flux of water vapor during a period of one year must reflect an excess or deficit of evaporation as compared to precipitation aside from minor secondary considerations, the present subject offers for the first time an independent means of checking the global precipitation and evaporation balance elaborated by various workers in the science of climatology during the past several decades. The impor-

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tance of this entire field of endeavor for a number of oceanographic considerations hardly needs to be dwelt upon here.

PROCEDURE FOLLOWED

Water may be transported by atmospheric circulations in the solid, liquid and vapor form. For the purposes of our study, the transport in vapor form only is considered, the probability being that contributions from the solid and liquid forms are small in comparison, except as noted later. The flow of water vapor northward across a conical vertical wall along a complete circle of latitude from the surface to great heights and for any desired time period may be expressed as an integral of the form

$$\frac{1}{g}\int\int\int q\, v\, dx\, dt\, dp\,, \tag{1}$$

where g is the acceleration of gravity, q the specific humidity, v the northward component of velocity, dx an element of linear eastward distance, dt an element of time and dp an element of pressure taken vertically. The assumption of hydrostatic equilibrium is made use of in formulating this expression.

In recent years it has become feasible to evaluate the water vapor flux integral by using direct wind and moisture observations from a hemispheric network of upper air stations. While the quantity and quality of these observations are not yet adequate for reliable evaluation of the instantaneous transports on individual days, the mean flux over a large number of days can be approximated rather satisfactorily. This situation is comparable to the success with which the meridional flux of other quantities, notably of angular momentum, was measured by us in previous work to which reference has already been made. Certain details of the technique common to these studies, together with discussions of specific questions concerning the data used and other pertinent matters, are also to be found there. In view of this circumstance, only a brief statement of the methods is entered here.

The geographical distribution of the key stations is shown in Fig. 1 and is substantially the same as that used for the investigation of the hemispheric angular momentum balance (see Starr and White, 1954). This station network was again divided into five latitude zones centered in the vicinity of 13°, 31°, 42.5°, 55° and 70° N, as indicated.

The measurements of specific humidity were obtained from the reported dewpoints and temperatures. Because of the low moisture

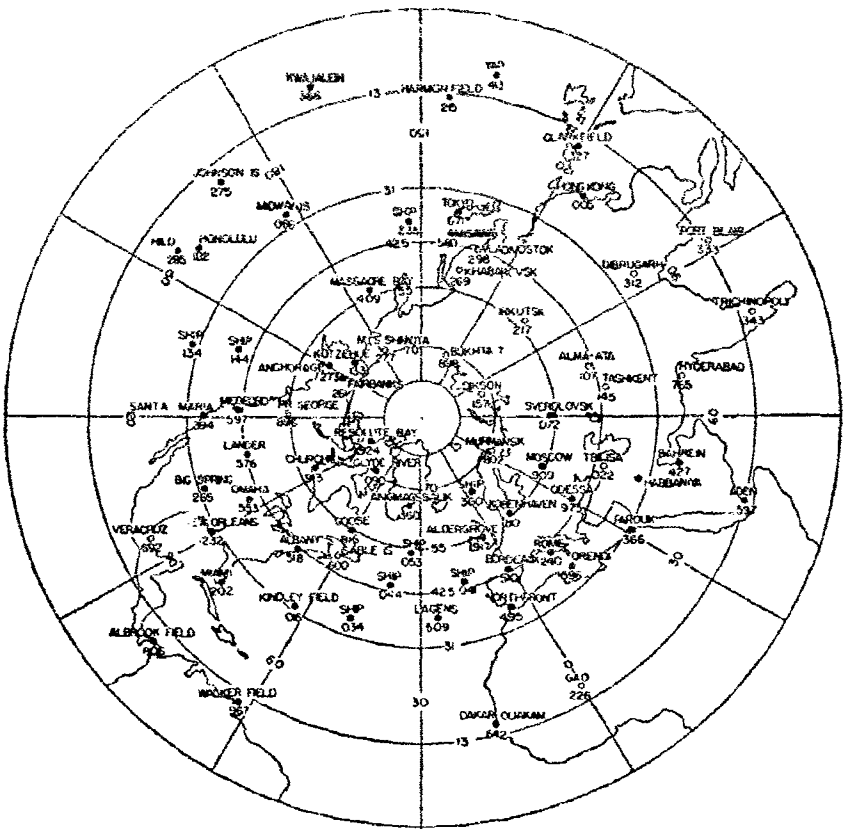

Figure 1. The distribution of key stations over the northern hemisphere used in the investigation of the atmospheric water vapor flux. Pilot-balloon wind stations shown by open circles, radio wind stations by dots.

content above 500 mb, the computations were restricted to the four standard pressure levels, 1000, 850, 700, 500 mb.[1] The flux of water vapor was evaluated at each latitude and level by first forming the product of the northward component of the wind and the specific humidity at each station for each day. Simple longitudinal daily

[1] In the processes of making radiosonde observations of humidity, an instrumental phenomenon known as "motorboating" occurs under certain definite atmospheric conditions. It indicates in general that the moisture content is too low to be measured and takes place at quite low temperatures ordinarily. An upper limit to the moisture amount may still be specified, however, and such values were used in these cases (see appropriate instructions for observers).

means were then made, and these in turn were averaged for the entire year. Finally, vertical integrals with respect to mass were calculated and converted into moisture flux in grams per second across each of the five complete latitude circles.

SUMMARY OF RESULTS

The figures obtained for the five latitudes are given in the fourth column of Table I and by the black dots in Fig. 2 in terms of 10^{11} grams per second. The largest positive (northward) flux is across 42.5° N, although it is likely that a continuous curve would show the maximum to be practically at 40° N; this would be in agreement with climatological estimates of the latitude which separates the zone of precipitation excess over evaporation to the north from the zone where the reverse condition obtains immediately to the south. At 13° N the flux is southward, thus indicating a great divergence of moisture flow out of the zone occupied by the subtropical anticyclones and again corroborating conclusions drawn from climatological considerations.

Table I. Numerical values of water vapor flux across the specified latitudes as given by Conrad and by Benton, both based on climatological data compiled by Wüst, together with directly measured values obtained by Starr and White for 1950 from the number N of observations in the last column. The fluxes are in units of 10^{11} grams per second.

Latitude	*Conrad-Wüst*	*Benton-Wüst*	*Starr and White*	*N*
70.0	+0.8	+0.5	+1.4	8,463
55.0	+3.6	+3.2	+4.4	12,845
42.5	+7.5	+5.5	+5.6	14,916
31.0	+6.2	+3.5	+4.6	20,748
13.0	−4.0	−4.6	−2.9	12,729

Numerical estimates of the water vapor flux requirements have been given by Conrad (1936) and by Benton and Estoque (1954), both of these being based on original data concerning precipitation and evaporation presented by Wüst (1922); these are shown in the first two columns of Table I and by the two curves in Fig. 2. Perhaps with some reservations relative to the situation in the tropics, which will be treated further below, it appears that agreement of measurements with requirements as portrayed by the curves is all that could reasonably be expected.

Since only five latitudes were sampled in making the computations, values of the flux at regular ten degree intervals cannot be interpolated without a certain subjectivity. Nevertheless, when this is done as

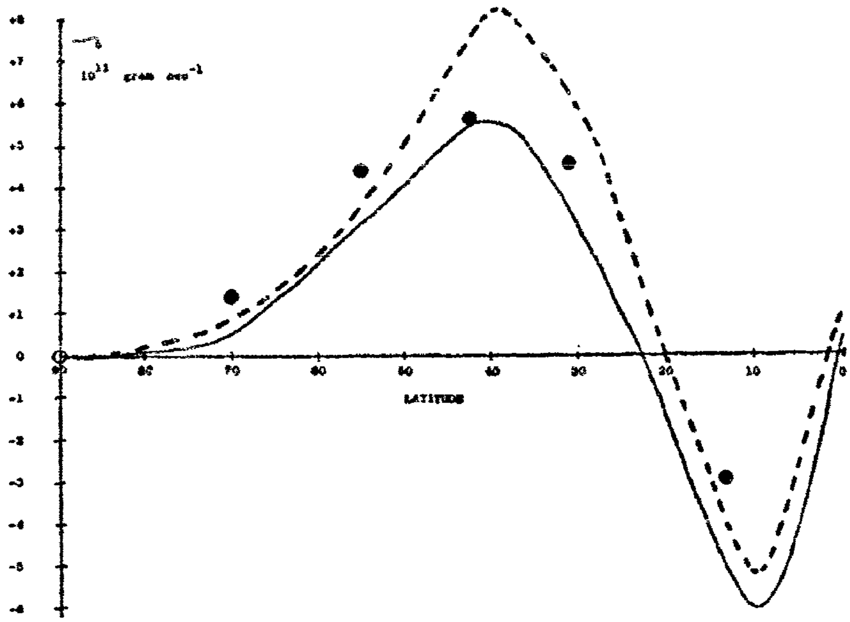

Figure 2. The meridional distribution of the poleward water vapor flux in the atmosphere. The dashed curve and the solid curve represent estimates of this flux deduced from evaporation and precipitation by Conrad (1936) and Benton (1953) respectively, both after data compiled by Wüst (1922). The dots represent the flux computed from atmospheric data for the year 1950. The units are 10^{11} gm sec^{-1}.

best one can, let us say from a smooth curve, it becomes possible to express the comparison of the results with climatological information in terms of the depth in centimeters of precipitation minus

Table II. The meridional distribution of the zonally averaged difference between precipitation minus evaporation by ten degree latitude belts for the northern hemisphere. The figures according to the indicated investigators are given in terms of cm per year.

Latitude Belt	*Conrad-Wüst*	*Benton-Wüst*	*Starr and White*
90–80°	+ 8	+16	—
80–70°	+22	+14	+27
70–60°	+25	+25	+35
60–50°	+32	+22	+19
50–40°	+32	+19	+ 7
40–30°	−19	−24	−11
30–20°	−48	−35	−30
20–10°	−37	−34	−34
10– 0°	+43	+46	—

evaporation on an annual basis by ten degree latitude belts. Such an arrangement of the comparison is given in Table II, where the figures obtained by the procedure here given are found in the fourth column while the requirements quoted from Conrad and Benton are found in the second and third columns.

It would add materially to the interest of Table II if the last entry were available. It is regrettable that upper air observation stations are not numerous enough as yet to permit water vapor flux measurements across the equator and thus provide this information.

SUPPLEMENTARY AND CRITICAL REMARKS

There are a number of points which may be raised concerning both the data and the computation techniques which doubtless contribute to the discrepancies between the three sets of results. It would scarcely be fitting or convenient in a discussion of this scope to engage in an evaluation of the techniques used by Wüst, Conrad and Benton. Therefore only certain of those factors which may lead to inaccuracies of the present calculation are touched upon here; specifically the more important considerations are the following:

(a) The direct computations of the flux are for the single specific year 1950, while the figures quoted from Wüst, Conrad and Benton represent long-term normals. It is highly probable that significant departures from normal do occur in individual years, although it is unlikely that the essential character of the meridional distribution of the water vapor flux changes radically from year to year. On the whole it would be highly coincidental if the flux distribution for 1950 were exactly normal.

(b) Fig. 1 shows the distribution of key stations used. In addition to these, numerous alternate stations were added as is described in the references already given. Since a certain fraction of the wind reports were obtained from pilot-balloon soundings, it is to be expected that some bias might thus be introduced because of the impossibility of making such soundings when cloudiness obscured the balloons. At least in middle latitudes this factor might lead to fluxes which are spuriously small, since the cloudiness is more prevalent on the eastward (more moist) sides of cyclones. Actually it is rather easy to overstress the importance of this circumstance, because the factors involved do not possess sufficient regularity. From an examination of the computations for various stations while the work was in progress, the impression is gained that not much error results from this source for the station network actually employed.

Another feature of the station network is that, generally speaking, it is more dense over land areas. This could lead to an insufficient

sampling of conditions over ocean areas where the moisture content is larger. It is not thought, however, that this factor has a very appreciable effect on the results.

(c) The assumption that the surface pressure has everywhere the constant value of 1000 mb is probably the most serious one made in the study. However, it is not unremediable, although its removal would involve a considerable increase in labor. The importance of this factor is probably greatest in the tropics, where surface winds having a component toward the equator are found in the mean and where the normal pressure is in the vicinity of 1010 mb, thus no doubt leading to an underestimate of the southward flux of water vapor. This same condition in the tropics probably represents also an instance where the effect of the transport of water southward in the form of liquid cloud droplets may be of some importance as well.

In terms of relative significance it appears that the neglect of the contributions to the flux from the atmosphere above 500 mb is of minor importance.

APPENDIX

It has been our custom in previous articles to present the results of flux calculations of various quantities in the form of a special table giving various details as to the classification of the atmospheric eddies which accomplish the transport. In the case of the flux of water vapor, such a table has been presented for each of the four latitudes 31°, 42.5°, 55° and 70° N by Starr and White (1954). In order to complete the series, the appropriate corresponding information for 13° N is here given in Table III. For a complete discussion of the terminology and symbols appearing in the column headings, see Starr and White (1954). For the purpose at hand it suffices to indicate that square brackets signify averaging with respect to longitude, a bar signifies averaging with respect to time, curly brackets indicate averaging over the total number of observations N, while n is the number of days with available observations and r is the coefficient of linear correlation between the northward component of wind velocity v and the specific humidity q. Primes denote the deviations from time or longitude averages, depending on the quantity to which they are affixed.

Note that, in the case of water vapor transport, the contribution of the so-called meridional cell component as given in column 6 at 1000 mb (due to net southward air motion) is large enough to give dominant importance to the vertical integral at the foot of the column. This is apparently due to the large concentrations of water vapor

Table III. Numerical analysis of water vapor flux data for year 1950 at latitude 13°N. All velocities are in m sec⁻¹, humidities in gm kg⁻¹. The levels are in mb.

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Level	$\overline{[q]}$	$\overline{[v]}$ *	$\overline{[q]\,[v]}$ *	$\overline{[qv]}$ *	$[\bar{q}]\,[\bar{v}]$	$\overline{[q]'\,[v]'}$	$\overline{[q'v']}$ *	n	$\{q\}$	$\{v\}$	$\{qv\}$	$\{qv\}-\{q\}\,\{v\}$	r	N
500	+ 1.6	−0.15 ±0.18	− 0.0 ± 0.3	+ 0.4 ± 0.3	− 0.2	+0.2	+0.5 ±0.2	365	+ 1.6	−0.16	+ 0.4	+0.7	+0.14	2873
700	+ 5.0	−0.06 ±0.14	− 0.1 ± 0.7	+ 0.4 ± 0.7	− 0.3	+0.2	+0.6 ±0.3	365	+ 5.1	−0.08	+ 0.6	+1.0	+0.10	3461
850	+ 9.5	−0.12 ±0.14	− 0.5 ± 1.3	+ 0.7 ± 1.4	− 1.1	+0.6	+1.2 ±0.4	365	+ 9.6	−0.11	+ 0.7	+1.8	+0.15	3171
1000	+15.6	−1.08 ±0.19	−15.7 ± 2.8	−13.8 ± 2.9	−16.9	+1.1	+2.0 ±0.5	365	+15.7	−1.05	−13.4	+3.1	+0.22	3224
Integral (10^3 CGS units)				− 0.7	− 1.6	+0.3	+0.6				− 0.7	+0.8	Sum	12729

* Confidence limits are twice the standard error of the mean.

near the surface and is in striking contrast to the situation in regard to the flux of angular momentum where the corresponding integral accounts for only a small percentage of the total flux which is itself small when compared to latitudes a little farther removed from the equator (see Starr and White, 1952c).

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ON THE MERIDIONAL FLUX OF WATER VAPOR IN THE NORTHERN HEMISPHERE

by V. P. STARR (*), J. P. PEIXOTO (**) & G. C. LIVADAS (***)

Summary — Maps of the meridional vertically integrated flux of atmospheric water vapor over the northern hemisphere for summer, winter and the entire year of 1950 are presented. These results are derived from all available meteorological soundings of humidity and winds. A corresponding set of three maps showing the average vertically integrated values of the moisture content are included. Tables and graphs of zonally averaged numerical values extracted from these maps are reproduced and discussed in the light of various meteorological considerations.

Résumé — Dans cet article les auteurs presentent des cartes du flux meridional de la vapeur d'eau integré suivant la verticale, pour l'été, l'hiver et pour toute l'année de 1950. Ces resultats ont été derivés à partir, de tous les radiosondages disponibles de l'humidité et des vents. On presente d'abord un ensemble correspondant de trois cartes avec l'analyse des valeurs moyennes du teneur en humidité integrées suivant la verticale. Finalement on reproduit des tableaux et des graphiques avec les valeurs moyennes zonales calculées d'après ces cartes et dont on fait une discussion a là lumière de diverses considerations météorologiques.

1. *Introduction* — One mode of approach to the study of general circulation of the atmosphere is to examine certain integral requirements deduced from dynamical principles governing the motion of the atmosphere, formulated in terms of physical properties, such as energy, momentum, mass or water content, etc.

The present paper intends to give some aspects of the results obtained in the study of the water balance requirements of the atmosphere. According to the prin ple of conservation of mass, water substance cannot be created or destroyed within the atmosphere. Accordingly a local change of water content can be brought about only through the addition or abstraction of water. The water balance therefore may be taken as a constraint for the general circulation.

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(***) University of *Thessalonica*.

The necessity for the transport of water in the atmosphere arises from the existence of an excess of precipitation over evaporation over certain regions with a reversal of prevailing conditions in other areas. However, over a sufficiently long period of time, the amount of precipitable water does not change appreciably, which means that the storage effects are small enough so that the deficits and excesses must be made up through the transport of water by atmospheric circulations, since there can be no significant net inflow or net outflow of water in the atmosphere as a whole.

This transport, accomplished by atmospheric circulations, may occur in any one of the three phases, but the transport in the solid and liquid phases is probably very small compared to the transport in the vapor phase. However, in the tropics the southward flow of water in the liquid phase (clouds) may be of some importance as well. From a thermodynamical point of view we are facing then, a monophasic, heterogeneous and plurivariant system. The flux of the component water is accomplished by the exchange of nearly equal masses of moist air with different concentrations in water.

With the great improvement in the network of aerological stations distributed over the northern hemisphere, it has been possible now to treat many new problems as regards the behavior of the atmosphere, using the values of the observations, directly. These observational studies on an extensive hemispheric scale are of decisive importance in order to secure a correct framework for the discussion and further study of the mechanisms of the general circulation (STARR, 1951; SUTCLIFFE, 1956).

Along this line of thought, the first attempt to measure directly from wind and humidity observations, on a hemispheric scale, the effects of atmospheric motions in transporting water vapor across latitude walls was that of WHITE (1951), for the purpose of including the contribution of latent heat in the study of the energetics of the earth-atmosphere system. Later, these studies were much amplified by STARR & WHITE (1954), again with regard to the energy balance. Some oceanographic and climatological aspects of this study were also given by these writers (1955). See also BENTON & ESTOQUE (1954).

Following a procedure similar to the one used by BUCH (1954) in the study of wind conditions, it was recognized that the aerological humidity data for the northern hemisphere are sufficient not only for obtaining the zonal average of the humidity flux, but also for drawing of hemispheric maps of several quantities involved in the observational study of this flux. It was decided to use the same time intervals as BUCH, namely the calendar year 1950, with a half-year summer and a half-year winter season.

This task has now been accomplished and all of the results are to be published *in extenso* later. Some special aspects of this extensive study have been treated by STARR & PEIXOTO (1956), PEIXOTO (1957) and PEIXOTO & SALTZMAN (1957). The present paper, which may be regarded as an extension of the previous study given by STARR & WHITE (1955) deals with still another one of these special aspects. It constitutes an attempt to obtain a direct measure of the total meridional water vapor flux and of the moisture content in the atmosphere, through the averaging of the daily observations at individual stations. Levels up to and including 500 mb for 90 aerological stations distributed throughout the northern hemisphere as presented by STARR & PEIXOTO (1956) were used for this purpose.

2. *Procedures and results* — Since we may assume that to a very high degree of accuracy, the atmosphere is in a state of hydrostatic equilibrium, we shall take the pressure p as the vertical coordinate and use a (λ, Φ, p, t) coordinate system. Here λ is the longitude, Φ the latitude and t time.

The total horizontal flux of water vapor above a point on the earth's surface for a given interval of time defines a two-dimensional vector field given by

$$\vec{Q}(\Phi, \lambda) = \frac{1}{g} \int\int q\vec{V} dp dt . \tag{1}$$

The respective zonal and meridional components in the (λ, Φ, p, t) system are represented by the expressions

$$\left\{ \begin{aligned} Q_\lambda &= \frac{1}{g} \int\int qu dp dt ; \\ Q_\Phi &= \frac{1}{g} \int\int qv dp dt ; \end{aligned} \right. \tag{2}$$

where g is the acceleration of gravity, q the specific humidity, u and v the zonal and meridional components of the wind field $\vec{V}$, at a given level p, counted positive eastward and northward, dp an element of pressure in the vertical and dt an element of time.

Similarly, the precipitable water vapor contained in a unit column of air at a given instant at a point on the earth's surface is expressed by

$$W(\Phi, \lambda, t) = \frac{1}{g} \int_0^{p_0} q dp . \tag{3}$$

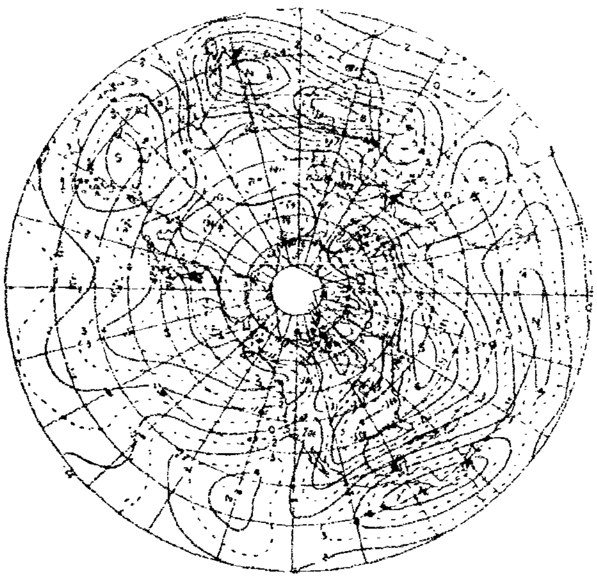
The pressure integrations in (1), (2) and (3) extend from zero to the value of the pressure at the surface.

For an interval of time τ, expressions (1), (2) and (3) may be averaged with respect to time, following the customary procedure, leading to the corresponding mean values $\overline{\vec{Q}}$, $\overline{Q_\lambda}$, $\overline{Q_\Phi}$ and $\overline{W_{\Phi\lambda}}$, where the bar defined by the operator

$$\overline{(\ )} = \frac{1}{\tau} \int_0^\tau (\ ) dt \tag{4}$$

denotes, then, the time average. In the calculations three values of the time domain τ were used, i.e., one winter half-year, one summer half-year and the whole year 1950.

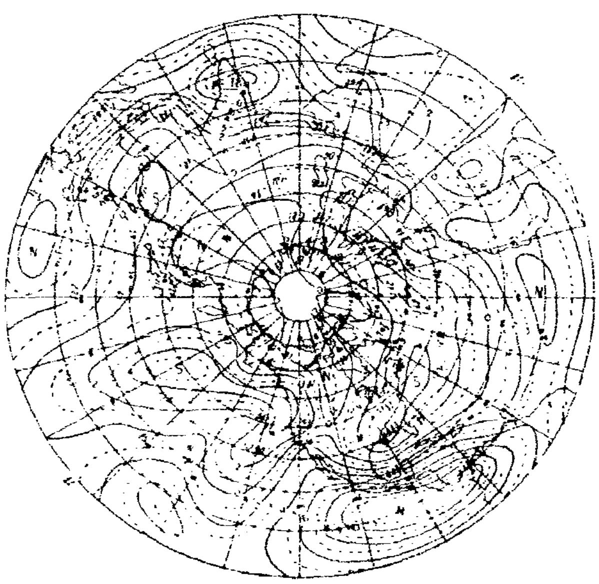
In the evaluation from the data the network presented by STARR & WHITE (1955) was much extended. In order to facilitate the analysis of maps, some stations near the equator in the southern hemisphere were also added. The evaluation of the expressions (2) and (3), and the corresponding time means, were carried out independently for each station, using actual wind and humidity values reported daily for the levels surface, 1000, 850, 700, 500 mb, except where the elevation of the earth's surface interfered.



YEAR

Fig. 1-a - *Time average of the vertically integrated meridional transport of moisture in 10^2 gm cm^{-1} sec^{-1} for the year 1950. S denotes transport from south to north.*

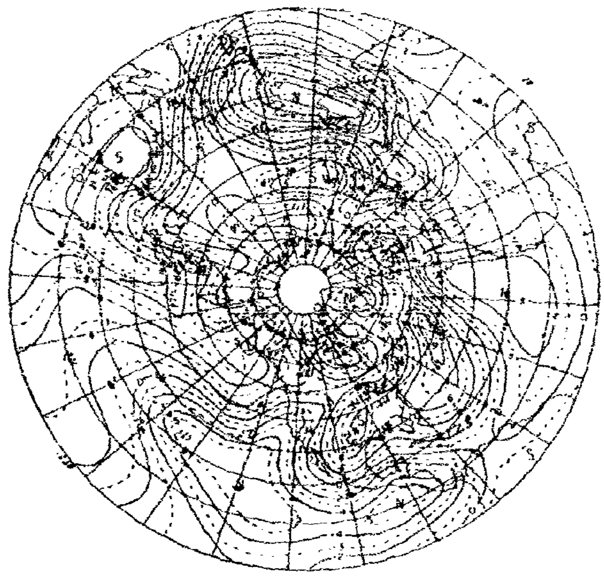
The exact upper level at which the integrands could be assumed to vanish was found to make but little difference for the total flux and water vapor content. It was seen from trial calculations for several stations that levels higher than the



WINTER

Fig. 1-b -*Time average of the vertically integrated meridional transport of moisture in 10^2 gm m^{-1} sec^{-1} for the winter months* 1950.

Fig. 1-c - *Time average of the vertically integrated meridional transport of moisture in 10^2 gm cm^{-1} sec^{-1} for the summer months* 1950.



500 mb are unnecessary. Despite high wind velocities beyond this level the specific humidities are generally so low that the moisture and its transport are much

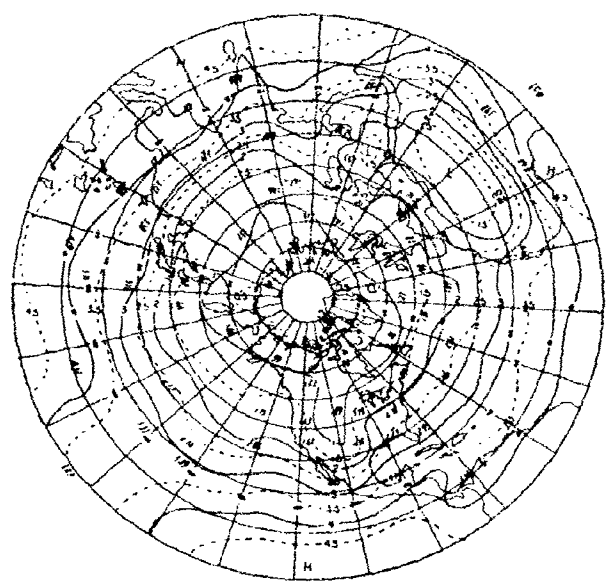


Fig. 2-a - *Time average of the vertically integrated values of specific humidity (precipitable water) in gm per cm^2 for the year* 1950.

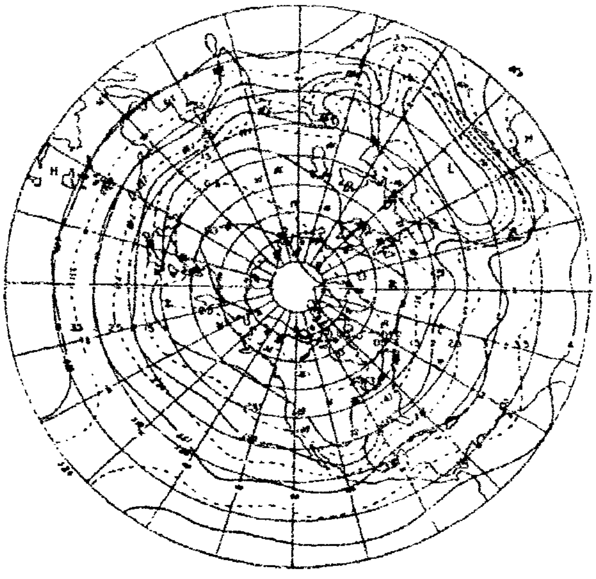


Fig. 2-b - *Time average of the vertically integrated values of specific humidity (precipitable water) in gm per cm² for the winter months* 1950.

too small to be of practical concern here. The integrations with respect to pressure were evaluated numerically by applying the trapezoidal rule.

For each of the quantities $\bar{Q}_\Phi(\lambda, \Phi)$, $\bar{Q}_\lambda(\lambda, \Phi)$ and $\bar{W}(\lambda, \Phi)$ maps were constructed for the entire calendar year 1950, for the summer season (April-Septem-

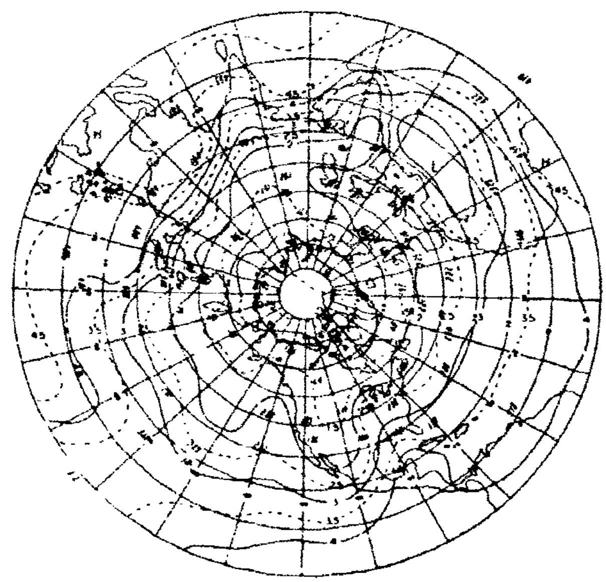


Fig. 2-c - *Time average of the vertically integrated values of specific humidity (precipitable water) in gm per cm² for the summer months* 1950.

ber) and for the winter season (January-March; October-December). Here only the $\bar{Q}_\Phi$ and $\bar{W}$ maps are reproduced, as shown in Figs. 1-a, 1-b, 1-c; 2-a, 2-b, 2-c. It seems that the mean monthly or even mean seasonal (3 months) conditions cannot be examined on the basis of only one year's data.

Using data read off from these maps for every point of a grid 10 × 10 degrees in longitude and latitude it was possible to evaluate numerically the quantities defined as follows

$$
(5) \qquad \begin{cases} 2\pi\,[\bar{Q}_\Phi(\Phi)] = \oint \bar{Q}_\Phi(\lambda, \Phi)\, d\lambda \\ [\bar{W}(\Phi)] = \dfrac{1}{2\pi} \oint \bar{W}(\lambda, \Phi)\, d\lambda\,. \end{cases}
$$

Numerical estimates of the latitudinal distributions of the poleward water vapor flux and the mean precipitable water in the atmosphere were thus obtained. These values are given in tables I and II and the corresponding curves are shown in Figs. 3 and 4 below (*). The units are respectively 10^{11} gm sec^{-1} for the flux and gm cm^{-2} for $[\bar{W}]$.

In the (λ, Φ, p, t) coordinate system the horizontal divergence of $\vec{Q}(\lambda, \Phi)$ is expressed by

$$
(6) \qquad \nabla \cdot \vec{Q} = \frac{1}{R \cos \Phi} \left(\frac{\partial Q_\lambda}{\partial \lambda} + \frac{\partial}{\partial \Phi} (Q\Phi \cos \Phi) \right)
$$

where R is the radius of the earth assumed constant (6.371×10^8 cm). The divergence by ten degree latitude belts was also computed by the expression

$$
\nabla \cdot \bar{\vec{Q}}(\Phi) = \frac{1}{R \cos \Phi} \frac{\partial}{\partial \Phi} \oint (\bar{Q}_\Phi(\Phi, \lambda) \cos \Phi)\, d\lambda\,,
$$

since

$$
\oint \frac{\partial \bar{Q}_\lambda}{\partial \lambda}\, d\lambda = 0\,,
$$

where the derivatives were evaluated by finite difference approximation. These values give, by virtue of the continuity principle, the meridional distribution of the zonally averaged difference between precipitation minus evaporation. The corresponding values in cm per year are given in table III. Basically, the curves that represent this divergence are the derivatives with respect to the latitude of the profiles given in Fig. 3, and we therefore do not give their graphical representations.

(*) Due to a somewhat unequal partition of the number of observations between summer and winter the yearly means are not exactly the average of the individual seasonal means.

TABLE I: *Numerical values of water vapor flux across the specified latitudes for the northern hemisphere in units of 10^{11} grams per second.*

Latitude	80	70	60	50	45	40	30	20	10	0
Winter	—0.10	+0.64	+2.24	+4.48	+5.04	+5.36	+3.81	—3.68	—14.36	—9.32
Summer	—0.12	+0.48	+2.44	+6.80	+6.92	+5.64	+2.11	—1.84	+1.92	+9.08
Year	—0.11	+0.54	+2.36	+5.32	+5.80	+5.46	+2.91	—2.70	—6.16	0.0

TABLE II: *The meridional distribution of zonally averaged precipitable water vapor content at specified latitudes in grams per square centimeter.*

Latitude	80	70	60	50	45	40	30	20	10	0
Winter	0.22	0.34	0.55	0.77	0.96	1.19	1.94	2.61	3.58	4.22
Summer	0.78	1.02	1.33	1.78	2.04	2.24	2.81	3.39	3.98	4.48
Year	0.48	0.61	0.96	1.33	1.53	1.77	2.40	3.08	3.83	4.39

Although the figures given in tables I and III agree fairly well with normals deduced through other methods by several climatologists, as quoted by STARR & WHITE (1955) and reproduced below, it must be remembered that the actual values obtained here are for only one specific year. Furthermore, a number of rather

TABLE III: *The meridional distribution of zonally averaged difference between precipitation minus evaporation by ten degree latitude belts for the northern hemisphere as computed from atmospheric transport data. The units are cm per year. Independent previous estimates of normals made by climatologists named appear in the two last lines.*

Latitude belt	90-80	80-70	70-60	60-50	50-40	40-30	30-20	20-10	10-0
Winter	— 8.0	+ 20.0	+26.5	+27.3	+ 8.7	—13.2	—57.8	—77.4	+35.4
Summer	— 9.6	+15.7	+32.5	+53.2	—11.5	—30.2	—30.5	+27.2	+50.3
Year	— 8.8	+17.3	+30.1	+36.1	+ 1.4	—21.8	—43.3	—25.1	+43.3
CONRAD-WÜST	+ 8	+22	+25	+32	+32	—19	—48	—37	+43
BENTON-WÜST	+16	+14	+25	+22	+19	—24	—35	—34	+46

obvious shortcomings involved both in the data and in computation techniques must be taken into account, as pointed out by STARR & WHITE (1954, 1955). Nevertheless, it seems that the essential characteristics of the total meridional water vapor flux and distribution of water vapor are well portrayed in the present study.

3. *Some general comments* — An inspection of Figs. 1-a, 1-b, 1-c shows immediately that the most prominent feature in the poleward water vapor flux is the nonzonal assymmetry revealed by the existence of northward (positive) and south-

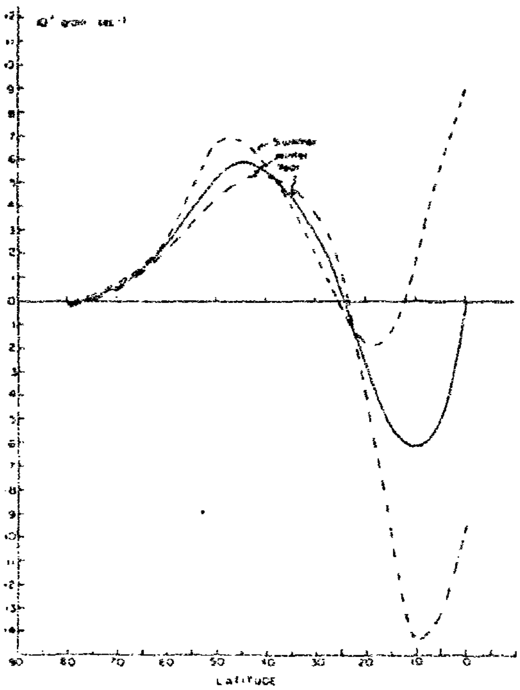


Fig. 3 - *The meridional distribution of zonally averaged water vapor flux in the atmosphere computed from atmospheric data.* The solid curve represents the yearly distribution, the dotted-dashed winter and the dashed the summer distribution. The units are 10^{11} gm sec^{-1}.

ward (negative) centers which differ in shape, size, location and intensity. The general pattern is of course in a large measure controlled by the prevailing atmospheric circulation, as follows from synoptic experience.

This state of affairs shows how cautious one must be when extending the results obtained from a single station, or from a number of stations in a limited geographical region, to the whole hemisphere in order to deduce the behavior of the general circulation. This procedure might lead to a fictitious kind of symmetrical model that oversimplifies the true conditions.

By-and-large a northward flux prevails over the middle latitude regions, whereas a southward transport predominates over the lower latitudes and probably to a slight extent over a small area in the polar region. The annual map shows that, on the average, over the equator, regions of positive and negative flux balance each other, so that the net moisture flow is practically zero.

When summer and winter maps are compared, some striking differences appear in the position and intensity of the centers of flux. An interesting feature exhibited is the seasonal reversal of direction of the flow over the larger part of the Bay of Bengal, the south China Sea, the equatorial portion of the western Pacific, and over the Gulf of Guinea and the adjacent African region. This oscillation is responsible for a net positive moisture flow across the equator during summer and an opposite flow during winter, and is a reflection of the monsoon effects and the well known shifting toward the north of the intertropical convergence region, during summer. On all three maps the region of maximum northward flux is clearly associated with the mean position of the polar front in each case.

Unlike the meridional flux, the distribution of precipitable water vapor shown in Fig. 2 is more regular, and the general pattern of the analysis does not differ greatly from one map to another. There is, however, a general increase at all the latitudes of the moisture content from winter to summer, which is clearly seen from the areas limited by the isolines of 1.0 gm cm^{-2} and 3.0 gm cm^{-2}, for instance, in each season. The configuration of the 1.0 gm cm^{-2} isoline is very interesting since it reveals the effects of the influence of continents and oceans. Low values are also observed over the deserts in general, e.g., northern Mexico and southwestern United States, Sahara, Arabia, Iraq and Iran, etc.

Figs. 3 and 4 summarize the main characteristics of the meridional flux and the occurrence of water vapor in the northern hemisphere with respect to latitude. It is obvious that these curves depict many of the points already stated in the discussion of the maps. The largest positive value of the flux is in the neighborhood of 45° and the largest negative value, much stronger in winter than in summer, occurs in the neighborhood of 10° with a northward displacement in summer. The subtropical regions always act as a source of moisture ($\partial[Q_\Phi]/\partial\Phi > 0$), with a considerable seasonal variation, while the equatorial and middle latitude regions act primarily as sinks ($\partial[Q_\Phi]/\partial\Phi < 0$). Probably the polar region acts as a source as seems to be indicated by the position of the zero isoline around that area (Figs. 1-a, 1-b, 1-c).

The magnitude of the maximum positive flux is higher during summer than during winter. On the other hand, air motions in middle latitudes are much less vigorous in summer on the average. It must therefore follow that the higher summer transports reflect the effect of much higher moisture values and more intense, moisture fluctuations in summer.

As regards the magnitude of the mean precipitable water content, the latitudinal profile (Fig. 4) also shows a seasonal variation with lower values during the winter as expected, and a continuous decrease northward ($\partial[\overline{W}(\Phi)]/\partial\Phi < 0$) in all cases. The storage of the atmosphere is very small. Thus a simple computation shows that the entire atmospheric water content could be removed within a period of about ten days by the mean rainfall acting alone.

The occurrence and transport of moisture in the atmosphere play an impor-

tant role in many atmospheric processes. Particularly the transport of energy in latent heat form, constitutes an important feature of the energetics of the atmosphere. The efficiency of this energy transport is best seen when we express the figures given in table I in energy units by multiplying them by the latent heat

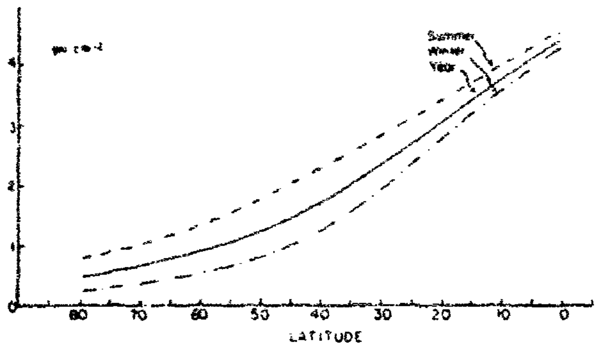


Fig. 4 - *The meridional distribution of zonally averaged precipitable water vapor content computed from atmospheric data.* The solid curve represents the yearly distribution, the dotted-dashed winter and the dashed the summer distribution. The units are gm per cm².

of water vapor. The magnitude of the results is such as to be comparable with meridional transports of other forms of energy (see, e.g., STARR & WHITE, 1954).

It is worthwhile to point out that a high value of the water vapor flux is not the *quantum satis* for the occurrence of precipitation, but rather the precipitation is more directly associated with the field of *convergence* (STARR & PEIXOTO, 1956). A high value of moisture convergence into a given region is a sufficient condition for the maintenance of precipitation, once it is started. On the other hand, although it may happen that the moisture for the occurence of precipitation over some limited area is derived from evaporation *in situ*, this sort of balance is not generally present.

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On the Global Balance of Water Vapor and the Hydrology of Deserts

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(Manuscript received December 15, 1956)

Abstract

An attempt is made to measure the horizontal divergence of the vertically integrated moisture flux over the northern hemisphere for the calendar year of 1950. This is done through the evaluation of daily data for some 90 upper air stations at several levels up to 500 mb. The final map obtained indicates strong maxima of divergence not only over certain oceanic regions, but also over three separate arid regions. Such a result implies an unexpectedly large excess of evaporation over precipitation in these latter areas, which must be balanced by convergence of liquid water on and below the ground surface, unless the year studied was rather completely different from normal, this not being so likely, however.

1. Introduction

With the advent of more plentiful observations concerning the troposphere from stations distributed over the northern hemisphere, it has become feasible to treat many problems associated with the terrestrial hydrological cycle on a hemispheric basis. Previously many such questions could be approached, if at all, only on a regional basis with corresponding limitations in the significance of results when the general circulation processes and other global questions are at issue. Even when more local investigations are intended the simultaneous consideration of hemispheric conditions imparts a sense of perspective not otherwise obtainable. The importance of the last remark is illustrated by various features of the material which follows.

Probably one of the first truly hemispheric attempts to measure directly from winds and humidities the effect of atmospheric motions in transporting water vapor across latitude circles was that of WHITE (1951), in an effort to include the contribution of latent heat to the earth's energy balance. These efforts were much amplified by STARR and WHITE (1954), again especially from the viewpoint of the energy balance of the general circulation. Oceanographical and climatological aspects were later touched upon in some still further extensions of this work by STARR and WHITE (1955).

All of the papers referred to involve the assessment of the net vapor flux across latitude circles, following the pattern of tech-

[1] The research resulting in this work has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, under Contract No. AF 19 (604)-1000.

niques evolved for the similar evaluation of the momentum flux in general circulation studies. In the case of these latter endeavors, BUCH (1954) has shown that air data for the northern hemisphere are sufficient not only for the calculation of zonal averages of the momentum flux, but also for the drawing of reasonable hemispheric yearly and seasonal maps of a number of quantities involved in the observational treatment of this flux. It was therefore decided that the comparable maps for the water vapor flux problem should be drawn for the same time interval as was used by Buch, namely the calendar year 1950. The feasibility of this plan was also suggested by the studies of moisture flux over North America made by BENTON and ESTOQUE (1954). This aim necessitated extensive recomputation and augmentation of data tabulations previously compiled by the General Circulation Project at the Massachusetts Institute of Technology.

At the present writing this task has been accomplished, and the results will be published *in extenso* as soon as time permits. In the meantime, a survey of the material is being presented by PEIXOTO (1958). Also, certain special aspects are treated by STARR, PEIXOTO and LIVADAS (1958), and by PEIXOTO and SALTZMAN (1957). The present paper is still another such discussion dealing with the hydrological significance of the results, especially for some of the principal desert areas of the hemisphere.

2. Procedures and results

The zonal and meridional components of the horizontal total flux of water vapor above a point on the earth's surface may be represented by the expressions

$$Q_\lambda = \frac{1}{g}\iint qu\, dp\, dt; \qquad Q_\varphi = \frac{1}{g}\iint qv\, dp\, dt, \quad (1)$$

where g is gravity, q specific humidity, p pressure, t time and u and v are the zonal and meridional wind components counted positive eastward and northward. Use has here been made of the hydrostatic relation. The pressure integration extends from zero to the value of pressure at the surface, while the time integration extends for a period of one year, in the present instance. It is clear that the expressions (1) are the components of a vector, conveniently denoted by $\vec{Q}$, which defines a two-dimensional vector field over the earth's surface. It follows that one may take the (horizontal) divergence of $\vec{Q}$ and express it in terms of a convenient system of coordinates—spherical polar in our case. We thus have

$$\nabla \cdot \vec{Q} = \frac{1}{R \cos \phi} \times \left\{\frac{\partial}{\partial \lambda}(Q_\lambda) \frac{\partial}{\partial \phi}(Q_\varphi \cos \phi)\right\} \quad (2)$$

Here R is the earth's radius assumed constant, λ is longitude and ϕ latitude.

From the magnitudes encountered in the atmosphere, such as that for the precipitable water vapor contained in a typical column of air, it must be concluded that the storage effects are sufficiently small so that the water lost in regions of positive divergence must be resupplied and that concentrated into regions of convergence must be removed. Since such corrections as the horizontal transport of liquid water are small, the divergence must, for all practical purposes then be numerically equal to the annual excess of evaporation over precipitation.

In the evaluations from data, the station network presented by STARR and WHITE (1955) was significantly extended especially in Africa and elsewhere. Some stations near to the equator in the southern hemisphere were used as an aid in the drawing of maps. The augmented key station distribution is shown in Fig. 1. Many of the procedures followed are the same as those described in the reference given. Thus, for instance, the advection was computed wherever possible from radio winds although a number of pilot-balloon stations again had to be resorted to. Unlike the previous study, the evaluation of the expressions (1) was carried out independently for each station, using wind and humidity data reported daily from the levels: surface, 1000 mb, 850 mb, 700 mb and 500 mb, except where elevation of the earth's surface interfered. Above 500 mb the moisture transports are much too small to be of practical concern.

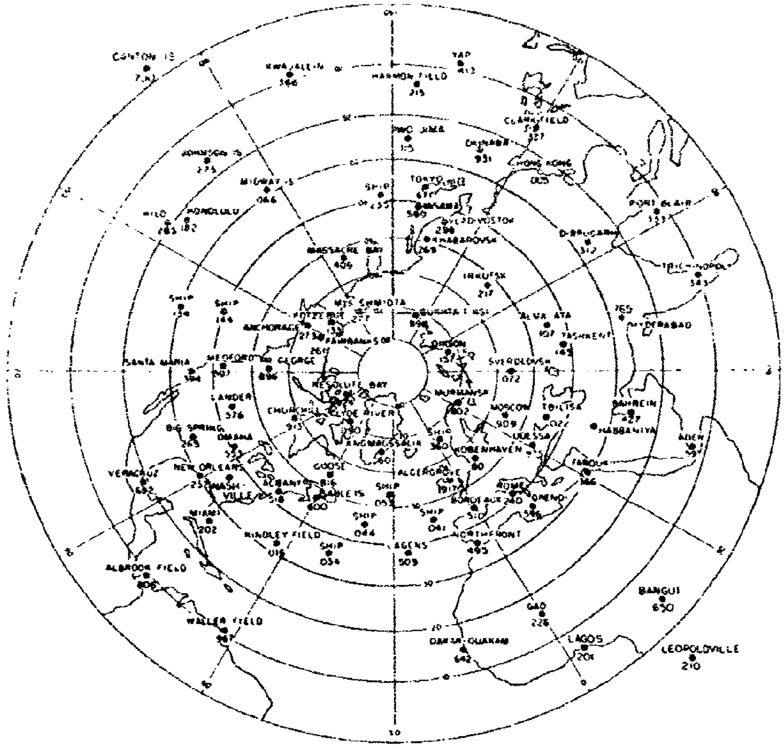

Fig. 1. The distribution of key stations over the northern hemisphere (and slightly beyond the equator) used in the investigation of the atmospheric water vapor transport. A few additional stations which are not shown were also used at times as alternates.

Two maps were then constructed and analyzed, one for each of the quantities Q_λ and Q_φ. Using data read off from these charts for every 10° of latitude and longitude, $\nabla \cdot \vec{Q}$ was computed through the use of equation (2), measuring the derivatives by finite difference approximations. In this way a value of the divergence was obtained for each 10° interval zonally and meridionally. These were plotted on a third map and analyzed, the resulting picture being that shown in Fig. 2. An approximate representation of the field of $\vec{Q}$ itself is given in Fig. 3.

3. General discussion

It is to be understood at the outset that the character of observations at our disposal, and the consequent methods which must be resorted to in performing calculations of the present kind, prevent the reproduction of a vast amount of detail which, without a doubt, is present in the true picture. Viewing the result then only as a general, first approximation to actuality, the writers have concluded that it nevertheless possesses much that challenges one's scientific imagination and raises many questions which deserve something more than mere facile answers. The following list of topics includes several such subjects.

a. By and large, the areas of divergence and convergence balance for the hemisphere so that there is but little net flow across the equator. Zonal averages along other latitudes agree with corresponding climatological estimates secured by independent means (see STARR, PEIXOTO and LIVADAS 1958).

b. Large centers of convergence are to be found in the general vicinity of the headwaters and drainage basins of many large rivers. Such convergence is found over the northwestern United States and western Canada (Columbia,

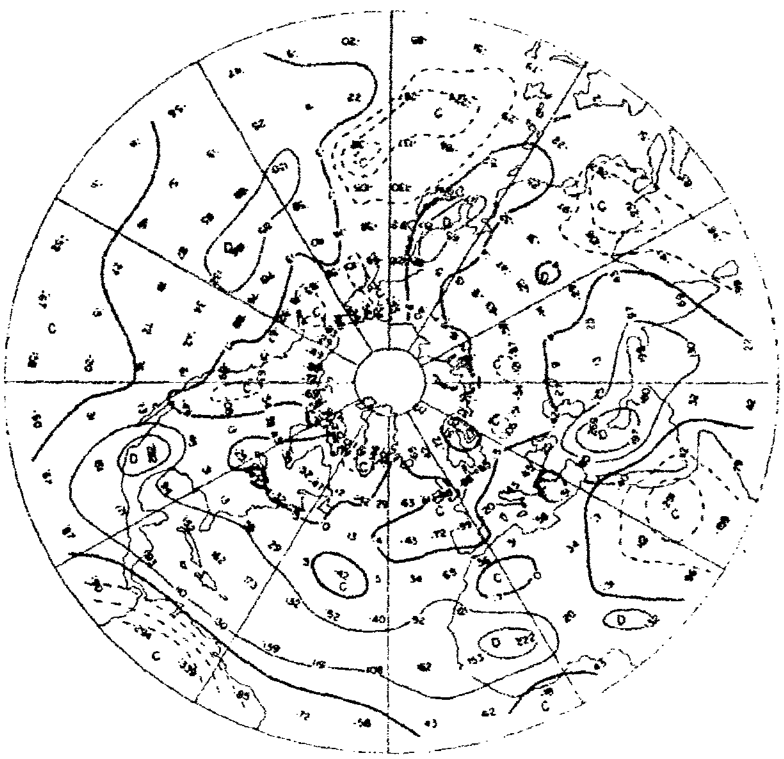

Fig. 2. Distribution of the horizontal divergence of the vertically integrated total annual flux of water vapor for the year 1950. The units are the equivalent depth of liquid water in centimeters per year. The isopleths (full lines for divergence and dashed for convergence) are entered for intervals of 100 cm per year.

Mississippi, Colorado, Mackenzi, Saskatchewan and Rio Grande rivers); over northern South America (Magdalena, Orinoco, and Amazon rivers); over eastern Africa (Nile, Congo, Juba and Scebeli rivers); over eastern India, Burma and Indo-China (Ganges, Brahmaputra, Irrawaddy, Salween, Yangtze, Si Kiang and Mekong rivers). In the case of some rivers of large size, no particularly marked convergence areas seem to manifest themselves; e.g., the Indus river—a quite possible result of deficient reports from central Asia where it rises.

c. Convergence centers are to be found over ocean areas as for example, the one in the western Pacific. Since the addition of such large amounts of fresh water are involved, the resulting dilution of the mineral salts in sea water is a matter of consequence in oceanographic studies.

d. Several areas of strong divergence are located over the oceans; notably, over the southern Atlantic and the Gulf of Mexico, and in the mid-Pacific. These then are major sources for atmospheric moisture as seems reasonable enough on general meteorological grounds. From the standpoint of oceanography, the effect of the resulting concentration of salinity in such locations has been considered.[3]

e. One of the more striking and puzzling features of Fig. 2 is the delineation of strong divergence centers over a number of (but not all) deserts. This is indicated over northern Mexico and the southwestern United States, over the western Sahara and over the general region of Arabia, Iraq and Iran. The eastern Sahara and the Gobi are exceptions, although the meager reports from central Asia may again be involved in the latter instance. Ac-

[3] From Fig. 2. by - and - large greater surface salinities should be expected in the North Atlantic than in the North Pacific. A more detailed study of this subject is now in progress.

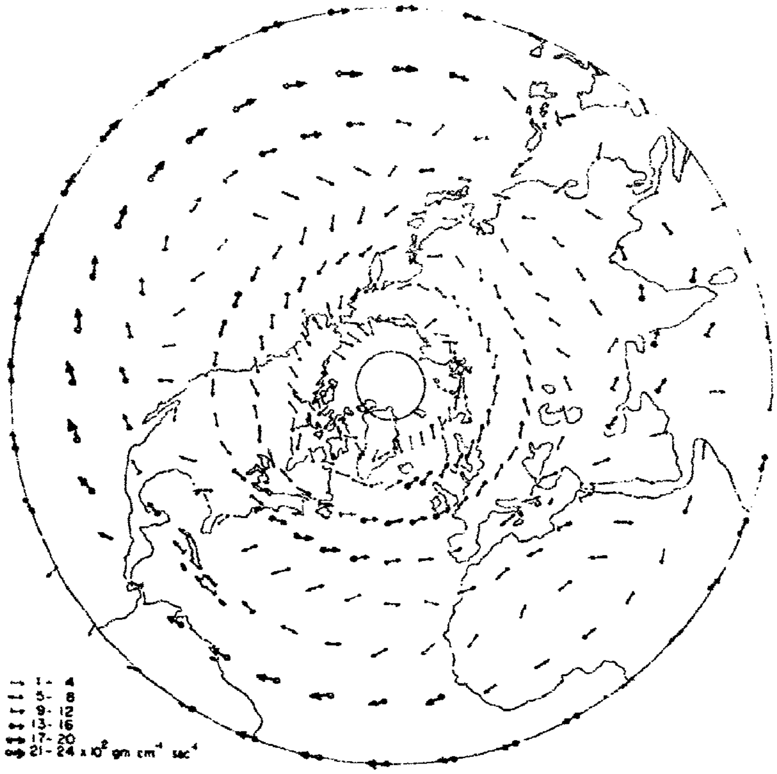

Fig. 3. Approximate distribution of the net moisture transport vector averaged for the year 1950. It should be stressed that the interdiurnal variability of this quantity is large and that because of this large standard deviation the rough streamlines suggested by this map cannot be regarded as being also the trajectories of individual water vapor particles. It is of pertinence to note the good agreement of this map with the analogous one drawn by Benton and Estoque (1954) for the North American sector from data for the calendar year 1949.

cepting the magnitudes shown, it is tempting to draw the somewhat startling conclusion that desert areas are on a par with oceans as major sources of moisture for our hemispheric regime of precipitation. Many questions are however raised immediately, and several of these will be enumerated in the following section.

f. The divergence pattern is most directly related to the difference between the evaporation and precipitation, as has already been explained. On the whole, some relation should exist with the (annual) precipitation itself, nevertheless. It follows that a better analyzed set of isopleths might have been drawn in Fig. 2 by making explicit use of a precipitation chart for the hemisphere. This was not done for several reasons, the lines having been sketched without at least deliberate deference to the rainfall distribution. Since the grid-point values are given, this allows each reader to make such adjustments as seem reasonable to him, the process being inherently subjective. In such attempts, it should be kept in mind that the divergence pattern for 1950 need not be exactly normal, and likewise that the precipitation for this one year may depart from the longer term average values at many points. The large area of convergence in the western Pacific may not, for example, be a normal feature.

g. To the extent that Fig. 2 resembles the long term mean pattern, it must be accompanied by an equal and opposite divergence of water substance below the base of the atmosphere in the hydrosphere and lithosphere, since continuity must be maintained and secular

changes in long term storage are, no doubt, slow. Liberation of juvenile and connate water and other such effects are to be reckoned with here, but the sum total of their contributions is doubtless small.

This compensatory lower circulation cannot be determined from a knowledge of its divergence alone. At best, the only component of it which can so be determined is an irrotational part, which might nevertheless be of some interest. This could be accomplished by inserting derivatives of a velocity potential for Q_λ and Q_φ in an expression of the form (2) and solving the resulting Poisson equation by finite difference procedures or otherwise. The stipulation of a given flow across the equator completes the specification of a Neumann type problem. The resulting flow would, of course, be indeterminate to the extent of a quite arbitrary additive divergence-free transport.

h. During the preparation of the vast amount of data which went into the end product shown in Fig. 2, averages were also made for shorter time periods. Such seasonal maps are not reproduced here, since it is thought that the annual mean condition already presents a sufficient array of problems without now considering less reliable short term pictures. Plans are now in progress to treat additional years of record, especially the Geophysical Year. Upon the completion of such longer period averages, it may be feasible to return to the consideration of seasonal means.

i. We may pause but briefly here in order to ask philosophical questions concerning our material. The ramifications of the subject are well nigh endless as even slight reflection will confirm. Evaporation fares best under low relative humidities effectively produced by descent of air. Why do we have some of the hottest air on the earth's surface descending? Why, in other places such as southeast of Japan does the warm air ascend so vigorously but not elsewhere over certain other ocean areas with seemingly similar air properties? Why does the moisture evaporate so intensely in the middle of the Pacific south of the Aleutians? Why over northern Korea and eastern Siberia?

Answers to these questions and others exist. In many cases, simple statements of considerable pertinence can, no doubt, be made. But, more generally, the subject is part and parcel of the general circulation problem; to be elucidated fundamentally not otherwise than through a better understanding of the whole system. Fortunately, more sophisticated approaches to general circulation theory are at length gaining popularity, and worthwhile results are forthcoming at an increasing rate.

4. Comments on desert hydrology

It has long been recognized that evaporation far exceeds rainfall in most deserts. What is surprising is the magnitude in Fig. 2 of the evaporation from those arid regions already enumerated. How does the water balance at and below the surface maintain itself in order to permit this large annual loss? Unfortunately, the state of our information here is such that it is possible only to speculate about the processes which may furnish the answer to this and other related questions. The following notes may, it is hoped, at least be suggestive, pending the corroboration of our results from independent data and through other means.

a. Desert areas generally contain evidence of evaporation processes. Thus, residues of salt, gypsum and other deposits left behind are common. Presumably, these materials were leached out from ground substances during the passage of water over or through the lithosphere, although at least occasional contributions from sea water cannot be ruled out. Special phenomena such as the deposition on exposed rock surfaces of "desert varnish" indicate the accumulation of a mineral coating, again through the agency of evaporation (see, e.g., HOLMES 1945).

b. A considerable amount of water is transported by surface flow from surrounding territories of more copious rainfall, since deserts tend to have an internal drainage. Various gorges and wadis mark the sites of ephemeral rivers which on occasion carry torrential streams into the central parts of the depressions formed by wind erosion. Such low areas often are below sea level. In some instances, permanent rivers like the Volga and the Jordan never find their way to the sea, their waters being eventually evaporated in toto. In other cases, as with the Nile and Rio Grande, a sizeable fraction of the flow is evaporated before the remainder is drained into the ocean.

c. The flow of ground water into deserts through aquifers and other means (underground rivers and such) can and, no doubt, usually does take place. If in a circular region one thousand kilometers in diameter, one meter of water is to be evaporated annually, the required inflow through porousmedia, distributed uniformly through a depth of one kilometer around the periphery would be much less than one millimeter per second.

That underground waters are abundant in deserts is commonly recognized because of the presence of oases and the fact that many wells have been constructed. It is said that the Nile receives water from such sources on its way across the arid portions of its course.

d. The problem of how underground water might rise from the water table to the surface either as a liquid or vapor is not known. In places where erosion by the wind has lowered the floor of depressions sufficiently, this question does not occur. Elsewhere, great thicknesses of sand intervene, and studies would have to be made in order to provide an answer. Much might perhaps be learned in this regard from laboratory experiments.

e. Since much remains to be verified as to the manner in which large evaporation in these arid regions may take place, it would be highly instructive to make extensive direct measurements of the vertical water vapor transfer near the ground by the standard methods. Even though these procedures involve questions as to the absolute accuracy which may be obtained, still the order of magnitude of the vertical transport can be determined. Due to the extremes of temperature and relative humidity likely to be encountered, special instrumental problems might have to be considered in some detail in order to achieve success.

f. Without further information it would seem from what has been said that if such intense inflows of water into certain deserts really do take place, some potential economic significance should be involved. The flow might thus be strong enough in order to make it profitable to install additional irrigation stations to intercept it at strategic places depending upon hydrological factors.

Also, it would appear that if a strong vertical flow of water in some form takes place through the ground from the water table, some means might be found for arresting it at or near the top surface. This might result in somewhat improved soil moisture conditions for plant growth.

The writers wish to express their thanks to Professor G. C. Livadas of the University of Thessaloniki for valuable assistance and discussions.

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HARMONIC ANALYSIS OF THE MEAN NORTHERN HEMISPHERE WATER VAPOR DISTRIBUTION FOR THE YEAR 1950 *

by José P. Peixoto (*) & Barry Saltzman (**)

Summary — Harmonic analyses along latitudes 30°, 45° and 60° of the mean northern hemisphere water vapor field are presented. The seasonal variations of these spectra are investigated and the relative contributions of the various scales of mean eddies to the meridional transport of water vapor are calculated. Of special interest is the finding that perturbations of wave number 2, corresponding to the great continents and oceans, are dominant at all three latitudes and of primary importance in effecting the northward transport of water vapor.

1. *Introduction* — In a previous paper (Saltzman & Peixoto, 1957) the writers presented the results of the harmonic analysis of the mean wind field for the year 1950. In the present study similar statistics for the mean water vapor distribution are presented, based again on data for the year 1950. This water vapor data has already been the subject of discussion in other connections by Starr & White (1955), Starr & Peixoto (1958a, b), Starr, Peixoto & Livadas (1958), and Peixoto (1958a, b). The analysis given here should, accordingly, be viewed as supplementary to these other studies, particularly to the papers by Starr & Peixoto (1958b) and Peixoto (1958b) in which the gross aspects of the yearly and seasonal mean water vapor field are discussed.

As in the wind field study our primary concerns in performing the harmonic analysis of the mean water vapor field are threefold:

1) to determine the scales of the dominant fluctuations (eddies) which comprise the mean hemispheric water vapor distribution and to measure quantitatively their amplitudes and phases,

2) to determine the seasonal variations in the amplitude and phase of these « standing » eddies, and

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3) to determine the role of the various scales of standing eddies in effecting the meridional transport of water vapor.

Harmonic analyses for components up to wave number 12 were performed along three latitudes (30, 45 and 60 deg) and for three pressure surfaces (850, 700 and 500 mb), based on data spaced at 10 deg intervals around the latitude circles (36 points). As in the previous study we denote the wave-number by n and the amplitude and phase of the harmonic components by the symbols $|F(n)|$ and $\varepsilon(n)$, respectively, in terms of which we may write the following expression for the Fourier expansion of an arbitrary function of longitude, $f(\lambda)$:

$$f(\lambda) = [f] + \sum_{n=1}^{\infty} |F(n)| \cos n [\lambda - \varepsilon(n)] \tag{1}$$

where

$$[f] = \frac{1}{2} |F(0)| = \frac{1}{2\pi} \int_0^{2\pi} f d\lambda \tag{2}$$

$$|F(n)| = [\mathfrak{F}_1^2(n) + \mathfrak{F}_2^2(n)]^{1/2} \tag{3}$$

and

$$\varepsilon(n) = \frac{1}{n} \arctan \left\{ \frac{\mathfrak{F}_2(n)}{\mathfrak{F}_1(n)} \right\}, \tag{4}$$

$\mathfrak{F}_1(n)$ and $\mathfrak{F}_2(n)$ being the real and imaginary parts of the complex Fourier coefficient,

$$F(n) = \frac{1}{\pi} \int_0^{2\pi} f(\lambda) e^{-in\lambda} d\lambda = \mathfrak{F}_1(n) - i\mathfrak{F}_2(n). \tag{5}$$

In the present case we shall consider the mean specific humidity, $\bar{q}$ whose complex Fourier coefficient is denoted by $\bar{Q}(n) = \bar{Q}_1(n) - i\bar{Q}_2(n)$.

The spectral function for the meridional eddy transport of water vapor across latitude Φ per unit pressure difference and per unit time, due to the standing eddies, is given by

$$\tau_q(n) = \frac{\pi a \cos \Phi}{g} [\bar{Q}_1(n) \bar{V}_1(n) + \bar{Q}_2(n) \bar{V}_2(n)], \tag{6}$$

where $\bar{V}_1$ and $\bar{V}_2$ are the real and imaginary parts of the complex Fourier coefficient of the mean meridional velocity (see SALTZMAN & PEIXOTO, 1957), a is the radius of the earth, and g is the acceleration of gravity. This expression satisfies the relation,

$$\frac{2\pi a \cos \Phi}{g} [\bar{q}'\bar{v}'] = \sum_{n=1}^{\infty} \tau_q(n) \tag{7}$$

(primes denote a deviation from the zonal average). The significance of this quantity in studies of the hemispheric balance requirements has been discussed by PRIESTLY (1949), STARR & WHITE (1952), LORENZ (1953) and BENTON & LA SUER (1953).

In order to study seasonal effects, the year was divided into two 6-month periods, the «summer» period being April-September and the «winter» period being January-March and October-December.

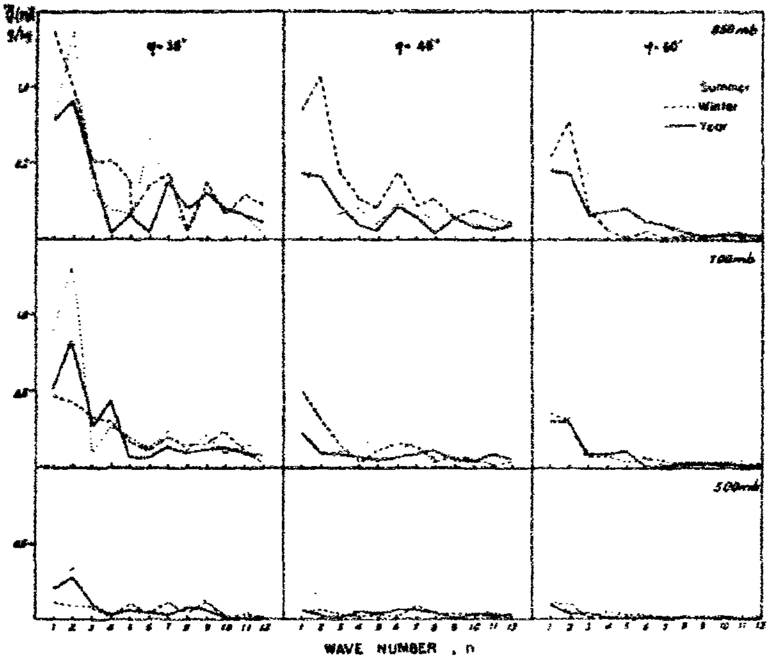

Fig. 1 - *Amplitude spectra of the yearly and seasonal mean specific humidity for the year 1950.*

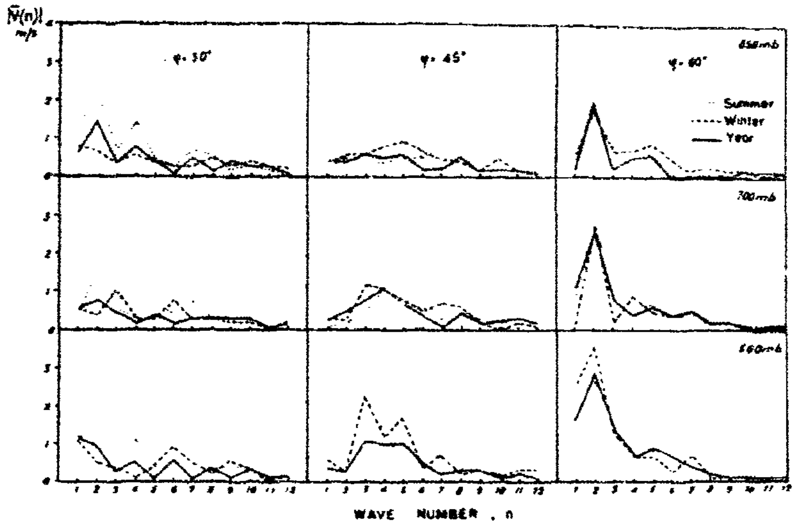

Fig. 2 - *Amplitude spectra of the yearly and seasonal mean meridional velocity component for the year 1950.*

2. ***Results*** — The amplitude spectra for the mean specific humidity are shown in Fig. 1 (*). It may be seen that the primary variations of the mean humidity fiield are of long wave length (e.g., $n = 1$ and 2), though in lower levels (850 mb) some significant variations are present in the intermediate wave length band ($n = 5$ to 9) at 30 and 45 deg latitude. As would be expected the greatest amplitudes are found at 850 mb and at 30 deg latitude.

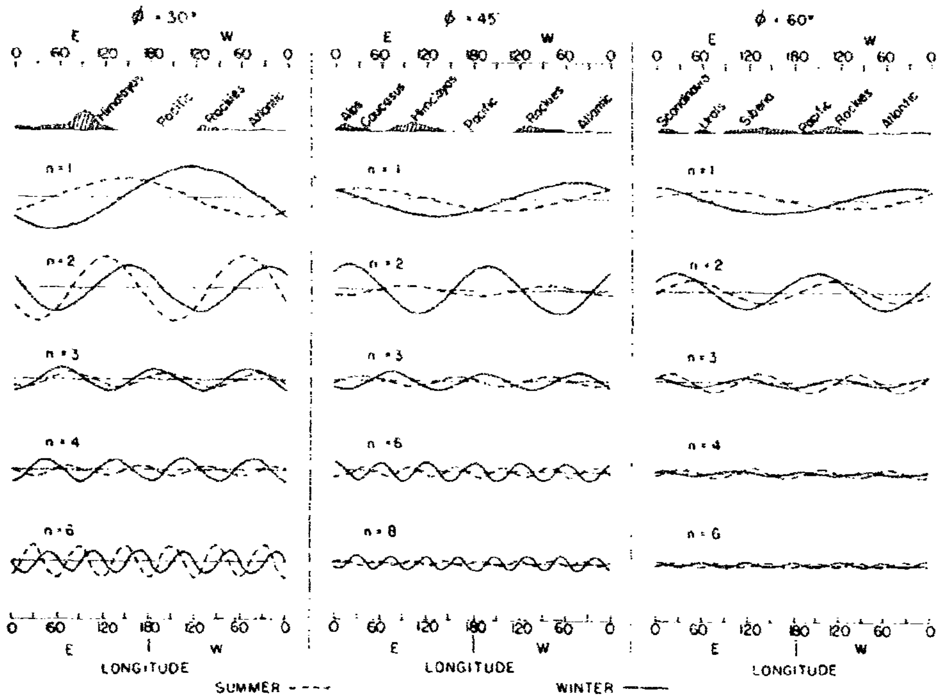

Fig. 3 - *Schematic representation of the prominent waves wich comprise the winter and summer mean fields of the specific humidity at 850 mb.*

In Fig. 2 the corresponding amplitude spectra for the mean meridional velocity component are presented. The 500 mb results shown in this figure have already been discussed in detail by Saltzman & Peixoto (1957). Further reference will be made to this figure in discussing the meridional transport of water vapor by the mean (i.e., standing) eddies.

The phase characteristics of the water vapor harmonics measured at 850 mb are shown in Fig. 3. Here the 6 harmonics of greatest amplitude at each latitude are drawn to scale showing their positions relative to the fixed geographical features and showing their seasonal variations. The seasonal changes depicted are related to the reversals in circulation which develop between continental and oceanic

(*) These and subsequent spectra presented are discrete line spectra and hence have meaning only for the integral values of wave number.

regions. For example, the striking phase shifts occurring at 30 deg in wave numbers 1 and 2 are in large part attributable to the pronounced Asiatic monsoon.

In Fig. 4 the spectra for the meridional transport of water vapor by the standing eddies are presented. As seen from (6) the transport effected by a given wave-length depends on, 1) the degree to which perturbations of that wave-length in

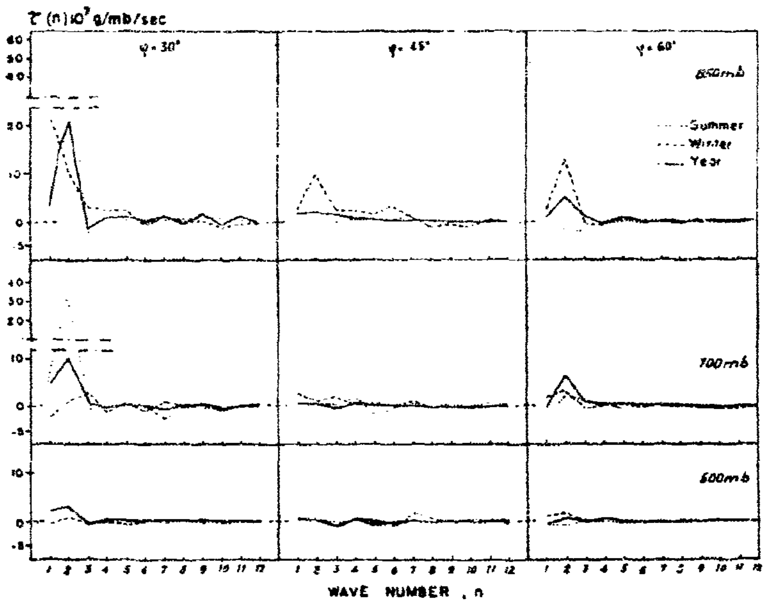

Fig. 4 - *Spectra of the meridional transport of water vapor per unit pressure difference and per unit time across latitudes 30°, 45° and 60° due to the standing eddies. A positive sign indicates a northward transport.*

the mean wind field (amplitudes shown in Fig. 2) are in phase with perturbations of the same wave length in the mean water vapor field, and 2) on the amplitudes of the waves. It is of interest, first, to estimate the fraction of the total meridional transport, accounted for by all of the standing eddies. This quantity, denoted by

$$R(\Phi) = \frac{\{[\bar{q}'\bar{v}']\}}{\{[\bar{q}\bar{v}]\}} = \frac{\sum_{n=1}^{12}\{\tau_q(n)\}}{2\pi a \cos\Phi g^{-1}\{[\bar{q}\bar{v}]\}},$$

where the curled brackets denote a vertical integral for the four pressure surfaces, 1000, 850, 700 and 500 mb, is given in table 1.

TABLE 1. — *Fraction of the total mean meridional transport of water vapor accounted for by the standing perturbations*, R (Φ).

Φ	Year	Summer	Winter
30°	.43	1.13	.23
45°	.04	.02	.16
60°	.11	-.03	.25

It may be seen that the effect of the standing eddies is of greatest significance in low latitudes ($\Phi = 30$) where the quasistationary disturbances are dominant. During the summer at this latitude they are clearly the most important factor in transporting moisture northward, the effect of the transient disturbances generally being small. At 45 deg, where the vigorous transient disturbances exist, the standing eddies play a very minor role, but again at 60 deg, their role increases, especially in winter when the « semi-permanent » lows are most intense.

The spectral distributions for 850 mb shown in Fig. 4 indicate that the perturbation of wave number 2 is the primary agency for the northward transport of water vapor by the mean field. This implies that the high amplitude variations of this scale in both $\bar{q}$ and $\bar{v}$ (see Figs. 1 and 2) tend to be in phase. The extremely high summer value at 30 deg is again a reflection of the strong monsoonal effects occurring on the scale of the two great continents and oceans.

To conclude, it is worth repeating (see STARR & PEIXOTO, 1958a) that the « explanation » for observations of the type presented here cannot be achieved by qualitative arguments alone, but must come from a complete dynamical theory of the general circulation which includes the effects of non-homogeneous surface conditions. The water vapor spectral distributions shown in Figs. 1 and 3, for example, cannot be accounted for without an understanding of the dynamical effects which maintain the major quasi-permanent features of the circulation.

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ON THE ZONAL FLUX OF WATER VAPOR IN THE NORTHERN HEMISPHERE

by V. P. STARR (*) & J. P. PEIXOTO (**)

Summary — In this paper data are presented concerning the zonal transport of water vapor at several levels in the atmosphere for winter, for summer and for the calendar year of 1950, over the northern hemisphere. Vertical integrals and zonal averages are included in the discussion.

In a previous paper (STARR, PEIXOTO & LIVADAS, 1958) the authors presented and discussed the results of the computation for the year 1950 of the mean *meridional* total flux of water vapor in the northern hemisphere. The present note, which may be regarded as an extension of that paper, deals with the results obtained in the study of the mean *zonal* flow of moisture. The approach and procedures followed were similar to those used and discussed by the writers in the above mentioned paper.

Since the atmosphere is to a high degree of accuracy in a state of hydrostatic equilibrium, the pressure p was taken as the vertical coordinate and a (λ, Φ, p, t) coordinate system was used, where λ denotes longitudes, Φ latitude and t time. At a given isobaric level the mean horizontal transport field of water vapor in the time interval τ is represented by

$$\vec{F}(\lambda, \Phi, p) = \frac{1}{g\tau}\int q\vec{v}\,dt \tag{1}$$

where g is the acceleration of gravity, q is the specific humidity, $\vec{v}$ is the (horizontal) vector wind with eastward and northward components u, v. Let us define the bar operator as time average for the time interval τ,

$$\overline{(\ )} = \frac{1}{\tau}\int (\ )\,dt\,. \tag{2}$$

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It follows that the time average of the zonal component of F is given by

$$\overline{F}_\lambda = \frac{1}{g\tau}\int qu\,dt = \frac{1}{g}\,\overline{qu} \tag{3}$$

which may be expressed in units of grams per centimeter per sec per millibar. The total zonal mean flux of water vapor above a point on the earth's surface is given by

$$\overline{Q}_\lambda = \frac{1}{g\tau}\iint qu\,dp\,dt \approx \int^{p_0} \overline{F}_\lambda\,dp \tag{4}$$

where p_0 represents the mean value of the surface pressure. $\overline{Q}_\lambda$ may, of course, be measured in grams per centimeter per second.

The evaluation of the expressions (3) and (4) were carried out independently for each station using daily actual wind and humidity measurements for the levels surface, 1000, 850, 700, 500 mb. The time means were computed for the calendar year 1950, for the summer season (April - September, inclusive) and for the winter

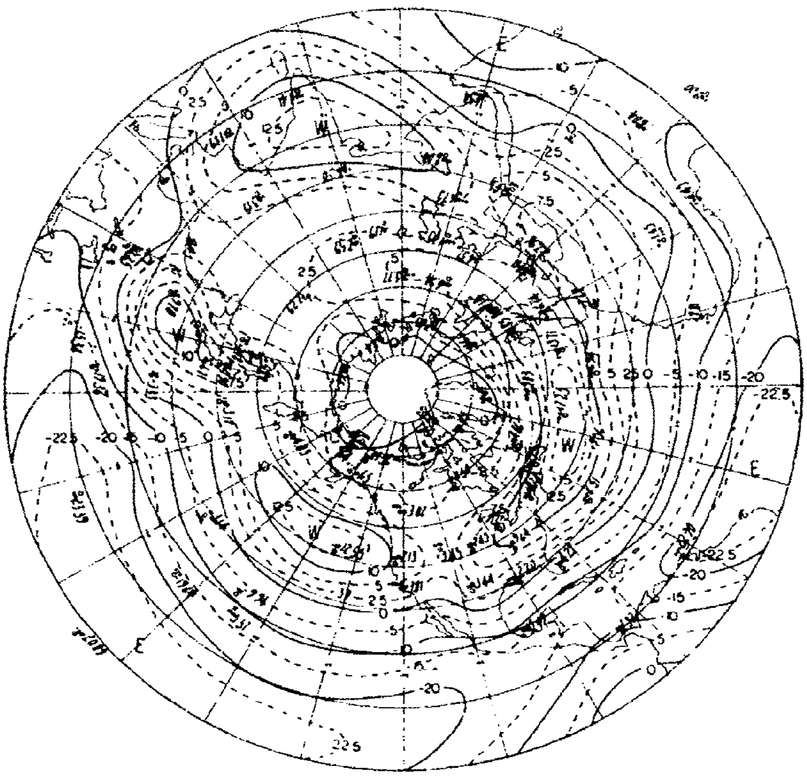

Fig. 1*a* - *Time average of the vertically integrated zonal transport of moisture in* 10^2 *grams per centimeter per second for the year* 1950. Isoline spacing (full curves) 10^3 gm cm^{-1} sec^{-1}. *W* denotes transport from west to east.

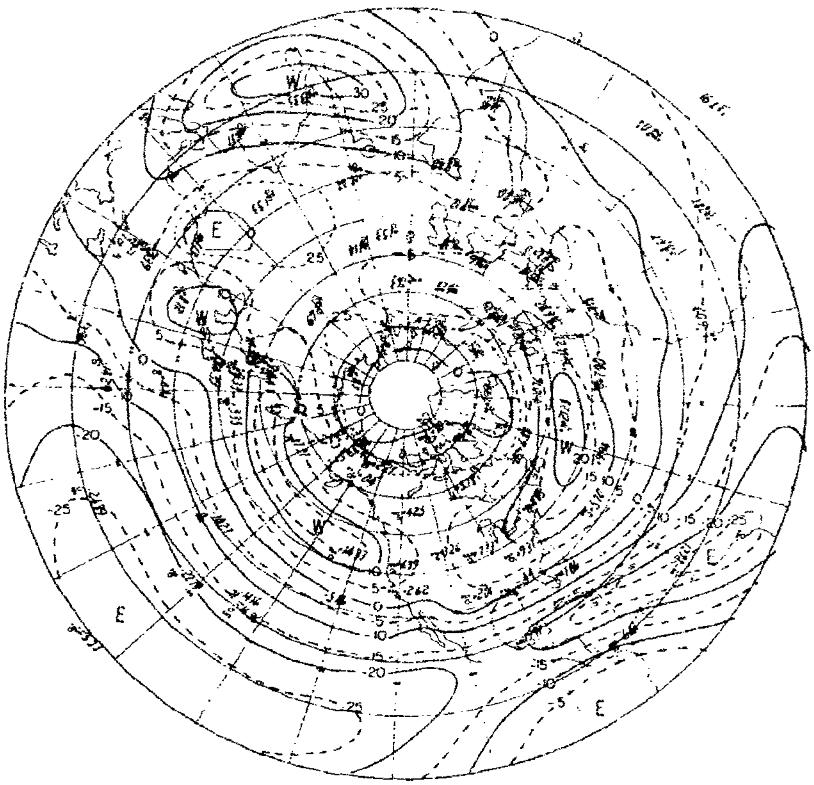

Fig. 1b - *Time average of the vertically integrated zonal transport of moisture in 10^2 grams per centimeter per second for the summer,* 1950.

season. The local values of $\bar{F}_\lambda$ and of $\bar{Q}_\lambda$ were plotted on hemispheric maps and the analysis of these fields was performed using standard methods. Here only the $\bar{Q}_\lambda$ maps are presented (Fig. 1a, Fig. 1b, Fig. 1c).

Using grid-point values of $\bar{F}_\lambda$ and of $\bar{Q}_\lambda$ read from the maps at every 10 degree gridpoint of longitude and latitude, it was possible to evaluate the respective zonal mean values, defined as follows:

$$(5) \qquad \begin{cases} [\bar{F}_\lambda] = \dfrac{1}{2\pi} \oint \bar{F}_\lambda \, d\lambda \\ [\bar{Q}_\lambda] = \dfrac{1}{2\pi} \oint \bar{Q}_\lambda \, d\lambda \, . \end{cases}$$

The meridional distribution of $[\bar{Q}_\lambda]$ for the year, for summer and for winter together with $[\bar{F}_\lambda]$ for the year are given in tables I and II. The corresponding curves of $[\bar{Q}_\lambda]$ are presented in Fig. 2.

The predominant features of the $\bar{Q}_\lambda$ maps are consistent with the distribution of the mean zonal wind current. Thus, a broad band of mean westerly transport occurs in the middle latitudes with several centers of maximum value. Bands o

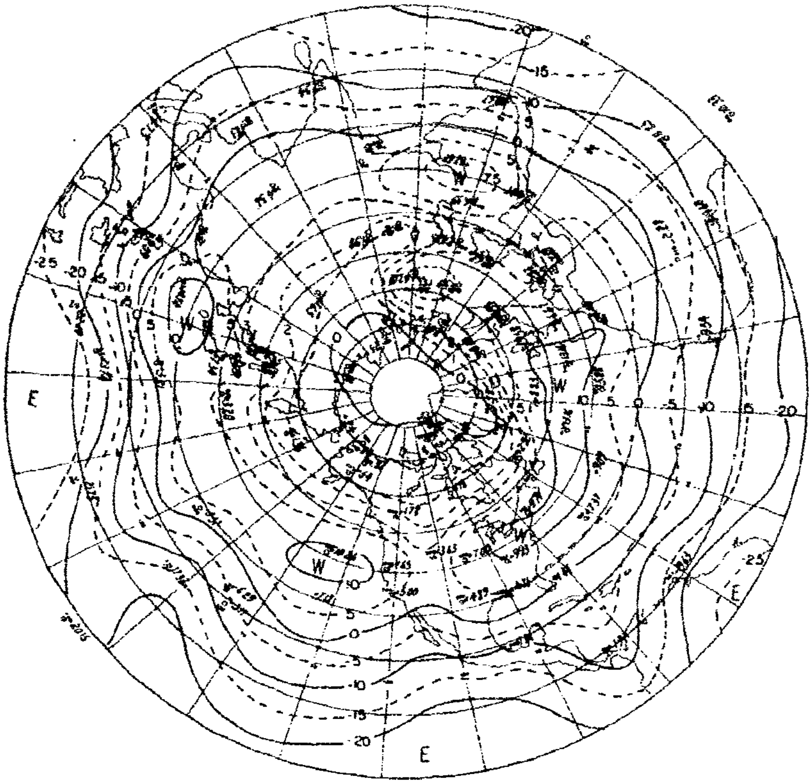

Fig. 1c - *Time average of the vertically integrated zonal transport of moisture in 10^2 grams per centimeter per second for the winter,* 1950.

Table I - *Numerical values of the total zonal flux of water vapor* $[\overline{Q_\lambda}]$ *in units of* 10^2 *grams per centimeter per second.*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Year	+0.78	+3.19	+6.60	+7.07	+6.58	+2.78	−4.30	10.34	−14.44
Summer	+0.99	+4.31	+9.15	+8.99	+6.83	+1.11	−5.52	7.81	−8.98
Winter	+0.42	+2.23	+4.19	+5.44	+6.51	+5.00	-3.36	14.00	−20.09

Table II - *Zonally averaged values of the mean zonal transport of water vapor* $[\overline{F}_\lambda]$ *in grams per centimeter per second per millibar for yearly data.*

Latitude	65°	55°	45°	35°	25°	15°	5°
Pressure							
1000	+0.09	+0.81	+0.87	0.04	1.44	−3.19	−4.18
850	+0.44	+1.40	+2.08	+1.41	−0.04	-2.15	−3.41
700	+0.42	+1.10	+1.66	+1.58	+0.72	0.97	−2.44
500	+0.17	+0.52	−0.88	−0.93	−0.62	0.24	−0.78

easterly transport are present to the north and south. The positions of the zero isolines which separate the westerly from the easterly flux, shift to the north during the summer following the mean position of the centers of the sub-tropical anticyclones and sub-polar lows.

The westerly transport of water vapor as shown in table II attains its maximum value in the lower troposphere, around the 800-mb level, whereas the maximum easterly flux observed in low latitudes occurs near the surface, being stronger in winter on the average.

Figure 2 summarizes the main characteristics of the zonal flux. The largest

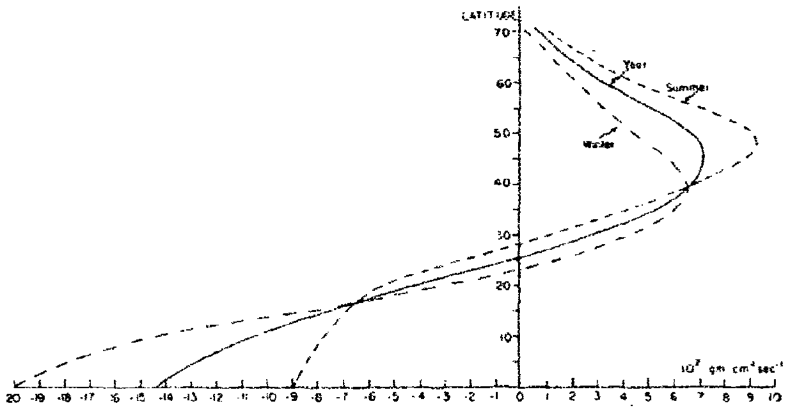

Fig. 2 - *The meridional distribution of the mean zonal flux of water vapor in the atmosphere computed from atmospheric data.* The solid curve represents the yearly distribution, the dotted-dashed winter, and the dashed the summer distribution. The units are 10^3 gm cm^{-1} sec^{-1}.

positive value of the flux is in the neighborhood of 45°, with a northward displacement in summer. The largest negative value, much stronger in winter than in summer, occurs in the equatorial region. The magnitude of the maximum positive flux is higher during summer than during winter.

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ON THE GLOBAL WATER VAPOUR BALANCE AND THE HYDROLOGICAL CYCLE

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Introduction

Quantitative descriptions of the water balance at the earth's surface have in the past been based mainly upon the study of the terrestrial branch of the hydrological cycle through the use of the so-called equation of hydrology. However, the great difficulties in obtaining reliable measurements of the evaporation and, to some extent, of the precipitation and of the storage have always limited the application of this equation. A new approach to the study of the hydrological problems which bypasses some of these difficulties has recently started to emerge with the realization that it is possible to measure with fair accuracy the field of water vapour transport in the atmosphere, directly from the aerological observations. In this manner, the difference between evaporation and precipitation over a given region is inferred by continuity requirements from a knowledge of the upper-air branch of the hydrological cycle. Use of water vapour transport data to evaluate the water balance of a given region of the atmosphere has been made by Wundt [15], Benton and Estoque [1], Starr and White [13] among others. The first attempt to measure water vapour transport on a hemispheric scale directly from the wind and humidity observations was that of White [14] for the purpose of including the contribution of latent heat in the study of the energetics of the earth-atmosphere system. Later, these studies were much amplified by Starr and Peixoto [10].

In view of the improved quality and quantity of aerological date for the northern hemisphere, such as formed the basis for Buch's atlas of hemispheric wind conditions [2], a complete set of hemispheric maps of several quantities of importance in the study of the water vapour and water vapour transport fields was prepared and published *in extenso* by the General Circulation Project at M.I.T. [4, 6, 7].

Some special aspects of this study have been treated on several occasions elsewhere by the author, in collaboration with others. The present paper constitutes an attempt to study the hydrological implications of the field of divergence of water vapour flux, as evaluated by Starr and Peixoto [10], stressing the importance of such an approach in studying the hydrology of broad regions. Special attention will be given in this discussion to the water divergence field in the northern part of the continent of Africa.

The water vapour balance equation as an equation of hydrology

In view of the principle of conservation of mass, water cannot be created or destroyed within the atmosphere. Moreover, there is no significant net inflow or net outflow of water in the atmosphere as a whole. The necessity for the transport of water in the atmosphere arises from the existence of an excess of precipitation over evaporation in certain areas of the globe with a reversal of these conditions in other regions. Since in the long time average the storage effects of the atmosphere for the water vapour are small enough to be disregarded, and since for a sufficiently long interval of time the amount of precipitable water does not change appreciably, the excess and the deficits must be balanced by the transport of water by atmospheric circulations. This transport may occur in any one of the three phases of water but the transport in solid and liquid phases appears to be quite small compared to the transport in the vapour phase except possibly in the tropical regions where the southward transport of water in liquid phase may be of some importance.

Thus the flux of water is accomplished mainly by the exchange of nearly equal masses of moist air with different concentrations of water. Since we may assume that, to a very high

degree of accuracy, the atmosphere is in hydrostatic equilibrium, we shall take the pressure p as the vertical coordinate and use a (λ, ϕ, p, t) coordinate system where λ denotes the longitude, ϕ the latitude and t the time. If we consider at each point of the earth's surface a unit column of air which extends from the surface to the top of the atmosphere, the total horizontal flux of water vapour above that point, for a given interval of time τ, defines a two-dimensional vector field given by

$$\mathbf{Q}\,(\lambda, \phi) = \frac{1}{g} \iint q\mathbf{v} \, . \, dp \, . \, dt \tag{1}$$

The respective zonal and meridional components in the (λ, ϕ, p, t) system are represented by

$$\left\{ \begin{aligned} Q_\lambda &= \frac{1}{g} \iint qu \, . \, dp \, . \, dt \\ Q_\phi &= \frac{1}{g} \iint qv \;\; dp \;\; dt \end{aligned} \right. \tag{2}$$

where g is the acceleration of gravity, q the specific humidity, u and v the zonal and meridional components of the wind field $\mathbf{v}$ $(= u\,\mathbf{i} + v\,\mathbf{i}\,\phi)$, at a given level p, positive eastward and northward, dp an element of pressure in the vertical and dt an element of time.

The precipitable water contained in a unit column of air at a given instant at a point on the earth's surface is expressed by

$$W\,(\lambda, \phi, t) = \frac{1}{g} \int q \, dp \tag{3}$$

In writing the expressions (1), (2) and (3), use has been made of the hydrostatic equilibrium condition. The pressure integrations extend from zero to the value of the pressure at the surface. Expressions (1), (2) and (3) may be averaged with respect to time over the interval τ, leading to the corresponding mean values $\bar{\mathbf{Q}}$, $\bar{Q}_\lambda$, $\bar{Q}_\phi$ and $\bar{W}$, where the bar denotes the operator:

$$\overline{(\quad)} = \frac{1}{\tau} \int_\tau (\qquad) \, dt \tag{4}$$

Using the continuity equation, the water vapour balance equation, at a given point of the atmosphere for an instant t, can be expressed formally by a general equation of balance, as follows:

$$\frac{\partial q}{\partial t} + \nabla \cdot q\mathbf{v} + \frac{\partial qw}{\partial p} = \frac{dq}{dt} \equiv \sigma\,(q) \tag{5}$$

where σ (g) represents the rate of generation of water vapour in the unit mass of the atmosphere and $w = \frac{dp}{dt}$ the individual rate of change of pressure. The balance equation (5) may be integrated with respect to pressure and in the (λ, ϕ, p, t) system the resulting equation assumes the form:

$$\frac{\partial W}{\partial t} + \frac{1}{a^2 \cos\phi} \left\{ \frac{\partial}{\partial \lambda} (a\,Q_\lambda) + \frac{\partial}{\partial \phi} (a \cos\phi\, Q_\phi) \right\} = \Sigma\,(q) \tag{6}$$

since $\int_0^{p_0} \frac{\partial q}{\partial p} w \, dp = 0$. In equation (6) a represents the mean value of the radius of the

earth and Σ (q) the total rate of generation of water vapour in the unit column of the atmosphere at a given point of the globe.

Since the storage effects of the atmosphere for the water vapour are very small (Starr and Peixoto [10]) (at any rate much smaller than the ground water storage) the local rate of change of precipitable water in the atmosphere is so small that $\frac{\partial W}{\partial t}$ may be set at zero. The only sizeable sources and sinks of water vapour in the atmosphere are those due to evaporation and condensation and this is mainly due to effects at the earth's surface rather than from effects involving clouds within the atmosphere.

Thus, the rate of generation of water vapour inside the unit column of the atmosphere may be taken accurately enough as:

$$\Sigma\,(q) = \frac{1}{g}\int \sigma\,(q)\,dp = E - P \tag{8}$$

As noted in the introduction, the water transports in solid and liquid phases by the clouds are negligible when compared to the transport of water in the vapour phase. It is generally accepted that the amount of water in solid and liquid phases of a cloud is considerably small when compared to the water vapour content within the corresponding volume in the atmosphere.

Therefore, in applying the above-mentioned assumptions to the water vapour balance equation (6), we are led to an equation of balance for the *water component*, which reads as follows:

$$\frac{1}{a\cos\phi}\left\{\frac{\partial Q_\lambda}{\partial\lambda} + \frac{\partial}{\partial\phi}\left(Q_\phi\cos\phi\right)\right\} = E - P \tag{9}$$

If the balance equation (9) is to be applied to a region of the atmosphere bounded by a vertical cylindrical wall which defines an area A on the earth's surface bounded by a closed curve c (e.g. a river drainage basin, etc.) equation (9) may be transformed by the Ostrogradsky-Gauss theorem into:

$$\frac{1}{A}\oint (\mathbf{Q}, \mathbf{n}_c)\,dc\;E - P \tag{10}$$

where $\mathbf{n}_c$ denotes the outward normal vector at any point on c.

Equations (5), (6), (9) and (10) may be averaged with respect to time by applying the bar operator as defined by (4) to both members of these equations. Since the "bar" and divergence operators are permutable and the order of integration in the resulting equation (10) is immaterial, we can write the balance equations as follows:

$$\frac{1}{a\cos\phi}\left\{\frac{\lambda}{\partial\lambda}\bar{\mathbf{Q}}_\lambda + \frac{\partial}{\partial\lambda}\left(\bar{\mathbf{Q}}_\phi\;\cos\;\phi\right)\right\} = \overline{E - P} \tag{9a}$$

and

$$\frac{1}{A}\oint (\bar{\mathbf{Q}}\cdot\mathbf{n}_c)\,dc = \bar{E} - \bar{P} \tag{10a}$$

Equation (9) or (9a) may be regarded as an *equation of hydrology* for the atmospheric branch of the water cycle which affords the means of obtaining reliable mean values of the difference between the evaporation minus the precipitation without the uncertainty involved, if measured separately by the direct instrumental methods. This equation (9a) or (10a) provides us

with an independent means of estimating the water economy of a given region and once the characteristic behaviour of the water vapour flow has been studied, estimates of run-off, storage effects, evapotranspiration and other related concepts may be considerably improved.

In considering equations (9a) and (10a) special attention must be paid to the correct meaning of $\mathbf{Q}\,(Q_\lambda\,\mathbf{e}_\lambda + Q_\phi\,\mathbf{e}_\phi)$. As follows from equations (2), application of the bar operator will lead the integrands to expressions such as $\overline{qu}$ and $\overline{qv}$, which may be expanded in the forms:

$$\begin{cases} \overline{qu} = \bar{q}\,\bar{u} + \overline{q'u'} \\ \overline{qv} = \bar{q}\,\bar{v} + \overline{q'v'} \end{cases} \tag{11}$$

where the primes denote the deviations from time averages. In these expressions the second terms, representing the covariance of the instantaneous values of q and u and of q and v at any level and at any point, are proportional to the local zonal and meridional eddy transports of water vapour associated with the "transient" large-scale horizontal eddies, while $\bar{q}\,\bar{u}$ and $\bar{q}\,\bar{v}$ measure the transports due to the mean or "standing" fields of water vapour and wind.

The zonal and meridional mean components of the total vertically integrated eddy transport due to the transient eddies, given by the expressions

$$\begin{cases} Q'_\lambda = \dfrac{1}{g}\displaystyle\int \overline{q'u'}\,dp \\ Q'_\phi = \dfrac{1}{g}\displaystyle\int \overline{q'v'}\,dp \end{cases} \tag{12}$$

have been computed by Starr and Peixoto [10] and it is found that they cannot be disregarded. The meridional eddy component Q'_ϕ, for example, is of the same order of magnitude as the *total* meridional transport. Thus the approach that at times has been followed by some authors of estimating the total meridional transport by the product of the mean vertically integrated values of $\tilde{q}$ and $\tilde{v}$ leads to a very poor estimate.

A clearer appreciation of these facts is gained when one considers an even fuller expansion of $\overline{qv}$ than is given by (11), and tries to get a quantitative assessment of the different terms and the role that they play in building up the total mean water vapour transport field. Thus we consider the expansion:

$$\begin{aligned} \bar{Q}_\phi &= \frac{1}{g}\int \overline{qv}\,dp = \frac{1}{g}\int \bar{q}\,\bar{v}\,dp + \frac{1}{g}\int \overline{q'v'}\,dp = \\ &= \frac{1}{g}\left[\int \bar{q}\,dp \int \bar{v}\,dp\right] + \frac{1}{g}\int \bar{q}^{*}\,\bar{v}^{*}\,dp + \frac{1}{g}\int \overline{q'v'}\,dp \end{aligned} \tag{13}$$

where the asterisks denote the deviations of point mean values of $\tilde{q}$ and $\tilde{v}$ from the corresponding vertically averaged values.

Actual evaluation of the eddy components given by the second and third terms of right-hand side of expansion (13) shows it to be of too great importance in most of the cases to be disregarded, Starr and Peixoto [10]. In considering only the expression between brackets in (13) one is running the risk of not even obtaining the right sign for the "estimate" of total water vapour transport.

Data and procedure

Using the grid point values of $\overline{Q_\lambda}$ and of $\overline{Q_\phi}$ for every point of a grid 10 × 10 degrees in longitude and in latitude, the mean value of the divergence of water vapour flux for each 10 × 10-degree square was computed by applying finite difference methods, the expression used is:

$$\nabla \cdot \bar{\mathbf{Q}} = \frac{18}{\pi\, a \cos\phi}\left[\delta\bar{\mathbf{Q}}_{\lambda t} + \delta\,\bar{\mathbf{Q}}_{\phi k}\cos\phi_k\right]$$
$$= 1{\cdot}57 \times 5{\cdot}73 \times \frac{10^{-7}}{\cos\phi_k}\left[\delta\bar{\mathbf{Q}}_{\lambda k} + \delta\bar{\mathbf{Q}}_{\phi k}\cos\phi_k\right] \text{ g/cm.}^2\text{s.} \quad (14)$$

where δ represents centred differences, e.g.

$$\delta\bar{\mathbf{Q}}_{\lambda k} = \bar{\mathbf{Q}}_{\lambda k+\frac{1}{2}} - \bar{\mathbf{Q}}_{\lambda k-\frac{1}{2}}$$

In the evaluation from the data a network of 90 aerological stations scattered all over the northern hemisphere has been used. In order to facilitate the analysis of maps some stations near the equator in the southern hemisphere were also used. See Fig. 18.1. The evaluation of Q_λ and Q_ϕ given by the expressions (2) and the corresponding time means was carried out independently for each station using actual wind and moisture data reported daily during the whole year 1950 for the surface levels 1,000, 850, 700, 500 mb., except where the elevation of the earth's surface interfered. Unlike most of the studies previously made, which were on a much smaller scale, in the present study the geostrophic wind approximation has not been used.

The exact upper level at which the integrands of (2) and (3) could be assumed to vanish was found to make but little difference for the total flux and water vapour content. It was found that levels higher than 500 mb. were generally unnecessary. The values of the divergence have been computed using the expression (14) and converted into the equivalent differences $\overline{E - P}$ in centimetres per year. They were plotted on hemispheric maps and the analysis of the divergence field was performed by the standard procedure drawing the isothims smoothly, with as close a fit to the data as possible. The isothims were entered for intervals of 100 cm. per year.

General discussion

The analysis of the divergence field is shown in Fig. 18.1. This is the first hemispheric chart ever drawn from actual data treated in the manner described and was published by Starr and Peixoto [10]. A similar map for the southern hemisphere is not possible, due to the sparsity of upper-air observations at the present time.

The map shows the existence of divergence centres alternating with convergence centres. By and large the areas of divergence and convergence for the whole year balance for the hemisphere, so there is but a little flow of water vapour across the equator, in agreement with previous results obtained by Starr, Peixoto and Livadas [12]. However, summer and winter analyses not presented here differ substantially. In summer, the convergence area predominates, whereas in the winter the areas of divergence exceed those of convergence with a corresponding influx and outflux, respectively, across the equator. The monsoon effect, for example, is very well in evidence.

As results from the classic equation of hydrology, regions where $\nabla \cdot \mathbf{Q} = \overline{E - P}$ is negative must possess a means for the disposal of the excess water which falls. Besides some storage

and infiltration over land this generally implies drainage by rivers. Thus these regions should coincide with the drainage basins of large streams. On the map, this is generally true—an outcome which gives support to the reliability of the method used.

Large centres of convergence are found over the northwestern United States and western Canada (Columbia, Mississippi, Colorado, Mackenzie, Saskatchewan and Rio Grande rivers); over the northern part of South America (Madalena, Orinoco and Amazon rivers); over Eastern Africa (Nile, Congo, Juba and Scebeli rivers); over eastern India, Burma and Indo-China (Ganges, Brahmaputra, Irrawady, Salwen, Yangtse, Sikiang and Mekong rivers); over Russia (Volga, Don, Dnieper, Dniester, Onega, Duna and Obi rivers). As a possible result of inadequate information from central Asia, no particularly

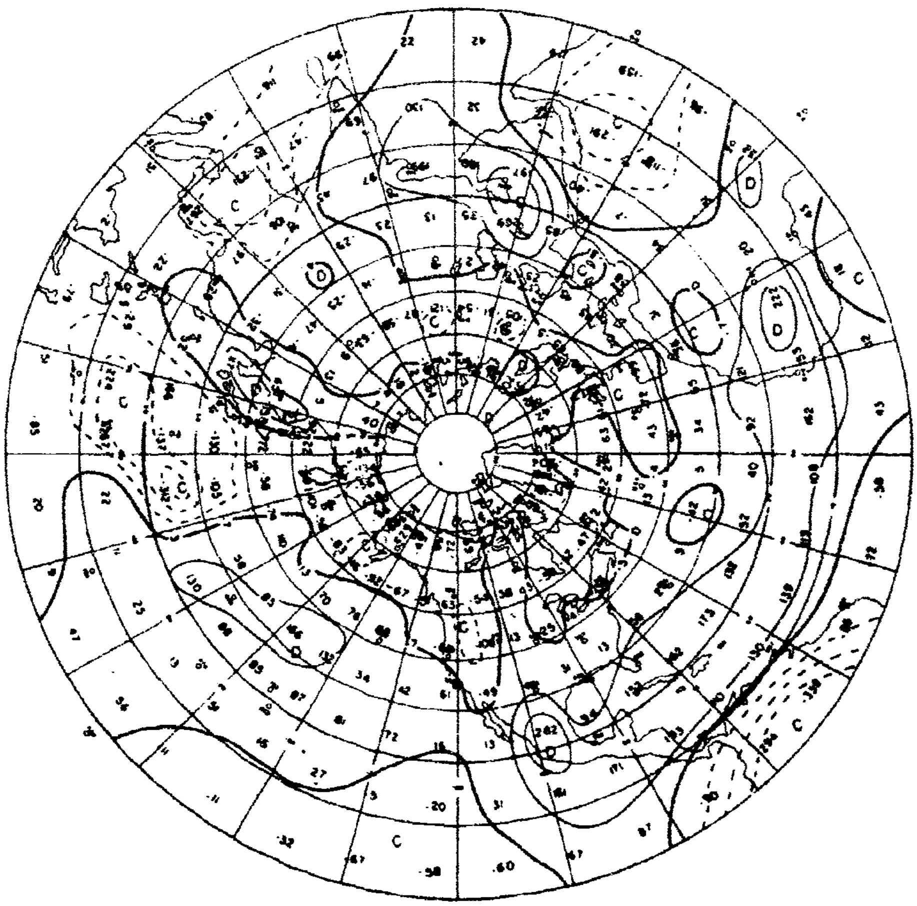

Fig. 18.1

Distribution of the horizontal divergence of the vertically integrated total annual flux of water vapour for the year 1950. The units are the equivalent depth of liquid water in centimetres per year. The isopleths (full lines for divergence and dashed for convergence) are entered for intervals of 100 cm. per year.

marked convergence areas seem manifest in the case of the Indus river. Some other areas of convergence are found over the oceans (northern part and southern part of North Atlantic Bay of Bengal, Eastern Pacific).

The areas of divergence (i.e. the areas in which $\overline{E-P}$ is positive) are located over the sub-tropical regions, over the ocean areas (Gulf of Mexico, central part of North Atlantic, Mid-Pacific) and over a number of deserts (Northern Mexico, Western Sahara, Arabia, Iran, Iraq, Thar desert, etc.). Another centre of divergence extends along the eastern coast of Asia centred over the Sea of Japan.

When these regions of divergence occur over ocean surfaces there is no problem concerning the supply of water for the net evaporation. When they occur over land, as for example over the deserts, the situation is completely different. In such cases, it must be the surface and underground flow from less arid areas that supplies the necessary water that accounts for the observed predominance of evaporation.

The study of the underground flow in desert areas is an extensive and difficult subject, although of increasing practical importance and of great economical value, Starr and Peixoto [10].

If one computes the mean value of the divergence by ten degrees latitude belts, Peixoto [8], the values obtained compare very well with the corresponding climatological estimates of $\overline{E-P}$ secured by independent means. Starr, Peixoto, and Livadas [13] have previously presented an estimate of the divergence by ten-degree latitude belts, computed from the map of the mean meridional transport, $\overline{Q}\phi$, since

$$\langle \nabla \cdot \bar{\mathbf{Q}} \rangle = \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \oint \left[\bar{Q}_\phi (\lambda) \cos \phi \right] d \lambda$$

noting that the $\langle\langle\ \rangle\rangle$ operator is defined by:

$$\langle\langle\ \rangle\rangle = \frac{1}{2\pi} \int (\quad) \, d \lambda$$

and
$$\oint \frac{\partial Q_\lambda}{\partial \lambda} d\lambda = 0.$$

The values obtained agree with the values derived from the actual map.

The zonal averages summarize the main global characteristics of the divergence field of the water vapour in the northern hemisphere. The net divergence is positive in the sub-tropical latitudes between 14° and 47° N. with a maximum about 26° N., and perhaps to the north of 80° N. and negative in the equatorial and middle latitude regions. The sub-tropical region acts as a source of moisture for the atmosphere, while the equatorial and the middle latitude regions act primarily as sinks. The polar region probably acts as a source.

The general pattern of the distribution of the divergence of water vapour flux, as portrayed in Fig. 18.1, is in large measure controlled by the prevailing atmospheric circulations, as follows from synoptic experience.

Some comments on the hydrology of Africa

The net moisture transport vector field and the field of horizontal divergence of the total annual flux of water vapour for the year 1950* over the northern part of Africa, as drawn by Lufkin [6], are presented in Fig. 18.2.

*The great majority of wind and humidity values used in this study were observed during the year 1950. There are, however, some exceptions because for some of the stations of the basic network (Gao, Lagos, Bangui, Leopoldville and Nairobi) no data were available for that year. Instead, the observations taken during 1954 were used. This necessity for mixing the data from different periods is somewhat unfortunate, but in all probability the overall characteristics of the results are not affected seriously by the procedure.

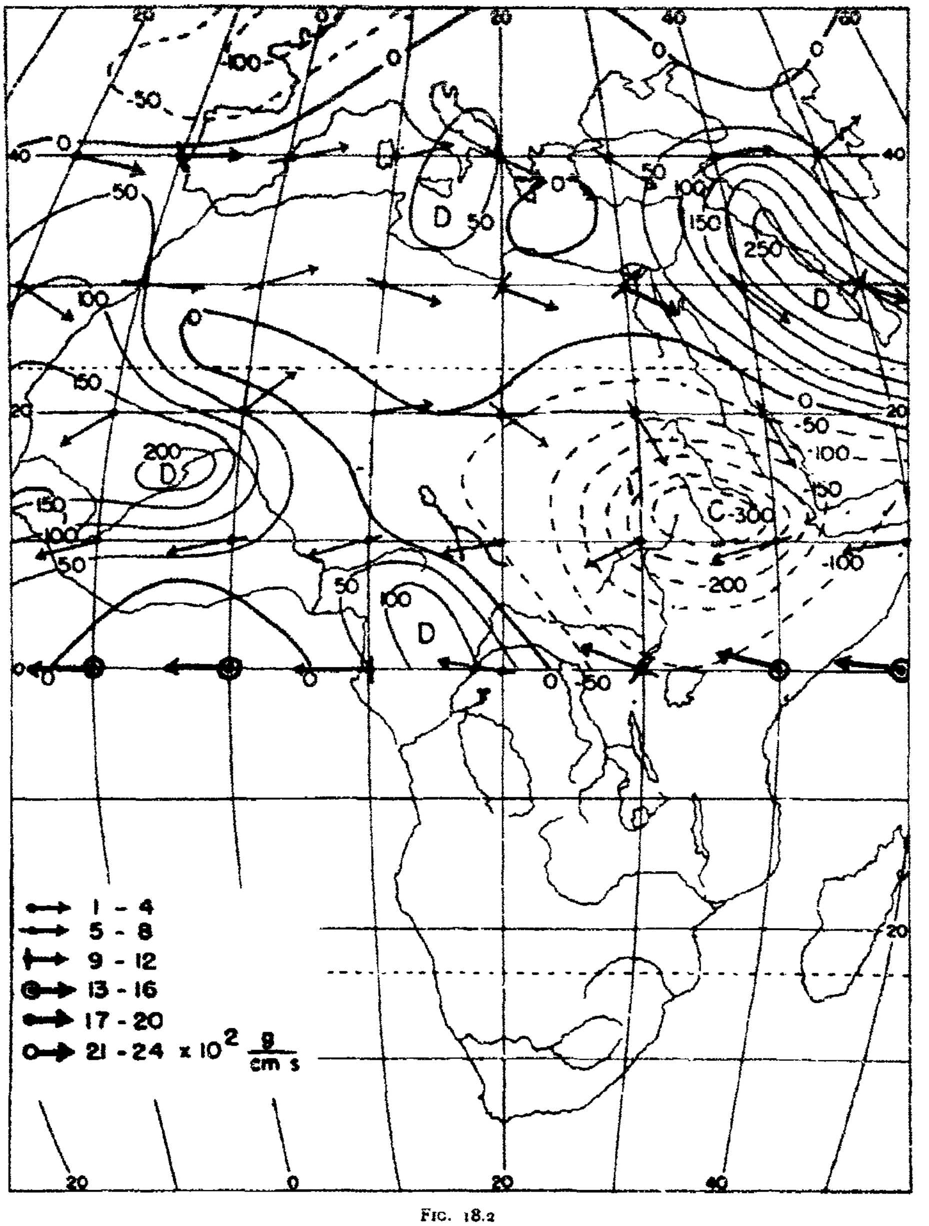

Fig. 18.2
Distribution of the net moisture transport vector and of the horizontal divergence of the total annual flux of water vapour for the year 1950 over the northern part of Africa. The isothims (full lines for divergence and dashed for convergence) are entered for intervals of 50 cm. per year.

The broad area of divergence that covers the sub-tropical region of the Atlantic Ocean splits into two large zones in the vicinity of the African Coast. One zone extends over the Mediterranean region and covers all North Africa and Northern Sahara. The other deflects southwestward towards the Congo region. In the northern branch the water vapour transport field is predominantly eastward, while in the southern branch the transport is towards the west.

The main centre of divergence is located over the French Sudan in the upper Niger, a region of small lakes and swamps, with a considerable drainage of water from the mountains of Guinea and highlands of Ghana where the precipitation is abundant. However, it seems that surface run-off does not account for the strong evaporation that seems to exist there. A part of the water evaporated over this divergence area is transported into the Central Sahara, where it probably accounts for the existing weak convergence zone that shows in Fig. 18.2.

Along the Mauritanean Coast much of the water evaporated comes from the highlands of Guinea, through Senegal and Gambia rivers, whose run-off, however, appears to be insufficient to maintain the high values of the evaporation which are to be observed. During the summer the water table of the lowlands of this region becomes so low that sea water penetrates deeply inland, both underground and along the dry streams, providing a considerable source for evaporation.

The other zone of divergence which extends to the south owes its existence, presumably, to the combined effect of the evaporation from the Congo and Ubangi rivers and of the strong evapotranspiration from the tropical forests which cover the northern part of the Congo territory. It is possible that, owing to boundary effects, the divergence values are overestimated in this area.

A zone of strong convergence is found over the highlands of Ethiopia, which forms the catchments area of the Nile System and of several smaller rivers which empty on the Somali Coast. This zone extends roughly to the edge of the Saharan plateau where the values of convergence are however much smaller.

Another zone of convergence which has already been mentioned is located on the Gulf of Guinea coast. The highlands of this region constitute the main part of the catchment area of Niger, Senegal, Gambia, Lagone and Sahari rivers. Here again boundary and smoothing effects tend to cause an underestimate of the values of the convergence which must be more intense.

Sparsity of soundings on the eastern part of Africa obscure details which would be of great hydrological interest. The divergence field over Nubian and Sudanese deserts, as well as over the Red Sea, fails to give a representation that would agree with the high values of the evaporation that prevails on these areas. Thus, it seems that the network of aerological stations in this region must be improved before a more reliable representation of the divergence of water vapour flux can be achieved.

It has long been recognized that evaporation far exceeds precipitation in most deserts. What seems surprising, however, is the magnitude of the evaporation from those arid regions, as shown in Fig. 18.2. Desert areas generally contain evidence of evaporation processes. Thus the *evaporites* formed by residues of salts are very common in Sahara, Algeria and Tunisia, where they occur in the "chotts". These materials were leached out from ground substances during the passage of water over or through the lithosphere although, at least in coastal regions, occasional contribution from sea water cannot be disregarded. Special phenomena, such as the deposit on exposed rock surfaces, known as "desert varnish" indicate the accumulation of a mineral coating consisting mainly of iron components. This "varnish" is the result of a strong evaporation followed by an active oxidation.

Since the deserts tend to have internal drainage, a considerable amount of water is transported by surface flow from surrounding regions where rainfall is more abundant. The existence of various gorges and wadis indicates the importance of the run-off caused by occasional torrential streams into the central parts of deserts, often below sea level.

In some instances of the permanent rivers already mentioned, a sizeable fraction of the flow is evaporated before the remainder is drained into the ocean. The flow of underground water into deserts through aquifers and subterranean rivers can, and usually does, take place. That vast quantities of underground water do exist in deserts is commonly recognized because of the presence of oases and of the many wells which have been constructed. It is accepted now that the Nile and the Cunene (Angola) rivers, for example, receive water from those sources as they cross the arid regions.

Another example of the possible influence of the underground flow may be found in the Chad Basin. The rainfall in this internal basin is known to be of the same order of magnitude as the value of the corresponding convergence which is obtained from Fig. 18.2. According to references given by Drouhin [3], although Chad Lake receives water from the Sahari and Lagone rivers, its area and its salinity remain constant. Without a sizeable subterranean drainage the salinity due to the combination of evaporation and the transport of salts by the fore-mentioned rivers ought to increase. Hydrological observations taken near Fort Lamy indicate that there is a flow towards north-northeast in an aquifer which extends beneath the lake.

The existence of similar underground flows has been suggested by Hellstrom [5] in a study dealing with the eastern Sahara near the Nile.

It would be highly desirable to make extensive direct measurements of the vertical water transfer near the ground by standard methods, because much remains to be verified concerning the manner in which large evaporation rates in these regions take place. For this problem special instrumental problems might have to be considered. Despite questions of accuracy, it seems reasonable that at least the order of magnitude of the vertical transfer can be obtained.

If such intense flows of underground water into certain deserts do take place, as seems to be the case, they can be considered of some potential economical significance.

Acknowledgements

The author wishes to express his thanks to Professor Amorim Ferreira and Professor Victor Starr for their interest in this work. Thanks are also due to Dr. Barry Saltzman for reading the manuscript and giving many valuable suggestions.

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DISCUSSION

F. M. ALI: With reference to Dr. Peixoto's statements about the existence of underground water in some places in the desert, I have the pleasure to refer to the important investigations and studies which have been carried out during the last four years by the Egyptian Region of the United Arab Republic, about the subsoil water and suitability of the land of the oases for agriculture.

The first results of these investigations prove that large amounts of water exist underground, not only in the areas occupied by these oases, but also in many desert areas which separate the oases. Furthermore, a good deal of the areas around and between the oases have a fertile soil which can be cultivated with important crops with the least possible effort.

Therefore arrangements are being made to dig a large number of wells and prepare the soil so that a new valley connecting these oases will be established in the least possible time and consequently a new source of wealth will be added to the economy of the country.

These investigations indicate also the existence of an underground flow passing by the oases, which is expected to be connected with the Nile River at unknown places.

L. J. TISON: Dr. Peixoto's lecture is all the more interesting to me, firstly because of its content, and secondly for the fact that it clearly establishes the necessity for very close co-operation between meteorologists and hydrologists.

Moreover, this document shows the impossibility of dissecting hydrology into several separate parts for which different organizations should be responsible. I believe that the hydrological considerations given in the lecture must be retained but we might ask ourselves, however, whether certain of the displacements described might not be of considerable importance.

I have in mind notably that which concerns a "negative" flow of the Niger in the lesser portion of its course (that is, a flow directed from the ocean towards the interior). The penetration of salt water from the sea into an estuary is not, in fact, proof of an entirely "negative" flow. This penetration can be due solely to the tide, and I am of the opinion that the penetration to which Dr. Peixoto refers should be subjected to verification, by reference, for example, to M. Jean Rodier, Director of Hydrological Services, ORSTOM.

H. FLOHN: In the general line of approach, I agree in nearly all details with Dr. Peixoto, and we should greatly appreciate the vast amount of work which has been done in this project. However, in the observational approach, we should take great care to avoid all possible sources of error. At first the large systematic differences in the radiosonde humidity data, secondly the linear interpretation between two stations, which appears to be dubious, since the humidity data of the radiosonde are frequently only representative of small or meso-scale weather problems. According to German investigations (Möller, Oeckel) the net divergence of water vapour transport, at least for individual months, is inconsistent with P-E from other sources of data. It seems most likely that the results of Figs. 18.1 and 2, insofar as they do not coincide with other considerations on the hydrological cycle, are strongly influenced by the above-mentioned sources of error, and by the lack of radiosonde data in low latitudes as early as 1950. In addition to the annual data, the seasonal changes of water vapour transported should be given.

H. T. MÖRTH: A question to Dr. Peixoto is whether he introduced an approximation when describing the vector field of Q in the $(\lambda\ \phi\ p\ t)$ system, as he only used principles based on an inertial system.?

J. P. PEIXOTO (in reply): I thank Dr. Ali for his most interesting information on the observations and studies of underground water occurrence and flow carried out in the desert regions of Egypt.

The remark brought out by Professor Tison seems to be very pertinent and I will try to get a verification of the statement following the suggestion kindly given.

I quite agree with Professor Flohn's comments, both on the large systematic differences in the humidity data and on the use of the linear interpolation used in the present study. Insofar as the discrepancies between the net divergence and the monthly mean values of P-E found by the investigations mentioned by Professor Flohn, I think that in the evaluation of the total water vapour transport only some of the terms given by the expansion (13) were taken into consideration. The mean seasonal fields have also been studied and some of the results have already been published (*Geofisica Pura e Aplicata*, 39, 174-185, *Revista Faculdade de Ciencias de Lisboa*—2a Serie—B. Vol. VII, *Scientific Report No. 3, General Circulation Project*, MIT).

We are aware of the uncertainties of some of the results in low latitudes, as Professor Flohn points out. I am glad to inform him that a new study, using all the data available for the I.G.Y. is already being done by the Planetary Circulation Project at MIT. It is hoped that both the improvements of the quality of the observations and the substantial increase of the number of stations to be used (i.e., approximately three times the number actually used) will be of inestimable value in the coming investigations.

With regard to the question raised by Dr. Mörth, the only approximation used in evaluating the vector field Q has been the assumption of hydrostatic equilibrium of the atmosphere. The balance equation, which is the basic equation used, holds, of course, for any kind of differential.

Nicht im Handel

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The Hemispheric Eddy Flux of Water Vapor and Its Implications for the Mechanics of the General Circulation[1]

By

Victor P. Starr and **José P. Peixoto**

With 8 Figures

Summary. A study of the vertically integrated eddy flux of water vapor in the northern hemisphere for summer, winter and the entire year of 1950 is presented. Tables and graphs of zonally averaged numerical values extracted from maps of the analyses of the zonal eddy and meridional eddy components for the hemisphere are reproduced and discussed in the light of various meteorological considerations. It is found that the southward transport of water vapor by the HADLEY type meridional circulation in the tropics does not imply a comparable importance of this cell for hemispheric momentum and zonal kinetic energy considerations as is assumed in classical general circulation theories.

Zusammenfassung. Es wird eine Untersuchung über die vertikale, integrierte Störungsbewegung von Wasserdampf in der nördlichen Halbkugel für den Sommer, für den Winter und für das ganze Jahr 1950 vorgelegt. Tabellen und graphische Darstellungen zonaler Durchschnittszahlenwerte, die aus analysierten Karten abgeleitet wurden, werden im Hinblick auf verschiedene meteorologische Beziehungen diskutiert. Es wurde festgestellt, daß der nach Süden gerichtete Transport von Wasserdampf durch eine meridionale Zirkulation in der HADLEYschen Art in den Tropen dieser Zelle keine besondere Bedeutung für die Betrachtungen der hemisphärischen Bewegung und der zonalen kinetischen Energie verschafft, wie im allgemeinen in der klassischen Zirkulationstheorie angenommen wird

[1] The research reported in this paper has been made possible through the support of the U.S. Air Force Cambridge Research Laboratories under contract AF 19 (628)—2408.

Résumé. On présente une étude du transport perturbé de la vapeur d'eau intégré verticalement dans l'hémisphère nord pour l'été, l'hiver et l'année 1950 entière. Les tableaux et diagrammes des valeurs numériques moyennes zonales, extraits des cartes d'analyses des transports zonal et méridional perturbés pour l'hémisphère, sont reproduits et discutés à la lumière des diverses considérations météorologiques. On a trouvé que le transport de la vapeur d'eau effectué par la circulation méridionale du type HADLEY dans les régions tropicales n'implique pas une importance comparable à celle de la même cellule quand on considère la quantité du mouvement et de l'énergie cinétique zonale comme il est assumé dans les théories classiques de la circulation générale.

1. Introduction

In previous papers (STARR and PEIXOTO [11], STARR, PEIXOTO and LIVADAS [13], STARR and PEIXOTO [12]) the writers presented and discussed the results of the meridional and the zonal components of the mean total water vapor transport field for the northern hemisphere, during the year 1950. In the present paper the study of the eddy transport field of water vapor is presented using again the same basic data for the year 1950. The analyses given now should be viewed as complementary to the studies mentioned and particularly to the monograph prepared by PEIXOTO [4] in which the gross aspects of the yearly and seasonal mean water vapor distribution and transport fields are discussed.

In the present paper use is made of equiscalar analysis to give the space distribution of the mean zonal and meridional eddy flux components of water vapor over the northern hemisphere and to deduce from the analysis the corresponding zonal averages. This approach differs therefore from the procedure used previously by STARR and WHITE [15, 16] in which the individual values at various stations distributed in the neighborhood of given latitude circles were combined to estimate the mean zonal values of the meridional eddy transport. The chief final aim of this paper is to throw additional light by such means upon the mechanics of the general circulation. Some aspects of the mean meridional standing eddy transport of water vapor have been treated previously and published elsewhere (PEIXOTO and SALTZMAN [6], PEIXOTO [4, 5, 7]).

2. Formulation of the Problem

To a very high degree of accuracy the atmosphere may be considered in a state of hydrostatic equilibrium so the pressure p is taken as the vertical coordinate. Thus a (λ, φ, p, t) reference system is used in which λ denotes the longitude, φ the latitude, and t time. At a given isobaric level the horizontal transport field of water vapor per unit pressure difference is represented by

$$\vec{F}(\lambda, \varphi, p, t) = \frac{1}{g} \vec{V} q \tag{1}$$

where g is the acceleration of gravity, q is the specific humidity and $\vec{V} = u\,\vec{i}_\lambda + v\,\vec{j}_\varphi$ is the horizontal wind field with eastward and north-

ward components u and v. Let us define the bar operator as a time average for the time interval τ, so that

$$\overline{(\ )} = \frac{1}{\tau}\int_0^{\tau} (\ )\,dt \tag{2}$$

It follows that the time average transport field of the water vapor in the (λ, ϕ, p, t) coordinate system can be written $\vec{F} = \overline{F}_\lambda \vec{i}_\lambda + \overline{F}_\varphi \vec{j}_\varphi$ where

$$\overline{F}_\lambda = \frac{1}{g}\overline{q\,u} \tag{3}$$

and

$$\overline{F}_\varphi = \frac{1}{g}\overline{q\,v} \tag{4}$$

The total horizontal mean flux of water vapor above a point on the earth's surface for the time interval τ, defines a two-dimensional vector field $(\overline{\vec{Q}})$ given by

$$(\overline{\vec{Q}}) = \frac{1}{g}\int_0^{p_0} (\overline{\vec{F}})\,dp = \overline{Q}_\lambda \vec{i}_\lambda + \overline{Q}_\varphi \vec{j}_\varphi \tag{5}$$

where p_0 denotes the mean value of the surface atmospheric pressure, and $\overline{Q}_\lambda$ and $\overline{Q}_\varphi$ are the corresponding zonal and meridional components. The quantities $\overline{qu}$ and $\overline{qv}$ may be expanded according to the REYNOLDS scheme as discussed for example by STARR and WHITE [14].

$$\overline{qu} = \overline{q}\,\overline{u} + \overline{q'\,u'} \tag{6}$$

$$\overline{qv} = \overline{q}\,\overline{v} + \overline{q'\,v'} \tag{7}$$

where the primes denote the local instantaneous deviations from time averages. The term $\overline{q'u'}$ and $\overline{q'v'}$ are the covariances of the instantaneous local values and are proportional to the local zonal and meridional mean eddy transports of moisture associated with the transient horizontal eddies.

If we define the total local eddy transport field by

$$(\vec{Q}') = Q'_\lambda \vec{i}_\lambda + Q'_\varphi \vec{j}_\varphi \tag{8}$$

where

$$Q'_\lambda = \frac{1}{g}\int_0^{p_0} \overline{q'\,u'}\,dp = \frac{1}{g}\int_0^{p_0} F'_\lambda\,dp \tag{9}$$

$$Q'_\varphi = \frac{1}{g}\int_0^{p_0} \overline{q'\,v'}\,dp = \frac{1}{g}\int_0^{p_0} F'_\varphi\,dp \tag{10}$$

8*

we can write for the total transport

$$(\vec{\bar{Q}}) = \frac{1}{g}\int_0^{p_0} \{\bar{q}\,\bar{u}\,\vec{i}_\lambda + \bar{q}\,\bar{v}\,\vec{j}_\varphi\}\,dp + (\vec{\bar{Q}}') \tag{11}$$

Taking the zonal averages of the expressions (6) and (7), the following expansions are obtained

$$[\overline{q u}] = [\bar{q}]\,[\bar{u}] + [\bar{q}^*\bar{u}^*] + [\overline{q' q'}] \tag{12}$$

$$[\overline{q v}] = [\bar{q}]\,[\bar{v}] + [\bar{q}^*\bar{v}^*] + [\overline{q' v'}] \tag{13}$$

the brackets denoting the zonal average operator defined by

$$[(\ \ )] = \frac{1}{2\pi}\oint (\ \ )\,d\lambda \tag{14}$$

and the asterisks the deviations of the local mean values $\bar{q}$, $\bar{u}$ and $\bar{v}$ from the zonally averaged values $[\bar{q}]$, $[\bar{u}]$ and $[\bar{v}]$. The terms $[\bar{q}^*\bar{u}^*]$ and $[\bar{q}^*\bar{v}^*]$ are the spatial covariances of the time mean values $\bar{q}$, $\bar{u}$ and $\bar{q}$, $\bar{v}$ at individual points along the latitude circle and measure the transport associated with the *standing* large scale horizontal eddies (STARR and WHITE, loc. cit.).

The terms $[\bar{q}]\,[\bar{u}]$ and $[\bar{q}]\,[\bar{v}]$ measure the advection of the mean zonal specific humidity by the zonally averaged mean wind field. The second of these measures the contribution of the mean meridional circulation.

The zonal and meridional components of the vertically integrated eddy transport associated with the standing eddies are given by the expressions

$$\frac{1}{g}\int_0^{p_0} [\bar{q}^*\,\bar{u}^*]\,dp \tag{15}$$

$$\frac{1}{g}\int_0^{p_0} [\bar{q}^*\,\bar{v}^*]\,dp \tag{16}$$

Thus we may write for the components of the total water vapor transport

$$[\bar{Q}_\lambda] = \frac{1}{g}\int_0^{p_0} [\bar{q}]\,[\bar{u}]\,dp + \frac{1}{g}\int_0^{p_0} [\bar{q}^*\,\bar{u}^*]\,dp + [\overline{Q'_\lambda}] \tag{17}$$

$$[\bar{Q}_\varphi] = \frac{1}{g}\int_0^{p_0} [\bar{q}]\,[\bar{v}]\,dp + \frac{1}{g}\int_0^{p_0} [\bar{q}^*\,\bar{v}^*]\,dp + [\overline{Q'_\varphi}] \tag{18}$$

3. Data and Procedures

As in the previous studies mentioned, the present study is again based entirely upon direct measurements of moisture and winds observed at different levels; no use was made of the geostrophic approximation.

The evaluation of the various quantities involved the tabulation of daily (0300 Z) values of moisture and zonal and meridional components

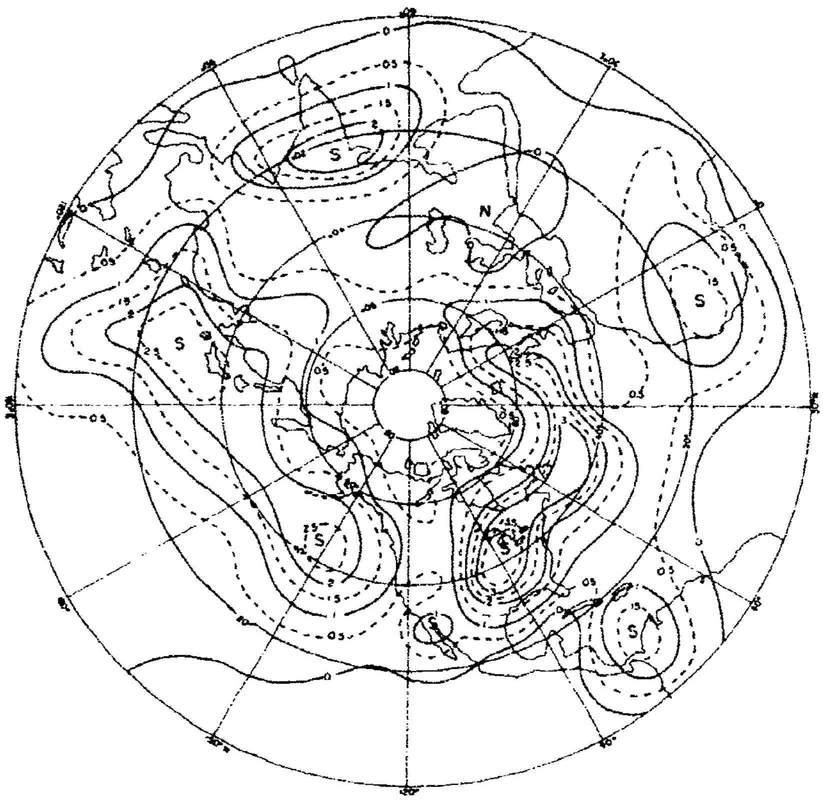

Fig. 1a. Time average of the vertically integrated meridional eddy transport of water vapor Q'_φ in 10^2 grams per centimeter per second, for the year 1950. S denotes transport from the south

of the wind field for each of the stations of the basic network at 1000, 850, 700, and 500 mb. The advantages and the limitations of the approach adopted in studies of this nature have been discussed previously by the authors.

In computing the average over the time interval τ the following expressions were used:

$$\bar{X} = \frac{1}{N}\sum_{1}^{N} X_i \tag{19}$$

$$\overline{x'y'} = \overline{xy} - \bar{x}\,\bar{y} \tag{20}$$

where x and y denote any variable q, u, v, and N is the number of observations for each station at each level in the time interval τ.

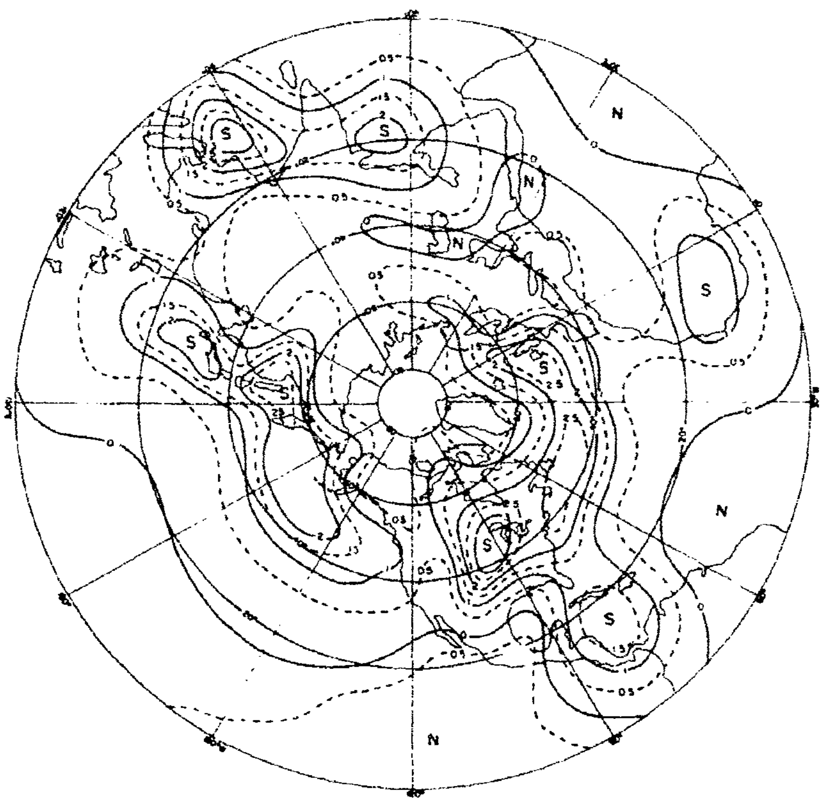

Fig. 1b. Time average of the vertically integrated meridional eddy transport of water vapor Q'_φ in 10^2 grams per centimeter per second, for the summer 1950

The vertically integrated expressions (9), (10), (15) and (16) were computed by applying the trapezoidal rule to the values at the isobaric level studied. The time means were computed for the calendar year 1950, for the summer season (April–September) and for the composite winter season. The local values of F'_φ, F'_λ and of Q'_φ and Q'_λ were plotted on hemispheric maps, together with the number of observations in order to allow a direct estimate of the reliability of each quantity. The spacial

distribution of the different quantities was obtained using the standard methods of isoline analysis. Here only the Q'_φ and Q'_λ maps are presented (Figs. 1 a, 1 b and 1 c; Figs. 2 a, 2 b and 2 c).

Zonally averaged values of Q'_φ, Q'_λ, F'_φ and F'_λ are given as follows:

$$[Q'_\varphi]_\varphi = \frac{1}{2\pi}\oint Q'_\varphi(\varphi,\lambda)\,d\lambda \approx \frac{1}{36}\sum_{\alpha=1}^{36} Q'^{\alpha}_{\varphi} \tag{21}$$

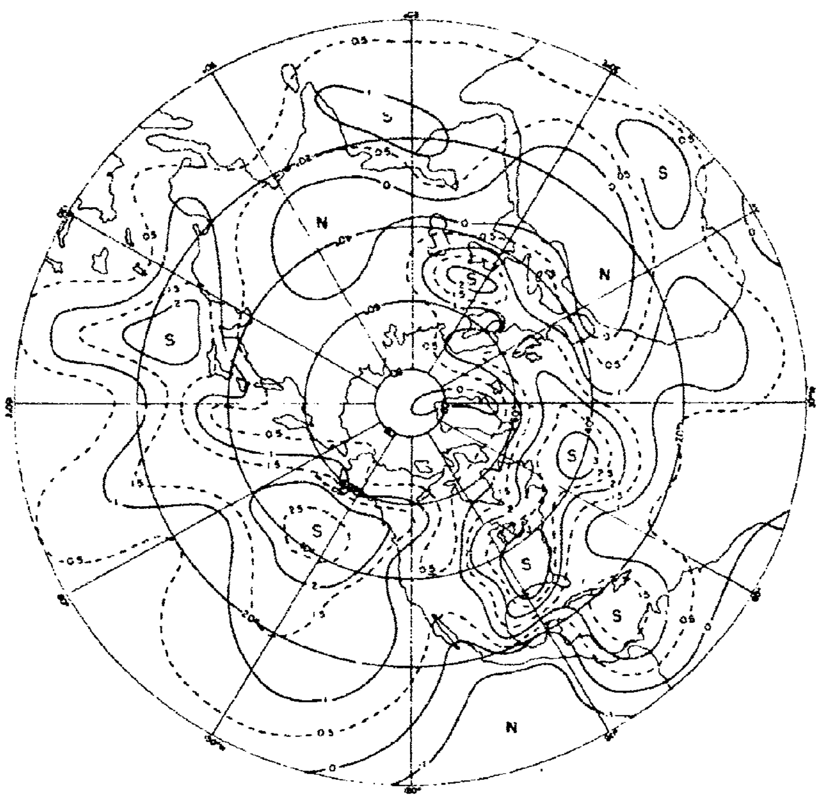

Fig. 1 c. Time average of the vertically integrated meridional eddy transport of water vapor Q'_φ in 10^2 grams per centimeter per second, for the winter 1950

$$[Q'_\lambda]_\varphi = \frac{1}{2\pi}\oint Q'_\lambda(\varphi,\lambda)\,d\lambda \approx \frac{1}{36}\sum_{\alpha=1}^{36} Q'^{\alpha}_{\lambda} \tag{22}$$

$$[F'_\varphi]_\varphi = \frac{1}{2\pi}\oint F'_\varphi(\varphi,\lambda,p)\,d\lambda \approx \frac{1}{36}\sum_{\alpha=1}^{36} F'^{\alpha}_{\varphi} \tag{23}$$

$$[F'_\lambda]_\varphi = \frac{1}{2\pi} \oint F'_\lambda (\varphi, \lambda, p)\, d\lambda \approx \frac{1}{36} \sum_{\alpha=1}^{36} F'^{\alpha}_{\lambda} \tag{24}$$

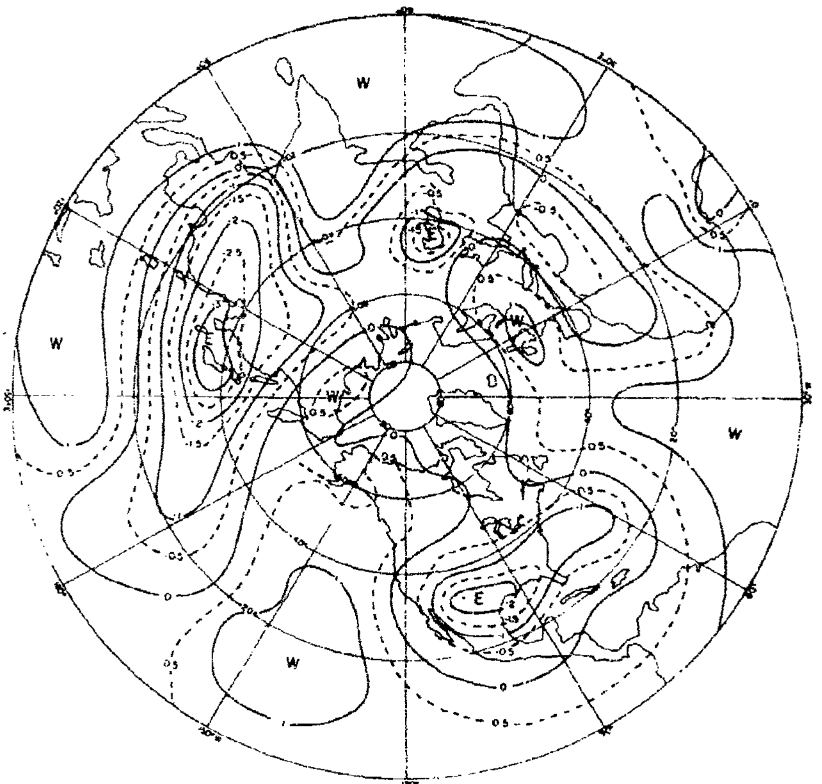

Fig. 2a. Time average of the vertically integrated zonal eddy transport of water vapor Q'_λ in 10^2 grams per centimeter per second, for the year 1950. *W* denotes transport from the west

Table 1. ***Zonally Averaged Values of the Mean Total Meridional Transient Eddy Transport of Water Vapor $[Q'_\varphi]$ in Units of 10^{11} Grams per Second for Yearly and Seasonal Data at Specified Latitudes***

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	+ 0.47	+ 1.41	+ 3.37	+ 3.73	+ 3.80	+ 3.33	+ 2.39	+ 2.04	+ 0.22
Summer	+ 0.77	+ 2.36	+ 3.96	+ 3.77	+ 3.29	+ 2.03	+ 1.92	+ 1.34	− 0.28
Year	+ 0.68	+ 2.17	+ 4.30	+ 4.68	+ 4.09	+ 2.90	+ 2.58	+ 1.44	− 0.11

Table 2. *Zonally Averaged Values of the Mean Total Meridional Standing Eddy Transport of Water Vapor* $\frac{1}{g}\int_0^{p_0} [\bar{q}^* \bar{v}^*]\, dp$ *in Units of* 10^{11} *Grams per Second for Yearly and Seasonal Data at Specified Latitudes*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	+ 0.09	+ 0.50	+ 0.70	+ 0.64	+ 0.40	+ 0.77	+ 1.29	+ 0.32	+ 0.03
Summer	+ 0.07	+ 0.02	+ 0.10	+ 0.08	+ 0.20	+ 2.18	+ 2.88	+ 0.29	+ 0.02
Year	+ 0.07	+ 0.33	+ 0.20	+ 0.16	+ 0.30	+ 1.02	+ 1.82	+ 0.24	+ 0.00

Table 3. *Zonally Averaged Values of the Mean Total Meridional Eddy Transport of Water Vapor* $[Q'_\varphi] + \frac{1}{g}\int_0^{p_0} [\bar{q}^* \bar{v}^*]\, dp$ *in Units of* 10^{11} *Grams per Second for Yearly and Seasonal Data at Specified Latitudes*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	+ 0.56	+ 1.91	+ 4.07	+ 4.37	+ 4.20	+ 4.10	+ 3.68	+ 2.36	+ 0.25
Summer	+ 0.84	+ 2.36	+ 4.03	+ 3.84	+ 3.49	+ 4.21	+ 4.80	+ 1.65	− 0.26
Year	+ 0.75	+ 2.50	+ 4.06	+ 4.84	+ 4.39	+ 3.92	+ 4.40	+ 1.60	− 0.11

Table 4. *Zonally Averaged Values of the Vertically Integrated Mean Zonal Transient Eddy Transport of Water Vapor* $[Q'_\lambda]$ *in Units of* 10^2 *Grams per Centimeter per Second for Yearly and Seasonal Data at Specified Latitudes*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	+ 0.10	+ 0.31	+ 0.25	+ 0.01	− 0.16	− 0.09	+ 0.18	+ 0.52	+ 0.69
Summer	− 0.11	+ 0.05	+ 0.12	+ 0.13	+ 0.01	− 0.12	+ 0.09	+ 0.13	− 0.06
Year	0.01	+ 0.01	− 0.01	− 0.26	− 0.56	− 0.52	+ 0.28	+ 0.96	+ 0.99

These were evaluated numerically by using the data read off from the corresponding maps at every 10-degree gridpoint of latitude and longitude. The numerical estimates of the meridional distribution are presented in Tables 1, 4, 6 and 8, and the corresponding curves are shown in Figs. 3 and 6.

Table 5. *Zonally Averaged Values of the Vertically Integrated Mean Zonal Standing Eddy Transport of Water Vapor* $\frac{1}{g}\int_0^{p_0} [\bar{q}^* \bar{u}^*]\, dp$ *in Units of* 10^2 *Grams per Centimeter per Second for Yearly and Seasonal Data at Specified Latitudes*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°
Winter	+ 0.01	+ 0.13	—	− 0.13	—	− 0.11	− 0.45	− 0.15
Summer	+ 0.04	+ 0.20	—	− 0.31	—	+ 0.07	+ 0.26	+ 0.10
Year	+ 0.07	+ 0.09	—	− 0.06	—	− 0.08	+ 0.21	+ 0.19

Table 6. *Zonally Averaged Values of the Mean Meridional Transient Eddy Transport of Water Vapor $[F'_\varphi]$ in Units of Gram Meter per Kilogram per Second for Yearly Data at Specified Latitudes. The Levels are Given in Millibars*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
1000 mb	+ 0.2	+ 1.5	+ 3.0	+ 3.6	+ 3.9	+ 3.5	+ 2.7	+ 1.3	+ 0.2
850	+ 1.3	+ 2.7	+ 3.9	+ 4.1	+ 3.5	+ 2.2	+ 1.6	+ 0.8	− 0.3
700	+ 0.9	+ 2.0	+ 2.9	+ 3.1	+ 2.7	+ 1.5	+ 0.8	+ 0.6	+ 0.3
500	+ 0.3	+ 0.6	+ 0.9	+ 1.0	+ 0.9	+ 0.7	+ 0.5	+ 0.2	− 0.4

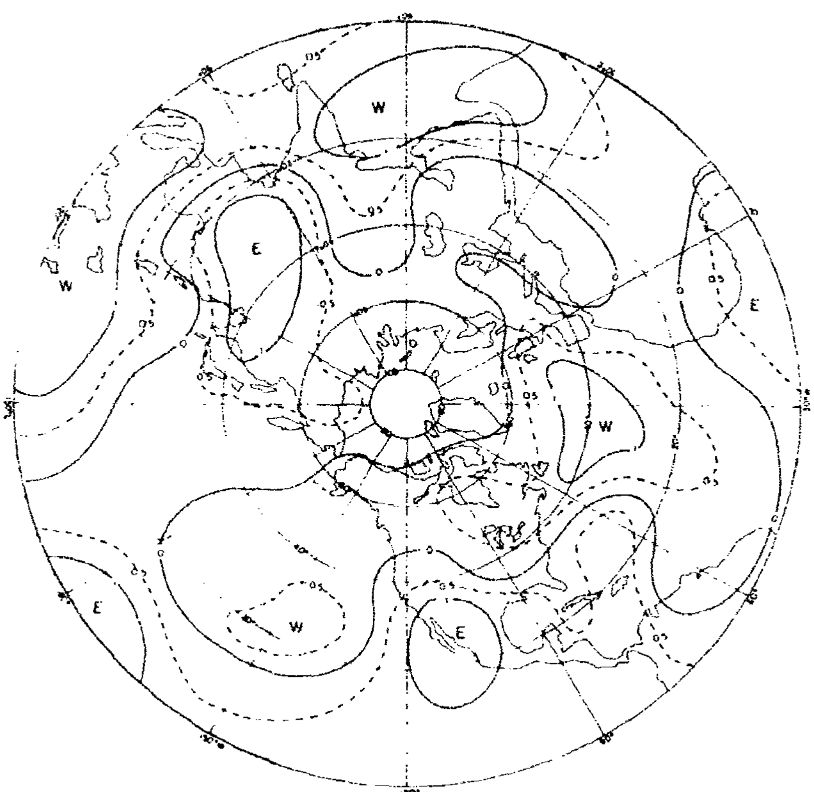

Fig. 2b. Time average of the vertically integrated zonal eddy transport of water vapor Q'_λ in 10^2 grams per centimeter per second, for the summer 1950

Table 7. *Zonally Averaged Values of the Mean Meridional Standing Eddy Transport of Water Vapor $[q^* v^*]$ in Units of Gram Meter per Kilogram per Second for Yearly Data at Specified Latitudes. The Levels are Given in Millibars*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
1000 mb	+ 0.3	+ 0.7	+ 0.5	+ 0.2	+ 0.4	+ 0.8	+ 1.1	+ 0.4	+ 0.4
850	+ 0.2	+ 0.3	+ 0.3	+ 0.3	+ 0.6	+ 1.0	+ 1.6	+ 0.5	+ 0.2
700	+ 0.0	+ 0.4	+ 0.3	+ 0.1	+ 0.2	+ 0.4	+ 0.8	+ 0.5	+ 0.0
500	+ 0.1	+ 0.0	+ 0.0	+ 0.0	+ 0.1	+ 0.2	+ 0.4	+ 0.2	+ 0.0

Table 8. *Zonally Averaged Values of the Mean Zonal Transient Eddy Transport of Water Vapor [F'_λ] in Units of Gram Meter per Kilogram per Second for Yearly Data at Specified Latitudes. The Levels are Given in Millibars*

Latitude	65°	55°	45°	35°	25°	15°	5°
1000 mb	— 0.3	— 0.0	— 0.5	— 0.3	+ 1.0	+ 1.6	+ 0.7
850	+ 0.4	+ 0.5	— 0.7	— 0.9	+ 0.3	+ 1.5	+ 1.2
700	+ 0.0	+ 0.0	— 0.7	— 1.8	— 1.0	+ 0.9	+ 2.2
500	+ 0.7	+ 0.6	— 0.4	— 1.2	— 1.4	— 0.6	+ 0.2

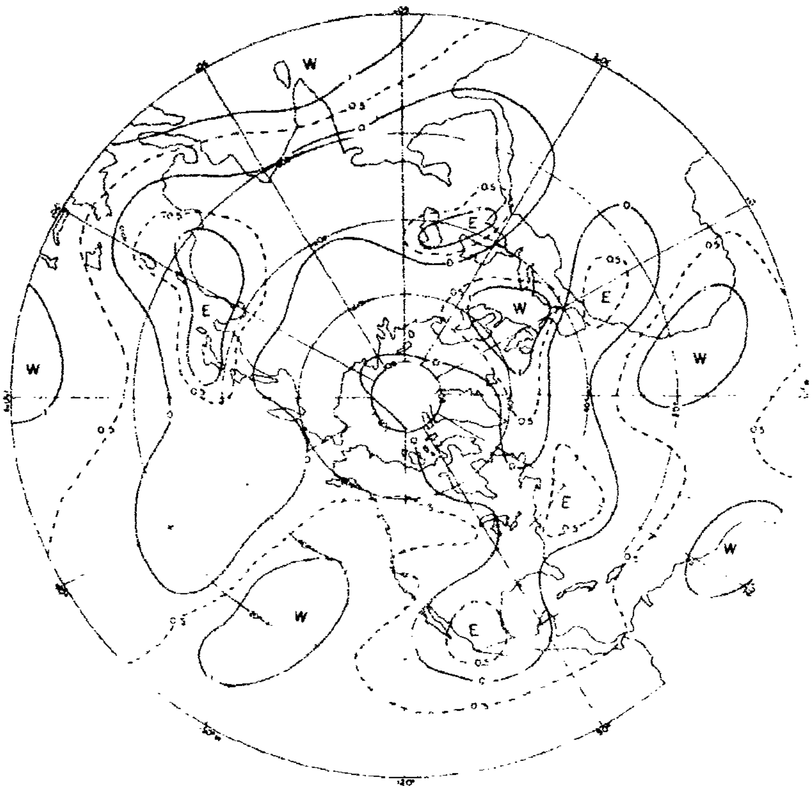

Fig. 2c. Time average of the vertically integrated zonal eddy transport of water vapor Q'_λ in 10^2 grams per centimeter per second, for the winter 1950

Table 9. *Zonally Averaged Values of the Mean Zonal Standing Eddy Transport of Water Vapor [$\bar{q}^* \bar{u}^*$] in Units of Gram Meter per Kilogram per Second for Yearly Data at Specified Latitudes. The Levels are Given in Millibars*

Latitude	70°	60°	45°	30°	20°	10°
1000 mb	+ 0.2	+ 0.2	+ 0.1	— 0.4	+ 0.8	+ 0.4
850	+ 0.2	+ 0.2	+ 0.2	— 0.4	+ 0.8	+ 0.4
700	+ 0.2	+ 0.2	— 0.4	+ 0.0	+ 0.2	+ 0.5
500	+ 0.0	+ 0.1	— 0.1	+ 0.4	0.0	+ 0.2

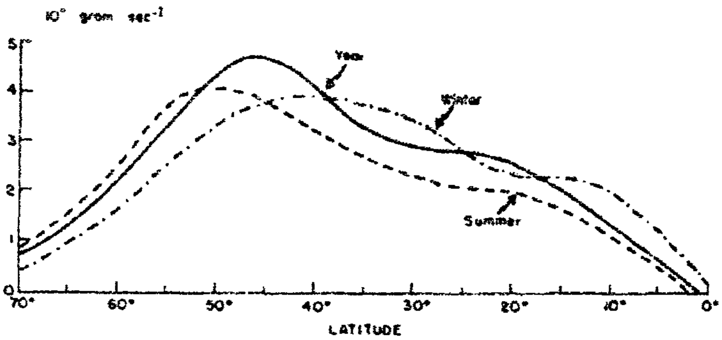

Fig. 3. The meridional distribution of the meridional water vapor flux in the atmosphere associated with transient eddies, computed from atmospheric data. The units are 10^{11} gm sec^{-1}

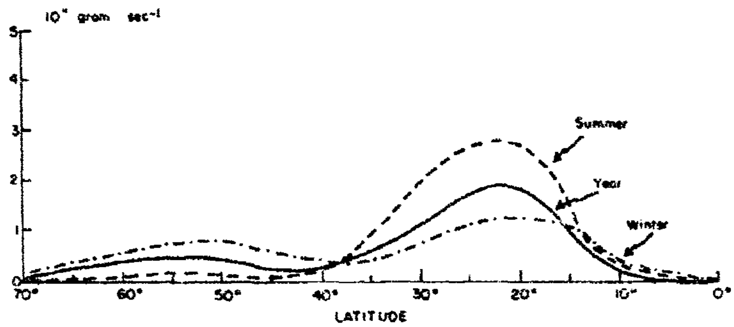

Fig. 4. The meridional distribution of the meridional water vapor flux in the atmosphere associated with standing eddies, computed from atmospheric data. The units are 10^{11} gm sec^{-1}

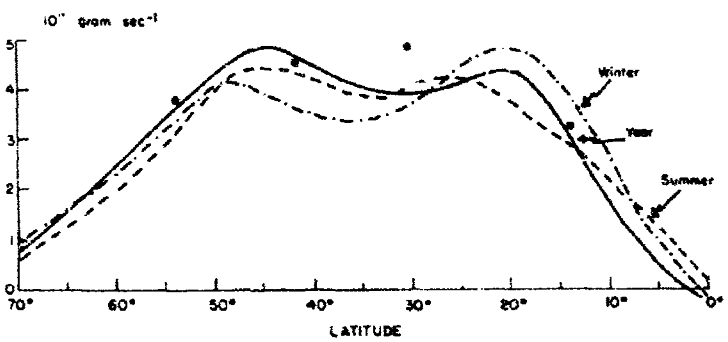

Fig. 5. The meridional distribution of the total meridional eddy flux of water vapor in the atmosphere. The units are 10^{11} gm sec^{-1}. The dots represent the estimates of the total meridional eddy flux across specific latitude circles obtained by STARR and WHITE [15, 16]

Table 10. *Zonally Averaged Values of Mean Meridional Transport of Water Vapor $[\bar{F}_\varphi]$ in Units of Gram Meter per Kilogram per Second for Yearly Data at Specified Latitudes. The Levels are Given in Millibars*

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
1000 mb	+ 1.0	+ 3.0	+ 6.0	+ 6.9	+ 5.4	— 1.6	— 12.7	— 6.4	+ 1.0
850	— 0.4	+ 2.6	+ 6.4	+ 6.5	+ 6.3	+ 0.7	— 2.9	— 1.2	+ 0.4
700	+ 0.4	+ 2.6	+ 4.8	+ 4.8	+ 4.4	+ 2.0	+ 0.3	— 2.4	— 0.8
500	— 0.1	+ 0.7	+ 1.1	+ 1.0	+ 0.9	+ 0.9	+ 0.6	— 0.1	— 0.5

Table 11. *Zonally Averaged Values of Mean Zonal Transport of Water Vapor $[\bar{F}_\lambda]$ in Units of Gram Meter per Kilogram per Second for Yearly Data at Specified Latitudes. The Levels are Given in Millibars*

Latitude	65°	55°	45°	35°	25°	15°	5°
1000 mb	+ 0.9	+ 7.9	+ 8.5	— 0.4	— 14.1	— 37.4	— 41.0
850	+ 4.9	+ 13.7	+ 20.4	+ 13.8	— 0.4	— 21.1	— 33.4
700	+ 4.0	+ 10.8	+ 16.3	+ 15.5	+ 7.1	— 9.5	— 23.9
500	+ 1.7	+ 5.1	+ 8.6	+ 9.1	+ 6.1	— 2.3	— 7.6

Table 12. *Zonally Averaged Values of the Total Water Vapor Transport Across Latitude Circles $[Q_\varphi]$ for the Northern Hemisphere in Units of 10^{11} Grams per Second. The Lower Number Give the Component of the Total Due to Mean Meridional Cells*

Latitude	80°	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	— 0.10	+ 0.64	+ 2.24	+ 4.48	+ 5.04	+ 5.36	+ 3.81	— 3.68	— 14.36	— 9.32
		+ 0.08	+ 0.33	+ 0.41	+ 0.67	+ 1.16	— 0.29	— 7.36	— 16.72	— 9.57
Summer	— 0.12	+ 0.48	+ 2.44	+ 6.80	+ 6.92	+ 5.64	+ 2.11	— 1.84	+ 1.92	+ 9.08
		— 0.36	+ 0.08	+ 2.77	+ 3.08	+ 2.15	— 2.10	— 6.64	+ 0.27	+ 9.34
Year	— 0.11	+ 0.54	+ 2.36	+ 5.32	+ 5.80	+ 5.46	+ 2.91	— 2.70	— 6.16	+ 0.00
		— 0.21	— 0.14	+ 1.26	+ 0.96	+ 1.07	— 1.01	— 7.10	— 7.76	+ 0.11

Using the gridpoint values of $\bar{q}$ (PEIXOTO [4]) and of $\bar{u}$ and $\bar{v}$ (BUCH [2]) for each latitude circle, the zonally averaged mean meridional and zonal transports of water vapor due to standing eddies were computed using the expression:

$$[\bar{q}^* \bar{v}^*]_\varphi = \frac{1}{36} \left\{ \sum_{\alpha=1}^{36} \bar{q}_\alpha \bar{v}_\alpha - \frac{1}{36} \sum_{\alpha=1}^{36} \bar{q}_\alpha \sum_{\alpha=1}^{36} \bar{v}_\alpha \right\} \tag{25}$$

and a similar expression for the mean zonal standing eddy transport.

The results obtained, as well as the total vertically integrated values, are presented in Tables 2, 5, 7 and 9 and in Figs. 4 and 7. The values of the mean zonal total meridional eddy flux of water vapor are shown in Table 3 and in Fig. 5 in units of 10^{11} grams per second for winter,

summer and for the whole year. The values of $\frac{1}{g}\int_0^{p_0} [\bar{q}^* \bar{v}^*]\, dp$ in units of 10^2 grams per centimeter per second for yearly and seasonal data are presented in Table 5 and in Fig. 7.

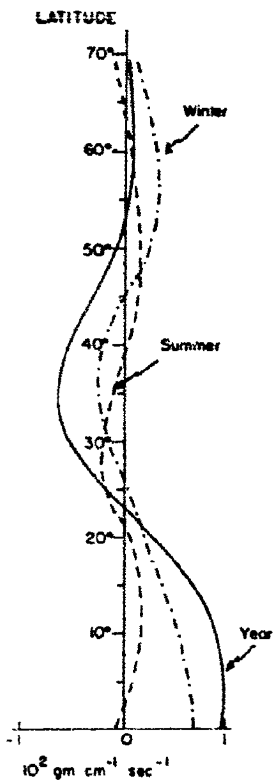

Fig. 6

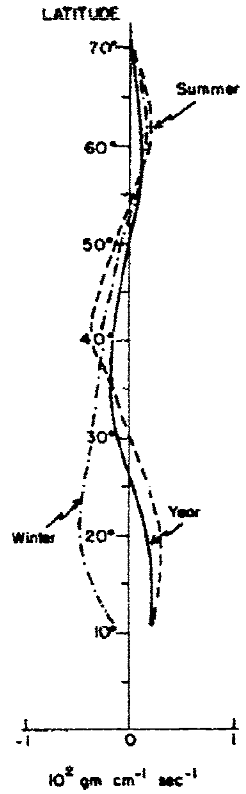

Fig. 7

Fig. 6. The meridional distribution of the mean zonal flux of water vapor associated with transient eddies, computed from atmospheric data. The units are $10^2\,\mathrm{gm\,cm^{-1}\,sec^{-1}}$ and positive values indicate eastward flux

Fig. 7. The meridional distribution of the mean zonal flux of water vapor associated with standing eddies. The units are $10^2\,\mathrm{gm\,cm^{-1}\,sec^{-1}}$ and positive values indicate eastward flux

Tables 10 and 11 giving the zonally averaged values of the mean meridional and zonal components of the transport field of water vapor at different levels, $[\bar{F}_\varphi]$ and $[\bar{F}_\lambda]$ are also included in the present paper, because they have not been published before, and will be needed in the present discussion.

4. Results

An inspection of Figs. 1 a, 1 b and 1 c and Figs. 2 a, 2 b and 2 c shows immediately the lack of zonal symmetry of the distribution of the total mean eddy transport field of water vapor.

As was previously mentioned the F'_φ and Q'_φ fields account only for the meridional transport accomplished by the *transient* eddies. The meridional eddy flux is predominently positive (from South) over the northern hemisphere. There are nevertheless isolated centers of negative eddy flux (from North), but the influence of physiographic factors is very clear (Mediterranean Sea, Black Sea, Caspean Sea, etc.). The most prominent feature in the hemispheric analysis of the maps is the existence of a belt of maxima located in middle latitudes. These maxima are clearly associated with the mean position of the polar front, as is to be expected in view of the role of the baroclinic perturbations in the eddy meridional transport processes. There are other maxima over the intertropical region which are associated with the perturbations in the intertropical convergence zone, and which are in general more intense during the summer. It is interesting to note the good agreement of the present analysis with the corresponding analysis for the North American sector obtained by BENTON and ESTOQUE [1], for the calendar year 1949.

The zonally averaged values of Q'_φ presented in Table 1 and Fig. 3, summarize the main characteristics of the mean meridional eddy flux. The values are predominantly positive in all cases, and the maximum shifts northward during the summer. The yearly maximum which occurs near latitude 47.5° exceeds the winter and the summer maxima, as was to be expected because the total covariance of the yearly values, assuming that the summer and winter populations are equal, is given by

$$\overline{\overline{q'}\,\overline{v'}}_y = \frac{1}{2}\left\{\overline{q'v'}_w + \overline{q'v'}_s + \frac{1}{2}\,(\bar{v}_w - \bar{v}_s)\,(\bar{q}_w - \bar{q}_s)\right\} \tag{26}$$

where the indices y, w and s denote yearly, winter and summer values respectively.

The distribution of $[F'_\varphi]$ with pressure and latitude for the year presented in Table 6, shows that the values are significantly positive, with two minor exceptions, and reaches a maximum in the middle latitude regions near the 850 mb level.

From a comparison of the maps for Q'_φ and those for $\bar{Q}_\varphi$ (STARR, PEIXOTO and LIVADAS [13]) it is seen that in the middle latitudes the maxima are almost coincident. The relative importance of the contribution to the total meridional transport by the transient eddies may be inferred from the data presented in previous papers by the writers and from the data given in Tables 1, 6 and 10 of the present study. From these results it appears that the meridional transient eddy transport forms the major part of the total meridional transport of water vapor in the middle latitude regions.

The zonally averaged values of the mean total meridional eddy flux of water vapor $\frac{1}{g}\int_0^{p_0} [\bar{q}^* \bar{v}^*]\,dp$ associated with the quasi-permanent features of the atmospheric circulation, as given in Table 2 and in Fig. 4, are always positive (northward) and in general smaller than the corresponding values of $[Q'_\varphi]$ associated with the transient eddies. The values obtained for the different latitudes present a well defined maximum at 20°N, associated with the semi-permanent subtropical anticyclones and another maximum not so well defined and much less intense around 55°N associated with the semi-permanent lows prevailing in this region.

The vertical distribution of the meridional standing eddy flux of water vapor $[\bar{q}^*\bar{v}^*]$ given in Table 7, shows that one maximum occurs in the neighborhood of 22.5°N at 850 mb and another around 55°N near the surface. It is interesting to point out that the lowest values occur at 45°N, where the largest values of $[F'_\varphi]$ are observed (Table 6).

The zonally averaged values of the mean total meridional eddy flux of water vapor shown in Table 3 and in Fig. 5, are predominantly positive (northward) in agreement with the well known fact that winds from the south generally contain more moisture than the winds from the north. The latitudinal distribution presents two maxima, resulting from the combination of the latitudinal distribution due to the transient and to the standing eddies. The present yearly values compare well with those computed by STARR and WHITE [*15*, *16*] for five latitude circles, using a different approach and another type of expansion. The discrepancy shown in Fig. 5 at 31°N may be due to the unavoidable difficulties inherent in the sampling process used by STARR and WHITE.

In view of the good agreement obtained by the two methods at other latitudes, the discrepancy at 31°N causes one to suspect a numerical error. Although considerable checking was performed, no such error has been located as yet.

It may be seen that the effect of the standing eddies is of greatest significance in low latitudes ($\varphi = 20°$) where the quasi-stationary disturbances are dominant. During the summer at this latitude, they are clearly the most important factor in transporting moisture northward, the effect of the transient disturbances being small. At middle latitudes ($\varphi = 45°$) the vigorous transient eddies predominate, and the standing eddies play a minor role. However, at 60°N their importance increases again, especially in winter when the semi-permanent lows are most intense.

The maps of the mean zonal transient eddy transport of water vapor Q'_λ (Figs. 2 a, 2 b and 2 c) show very little similarity with the corresponding maps for the total zonal transport $\bar{Q}_\lambda$ (STARR and PEIXOTO [*12*]). The zonally averaged values of Q'_λ presented in Table 4 and in Fig. 6, when compared with the values of $[\bar{Q}_\lambda]$ previously evaluated by the writers (1960), show that the mean zonal transient eddy flux of water vapor accounts for only a small percentage of the total zonal transport.

The latitudinal distribution of the zonal eddy component of the transport field of moisture associated with the standing eddies, $\frac{1}{g}\int_0^{p_0}[\bar{q}^*\bar{u}^*]\,dp$ as shown in Fig. 7, does not differ substantially from the corresponding distribution of $[Q'_\lambda]$. The values are alternately positive (eastward) and negative (westward) in both cases, which is related mainly to the major air mass movements and to the geographical features of the globe.

Inspection of Tables 8 and 9 with the vertical distribution of the eddy zonal transport, when compared with the corresponding estimates in Table 11 leads to the same conclusion. The absolute values of the zonal eddy transport are in general smaller than the corresponding meridional eddy components. The zonal eddy transport of water vapor is very small when compared with the total zonal flow. The total zonal flux can be almost entirely explained by the transport of the mean humidity distribution by the mean zonal wind, in striking contrast to the situation of the total meridional transport, where the eddies constitute the most important factor in the total transfer process in middle latitudes.

5. Some Scientific Implications of the Results

The relative importance of the contribution of the eddies to the total transfer of water vapor in the meridional and in the zonal directions shows that the hypothesis of local horizontal isotropy must be taken with reserve in studies of the large scale motions of the atmosphere.

When maps of the total eddy transport vector $\vec{Q}' = Q'_\lambda \vec{i}_\lambda + Q'_\varphi \vec{j}_\varphi$ are compared with the maps of the precipitable water, $\overline{W}$ (Starr, Peixoto, Livadas [*13*]), it is evident that there is little correspondence between regions of mean eddy outflow and regions of high mean water vapor content. Similar conclusions arise when vector eddy transport fields, defined at each point by $\vec{F}' = F'_\lambda \vec{i}_\lambda + F'_\varphi \vec{j}_\varphi$ are compared with the distributions of the mean specific humidity $\bar{q}$ at various isobaric levels (Peixoto [*4*]). On a broader planetary scale this fact becomes still more evident in comparing the zonally averaged values of the mean total meridional eddy transport, as given in Table 3 or in Fig. 5, with the latitudinal distribution of the zonally averaged precipitable water vapor content $[\overline{W}]_\varphi$ in the atmosphere (Starr, Peixoto and Livadas [*13*]). Furthermore the bimodal distribution of the zonally averaged transient plus standing eddy transport does not seem to bear any simple relation to the monotonic decreasing function $[\overline{W}]_\varphi$ or to its derivatives. Thus, it is evident that the "Austausch-Koeffizient" and the mixing-length formalisms are not adequate for describing or explaining the large scale "turbulent flow" of water vapor in the atmosphere, a conclusion in agreement with those found by Starr *et al.* in studying the behaviour of other atmospheric quantities.

As appears from inspection of the expression (18), the comparison of the total meridional transfer of water vapor as given by Starr, Peixoto and Livadas [13] and reproduced here in Table 12 and Fig. 8, with the actual values of the total meridional eddy flux presented in Table 3 and in Fig. 5, leads to an estimate of the contribution of the so-called meridional

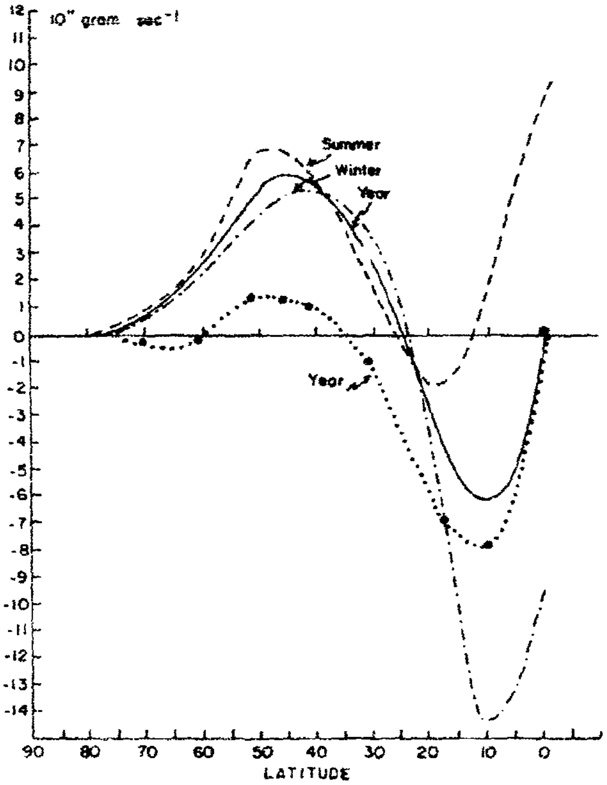

Fig. 8. The meridional distribution of the total water vapor transport across latitude circles in the atmosphere. The dotted curve represents the component of the total due to mean meridional cells. The units are 10^{11} gm sec^{-1}

cell component to the total meridional transport of water vapor, also given in Table 12 and Fig. 8 for the year. This comparison offers the important indirect evidence of the existence of the three-cell regime with two direct cells and one indirect cell. The contribution of the Hadley cell for the total southward transport of water vapor becomes dominant, whereas the contribution of the other two cells is only a small fraction of the total meridional flux of water vapor, the eddies being the more important factor in the total process.

The southward transport of moisture in the tropical and equatorial regions over the northern hemisphere, has in the past been estimated from climatological considerations by comparing the excess of precipitation over evaporation. These rough evaluations have been taken by some authors as a measure of the intensity of the mean tropical meridional cell and thereafter used as a general argument to claim the over-all predominance of the HADLEY regime in the dynamics of the general circulation of the atmosphere. Although these arguments in their extreme form are no longer generally credited because of the demonstrated effectiveness of eddy processes, the classical schemes for the general circulation cannot be salvaged by these arguments.

It should be stressed in this connection that indirect estimates of the character and intensity of the tropical mean meridional cell by these means is beset by serious difficulties and requires the introduction of assumptions whose validity is hard to prove. On the other hand the direct objective calculations give moisture transports compatible with available climatological information, while the same wind data give a momentum and energy balance which involves a rather small cellular contribution for the hemisphere. This is corroborated by the same general result for the energy cycle in the southern hemisphere as obtained from preliminary studies.

The contention that the actual wind data overlook a large equatorward transport of mass within the first kilometer or two above the surface is not corroborated by estimates and checks made by us from time to time during the progress of various calculations. It should be stressed that such net mass transports cannot be estimated alone from the notion of cross-isobar air flow due to friction, the theory for which is itself grossly deficient for this purpose. These cross-latitude flows may and probably do involve geostrophic and gradient wind contributions, besides deviations from these, other than frictional ones.

The reason why the tropical mean meridional circulation as contained in our data gives a net transport of moisture equatorward and is not of much effect in causing a net transport of angular momentum lies in the high concentration of moisture in the low layers. This effect seemingly has served to delude generations of meteorologists into the pursuit of general circulation schemes which, to say the least, missed perhaps the most important other processes as we now know, namely the negative eddy viscosity phenomena so to speak. The essentials of these alternative ideas in our writings are contained in STARR [*8*, *9*, *10*] and in PEIXOTO [*4*, *7*]. In addition a summary of various results for both hemispheres is to be found in OBASI [*3*].

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Hemispheric water balance for the IGY

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ABSTRACT

A study of the hemispheric water balance over the northern hemisphere during the IGY covering the mean conditions for the calendar year 1958 is presented. The study includes analyses of the amount of precipitable water, of the vertically integrated water vapor transport vector field and of the divergence of water vapor transport for the hemisphere. Some implications of the water vapor divergence field are deduced which are important for hydrology and oceanography. Finally, the water vapor balance is discussed in the light of various meteorological considerations.

1. Introduction

The impetus of the modern approach to studies of the general circulation of the atmosphere as outlined for example by STARR (1951) resulted *inter alia* in a number of extensive investigations of the northern hemisphere water balance and its relation to the general circulation. The more important of these are STARR & WHITE (1955), STARR, PEIXOTO & LIVADAS (1958), STARR & PEIXOTO (1958), PEIXOTO (1958, 1960) and STARR & PEIXOTO (1964). All these studies were based upon aerological data for the year 1950 and included various evaluations from 90 daily upper-air sounding stations at several levels up to 500 mb over the entire northern hemisphere. Encouraged by the results and by the continuing growth of the hemispheric network of upper-air sounding stations, the authors of this paper have extended the studies for the IGY year 1958.

2. Formulation of the problem

Since the formulation of the problem and the procedures followed in the present paper are the same as those described in the previous study by STARR & PEIXOTO (1958), it seems sufficient to present only a general review of the approach followed. The basic quantities used in this study are the specific humidity, q, the eastward wind component u and the northward component v, the total wind being $\mathbf{V}$.

To a high degree of accuracy the atmosphere may be considered in a state of hydrostatic equilibrium so the pressure p is taken as the vertical coordinate. Thus a coordinate system (λ, ϕ, p, t) is used in which λ denotes the longitude, ϕ the latitude and t the time. The precipitable water contained in a unit column of air at a given instant above a point on the earth's surface is expressed by

$$W(\lambda, \phi, t) = \frac{1}{g}\int_0^{p_s} q\,dp, \qquad (1)$$

where g is the acceleration of gravity and p_s the mean value of the surface pressure. The total horizontal transport of water vapor above a point on the earth's surface defines a two-dimensional vector field, $\mathbf{Q}(\lambda, \phi, t)$, expressed by

$$\mathbf{Q}(\lambda, \phi, t) = \frac{1}{g}\int_0^{p_s} q\mathbf{V}\,dp. \qquad (2)$$

The zonal and meridional components of the vector field are given by

$$Q_\lambda = \frac{1}{g}\int_0^{p_s} qu\,dp, \qquad (3)$$

$$Q_\phi = \frac{1}{g}\int_0^{p_s} qv\,dp. \qquad (4)$$

Expressions (1), (2), (3) and (4) may be averaged with respect to time over the interval τ, leading

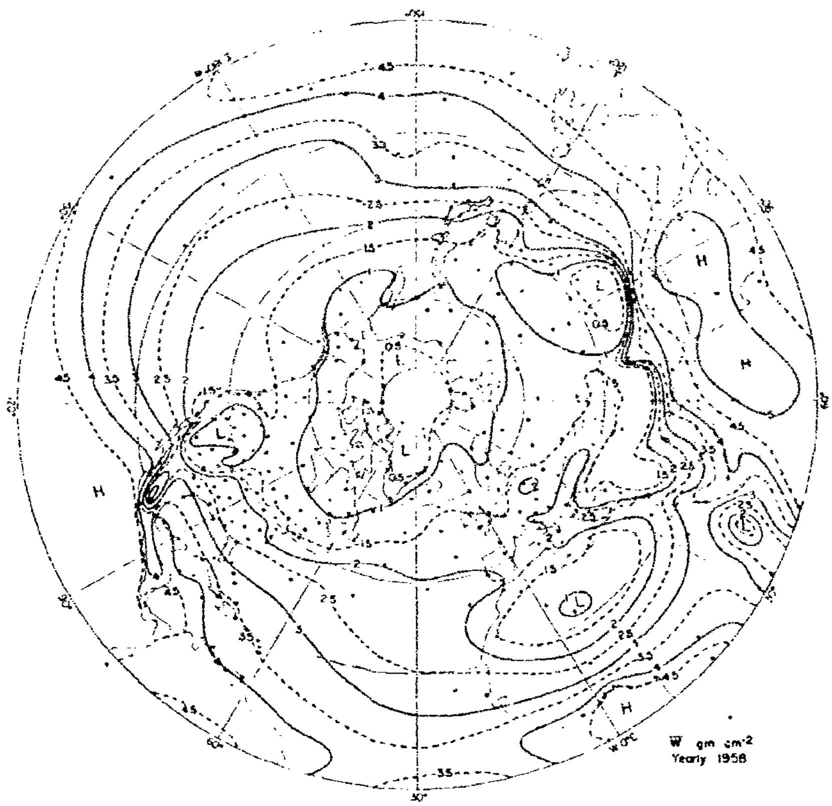

FIG. 1. Time average of the vertically integrated values of specific humidity (precipitable water) W, in gm per cm² for the year 1958. Isoline spacing (full curves) 1 gm cm^{-2}. The dots indicate the distribution of primary stations used in the present investigation.

to the corresponding mean values $\overline{W}$, $\overline{\mathbf{Q}}$, $\overline{Q}_\lambda$, $\overline{Q}_\phi$, where the bar denotes the operator

$$\overline{(\ \ )} = \frac{1}{\tau}\int_0^\tau (\ \ )\,dt. \qquad (5)$$

In this study τ represents the calendar year 1958. The divergence of the yearly mean field $\nabla \cdot \overline{\mathbf{Q}}(\lambda, \phi)$ is given in the (λ, ϕ, p) coordinate system by the expression

$$\Delta \cdot \overline{\mathbf{Q}}(\lambda, \phi) = \frac{1}{R\cos\phi}\left[\frac{\partial \overline{Q}_\lambda}{\partial \lambda} + \frac{\partial}{\partial \phi}(\overline{Q}_\phi \cos\phi)\right], \qquad (6)$$

where R denotes the radius of the earth.

For a unit column of air extending from the earth's surface (pressure, p_0) at each point to the top of the atmosphere (pressure, $p = 0$), the water vapor balance equation can be written

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathbf{Q} = \Sigma, \qquad (7)$$

where Σ represents the net sources of water substance in the atmospheric column. The sources and sinks of water vapor in the atmosphere are due primarily to evaporation, E, and to precipitation P. The transport of water in the solid or liquid phases is very small compared with the flux of water vapor in the atmosphere. For all practical purposes Σ is given by the excess of evaporation over precipitation, $E-P$. Thus, taking the time average for the given time interval (one year), the equation for atmospheric water vapor balance becomes,

$$\frac{1}{R\cos\phi}\left[\frac{\partial \overline{Q}_\lambda}{\partial \lambda} + \frac{\partial}{\partial \phi}(\overline{Q}_\phi \cos\phi)\right] = \overline{\Sigma} \equiv (\overline{E} - \overline{P}) \qquad (8)$$

because for this time interval $\partial W/\partial t$ may be taken as zero. Positive values of divergence show areas where the total evaporation exceeds the precipitation whereas negative values show areas where the total evaporation is exceeded by the precipitation.

3. Data and procedures

The basic data used in this study were taken directly from aerological observations made during the calendar year 1958. An extensive coverage of 321 selected weather stations, indicated by dots in Fig. 1 provided the data over the northern hemisphere. Where a choice was possible, the most reliable and meteorologically representative stations were selected. In areas where observations were sparse, all available data were used. The total of 321 stations was separated into 285 primary and 36 secondary stations. The upper-air data for the primary stations were obtained on punched cards or magnetic tape, while the secondary ones were taken from IGY microcards. All these data were checked and processed by electronic means. All rawinsonde data available for each primary station were used; rawinsonde data for most stations were available at least once each day. A majority of these stations provided two soundings each day, some three and even four. Statistical computations were based upon all the data available at each station. The data handling and machine processing were accomplished by the Air Weather Service Climatic Center, at Asheville, North Carolina.

The secondary stations were used principally in critical areas not covered by the primary station network and also a few of them were chosen to fill in gaps at the equatorial border. The data from these stations were obtained with either radiosonde, radio-wind, pilot balloon, rawinsonde or a combination of these methods. Although only seven pilot balloon stations were used, in general the corresponding data were not so reliable as those from the primary stations.

In spite of generally excellent coverage over the northern hemisphere and near the equatorial border in the southern hemisphere, there were some areas of little or no data; the Amazon River Basin in South America, the eastern Pacific Ocean from Central America to the Hawaiian Islands and the Indian Ocean.

The over-all coverage of reliable data over the Arctic and middle latitudes in the northern hemisphere was excellent. The data from arctic stations were fairly complete up to 80 degrees latitude. The coverage over North America was especially dense over the United States; all stations in this area were used except a few superfluous ones. The good coverage over China, Mongolia and especially the Tibetan plateau was most helpful.

The procedures and the methodology of the several computations were presented and discussed on several occasions by the writers. Briefly, the yearly mean values $\bar{q}$, $\overline{qu}$, and $\overline{qv}$ were computed for each station at the four standard pressure surfaces of 1000, 850, 700 and 500 mb. The vertical integrations required to compute $\bar{W}$, $\bar{Q}_\lambda$ and $\bar{Q}_\phi$ were performed numerically applying the trapezoidal rule. Contributions to the vertical integrals were disregarded above 500 mb and between 1000 mb and the surface and the various integrated fields are in some cases underestimated.

The values of specific humidity are, in general, small above 500 mb over middle and high latitude regions. Although the wind speeds are generally high, the water vapor transports remain relatively small. However, these contributions are likely to be greater in the tropical and equatorial regions and over extensive areas of high terrain. The contribution of higher layers has already been taken into consideration by the writers in studying the humidity conditions over Africa.

As mentioned the lower boundary was set at the 1000-mb pressure surface whenever possible. In cases where the mean surface pressure, p_0, for the yearly period is greater than 1000 mb, this procedure underestimates the total vertically integrated values. It was found in previous studies that, with the exception of tropical areas, the contribution of the thin layer between 1000 mb and the surface was of little relative significance for the total integrated values. The largest differences probably occur over the trade wind regions, where low-level humidities are high. In cases where the mean surface pressure is less than 1000 mb or where the surface topography normally extends above the 1000-mb surface, the actual surface values of humidity and wind were used.

The yearly mean values of $\bar{W}$, $\bar{Q}_\lambda$, and $\bar{Q}_\phi$ for each station were plotted on separate charts

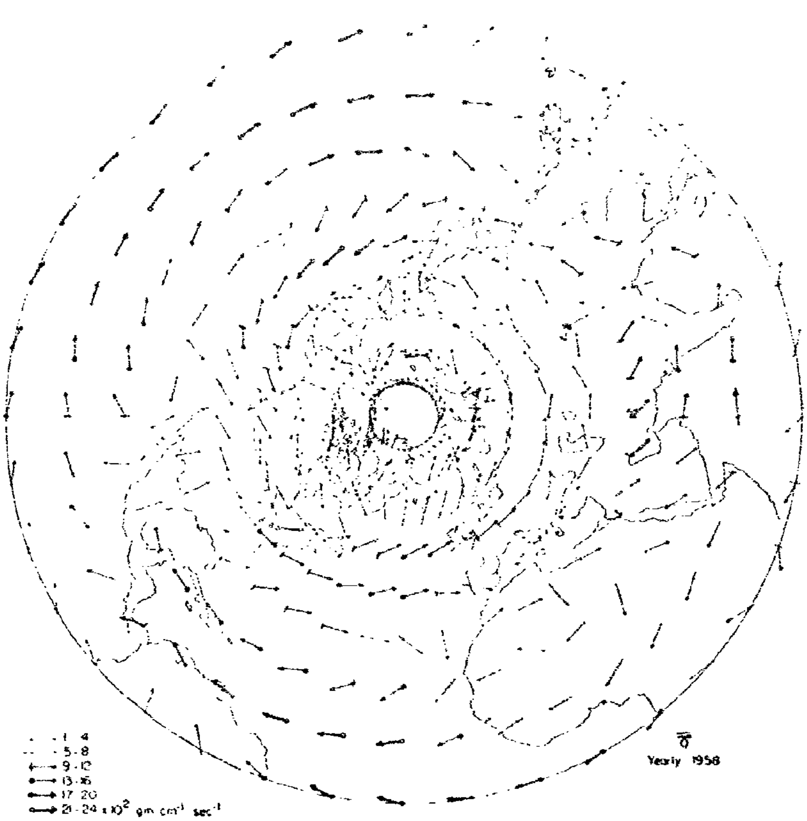

FIG. 2. Distribution of the vertically integrated moisture transport vector field averaged for the year 1958.

using polar stereographic projection maps and the corresponding fields were analysed using standard procedures. A five-degree latitude longitude grid was used to extract the corresponding gridpoint values. From these the total mean horizontal vector field of water vapor transport $\bar{\mathbf{Q}}(\lambda, \phi)$ was computed. The horizontal divergence $\nabla \cdot \bar{\mathbf{Q}}(\lambda, \phi)$ was calculated by finite difference methods using the expression (6).

4. Analysis and interpretation of results

The spatial distribution of the mean precipitable water vapor content, $\bar{W}$, is represented in Fig. 1. In general, there is a continuous decrease of precipitable water vapor content from the equator to the north pole. The maritime and continental influences are evident. The Sahara, the desert areas of the Middle East south of the Caspian Sea, and north of Tibet are dry. In addition, the effects of high terrain are illustrated by the very dry areas (less than 1.0 gm cm^{-2}) over the western United States, central Mexico, the Himalayas and the plateaus of Tibet and Central Asia and Central Africa.

Over the western portions of the subtropical oceanic anticyclones the water vapor content is generally higher than over the eastern portions, as is evident in the Pacific. This agrees with the concept of general convergence and divergence, respectively, in the western and eastern portions of these semi-permanent, large-scale features of the general circulation. The areas of highest water vapor content are the equatorial region of South America, the equatorial eastern and western Pacific Ocean, the Indian Ocean (especially south and east of India, including the Bay of Bengal) and equatorial West Africa. The driest area is in the Arctic, where the yearly mean precipitable water vapor content is less than 0.5 gm cm^{-2} north of 80°N. The 1.0 gm cm^{-2} isoline is found generally at or near 60°N. It dips south of 60°N over the regions of most frequent outbreaks of cold, dry polar continen-

tal air (eastern Siberia, the Bering Sea, Hudson Bay, etc.), and it extends slightly north of 70°N over Jan Mayen and northeast of Iceland, due to the Gulf Stream and the moist air masses frequently carried northeastward across the North Atlantic.

It must be pointed out that once again our analysis shows the mean water vapor storage in the atmosphere to be very small. The analysis of the mean precipitable water vapor content provides more detail and accuracy than heretofore available. Studies of the precipitable water vapor have important application to investigations of the radiation and heat balance in the atmosphere. Many specific applications of infrared radiation technology, however, require instantaneous information concerning atmospheric moisture.

A chart showing the total mean horizontal transport of water, $\vec{Q}$, in vector form is given in Fig. 2. This chart gives a general idea of the main features of the mean total transport of water vapor in the atmosphere. It shows good agreement with a similar one published previously by STARR & PEIXOTO (1958) and also supports their conclusion that the net moisture flow across the equator for the year is practically zero.

5. Water vapor balance

STARR & PEIXOTO (1958) calculated the mean E-P field over the northern hemisphere for the year 1950, inferred from the horizontal divergence of the water vapor transport using a ten-degree latitude-longitude grid. A similar procedure was used in this study to compute the mean E-P field for 1958. As mentioned before, in view of the greater amount of data available for this study, a basic five-degree, latitude–longitude grid was used. The analysis of the distribution of the mean total horizontal divergence for 1958 in cm per year is presented in Fig. 3.

This analysis shows the existence of divergence centers alternating with convergence centers and exhibits considerable detail. In the areas of dense and representative data coverage the detail obtained in the five-degree grid computations is undoubtedly justified. However, in areas of sparse data coverage some of it may not be reliable. In such doubtful areas the analysis was smoothed slightly. Otherwise the field of divergence was analyzed so as to fit the numerical values.

The equatorial regions of the Atlantic and Pacific oceans show a general convergence indicating an excess of precipitation over evaporation due to the convergence of the trade winds from both hemispheres. Marked centers of strong convergence are found just south of Panama and off the east coast of South America near the equator; both of these areas are known to have excessive precipitation. Although the data supporting the divergence south of the Gulf of Maracaibo in South America is sparse, this area does have rather scanty precipitation compared with the Amazon River basin and water shed farther to the south and east. Another area of very strong divergence is found over the Arabian Sea. Even though the data supporting the water vapor transport analysis in this area were peripheral and the analysis relied heavily on mean winds, this divergence area can be associated with the high salinity of the Arabian Sea caused essentially by the excessive evaporation.

PEIXOTO (1959, 1960), LUFKIN (1959) and JACOBS (1948) have derived separately, in slightly different ways, empirical relations between the sea-surface salinity and the field of E-P for areas of the oceans where the effects of horizontal transport of surface water salinity are negligible. Earlier SVERDRUP (1942) had established empirically a rather simple linear relation between surface water salinity and E-P. The simple relationship indicates that transport of salinity by ocean currents is of minor importance for average conditions over long periods of time, whereas, the difference between evaporation and precipitation is of primary importance. Since the field of E-P is intimately related to the general circulation of the atmosphere, it can be concluded that the average values of sea surface salinity are controlled by the atmospheric circulation.

SVERDRUP (1942) includes a chart showing surface salinity of the oceans in northern summer; this chart shows excellent agreement with Fig. 3 over the oceans. More recently both DEFANT (1961) and VON ARX (1962) have emphasized the important relationship between atmospheric circulation and sea surface salinity.

Two areas of strong convergence bordering the equatorial and subtropical regions are worthy of special note. One extends from south-

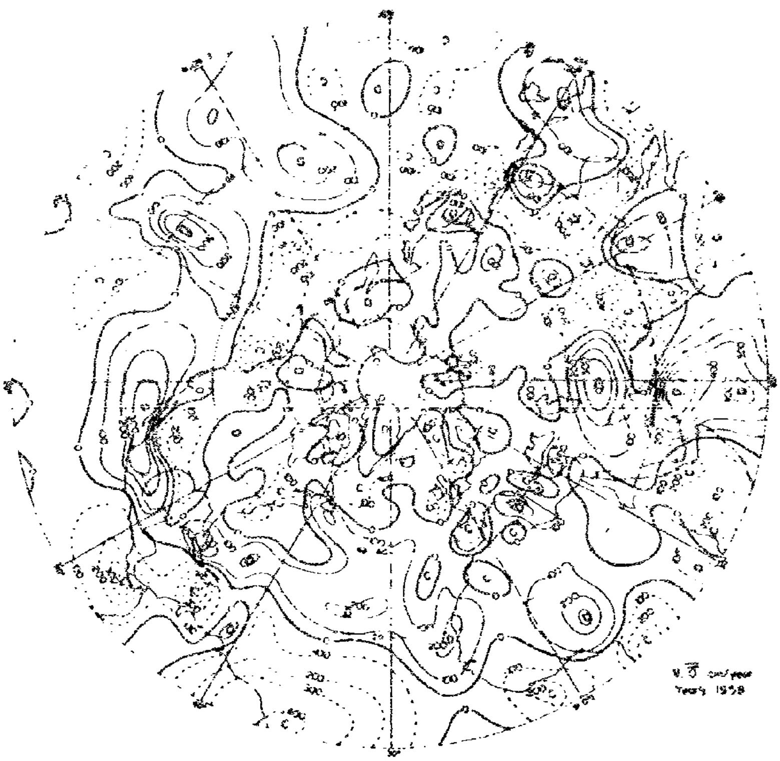

FIG. 3. Distribution of the horizontal divergence of the vertically integrated total annual flux of water vapor $\nabla \cdot \vec{Q}(\lambda, \phi)$ for the IGY in grams per cm² per year. The isopleths (full lines for divergence and dashed for convergence) are entered for intervals of 100 cm year^{-1}.

western Arabia near Aden generally westward and southward across equatorial eastern and central Africa and the other is over north-central India. The first area over Africa contains the headwaters of the Blue Nile and several tributaries of the White Nile; there are several rivers flowing southward from this area through Somaliland and Kenya. Upper parts of the Congo and Ubangui rivers are also in the area. The heavy rains over the areas referred to in India are well known. With more detailed data coverage close to and within the Himalayan mountains the water vapor transport analyses would undoubtedly support more accurately the location of the center, or possibly two centers of convergence, one further north and elongated along the mountain mass and one farther east over Assam. It seems very likely that even greater detail could be obtained in the divergence analysis over India if there were a high density of reporting stations located so as to delineate clearly the tremendous convergence of water vapor transport associated with the well-known summer monsoon. Nevertheless, the analysis shown on Fig. 3 appears to be consistent with the more general known facts of the earth's water balance over India. It is interesting to note the extension of this prominent convergence northward through Kashmir to the Pamirs and Altai Mountains west of Sinkiang where rain and snow provide the head waters of the Indus River and also several other smaller rivers flowing into the Tarim Basin of Sinkiang where they disappear. Actually there is a further extension of the convergence over the marshy west Siberian lowlands. It should also be noted that this entire large area of convergence over India covers the vicinity of the headwaters of several extensive river systems: Indus, Ganges, Brahmaputra, Salween, Mekong, and Yangtze.

The subtropical regions of the Atlantic and Pacific Oceans show rather strong and extensive areas of divergence. In the Atlantic the

divergence pattern is elongated in an east-west direction and generally uninterrupted. In the Pacific the divergence extends westward from Mexico to Marcus Island; it shows several centers of marked divergence interrupted by areas of weak convergence or less marked divergence. This feature of the analysis may be due to a cellular structure in the Pacific anticyclone. Over the western portion of the Pacific anticyclonic belt the pattern is somewhat complex, but, in general, convergence predominates; the area south and west of Japan shows rather strong convergence as might be expected because the mean position of the polar front is in this region. Closely associated with the strong divergence of the subtropical oceanic anticyclones are three other interesting areas of divergence; one over the central Mediterranean Sea, another over Iran, and the third over Mauritania in west Africa. The central Mediterranean divergence extends southward over the desert areas of Libya and Algeria and actually joins to the east through the Syrian Desert with the divergence over Iran; this whole area is known for its dryness and is an important source of atmospheric moisture; also the Mediterranean is known for its high salinity, which is associated with high positive mean values of $E-P$. There are centers of convergence in southern and central Europe and in North Africa (Atlas Mountains, Tunisia). These centers are associated with the frontal perturbations and with the topography. The center over the Iberian Peninsula is somewhat displaced to the south. However, it is well known that the northern part of the Iberian Peninsula is one of the regions of highest mean rainfall in Europe, and is the source of important rivers: (e.g., Tagus, Douro, Ebro, etc.). The divergence over west and Central Africa coincides with scanty precipitation and with the cold Canary or North African Current. The dryness of the Cape Verde Islands is well known. It is not difficult to recognize and to accept that the subtropical ocean areas which show strong divergence of water vapor transport are, in fact, major sources of atmospheric moisture. But it is more difficult to conceive of deserts in West and Central Africa, Arabia, the Middle East, and Iran as contributing sources of atmospheric moisture. Nevertheless, the divergence of atmospheric water vapor transport shows this to be the case. STARR & PEIXOTO (1958) have already commented on this point, since their study of 1950 showed similar divergence over these same deserts. BARNES (1963) also has discussed atmospheric water vapor divergence and certain applications of such information for climatic modification. The strong, positive divergence of water vapor transport over dry, desert areas and the attendant interesting speculations aroused thereby are certainly worthy of further study from a climatic and hydrologic viewpoint.

The mid-latitude regions around the northern hemisphere show many areas of divergence and convergence. The most prominent are areas of convergence associated with the extra-tropical storm tracks across the North Atlantic and North Pacific oceans. The convergence between Iceland and Greenland, and the other rather strong and marked areas of convergence in the North Atlantic region are clearly related to polar front storms; this is especially evident over the eastern United States and over the Gulf Stream and also in the vicinity of the western and coastal regions of Norway and Sweden. A long and extensive area of convergence extends from the East China Sea northeastward over the Japanese islands and Sakhalin then eastward across the entire northern Pacific Ocean to the west coast of North America. Here, in the vicinity of the Queen Charlotte Islands off the coast of British Columbia, is found a strong area of convergence extending northward and southward along the coastal mountain ranges. This area is known to have copious and regular precipitation year after year. An area of weak convergence is found inland of the coastal mountains, and divergence is actually shown over the desert areas of Nevada and southern California including Death Valley and the Salton Sea; farther inland over the Rocky Mountains is found another area of moderately strong convergence. Within this general area of convergence are the headwaters of several large river systems: Columbia, Missouri, Colorado, Arkansas, and Rio Grande. The details of other small areas of weak convergence and divergence over the United States and Canada can be supported by excellent data coverage.

There are two rather strong and marked regions of divergence in the mid-latitudes that should be mentioned, although the over-all picture is one of general convergence. One area is found just south of Newfoundland and extends southeastward into the Atlantic; the other

FIG. 4. Distribution of divergence for 1958 similar to preceding figure, but prepared by an equal weighting of summer and winter conditions especially over India with free use of climatological information over that region.

area is found over the northern portion of the Yellow Sea, northern Korea, and the western portion of the Sea of Japan. The divergence over northern Korea was also found by STARR & PEIXOTO (1958); it may be associated with the long winter monsoon carrying cold, dry air across this region and increasing its moisture at the expense of the underlying surfaces, especially over the Sea of Japan. It should be noted that the pattern abruptly changes to one of convergence along the western shores of the Japanese islands. The rather strong divergence near Newfoundland is more difficult to justify as a semi-permanent feature of the general circulation; it again may be possible that the outbreaks of cold, dry Canadian air masses over this region are responsible for it.

The arctic regions north of 60° N show a patchwork of small areas of weak convergence and divergence. Nevertheless, the data coverage north to 80° N was good, and at least the divergence pattern, complex as it may be, should represent conditions in 1958. It appears that there is a southward transport across 80° N, as is also the case across 70° N.

6. Final comments

The analysis was performed as objectively as possible and the use of preconceived ideas from climatology was avoided.

It should be noted, that except for differences already mentioned and various smaller details, the major features of the divergence analysis in this study agree quite well with those of the study made for 1950 by STARR & PEIXOTO (1958). We may consider the similarities between the two studies. The strong region of convergence over northern South America in 1950 is repeated in 1958 with more detail. The convergence center is associated with the heavy rainfall in the Amazon Valley. The divergence region, splitting northern South America on the 1958 map, was subsequently found to be as-

sociated with a dry region over Venezuela. This detail was not picked up on the 1950 map, although the zero is found along the northern coast of the continent. The strong convergence over the source region of the Nile was found for both years but with more detail in 1958. Divergence over the Mediterranean and over the western Sahara is found on both maps, although details are different.

Differences over India and Southeast Asia have not been completely reconciled. Supplemental stations used for the 1958 map altered the analysis significantly and made us question the representativeness of some of the mean values used in the 1950 analysis. Because of monsoonal effects over India, more numerous observations in one season than the other could greatly bias the transport vector field, and hence also its divergence.

As an experiment designed to investigate this last possibility somewhat further, our colleague Mr. E. Rasmusson kindly prepared an alternate map of the divergence for 1958. The distribution shown in Fig. 4 was obtained independently, through the analysis of the zonal and meridional transports for summer and winter separately. The seasonal distributions were then weighted equally and combined into the chart for the year. In the preparation of Fig. 4 general climatological information was used as an aid in the analysis over the region in question. It is clear that the hydrological phenomena of India such as the boundary of the Thar desert and the high rainfall inland along the Malabar coast and over the Western Ghats region are reflected to better advantage.

The divergence region present in both years along the east coast of Siberia and China is consistent with the dry air coming off the Asian continent as already mentioned.

The presence of the divergence in both 1950 and 1958 over the Mississippi-Missouri Valley, continuing up into Saskatchewan, indicates a significant item of agreement.

On the whole where the data are adequate, the main features of the divergence field seem to be repeated for the two years. Reasons for differences may be due to inadequate data to define the divergence field, and to differences between the mean state of the circulation and moisture content for the two years, such as the action of hurricanes and typhoons. The annual total precipitation can sometimes be dependent on just one such storm. Finally, differences may be due to the finer gridwork used for the 1958 study.

The present study indicates the necessity of extending the analysis throughout the southern hemisphere. It seems also desirable at this point to pursue equivalent studies on the water balance on a regional scale so that in performing a more detailed analysis physiographic influences and local factors can be taken more fully into consideration.

Acknowledgements

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ON THE ROLE OF WATER VAPOR IN THE ENERGETICS OF THE GENERAL CIRCULATION OF THE ATMOSPHERE (*) (**)

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ABSTRACT — The role of water vapor in the energetics of the general circulation of the atmosphere is studied and discussed. The water vapor is the most important absorber of solar energy in the atmosphere, and hence, its distribution influences the form of the energy input into the system. Through the release of latent heat it generates zonal available and eddy available potential energy. The available potential energy of the disturbances is partly supplied by the non adiabatic heating due to condensation. The implications of the meridional transport of latent heat and of its divergence are discussed in the light of various meteorological considerations. The total mean meridional transport of latent energy is southward in the equatorial regions and northward in middle and high latitude regions whereas the meridional transports associated to transient and standing disturbances are predominantly positive (from south). The contribution of Hadley cell for the total southward transport of latent energy becomes dominant in the lower troposphere of equatorial regions.

RÉSUMÉ — On présente une étude du rôle de la vapeur d'eau dans l'énergétique de la circulation générale de l'atmosphère. La vapeur d'eau est l'absorbant le plus important de l'atmosphère et sa distribution module «l'input» de l'énergie dans le système. En dégageant de la chaleur latente elle va générer l'énergie potentielle disponible zonalle et perturbée. L'énergie disponible des pérturbations est partiellement fournie par le rechauffement non adiabatique due à la condensation de la vapeur d'eau. On étudie d'abord les implications du transport méridional d'énergie latente et de sa divérgence à la lumière de diverses considérations météorologiques.

Le transport méridional total de l'énergie latente est dirigé vers le sud dans les régions équatoriales et vers le nord dans les régions des latitudes

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moyennes et élevées, tandis que le transport méridional associé aux pértur-bations stationaires et transientes est, d'une façon générale, positive. La contribution des circulations du type Hadley est dominante pour le transport vers le sud de l'énergie latente dans la basse troposphère des régions équatoriales.

1. *INTRODUCTION*

One of the possible approaches to the study of the general circulation of the atmosphere is to examine certain integral requirements deduced from dynamical principles governing the motion of the atmosphere, formulated in therms of physical properties such as energy, momentum, mass, water vapor content, etc.

This approach has been used extensively at the M. I. T. Planetary Circulations Project.

According to the principle of the conservation of mass, water substance cannot be created or destroyed within the atmosphere. The water balance may therefore be taken as a constraint on the general circulation. The necessity for the transport of water in the atmosphere arises from the existence of an excess of precipitation over evaporation in certain regions, with a reversal of prevailing conditions over other areas. Since storage effects of the atmosphere are small enough to be neglected, the excesses and deficits must be made up through the transport of water by atmospheric circulations, since there can be no significant net flux of water into or out of the atmosphere as a whole. Therefore, water vapor can be regarded as an indicator of the mechanisms which maintain the general circulation.

In dealing with the energetics of the atmosphere, one cannot ignore the existence of water component in its various phases. In the vapor phase, it is the most active constituent of the atmosphere with regard to radiative processes. It is a highly selective absorber of incoming solar energy; and also an important emitter of long wave radiation. The solid and liquid phases in the form of clouds have a profound influence upon the spatial distribution of planetary albedo, and consequently upon the amount of solar energy that is available for absorption by the earth. Furthermore clouds influence the long wave radiative balance, and through this, the vertical distribution of temperature in the atmosphere.

Since all its phases can occur within the usual range of the observed temperatures there are large amounts of energy associated with the phase changes that play an important part in the energy budgets of the earth and of the atmosphere.

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With the imposed horizontal flux of water vapor is associated a transport of energy in the form of latent heat which constitutes an important part of the energy balance of the atmosphere. The corresponding vertical transport serves to compensate for radiative effects which tend to cool the atmosphere as a whole (1) (2). Finally, the energy associated with phase changes alters the baroclinicity of the atmosphere thereby influencing the kinetic energy, momentum and vorticity fields.

The present paper intends to give some aspects of the results obtained in the study of the water balance requirements of the atmosphere, and their implications for the energetics and the mechanisms of the maintenance of the general circulation.

2. *NOTATIONS AND FORMULAE*

In the present study we shall use the following notations:

λ = longitude
φ = latitude
p = pressure
t = time
a = radius of the earth
$u = a \cos \varphi \, d\lambda/dt$ = eastward wind component
$v = a \, d\varphi/dt$ = northward wind component
$\mathbf{V} = u \, \mathbf{i} + v \, \mathbf{j}$ = horizontal wind vector
$\omega = dp/dt$ = «vertical velocity»
z = height of an isobaric surface
g = acceleration of gravity
$\Phi = gz$ = geopotential
α = specific volume of the air
ρ = density of the air
T = temperature
R = gas constant
c_p, c_v = specific heats at constant pressure and at constant volume
$k = R/c_v$
$\theta = T_p^{-k} p_o^k$ = potential temperature
q = specific humidity
L = latent heat of condensation

$$T_e = T + \frac{L}{c_p} q = \text{equivalent temperature}$$

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ν = zenith angle of the sun

$dm = a^2 \cos \varphi \, d\lambda \, d\varphi \dfrac{dp}{g}$ = mass element

P = precipitation

E = evaporation

$U = c_v \, T$ = internal energy

$H = c_p \, T$ = enthalpy

$\Phi^* = U + \Phi$ = total potential energy

Ω = angular velocity of the earth

$f = 2 \, \Omega \, sen \, \varphi$ = Coriolis parameter

$\mathbf{F}$ = frictional force $= F_\lambda \mathbf{i} + F_\varphi \mathbf{j}$

$\dot{Q}_F = \dfrac{dQ_F}{dt}$ = heating rate due to conduction and friction

$\dot{Q}_R = \dfrac{dQ_R}{dt}$ = heating rate due to radiation

$\dot{Q}_L = \dfrac{dQ_L}{dt}$ = heating rate due to condensation

$u = g^{-1} \int_0^{p_0} dp$ = seccional mass

$W = g^{-1} \int_0^{p_0} q \, d \, p$ = precipitable water

$\mathbf{Q} = g^{-1} \int_0^{p_0} q \, \mathbf{v} . \, dp = Q_\lambda \mathbf{i} + Q_\varphi \mathbf{j}$ = water vapor vector transport

$Q_\lambda = g^{-1} \int_0^{p_0} q \, u \, . \, dp$ = zonal transport of water vapor

$Q_\varphi = g^{-1} \int_0^{p_0} q \, v \, . \, dp$ = meridional transport of water vapor

S_q = source function for water vapor

Λ = availability of energy

$\Gamma = - \dfrac{\partial T}{\partial z}$ = lapse rate

$\Gamma_d = - \dfrac{g}{c_p}$ = dry adiabatic lapse rate

$\mathfrak{P} \equiv P_i^j = - \mu \left[\dfrac{2}{3} \delta_i^j \dfrac{\partial v^k}{\partial x^k} - \left(\dfrac{\partial v^i}{\partial x^j} - \dfrac{\partial v^j}{\partial x^i} \right) \right]$ = Navier-Stokes tensor

$\mathfrak{R} \equiv R_i^j = \overline{v_i' \, v'^j}$ = Reynolds tensor

$$\bar{x} = \tau^{-1}\int_0^\tau x\,dt = \text{time average of } x$$

$$x' = x - \bar{x} = \text{deviation from time average}$$

$$[x] = (2\pi)^{-1}\oint x\,d\lambda = \text{zonal average of } x$$

$$x^* = x - [x] = \text{deviation from zonal average}$$

$$\overset{\equiv}{x} = \pi^{-2}\int\int^{\pi/2} x\,d\lambda\,d\varphi = \text{hemispheric average of } x \text{ over an iso-}$$

baric surface

$$x'' = x - \overset{\equiv}{x} = \text{deviation from hemispheric average}$$

$$<x> = \frac{1}{sen\,(\varphi_j + 10) - sen\,\varphi_j}\int\int x \cos\varphi\,d\varphi\,d\lambda = \text{space average}$$

for the latitudinal belt φ_j

$$[x\,y]_E = [\overline{x'\,y'}] + [\bar{x}^*\,\bar{y}^*] = \text{total eddy covariance of } x \text{ and } y$$

$$[\overline{x'\,y'}] = \text{transient eddy covariance of } x \text{ and } y$$

$$[\bar{x}^*\,\bar{y}^*] = \text{standing eddy covariance of } x \text{ and } y$$

$$K_M = \frac{1}{2}\int([\bar{u}]^2 + [\bar{v}]^2)\,dm = \text{zonal kinetic energy}$$

$$K_E = \frac{1}{2}\int([\overline{u'^2} + \overline{v'^2}] + [\bar{u}^{*2} + \bar{v}^{*2}])\,dm = \text{eddy kinetic energy}$$

$$\gamma = \left\{\begin{matrix} -\alpha\left(\dfrac{1}{\theta}\dfrac{\partial\overset{\equiv}{\theta}}{\partial p}\right)^{-1} \\ \left(\overset{\equiv}{T} - \dfrac{p}{R}\,c_p\,\dfrac{\partial\overset{\equiv}{T}}{\partial p}\right)^{-1} \\ \left(\dfrac{T}{\theta}\right)^2 \dfrac{\overset{\equiv}{T}^{-1}\,g}{(\Gamma_d - \Gamma)} \end{matrix}\right\} = \text{static stability parameter}$$

$$A_M = \frac{c_p}{2}\int\gamma\,[\bar{T}]''^2\,dm = \text{zonal available potential energy}$$

$$A_E = \frac{c_p}{2}\int\gamma\,([\overline{T'^2}] + [\bar{T}^{*2}])\,dm = \text{eddy available potential}$$

energy

$$D(K) = \int\mathbf{v}\,.\,\mathbf{F}\,dm = \text{rate of frictional dissipation of kinetic energy}$$

due to small scale turbulence and to eddy stresses at the boundary

$G(A)$ = rate of generation of available energy due to non-adiabatic effects

$C(K_E, K_M)$ = rate of conversion from eddy kinetic energy to zonal kinetic energy by the eddy momentum transport

$C(A_M, A_E)$ = rate of conversion from zonal to eddy available potential energy by the eddy sensible heat transport

$C(A_M, K_M)$ = rate of conversion from zonal available potential energy to zonal kinetic energy by mean meridional circulations

$C(A_E, K_E)$ = rate of conversion from eddy available potential energy into eddy kinetic energy by large-scale eddy processes

$\nabla = \mathbf{i}\dfrac{\partial}{a\cos\varphi\,\partial\lambda} + \mathbf{j}\dfrac{\partial}{a\,\partial\varphi}$ = surface spherical gradient operator on an isobaric surface

In this discussion the primitive hydrostatic equations for the atmosphere are written as follows:

a) equations of motion

$$\frac{\partial u}{\partial t} + \mathbf{v}.\nabla u + \omega\frac{\partial u}{\partial p} - \left(f + \frac{u\,tg\,\varphi}{a}\right) + \frac{1}{a\cos\varphi}\frac{\partial\Phi}{\partial\lambda} - F_\lambda = 0 \qquad (1)$$

$$\frac{\partial v}{\partial t} + \mathbf{v}.\nabla v + \omega\frac{\partial v}{\partial p} + \left(f + \frac{u\,tg\,\varphi}{a}\right) + \frac{1}{a}\frac{\partial\Phi}{\partial\varphi} - F_\varphi = 0 \qquad (2)$$

b) equation of hydrostatic equilibrium

$$\frac{\partial\Phi}{\partial p} + \frac{R}{p}T = 0 \qquad (3)$$

c) equation of continuity

$$\frac{\partial\omega}{\partial p} + div_p\,\mathbf{v} = 0 \qquad (4)$$

d) equation of continuity for the water vapor

$$\frac{\partial q}{\partial t} + div_p\,q\,\mathbf{v} + \frac{\partial}{\partial p}q\,\omega = \frac{dq}{dt} \equiv S_q \equiv -\frac{\dot{Q}_L}{L} \qquad (5)$$

e) equation of the first law of thermodynamics

$$\frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta + \omega \frac{\partial \theta}{\partial p} - \frac{\theta}{T} \frac{(\dot{Q}_F + \dot{Q}_R + \dot{Q}_L)}{c_p} = 0 \tag{6}$$

3. *THE WATER VAPOR IN THE GLOBAL BALANCE OF TOTAL ENERGY OF THE ATMOSPHERE*

3.1. *The balance equation of global energy*

The atmosphere contains significant amounts of potential energy (gravitational), internal energy (heat), latent energy (heat of condensation) and kinetic energy of various scales of motion.

For a moist atmosphere the total amount of potential energy, Φ, the internal energy, U, and the latent energy, LW, in a unitary column of the atmosphere in a state of hydrostatic equilibrium is proportional to the mean equivalent temperature of the column or to the wheighted potential equivalent temperature

$$\begin{aligned} \Phi + U + LW &= g^{-1} \int c_p \, T_e \, dp \\ &= p_0^{-k} \, g^{-1} \int c_p \, p^k \, \theta_e \, dp \end{aligned} \tag{7}$$

where W is the precipitable water content of the column, L the latent heat of condensation, assumed to be constant and Θ_e the potential equivalent temperature. This total energy may be designated as ***total moist potential energy***.

For a dry atmosphere the total amount of potential energy and internal energy for a unitary column of the atmosphere in a state of hydrostatic equilibrium is proportional to the total enthalpy of the column or to the wheighted potential temperature, θ:

$$\begin{aligned} \Phi + U &= g^{-1} \int c_p \, T \, dp \\ &= p_0^{-k} \, g^{-1} \int c_p \, p^k \, \theta \, dp \end{aligned} \tag{8}$$

Since the generation and the destruction of both forms of energy (potential plus internal) occurs simultaneously it is costumary to consider the two forms of energy as a single form, the so called *total potential energy*, Φ^*.

MARGULES (1903) has firmly established that the maintenance of the atmospheric motions in a synoptic scale against the dissipation is due to the conversion of total potential energy into kinetic energy. The rate of generation of total potential energy (internal plus potential), which has to be resupplied depends upon non adiabatic heating including radiation, frictional heating, the release of latent heat, heating of contact of the atmosphere with the earth (transport of sensible and latent heat by turbulent difusion), etc.

The mechanism of conversion is basically a sinking of colder air and a rising of warmer air at same level. It is then required an horizontal gradient of temperature for the process to continue. Thus only a small fraction of the total potential energy is really available for conversion into kinetic energy of the actual atmospheric motions. The process of generating *available potential energy* is essentially through the heating of warm regions and the cooling of cooler regions at the same isobaric level which is equivalent to a local decrease of entropy.

The local balance equation for the total energy which expresses the conservation of total energy for the atmosphere, may be written (3). (4), (5) in the form:

$$\frac{\partial}{\partial t}\rho(U+\Phi+K)+div\left[\rho(c_p T+Lq+\Phi+K)\mathbf{v}-(\mathfrak{R}+\mathfrak{P})\cdot\mathbf{v}\right]=\rho\dot{Q} \qquad (9)$$

where $\mathfrak{P}$ is the Navier-Stokes viscosity tensor, $\mathfrak{R}$ is the Reynolds turbulence tensor.

If this equation is integrated over the volume of a polar cap τ, bounded by a wall Σ, the resulting equation after averaged in time over the period considered becomes:

$$\frac{\partial}{\partial t}\iiint_\tau \rho\overline{(U+\Phi+K)}\,d\tau+\iint_\Sigma \rho\overline{(c_p T+Lq+K+\Phi)\,v_N}\,d\Sigma-$$
$$-\iint_\Sigma\left[\overline{(\mathfrak{R}+\mathfrak{P})\cdot\mathbf{v}}\right]_N d\Sigma=\iiint_\tau \rho\,\overline{\dot{Q}}\,d\tau \qquad (10)$$

The local variation of the total energy in the polar cap results from:

a) the flux of total energy in the form of enthalpy $(\overline{c_p T})$, of latent heat $(\overline{Lq})$, of potential energy $(\Phi=\overline{gz})$ and of kinetic energy of existing motions $\left(\frac{\rho\overline{\mathbf{v}^2}}{2}\right)$, across the boundary Σ;

b) the flux of energy due to the action of frictional forces, which would ordinarily consist of a dissipation due to molecular viscosity and to small-scale turbulence which can produce no significant tangencial stresses;

c) a production of energy due to non-adiabatic heating ($\dot{Q}$).

It must be pointed out that kinetic energy of existing motions is very small compared with the other forms of energy.

3.2. *The meridional flux of moist latent energy*

Latent heat is one of the component of the flux of energy in equation (10). Analyses of the transport fields of water vapor can be regarded, in fact, as representations of the fluxes of latent heat. As was discussed in previous papers (4), (6), (7) the total mean horizontal flux of latent heat above a point on the earth's surface is given by

$$\overline{L\mathbf{Q}} = g^{-1}\int \overline{Lq\,\mathbf{v}}\,dp = L\,(\overline{Q}_\lambda\,\mathbf{i} + \overline{Q}_\varphi\,\mathbf{j}) \tag{11}$$

where the latent heat of condensation, L, is assumed to be constant.

From the hemispheric analyses of the quantities $\overline{Q}_\lambda$ and $\overline{Q}_\varphi$ (6) the values of the latter quantity have been computed and are shown in TABLE I. By comparing these values with those given by HOU-

TABLE I

Zonally averaged values of the total latent energy transport across latitude circles, $[L\,\overline{Q}_\varphi]$, for the Northern Hemisphere in units of 10^{14} cal/sec. The lower numbers give the component of the total due to Mean Meridional cells $Lg^{-1}\int [\bar{q}]\,[\bar{v}]\,dp$.

Latitude	80°	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	— 0,06	0,38	1,34	2,69	3,02	3,22	2,29	— 2,21	— 8,62	— 5,59
		0,05	0,20	0,25	0,40	0,70	— 0,17	— 4,42	— 10,03	— 5,74
Summer	— 0,07	0,29	1,46	4,08	4,15	3,38	1,27	— 1,10	1,15	5,45
		— 0,22	0,05	1,66	1,85	1,29	— 1,26	— 3,98	0,16	5,60
Year	— 0,07	0.32	1,42	3,19	3,48	3,28	1,75	— 1,62	— 3,70	0,00
		— 0.13	— 0,08	0,76	0,58	0,64	— 0,61	— 4,26	— 4,66	0,07

GHTON (1) and by BUDYKO (8), (9), (10) one can see that in some regions water vapor contributes more than 0,30 of the meridional heat flux required to maintain the radiative balance.

The total transport of latent energy can be accomplished by the mean circulations and by the large transient and standing perturbations of the general circulation (11). In order to find the relative contribution of the various processes, the transport components $\overline{qu}$ and $\overline{qv}$ at a given isobaric level may be expanded according to the generalised REYNOLDS scheme:

$$[\overline{qu}] = [\overline{q}]\ [\overline{u}] + [\overline{q}^*\ \overline{u}^*] + [\overline{q' u'}] \tag{12}$$

$$[\overline{qv}] = [\overline{q}]\ [\overline{v}] + [\overline{q}^*\ \overline{v}^*] + [\overline{q' v'}] \tag{13}$$

Thus we may write for the components of the mean zonal and meridional total latent heat transports, respectivelly, the following expressions:

$$L\,[\overline{Q}_\lambda] = L\,g^{-1}\int_o^{p_o}[\overline{q}]\,[\overline{u}]\,dp + L\,g^{-1}\int_o^{p_o}[\overline{q}^*\,\overline{u}^*]\,dp + L\,g^{-1}\int_o^{p_o}[\overline{q' u'}]\,dp \tag{14}$$

$$L\,[\overline{Q}_\varphi] = L\,g^{-1}\int_o^{p_o}[\overline{q}]\,[\overline{v}]\,dp + L\,g^{-1}\int_o^{p_o}[\overline{q}^*\,\overline{v}^*]\,dp + L\,g^{-1}\int_o^{p_o}[\overline{q' v'}]\,dp \tag{15}$$

The terms of these equations are associated with the mean advection of latent heat ($[\overline{q}]\ [\overline{u}]$) and with the mean meridional circulation ($[\overline{q}]\ [\overline{v}]$); with the standing large scale horizontal eddies ($[\overline{q}^*\,\overline{u}^*]$; $[\overline{q}^*\,\overline{v}^*]$) and finally with transient horizontal eddies ($[\overline{q' u'}]$); ($[\overline{q' v'}]$).

In the study of the atmosphere and the earth's energy budgets the meridional transport $L\,\overline{Q}_\varphi$ plays a much more important role than the zonal transport $L\,\overline{Q}_\lambda$. Therefore we will discuss in detail the behaviour of all the components of the meridional transport, $L\,\overline{Q}_\varphi$, and its latitudinal distribution.

The zonally averaged values of the mean total moist latent energy transport across latitude circles, $[\overline{L\,Q_\varphi}]$, for the Northern hemisphere are given in TABLE I. The vertical distribution of the zonally averaged values of meridional transport of latent energy at various latitudes is presented in TABLE II. The extreme northward and southward values occur in the low troposphere in the layer 1000/850 mb.

TABLE II

Zonally averaged values of mean meridional transport of latent energy $g^{-1}[\overline{qv}]$ in units of cal/(mb. cm. sec.) for yearly data at specified latitudes. The levels are given in millibars.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
1000 mb	6,02	18,06	36,12	41,54	32,51	— 9,63	— 76,45	— 38,53	6,02
850	— 2,41	15,65	38,53	39,13	37,93	4,21	— 17,46	— 7,22	2,41
700	2,41	15,65	28,90	28,90	26,49	1,20	1,81	— 14,45	— 4,82
500	— 0,60	4,21	6,62	6,02	5,42	5,42	3,61	— 0,60	— 3,01

The meridional transient eddy flux of moist latent energy, $Lg^{-1}\int_{0}^{p_0}[\overline{q'v'}]\,dp$, as shown in TABLE III, is predominantly positive

TABLE III

Zonally averaged values of the mean total meridional transient eddy transport of latent energy $g^{-1}L\int[\overline{q'v'}]\,dp$, in units of 10^{14} cal/sec for yearly and seasonal data at specified latitudes.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	0,28	0,85	2,02	2,24	2,28	2,00	1,43	1,22	0,13
Summer	0,46	1,42	2,38	2,26	1,97	1,22	1,15	0,80	— 0,17
Year	0,41	1,30	2,58	2,81	2,45	1,74	1,55	0,86	— 0,07

(northward) over the northern hemisphere It shows a yearly maximum which occurs near 47,5°N, shifting to the north in summer and to the south in winter. The maximum observed is clearly associated with the mean position of the polar front, as to be expected, in view of the role of the baroclinic perturbations in the eddy meridional transport. The transient eddy transport varies with the altitude and reaches a maximum in the middle latitude region in the lower troposphere near the 850 mb level (TABLE IV).

TABLE IV

Zonally averaged values of the mean zonal transient eddy transport of latent energy, $g^{-1} L \int \left(\overline{q'v'}\right) dp$, in units of cal/(mb. cm. sec) for yearly data at specified latitudes. The levels are given in millibars.

Latitude	65°	55°	45°	35°	25°	15°	5°
1000 mb	— 1,81	— 0,00	— 3,01	— 1,81	+ 6,02	+ 9,63	+ 4,21
850	+ 2,41	+ 3,01	— 4,21	— 5,42	+ 1,81	+ 9,03	+ 7,22
700	+ 0,00	+ 0,00	— 4,21	— 10,84	— 6,02	+ 5,42	+ 13,24
500	+ 4,21	+ 3,61	— 2,41	— 7,22	— 8,43	— 3,61	+ 1,20

The total meridional transport of latent energy associated with the standing-eddies, $L\, g^{-1} \int [\overline{q}^* \overline{v}^*]\, dp$, (TABLE V) is always positive

TABLE V

Zonally averaged values of the mean total meridional standing eddy transport of latent energy $g^{-1} L \int_0^{p_0} [\overline{q}^* \overline{v}^*]\, dp$ in units of 10^{14} cal/sec for yearly and seasonal data at specified latitudes.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	0,05	0,30	0,42	0,38	0,24	0,46	0,77	0,19	0,02
Summer	0,04	0,01	0,06	0,05	0,12	1,31	1,73	0,17	0,01
Year	0,04	0,20	0,12	0,10	0,18	0,61	1,09	0,14	0,00

(from south) and in general presents a well defined maximum at 20°N, associated with semi-permanent subtropical anticyclones and another maximum, much less intense, near 55°N associated with the semi-permanent lows prevailing in this region. The vertical distribution of the meridional standing eddy flux of latent heat (TABLE VI) shows that a maximum occurs in the neighbourhood of 22,5°N at 850 mb and another around 55°N near the surface. It is interesting to point

TABLE VI

Zonally averaged values of the mean meridional standing eddy transport of latent energy, $g^{-1}L[\bar{q}^* \bar{v}^*]$, in units of cal/(mb. cm. sec) for yearly data at specified latitudes. The levels are given in millibars.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
1000 mb	+ 1,81	+ 4,21	+ 3,01	+ 1,20	+ 2,41	+ 4,82	+ 6,62	+ 2,41	+ 2,41
850	+ 1,20	+ 1,81	+ 1,81	+ 1,81	+ 3,61	+ 6,02	+ 9,63	+ 3,01	+ 1,20
700	+ 0,00	+ 2,41	+ 1,81	+ 0,60	+ 1,20	+ 2,41	+ 4,82	+ 3,01	+ 0,00
500	+ 0,60	+ 0,00	+ 0,00	+ 0,00	+ 0,60	+ 1,20	+ 2,41	+ 1,20	+ 0,00

out that the lowest values occur at 45°N where the largest values of the transient eddy transport of latent heat are observed.

The latitudinal distribution of the total eddy meridional flux (TABLE VII) presents a bimodal distribution, resulting from the

TABLE VII

Zonally averaged values of the mean total meridional eddy transport of latent energy $g^{-1}L\int\{[\overline{q'v'}] + [\bar{q}^* \bar{v}^*]\}\, dp$, in units of 10^{14} cal/sec for yearly and seasonal data at specified latitudes.

Latitude	70°	60°	50°	45°	40°	30°	20°	10°	0°
Winter	0,33	1,15	2,44	2,62	2,52	2,46	2,20	1,41	0,15
Summer	0,50	1,43	2,44	2,31	2,09	2,53	2,88	0,97	— 0,16
Year	0,45	1,50	2,70	2,91	2,63	2,35	2,64	1,00	— 0,07

combination of the latitudinal distribution due to transient eddies ($L\, g^{-1} \int_0^{p_0} [\overline{q'v'}]\, dp$) and to standing eddies ($L\, g^{-1} \int_0^{p_0} [\bar{q}^* \bar{v}^*]\, dp$) associated with the quasi-permanent features of the atmosphere circulation.

Thus, we can conclude that the effect of the standing eddies is of greatest significance in low latitudes, where the quasi-stationary

disturbances are dominant. At middle latitudes the vigorous transient eddies predominate and the standing eddies play a minor role in the meridional transport of energy. However, at 60°N their importance increases again, especially in winter when semi-permanent lows are most intense.

The comparison of the values of total meridional transport of latent energy and those of the total eddy transport offers the important indirect evidence of the existence of the three-cell regime with two direct cells and one indirect cell. The values of the mean meridional transport of latent energy by the mean meridional circulations are shown for comparison in TABLE I.

The contribution of the Hadley cell for the total southward transport of latent heat in the equatorial region becomes dominant, whereas the contribution of the other two cells play a minor role in the process, the eddies being the major factor in the total meridional flux of latent heat.

3.3. *The water vapor and the generation of total potentital energy*

The rate of non-adiabatic heating, $\dot{Q}$, due to conduction and friction, $\dot{Q}_F$, to radiation, $\dot{Q}_R$, and to condensation, $\dot{Q}_L$, will be written *in extenso* by adding up the individual contributions corresponding to the different physical processes that participate in the total heat balance.

We shall use the operator $<(\quad)>$ to define the mean value of a quantity within a zone j of the atmosphere which extends from latitude φ_j to latitude $\varphi_j + 10°$.

Then, the mean rate of heating of the atmosphere $<\dot{Q}(z)>$ is given by

$$\begin{aligned} <\dot{Q}_j> = {} & <\overline{S}_j(\infty)\downarrow> + <\overline{G}_j(0)\uparrow\downarrow> + <\overline{L}_j(0)\uparrow> + \\ & + <\overline{C}_j(0)\uparrow> - <\overline{S}_j(0)\downarrow> - <\overline{G}_j(\infty)\uparrow> \end{aligned} \qquad (16)$$

where $<\overline{S}_j(z)>$ and $<\overline{G}_j(z)>$ are the intensities of the solar and long wave radiation respectively, and $<\overline{L}_j(0)\uparrow>$ and $<\overline{C}_j(0)\uparrow>$ are the latent heat and sensible heat transport at the lower boundary respectively. The arrows indicate the direction of the net flux.

We will proceed to show that all the terms are influenced direct or indirectly by the presence of water vapor in the atmosphere.

The quantity of radiant energy absorbed and scattered by the atmosphere at each point of the globe, $<\overline{S}_j(\infty)\downarrow> - <\overline{S}_j(0)\downarrow>$, is a function of the air mass, $u = g^{-1}\int dp$, and the precipitable water, $W = g^{-1}\int q\, dp$. According to HOUGHTON this quantity is given by:

$$<\overline{S}_j(\infty)\downarrow> - <\overline{S}_j(0)\downarrow> = 0{,}175\,(\overline{W.u})^{0,30}\cos\nu \;\; (cal/cm.^2 min) \qquad (17)$$

where ν is the zenith angle of the sun.

To show the dependence of long wave radiative balance, $<\overline{G}_j(0)\uparrow\downarrow> - <\overline{G}_j(\infty)\uparrow>$, upon water vapor content, one need only refer to any radiation chart (see, for instance, Elsasser radiation chart).

Studies of precipitable water content such as those published by STARR, PEIXOTO and CRISI (12) have importante application to investigation of radiation and heat balance in the atmosphere. The maps of precipitable water may be used to find the spatial distribution of time averaged solar energy absorption. Furthermore, from the spatial distribution of specific humidity at different levels, one can examine the three dimensional distribution of this effect. These maps, in conjunction with temperature analyses could thus be useful in the computation of long wave absorption and emission at a given point in the atmosphere. Many specific applications of infrared radiation technology, however, require instantaneous information concerning atmosphere moisture.

Let us analyse now the effect of the clouds in the disposition of the solar radiation.

The planetary albedo has a mean value of 0.34, with a minimum of 0.28 in the subtropical regions, which are relatively devoid of cloudiness, and a maximum of 0.67 in the polar regions due to the presence of snow cover (1). Hence, the latitudinal distribution of solar energy available for absorption has a maximum in the subtropical regions, around 20°N, as shown by BUDYKO *et al.* (9). However, because the zonally averaged amount of precipitable water in the atmosphere is a monotonically decreasing function of latitude (6) and clouds are not important as absorbers of solar energy, the atmospheric absorption does not show this maximum. When the earth and atmosphere are taken as a system, the subtropical maximum is still evident, though it is suppressed by the effects mentioned above.

In order to obtain the distribution of $<\overline{L_j(0)\uparrow}>$ one can combine the values $<\overline{P-E}>$ as obtained by various authors (12) with values of evaporation $<\overline{E}>$, such as those given by BUDYKO *et al.* (8), (9), so as to obtain the distribution of $<\overline{P}>$ (*). The latter, when multiplied by the proper constant, yields $<\overline{L_j(0)\uparrow}>$. The values of $<\overline{P-E}>$ can be obtained from the divergence of the water vapor transport field $\overline{\mathbf{Q}}$, as has been discussed on several occasions (4), (12).

In fact for a unit column of air extending from the earth's surface (pressure p_0) to the top of the atmosphere (pressure $p=0$), the water vapor balance equation can be written:

$$\frac{\partial \overline{W}}{\partial t} + div\ \overline{\mathbf{Q}} = \overline{S}_q \tag{18}$$

where $\overline{S}_q$ represents the net source of water substance in the atmospheric column. The source and sinks of water vapor in the atmosphere are due primarly to evaporation E from the surface of the earth and to precipitation P. For all pratical purpose $\overline{S}_q$ is given by the excess of evaporation over precipitation, $\overline{E-P}$. Thus taking the time average for the given time period (one year), the equation for atmospheric water vapor balance becomes:

$$\frac{1}{a\ cos\ \varphi}\left[\frac{\partial \overline{Q}_\lambda}{\partial \lambda} + \frac{\partial}{\partial \varphi}\left(\overline{Q}_\varphi\ cos\ \varphi\right)\right] = \overline{S}_q \equiv (\overline{E-P}) \tag{19}$$

because for this time interval $\frac{\partial \overline{W}}{\partial t}$ may be taken as zero.

The values of the water vapor transport field $\mathbf{Q} = (Q_\lambda \mathbf{i} + Q_\varphi \mathbf{j})$ have been discussed and computed (4), (6), (7) and the analysis of

(*) At this point one might raise the objections that, since the author consulted the work of BUDYKO to obtain values of $<\overline{E}>$, why did he not use the same source to obtain values of $<\overline{P}>$ directly, instead of going through the rather involved procedure described above. Justification for the procedure used rests on the fact that evaporation is a smoother function of space and time (almost monotonically decreasing function of latitude) than is precipitation and hence the former is more adaptable to the averaging techniques used in this type of study.

the distribution of the mean total horizontal divergence, $\nabla \cdot \overline{\mathbf{Q}}$, for the years of 1950 and 1958 have been already studied (12), (14), (15). The analysis of $\nabla \cdot \overline{\mathbf{Q}}$ show the existence of divergence centers alternating with convergence centers and exhibit considerable detail.

The divergence by ten degree latitude belts has been computed (12), using the expression:

$$< \overline{\nabla \cdot \mathbf{Q}} > = \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \oint \overline{Q}_{\varphi} \cos \varphi \, d\lambda = < \overline{E - P} > \qquad (20)$$

The zonal values show a strong meridional variation with negative values $(\overline{E - P} < 0)$ in the equatorial and middle latitude regions and with positive values $(\overline{E - P} > 0)$ in the subtropical latitudes, where the divergence field shows a belt of maximuma. From the previous relation it is obvious that, over long periods, evaporation must exceed precipitation in these regions.

Thus the subtropical regions always act as a source of moisture for the atmosphere as a whole, while the equatorial, middle and high latitude regions act primarly as sinks.

3.4. *The hydrologic cycle in the global energetics of the atmosphere*

Now let us consider the expression of the rate of non-adiabatic heating and the latitudinal variation of its various components.

The values of $< \overline{S}_j(\infty) \downarrow > - < \overline{S}_j(0) \downarrow >$, which measure the rate of absorption of solar energy by the atmosphere and the values of the longwave radiation emitted by the earth and the atmosphere $< \overline{G}_j(\infty) \uparrow >$ were gruperted and discussed by HOUGHTON (1). Both are slowly varying functions of latitude. The difference between these quantities $\{ < (\overline{S}_j(\infty) \downarrow > - < \overline{S}_j(0) \downarrow) > - < \overline{G}_j(\infty) \uparrow > \}$ which appears in equation (16) is still more uniform.

According to BUDYKO $< \overline{C}_j(0) \uparrow >$ is always positive, smaller than any of the other terms in (16), and slowly varying with latitude; also BUDYKO's given values of $< \overline{G}_j(0) \uparrow \downarrow >$ are practically constant with respect to latitude (8), (9). In contrast to these slowly varying functions, the values of $< \overline{L}_j(0) \uparrow >$ determined by the latitudinal distribution of precipitation have a much more pronounced latitudinal

variation with a maximum over equatorial regions, a secondary maximum over high latitudes and minima over polar and subtropical regions (14), (15).

Hence we conclude that the latitudinal distribution input of energy into the atmosphere through non-adiabatic heating which is the generation source for all the potential energy and responsable for the maintenance of atmospheric motions against dissipation is essentially modulated by the function $\mathscr{L}(\varphi) \equiv < \overline{L}_f (0) \dagger >$ as can be inferred from the inspection of equation (16). Thus the hydrological cycle has a profound influence upon the energetics of the general circulation of the atmosphere.

In the *evaporation-condensation-precipitation cycle*, virtually all evaporation takes place at the surface of the earth, and therefore the cooling involved in the process does not directly affect the atmosphere and does not affect the generation of available potential energy. On the other hand, condensation occurs principally within the atmosphere, where the release of latent heat has a direct influence.

Virtually all the condensate will eventually reach the ground in the form of precipitation but there may, of course, be further transports before this occurs. However, it is well established that the transport of water in the vapor phase far exceeds that in the liquid and solid phases in the atmosphere (6). This fact justifies the assumption, used in the present discussion, that precipitation is a measure of condensation. Precipitation, surface drainage and runoff complete the mass cycle, but are unimportant as far as the energetics of the atmosphere are concerned. Viewing the cycle as a whole, we see that it produces a net transfer of heat from the earth's surface to the atmosphere where it modulates the total meridional input of energy.

An assessment of the importance of the hydrological cycle in the energetics of the atmosphere requires the knowledge of geographical distribution of condensation, but not that of evaporation.

Since the hypothesis of evaporation-precipitation *in situ* cannot be accepted there must also be a transfer of heat from one geographical location to another and from one level to another. Thus we see that this cycle plays a part in regulating the temperature distribution of the earth's surface and atmosphere. Moreover, the transport of energy in the form of latent heat can be looked upon as a mechanism which the general circulation uses to adjust itself to the heterogeneous boundary conditions imposed by the differential absorption of solar energy.

The water cycle, from the point of view of mass and its implications on the field of hydrology has been discussed elsewhere (11), (12), (15).

4. *THE MAINTENANCE OF THE MEAN AND EDDY CIRCULATIONS IN THE ATMOSPHERE*

4.1. *Available potential energy and kinetic energy*

The concept of mean available potential energy presented by MARGULES (1903) (16) has been elaborated and discussed later by LORENZ (1955) (17) who was able to express it in terms of the temperature variance on an isobaric surface. The expression of the available potential energy is given by:

$$A = \frac{1}{2} c_p \int \gamma \, \overline{T''^2} \, dm \tag{21}$$

where γ is the stability factor:

$$\gamma = -\alpha \left(\frac{1}{\theta} \frac{\partial \overline{\overline{\theta}}}{\partial p} \right)^{-1} = \left(\overline{\overline{T}} - \frac{p}{R} \, c_p \frac{\partial \overline{\overline{T}}}{\partial p} \right)^{-1}$$

Through an analysis of variance of the temperature field the available potential energy $A = \frac{1}{2} c_p \int \gamma \, \overline{T''^2} \, dm$ may be partitioned into zonal available potential energy A_M and eddy available potential energy A_E

$$A = A_M + A_E \tag{22}$$

where

$$A_M = \frac{1}{2} c_p \int \gamma \, [\overline{T}]''^2 \, dm \tag{23}$$

and

$$A_E = \frac{1}{2} c_p \int \gamma \, [\overline{T'^2} + \overline{T}^{*2}] \, dm \tag{24}$$

Similarly, through an analysis of variance of the wind field the total kinetic energy $K = \frac{1}{2} \int \mathbf{v}^2 \, dm$ may be partitioned into zonal

kinetic energy, K_M, corresponding to zonally averaged motions of the atmosphere and into eddy kinetic energy, K_E, the eddy motions:

$$K = K_M + K_E \tag{25}$$

where:

$$K_M = \frac{1}{2}\int\left\{[\bar{u}]^2 + [\bar{v}]^2\right\} dm \tag{26}$$

and

$$K_E = \frac{1}{2}\int\left\{[\overline{u'^2} + \overline{v'^2}] + [\overline{u^{*2}} + \overline{v^{*2}}]\right\} dm \tag{27}$$

It is convenient to consider K_M, K_E, A_M and A_E as separate forms of energy. A balance equation for each of these forms can be obtained following the usual and well known procedure. These equations have some terms in common with opposite signs and will be regarded as conversion functions among the various forms of energy (4), with the understanding that we reserve the term conversion functions for the terms which express clearly physical mechanisms that take part in the atmosphere.

4.2. *Balance equations for the kinetic energy*

In the derivation of the balance equation for the zonal kinetic energy the equation (1) of zonal motion is first multiplied by $[\bar{u}]$, next averaged in time and finally integrated over the mass for a polar cap or all for the atmosphere. The resulting equation, expressing the balance between the various processes which take part in the maintenance of K_M, can be written under the simbolic form:

$$\frac{\partial K_M}{\partial t} = C(K_E, K_M) + C(K_{M,\varphi}, K_{M,\lambda}) + D(K_M) + A(K_M) + W_e(K_M) \tag{28}$$

where:

a)

$$C(K_E, K_M) = \int [\overline{u\,v}]_E \cos\varphi \frac{\partial}{a\,\partial\varphi}\left[\frac{[\bar{u}]}{\cos\varphi}\right] dm + \int [\overline{u\,w}]_E \frac{\partial [\bar{u}]}{\partial p}\, dm \tag{29}$$

is the conversion from eddy into zonal kinetic energy by horizontal

and vertical eddies, which depends upon the transport of angular momentum along the gradient of angular velocity;

b)

$$C\,(K_{M,\varphi}, K_{M,\lambda}) = \int f\,[\bar{u}]\,[\bar{v}]\,dm + \int [\bar{v}]\,[\bar{u}]^2 \frac{tg\,\varphi}{a}\,dm \tag{30}$$

is the conversion of energy from the mean meridional motions into mean zonal motions due to the Coriolis effect in an Hadley regime;

c)

$$D\,(K_M) = \int [\bar{u}]\,[\bar{F}]\;dm \tag{31}$$

is the viscous, turbulent and frictional dissipation;

d)

$$A\,(K_M) = \iint ([\bar{u}] + [\bar{\omega}])\,\frac{1}{2}\,[\bar{u}]^2 \frac{d\,\Sigma}{g} \tag{32}$$

is the advection of K_M across the boundary Σ by mean meridional overturnings; and finally:

e)

$$W_e\,(K_M) = \iint [\bar{u}]\,([\overline{u\,v}]_E + [\overline{u\,\omega}]_E)\,\frac{d\,\Sigma}{g} \tag{33}$$

is the work performed on the volume τ by the eddy stresses at the boundary Σ.

The balance equation for the eddy kinetic energy is derived following an analogous procedure. The zonal equation of motion is multiplied by $u' + \bar{u}^*$ and the meridional equation by $v' + \bar{v}^*$. After adding them together and averaging in time, the integration in space over the mass of a polar cap will lead to the balance equation:

$$\frac{\partial K_E}{\partial t} = -C\,(K_E, K_M) + C\,(A_E, K_E) + D\,(K_E) + A\,(K_E) + \\ + W_p\,(K_E) + W_e\,(K_E) \tag{34}$$

where:

a)

$C\,(K_E, K_M)$ is the rate of conversion from eddy into zonal kinetic energy;

b)

$C(A_E, K_E)$ is the rate of conversion from eddy available potential into eddy kinetic energy:

$$C(A_E, K_E) = -\int \frac{R}{p} \left([\overline{\omega' T'}] + [\bar{\omega}^* \bar{T}^*] \right) dm; \qquad (35)$$

c)

$D(K_E)$ is the dissipation of eddy kinetic energy due to friction:

$$D(K_E) = \int [\overline{u F_\lambda}]_E \, dm + \int [\overline{v F_\varphi}]_E \, dm; \qquad (36)$$

d)

$A(K_E)$ is the advection of eddy kinetic energy through the boundary Σ by mean meridional circulations:

$$A(K_E) = \iint_\Sigma \left\{ [\bar{v}] + [\bar{\omega}] \right\} \cdot \frac{1}{2} \left\{ [\bar{u}^2]_E + [\bar{v}^2]_E \right\} \frac{d\Sigma}{g} \qquad (37)$$

and

e)

$W_e(K_E)$ is the work performed on the layer by the eddy stresses at the boundaries.

A net conversion from zonal to eddy kinetic energy would occur if the eddies acted to transfer angular momentum from latitudes of high angular velocity to those of low angular velocity in the manner of large scale viscosity. However, observations indicate that such a conversion does not take place (18), (20), but instead eddy kinetic energy is converted into zonal kinetic energy and appears to be the main source for the maintenance of the zonal currents against dissipation by turbulence, by friction, etc.

4.3. *Balance equations for the available potential energy*

The balance equation for the available potential energy can be derived from the equation (6) of first law of thermodynamics when it is multiplied by $\gamma [\bar{\theta}]''$ where γ is the stability parameter and resolving the potential temperature into the various eddy components $\theta'' = [\bar{\theta}]'' + \bar{\theta}^* + \theta'$.

The resulting equation after averaged in time and next integrated in space for a polar cap or for all the atmosphere and neglecting the triple correlations, assumes for the zonal available potential energy the form:

$$\frac{\partial A_M}{\partial t} = - C(A_M, A_E) - C(A_M, K_M) + G(A_M) \tag{38}$$

This equation expresses that there is a balance between the various processes:

a) The conversion $C(A_M, A_E)$ from zonal into eddy available potential energy by horizontal and vertical eddy processes:

$$C(A_M, A_E) = - c_p \int \gamma \left\{ [\overline{v' T'}] + [\overline{v}^* \overline{T}^*] \right\} \frac{\partial [\overline{T}]}{a\, \partial \varphi} dm -$$
$$- c_p \int \gamma \left(\frac{T}{\theta}\right) \left\{ [\overline{\omega' T'}] + [\overline{\omega}^* \overline{T}^*] \right\} \frac{\partial [\overline{\theta}]''}{\partial p} dm \tag{39}$$

which depends upon the horizontal and vertical eddy transports of sensible heat $[\overline{v\,T}]_E$ and $[\overline{\omega\,T}]_E$ along the gradient of temperature. When eddies transport sensible heat against the temperature gradient (from warm to cold zones) there is a conversion from zonal available potential energy A_M into eddy available potential energy A_E in the manner of large scale conductivity.

b) The conversion $C(A_M, K_M)$ from zonal available potential energy into zonal kinetic energy by mean meridional circulations:

$$C(A_M, K_M) = - R \int p^{-1} [\overline{\omega}]'' [\overline{T}]'' dm \tag{40}$$

c) The generation $G(A_M)$ of zonal available potential energy by non-adiabatic heating:

$$G(A_M) = \int \gamma [\overline{T}]'' [\overline{Q}]'' dm \tag{41}$$

The low latitude regions with a warm troposphere are continuously heated by the surplus of incoming solar radiation over outgoing terrestial long wave radiation, whereas middle and high latitude

regions with lower temperatures are cooled by the same radiation energy balances.

This results in a large positive value of the covariance $\widetilde{[T]'' [Q]''}$ and consequently in a large generation of zonal available potential energy.

The equation of balance for the eddy available potential energy can be derived multiplying the equation of the first law of thermodynamics by $\gamma (\overline{\theta}^* + \theta')$ where γ is the static stability factor, next averaged with respect to time and then integrated in space. The final equation will be:

$$\frac{\partial A_E}{\partial t} = C (A_M, A_E) - C (A_E, K_E) + G (A_E) \tag{42}$$

where $G (A_E)$ is the generating function of eddy available potential energy and is given by:

$$G (A_E) = \int \gamma [\overline{T' Q'} + \overline{T}^* \overline{Q}^*] \, dm \tag{43}$$

and the other terms have the previous meaning.

As was mentioned in § 3.1 only a small part of the total potential energy is converted into kinetic energy in the atmosphere. The maximum possible value is the available potential energy. However, it may now be inferred that the principal via of conversion of available potential energy to kinetic energy is a conversion of eddy available potential into eddy kinetic energy accomplished by downward eddy motions of cooler air and upward eddy motion of warmer air.

4.4. *The energy cycle of the general circulation in the troposphere*

We are led to the following scheme of the energy cycle of the general circulation in the troposphere. The net heating of the atmosphere in low latitudes and the net cooling in high latitudes result in a continual generation of zonal available potential energy. The atmosphere is baroclinically unstable and virtually all this energy is converted into eddy available potential energy by the resulting eddies. Some of this energy may be dissipated in the eddies through the combined effects of radiation, condensation, evaporation, the heat flux near the ground and the heating of colder portions of the eddies

and the cooling of warmer portions; the remainder is partly converted into eddy kinetic energy through the sinking of colder air and the rising of warmer air in the eddies.

Some of the kinetic energy in the large scale eddies is dissipated in a cascade regime by generating smaller and smaller eddies and by friction; the remaining part of this energy is converted into kinetic energy for the zonal currents.

Most of the zonal kinetic energy is dissipated by turbulence and by friction; a small residual is converted into zonal available potential energy again by an indirect meridional circulation and this brings back to the beginning of the cycle. Schematically we accept that the energy cycle in the troposphere proceeds from A_M to K_M, through the following scheme:

$$A_M \longrightarrow A_E \longrightarrow K_E \longrightarrow K_M \longrightarrow A_M$$

It therefore appears that eddies play a crucial role in regulating the general circulation. It is this very basic fact that has laid down the foundation for the modern concepts on general circulation and has changed all the perspective of the dynamics of the atmosphere (STARR, 1958) (20).

5. *THE WATER VAPOR AND THE ENERGY CYCLE OF THE GENERAL CIRCULATION*

5.1. *Availability in a moist atmosphere*

The formulation of the energy cycle, as has been presented, does not incorporate directly the water component in the atmosphere. The fundamental reason lays in the difficulty in defining a ***reference state*** for such complex system as the moist atmosphere; the specification of the reference state is essential for the assessment of the ***availability*** of the energy of a system. The reference state has to be a ***dead state***, inert for any thermodynamical transformation, characterized by the most stable state of thermodynamical equilibrium, without local contrast in entropy, suposed to have the maximum possible value compatible with the constraints imposed to the total system, and so with a minimum total potential energy.

The availability measures the maximum va e of the *useful work*, W_u, corresponding to a state of a subsystem within an ideal atmosphere in a dead state and is denoted by Λ. Thus:

$$\Lambda \equiv \Phi^* - \Phi^*_{min} \tag{44}$$

It follows that for any state of any atmospheric subsystem in the ideal dead atmosphere

$$\Lambda > 0 \ .$$

For the most stable state of the subsystem, which is then a dead state,

$$\Lambda = 0$$

which corresponds to the identification of the subsystem with the ideal atmosphere. For a finite change from the subsystem in state a to the subsystem in state b

$$W_u = \int_{\widehat{ab}} dW < - \int_a^b d\Phi \tag{45}$$

or

$$W_u < \Phi^*_a - \Phi^*_b = \Delta \Lambda^b_a \tag{46}$$

It follows that it is impossible to transform into useful work (kinetic energy of the atmospheric motions) all the variation of the availability when the subsystem undergoes any transformation that brings it from a thermodynamic state to another.

The behaviour of the water component in the atmosphere makes it very difficult to define a «dead state» for the moist atmosphere. With the possible changes of phase there is a mass transfer from one phase to another within the atmospheric subsystem and eventually a transfer of mass out of the total system (the atmosphere) through precipitation. The atmosphere is an open system for the water component and, furthermore, its content in water vapor is not even statistically constant. All these processes and transformations alter profoundly the time and the spatial distribution of energy and entropy within the system and makes it extremely difficult to define and to find the balance of those quantities. Thus, any model of reference state for this system so complex has to be highly idealized and restrictive, involving rather gross simplifications and limitations when compared with the real moist atmosphere.

A possible reference state for a moist monophasic and trivariant atmosphere, useful perhaps under very simple conditions, would be that corresponding to an ideal atmosphere with a minimum total moist potential energy and with uniform isobaric distributions of temperature, entropy and moisture content, as given, for example, by the corresponding isobaric averaged values.

5.2. *Effects of water vapor on the general circulation*

In spite of the mentioned difficulties in taking into account explicitly the influence of the water component in the present theory of the general circulation, we can infer some effects of the water vapor on the energy cycle formulated for a dry atmosphere and discuss some implications due to the presence of water vapor in the atmosphere.

The rate of generation of total potential (internal plus potential) energy depends upon the total non-adiabatic heating. However, the generation of available potential energy is determined by the spatial distribution of non-adiabatic heating with respect to the temperature field. Zonal and eddy available potential energies are affected directly by the water vapor through absorption of solar radiation and the processes of long wave radiation, through the changes in the albedo and, finally, through condensation and, to a less extent, through evaporation.

Let us consider the heat energy stored in the moist atmosphere due to water vapor. The existence of the energy which we shall denote as *moist «available» potential energy* can play a part in the dynamics of the atmosphere only if it is converted into available potential energy, through the process of a phase change (mainly through condensation). Since the hemispheric distribution of water vapor is almost axially symetric with a well defined meridional gradient, there is in the atmosphere a large storage of ***total moist zonal «available» potential energy***. The distribution has, of course, a non symetric component, which suggest the concept of ***total moist eddy «available» potential energy***.

The rate of generation of available potential energy, $G(A)$, is proportional to the covariance of non-adiabatic heating and temperature. Thus, through the release of latent heat the water vapor plays a direct role in the production of zonal and eddy available potential energy, $G(A_M)$ and $G(A_E)$, as can be inferred from their corresponding equations (41) and (43).

It follows also from the mathematical expressions of the conversion terms of the various forms of energy, as presented in § 4, that the water vapor plays an indirect role in the conversion of the various forms of energy because the water vapor can introduce significant changes in the u, v, ω and T and γ fields.

The time averaged transport of latent heat by both transient and standing eddies is poleward at all latitudes as it is shown in the corresponding tables of § 4. Since the mean zonal temperature also decreases poleward (21), (22), this means that the eddies must, on the average, be converting latent zonal available potential energy into latent eddy available potential energy, $C(A_M, A_E) > 0$. We should expect this effect to be most intense in middle and high latitudes where there is a maximum of eddy activity. To the extent that at these latitudes condensation occurs predominantly near the frontal zones there is a generation of eddy available potential energy — that is, some of the moist available potential energy released — in the perturbations along the frontal zone. This is precisely where the conversion of available potential energy into kinetic energy, through baroclinic processes, is taking place.

In fact condensation $(\dot{Q}_L > 0)$ in middle latitude perturbations occurs generally with southerly warm ascending currents $(T' > 0)$, $(\omega < 0)$ and evaporation $(\dot{Q}_L < 0)$ with northerly cold subsiding currents $(T' < 0)$, $(\omega > 0)$. Since the correlation between $\dot{Q}_L$ and T is positive there is a generation of eddy available potential energy with the release of latent heat, $\{G(A_E)\}_L > 0$. However, when the radiation effects are considered the oposite takes place. The moist warm air from south $(T' > 0)$ in its movement towards higher latitudes is cooled $(\dot{Q}_F + \dot{Q}_R < 0)$, while the northerly cold air $(T' < 0)$ is warmed $(\dot{Q}_R + \dot{Q}_F > 0)$. In this case the covariance along the latitude circles between T and $\dot{Q}_R$ is negative which leads to a destruction of eddy available potential energy, $\{G(A_E)\}_R < 0$. The net value for $G(A_E)$ will depend upon the balance of these oposite effects; $G(A_E)$ is presumably negative (18), but it might happen that the release of latent heat could alter at times the sign of $G(A_E)$.

In these disturbances the covariance between T and ω is negative $(\overline{[\omega\, T]}_E < 0)$ and there is a conversion of eddy available potential energy into eddy kinetic energy, $C(A_E\, K_E) > 0$. The condensation process reinforces the vertical motion field by heating warm rising air, thus augmenting the rate of generation of eddy

kinetic energy by baroclinic processes. AUBERT (23) has shown the importance of this effect.

It is also interesting to point out that, since the large-scale condensation and evaporation processes are accompanied simultaneously by rising of warm air and sinking of cold air, respectively, the available energy so generated is not exposed to the dissipation through long wave radiation as it would happen, with the potential energy generated by the transport of sensible heat. Probably the efficiency of the conversion of available potential energy generated through the release of latent heat into kinetic energy is very high.

The strong precipitation observed in equatorial regions due to the Hadley cell contributes decisively to the production of zonal available potential energy because the release of latent heat occurs in regions where the temperature is already higher than average and almost zonally uniform $([\overline{\overline{T}}]''[\overline{Q}_L]'' > 0)$. It might appear, at first glance, that the secondary maximum in the curve of $\mathscr{L}(\varphi) = \langle \overline{L}_f(0) \uparrow \rangle$ at high latitudes is associated with the destruction of zonal available potential energy, because heating is taking place in a region where the zonally averaged temperature is relatively lower. However, in this case, the zonally averaged picture is misleading, for, as we have seen above, the heating due to the release of latent heat at these latitudes occurs selectively in the warm air giving rise, *de facto*, to the generation of eddy available potential energy, $G(A)_E > 0$. When we take the zonal average of the rate of generation of available potential energy, we find, in fact, that there is a relative maximum in these latitudes.

Let us consider still another aspect of the energy cycle where the effects of water vapor must be taken into account. All energy conversion processes involving available potential energy are dependent upon static stability. We note that static stability occurs in the denominators of equations (21), (23), (24), (39), (41), (43). The direction of conversion between available potential energy and kinetic energy depends upon the sign of this term, and the rate depends upon its numerical value, all other things being considered constant. If, initially, the reaction proceeded so as to generate kinetic energy, then eventually, the rising of warm air and sinking of cold air involved in the process, if unopposed by other factors, would stabilize the atmosphere, thus bringing the reaction to a halt. However, the release of latent heat alters this situation in the following manner. The condensation process occurs mainly near the 700 mb level, which is below the level of maximum rate of energy conversion. Hence, the

release of latent heat destabilizes the atmosphere in the regions where kinetic energy is being generated, thus contributing to the perpetuation of the conversion processes, because the dominant baroclinic waves become more unstable and the release of latent heat leads to an acceleration of their growth.

The adiabatic lapse rate Γ_d that figures in the stability parameter γ in various equations is applicable only in an unsaturated environment. In a saturated atmosphere it must be replaced by Γ_s, if the pseudo-adiabatic assumption is to be used. Hence, in a saturated atmosphere the rate of energy generation and conversion will have to be adjusted by the factor:

$$\frac{\Gamma_s}{\Gamma_d}\frac{(\Gamma-\Gamma_d)}{(\Gamma-\Gamma_s)}$$

5.3. *Heating of the atmosphere due to condensation*

A possible functional representation for $\dot{Q}_L$ can be obtained from the balance equation for the water component. At a given isobaric level p, the rate of heating, ${}_p\dot{Q}_L$, due to the condensation of dq grams of water vapor is given by:

$${}_p\dot{Q}_L = -L\frac{dq}{dt} \tag{47}$$

Using the water vapor continuity equation we can write the previous equation, averaging in time:

$${}_p\overline{\dot{Q}}_L = -L\left(\frac{\partial\bar{q}}{\partial t} + \overline{\mathbf{v}\cdot\nabla q} + \overline{\omega\frac{\partial q}{\partial p}}\right) \tag{48}$$

This equation transformed with the equation of continuity can be written as follows:

$${}_p\overline{\dot{Q}}_L = -L\left(\frac{\partial\bar{q}}{\partial t} + \nabla\cdot\overline{q\,\mathbf{v}} + \frac{\partial\overline{q\omega}}{\partial p}\right) \tag{49}$$

We should also include in this equation the vertical eddy difusion due to small scale turbulence. However, in large scale processes this effect can be neglected.

The vertical integration of this equation leads to the equation of the divergence of total water vapor transport referred in the previous paragraph:

$$\int_p^{p_0} {}_p\dot{Q}_L \, dp = \overline{\dot{Q}}_L = - L \frac{\partial \overline{W}}{\partial t} - L \, div \, . \, \overline{\mathbf{Q}} \qquad (50)$$

because the contribution of the terms in ω and q, due to the boundary conditions is zero.

The analysis of the divergence of water vapor transport, proportional to the heating associated with the release of latent heat, shows centers of convergence ($div \, . \, \overline{\mathbf{Q}} < 0$) alterning with centers of divergence ($div \, . \, \overline{\mathbf{Q}} > 0$) over all the northern hemisphere (12). Therefore the spatial distribution the heating of the atmosphere due to the release of latent heat is not uniform. This illustrates the need for considering, in all problems involving the generation of available potential energy, the actual spatial covariance of the divergence of water vapor and temperature, rather than the covariance of the zonally averaged values of the respective fields.

With the usual REYNOLDS expansion equation (49) will assume the form:

$${}_p\dot{Q}_L = - L \left(\frac{\partial \overline{q}}{\partial t} + \nabla \cdot \overline{q' \mathbf{v}'} + \frac{\partial \, \overline{\omega' q'}}{\partial p} + \nabla \cdot \overline{q} \, \overline{\mathbf{v}} + \frac{\partial \, \overline{\omega} \, \overline{q}}{\partial p} \right); \qquad (51)$$

In a (λ, φ, p, t) coordinate system this equation is written:

$$\begin{aligned} {}_p\dot{Q}_L &= - L \left(\frac{\partial \overline{q}}{\partial t} \right) - L \frac{1}{a \cos \varphi} \left(\frac{\partial \, \overline{q}}{\partial \lambda} + \frac{\partial \overline{q v} \cos \varphi}{\partial \varphi} + \frac{\partial \, \overline{\omega q}}{\partial p} \right) \\ &= - L \frac{\partial \overline{q}}{\partial t} - L \frac{1}{a \cos \varphi} \left(\frac{\partial \, \overline{u' q'}}{\partial \lambda} + \frac{\partial \, \overline{q' v'}}{\partial \varphi} \cos \varphi + \frac{\partial \, \overline{\omega' q'}}{\partial p} \right) - \\ &\quad - L \frac{1}{a \cos \varphi} \left(\frac{\partial}{\partial \lambda} \overline{u} \, \overline{q} + \frac{\partial}{\partial \varphi} \overline{q} \, \overline{v} \cos \varphi + \frac{\partial}{\partial p} \overline{\omega} \, \overline{q} \right) \end{aligned} \qquad (52)$$

In the long time average the local change $\frac{\partial \overline{q}}{\partial t}$ is very small and can be disregarded in comparison with the other terms. All the other terms, except $\frac{\partial \, \overline{q' \omega'}}{\partial p}$ and $\frac{\partial \, \overline{\omega} \, \overline{q}}{\partial p}$ can be computed from hemispheric humidity charts such as those already published (6).

The local values of the divergence associated with the transport of water vapor by the transient and standing eddies, $\nabla \cdot \overline{q' \mathbf{v}'}$, and $\nabla \cdot \bar{q}\, \bar{\mathbf{v}}$ are of the same order of magnitude. However, the mass integral for all the atmosphere gives in general higher values for the divergence associated with the standing eddies ($\nabla \cdot \overline{q^* \mathbf{v}^*}$) than those associated with the divergence of transient eddies ($\nabla \cdot \overline{q' \mathbf{v}'}$). The values of the divergence, $\nabla \cdot \bar{q}\, \bar{\mathbf{v}}$, decrease rapidly with the altitude because $\bar{q}$ becames very small as the height incrases. The values of $\nabla \cdot \overline{q' \mathbf{v}'}$ decrease also with height, with a maximum at 850 mb level.

The evaluation of the vertical divergence given by the terms $\frac{\partial \overline{q' \omega'}}{\partial p}$ and $\frac{\partial \bar{q}\, \bar{\omega}}{\partial p}$ depends on the knowledge of the ω-field at various levels. The values of ω could be obtained from the observations using the method suggested by BARNES (24). For a given layer of the atmosphere the vertical divergence of water vapor may have values comparable to those of the horizontal divergence (and perhaps of oposite sign). However, for all the atmosphere, since the integration has to be done in p, the horizontal divergence field will predominate in view of the boundary values of $\bar{q}$ and $\bar{\omega}$ at the bottom and at the top of the atmosphere.

5.4. *Generation of available potential energy due to release of latent heat*

Since the effects of water vapor are so complex we shall only discuss here the effects of phase changes of water vapor in the atmosphere. Simultaneously a method for computing the rate of generation of available potential energy due to release of latent heat wil be presented.

The generation of available potential energy due to the release of latent heat is obtained substituing $\dot{Q}$ by the value of $\dot{Q}_L$, as given by equation (51), in the expressions (41) and (43):

$$G(A_M) = \int \Upsilon\, [\bar{T}]''\, [\bar{Q}]''\, dm$$

$$G(A_E) = \int \Upsilon\, [\overline{T' Q'} + \bar{T}^* \bar{Q}^*]\, dm$$

The rate of generation of zonal available potential energy due to the release of latent heat is therefore:

$$G(A_M)_L = -L\int \Upsilon\,[\overline{T}]''\frac{\partial\,[\overline{q}]''}{\partial t}\,dm - L\int \Upsilon\,[\overline{T}]''\,\nabla\cdot[\overline{q\,\mathbf{v}}'']\,dm -$$
$$- L\int \Upsilon\,[\overline{T}]''\frac{\partial\,[\overline{q\,\omega}]''}{\partial p}\,dm \qquad (53)$$

The rate of generation of eddy available potential energy will be given by:

$$G(A_E)_L = -L\int \Upsilon\left[\overline{T'\frac{\partial q'}{\partial t}}\right]dm - L\int \Upsilon\,[\overline{T'\,\nabla\cdot(q\,\mathbf{v})'}]\,dm -$$
$$- L\int \Upsilon\left[\overline{T'\frac{\partial\,(q\,\omega)'}{\partial p}}\right]dm - L\int \Upsilon\left[\overline{T}^*\frac{\partial \overline{q}^*}{\partial t}\right]dm -$$
$$- L\int \Upsilon\,[\overline{T}^*\,\nabla\cdot\overline{q\mathbf{v}}^*]\,dm - L\int \Upsilon\left[\overline{T}^*\frac{\partial \overline{q\,\omega}^*}{\partial p}\right]dm \qquad (54)$$

For computation purposes it is more convenient to write the expressions of $G(A_M)_L$ and of $G(A_E)_L$ in the (λ, φ, p, t) coordinate system as we did above.

In principle all the terms of $G(A_M)_L$ and of $G(A_E)_L$ due to the release of latent heat could be computed by using time series of spatial isobaric distributions (charts) of the fields of T, q and of $\overline{q\,\mathbf{v}} = (\overline{q\,u}\,\mathbf{i} + \overline{q\,v}\,\mathbf{j} + \overline{q\,\omega}\,\mathbf{k})$. Through the analysis of these charts with a suitable grid point the calculations could be made using the standard finite differences method.

In longe time average the local change $\dfrac{\partial \overline{q}}{\partial t}$ is very small and can be neglected. However, for short time intervals, it may become significant, and cannot be disregarded in the evaluation of the covariances.

The contribution of the terms $\left[\overline{T'\dfrac{\partial\,(\omega\,q)'}{\partial p}}\right]$ and $\left[\overline{T}^*\dfrac{\partial\,(\overline{q\,\omega})^*}{\partial p}\right]$ for all the atmosphere is proportional to the difference of the surface covariances of $\overline{q\,\omega}$ and $\overline{T}$ over the boundary surfaces Σ_1 and Σ_2 of the atmosphere. In fact we may write for the last term:

$$\left[\overline{T}^*\frac{\partial\,(\overline{\omega q})^*}{\partial p}\right] = \left[\frac{\partial}{\partial p}(\overline{T}^*(\overline{\omega\,q})^*)\right] - \left[(\overline{\omega\,q})^*\frac{\partial\,\overline{T}^*}{\partial p}\right] \qquad (55)$$

The second term on the right hand side may be neglected because we accept as a good approximation that

$$\frac{\partial T}{\partial p} \simeq \frac{[T]}{\partial p} \tag{56}$$

For all the atmosphere, since $dm = g^{-1}\, dp\, d\Sigma$, the resulting contribution will be:

$$\begin{aligned}\iiint \left[\overline{T}^* \frac{\partial\, \overline{(\omega p)}^*}{\partial p}\right] dm &= \iint\int_0^{p_0} \frac{\partial}{\partial p}\, [\overline{T}^*\, (\overline{\omega q})^*]\, \frac{dp}{g}\, d\,\Sigma = \\ &= g^{-1} \iint_{\Sigma} [\overline{T}^*\, \overline{\omega q}^*]\, d\,\Sigma \Big|_0^{p_0} = \\ = g^{-1} \iint_{\Sigma_1} [\overline{T}^*\, \overline{\omega q}^*]\, d\,\Sigma_1 &- g^{-1} \iint_{\Sigma_2} [\overline{T}^*\, \overline{\omega q}^*]\, d\,\Sigma_2 \end{aligned} \tag{57}$$

The values of these covariances are presumably very small in view of the boundary conditions at the bottom and at the top of the atmosphere for both quantities $\overline{q}$ and $\overline{\omega}$.

It is expected that this suggested method for computating $G\,(A_E)_L$ will lead to good estimates of the rate of generation of eddy available potential energy in extratropical regions due to release of latent heat. In low latitudes the estimates are not so good, because the mesoscale phenomena, so important for the vertical transport of water vapor in this region, have not been taken into account.

6. *FINAL COMMENTS*

We have seen that water vapor plays a vital role in the energetics of the general circulation. It is the most important absorber of solar energy in the atmosphere, and hence, its distribution influences the form of the energy input into the system. The release of latent heat constitutes another important energy input. Through this process the water vapor distribution significantly influences the motions, and motions, in turn, deform the water field. This complex feedback mechanism constitutes what is probably the most important non--adiabatic effect in the general circulation.

To illustrate its importance let us consider the highly simplified situation where a mean meridional cell has formed in response to the temperature difference between the air at two latitudes. Let us examine what happens if there is evaporation at the surface below the descending part of the cell, a transport of water vapor by the

lower branch, and precipitation in the area of ascent. In a crude qualitative sense, this corresponds to the Hadley cell at low latitudes. The release of latent heat due to the moisture transport by the cell increases the temperature difference between the two latitudes, thus increasing the strength of the motions. In effect, the rising air is being warmed by the latent heat release, and is thus forced to rise more rapidly. Continuity demands that the entire cell be strengthened by this effect.

Until recently the many dynamical models which have been put forth in an attempt to explain the features of the general circulation have been formulated for a dry atmosphere. This approach has been necessitated by the difficulties which arise when one tries to describe analytically the mechanism which transport water vapor. Very close to the ground (in the lowest few meters) microscale effects predominate, while at higher levels, mesoscale motions affect the vertical transports, and horizontal transports are accomplished predominantly by macroscale motions. Further complications arise when one attempts to specify the necessary and sufficient conditions for the condensation process to take place. The subsequent precipitation process is also extremely difficult to describe, even if a pseudoadiabatic process is assumed. Although some of these difficulties have been dealt with sucessfully in a recent model put forth by the staff of the Geophysical Fluid Dynamics Laboratory, of ESSA (Environmental Sciences Services Administration), Washington (25), the understanding of the role of the water vapor in the dynamics of the atmosphere has not yet reached the stage where any single model can accurately simulate all the above processes (26).

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IV. SURVEYS AND SYNTHESES

THE PHYSICAL BASIS FOR THE GENERAL CIRCULATION

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Since time immemorial man has inescapably observed the atmosphere in which he lives and has his being. It would therefore seem reasonable to expect that at the present date the science of meteorology should be one of the most advanced fields of human endeavor. Yet, if a distinction is made between the mere collection of descriptive facts of observation on the one hand and interpretative work which aims to give a rational intellectual understanding of phenomena on the other, it must be confessed that our knowledge concerning the large-scale motions of the atmosphere is restricted mostly to the former category of information. Thus, for example, no one has as yet given a satisfactory rational explanation for one of the most outstanding features of the general circulation, namely the large belts of westerly winds in the temperate latitudes of each hemisphere. However, it must be recognized that it is only in the last few decades that anything approaching sufficiently complete global observations *for the checking of hypotheses* regarding the general circulation has become available, so that progress at a more accelerated pace should now be forthcoming.

The physiognomy of present-day meteorology bears much of the imprint imparted to it by the (so to speak) accidental way in which synoptic reporting networks developed and grew. In fairly recent times it was quite an achievement for a meteorologist to have at his command a network of reports large enough to depict an entire cyclone. The immediate temptation was then to treat this feature as an independent entity and to separate it artificially from its meteorological environment in seeking a rational explanation for it. The cyclone thus became a phenomenon that existed independently. More extensive observations now available point to the inadequacy of this tacit assumption. The cyclone constitutes a cogwheel in a larger mechanism and probably can be understood only in relation to and not independent of its atmospheric context.

A criticism which is the same in principle can be leveled against many other efforts to explain synoptic structures. Indeed meteorology is replete with attempts to formulate *ad hoc* explanations for individual details of the general circulation without due cognizance of their role as functioning parts of a global scheme.

In the more recent literature there are signs that we are outgrowing this restricted point of view; there are indications which emphasize the essential oneness of the atmosphere which must be studied as an internally integrated and coordinated unit. The general circulation presents a puzzle to be solved. We must learn how the various parts fit together into a whole if we are to understand it. With the hemispherical data now becoming available and the clues already at our disposal the avenue to sound progress is wide open and, at least as far as the writer is concerned, quite inviting.

It is therefore in the spirit of appraising our knowledge from the larger point of view that this commentary is written. Admittedly and intentionally the treatment reflects the writer's viewpoint gained through a number of years of concentration on the subject under consideration. As a candid personal sidelight it should be mentioned, however, that much of the material is in a sense a relatively recent culmination of the writer's striving toward a more unified and coherent conception of the global circulation. Nor is there currently any sign that this process has reached a state of final crystallization. This paper is therefore in the nature of an individualistic progress report designed to portray and to share with others a certain outlook and approach in the hope that really substantial and enduring progress may eventually be attained thereby. Within the space of these pages we cannot hope to give anything approaching an exhaustive exposition of various topics. For this reason only certain basically important highlights will be elaborated, and we shall rely upon the reader's competence to interpolate various items of information which are generally available in the literature. Furthermore, the selection of the subjects touched upon is not one dictated by an aim at logical completeness, but rather is limited to those aspects which in the writer's mind are most likely to be conducive to further results at the present stage of development of the science.

As an outstanding problem of paramount importance for human activities, it is rather astonishing that the global circulation of the atmosphere has not up to the present time received more consideration from physical scientists generally. This situation is probably due in part to the lack of proper observational information so necessary for the successful prosecution of research concerning the subject. The gaps in at least our gross factual information are currently being removed rather rapidly, with the consequence that questions regarding the proper interpretation of the data begin to be the major issues. We might, with benefit in this connection, digress for a moment and compare the development of meteorology with that of another physical science, namely astronomy. No one can dispute the claims that Galileo and Newton created a new and orderly conception of the solar system. This achievement was necessarily preceded not only by the laborious accumulation of observational knowledge by Tycho Brahe and others but also by the proper interpretation of these data by Kepler who, it might be said, defined in precise terms the dynamic puzzle to be resolved. In meteorology we find ourselves in what might be called

the Keplerian era. It behooves us to marshal and correlate our observations into as precise and consistent a scheme as possible in order that we may know in sufficient detail what is to be explained.

From what has been said it might seem to the reader that meteorology must still undergo a rather protracted period of development before results can be expected at the final fruition of this process. Although there is reason to expect this pattern of events in the philosophical aspects of the subject, it should not be forgotten that the main practical use of meteorological knowledge, the preparation of weather forecasts, is at present largely an empirical procedure. As such, every improvement in our empirical information about the atmosphere enhances in some degree the possibility of improved forecasts, even though satisfactory understanding still remains to be achieved. Here only the intellectual aspects of problems are touched upon, leaving any possible practical applications for treatment elsewhere.

Following the general plan implied in what has been said, let us begin by examining how the general circulation, as we observe it, *achieves internal dynamic consistency in several important respects*. As will be seen, this is merely the extraction from data and interpretation of certain information, and does not in any sense constitute an *explanation* of why the facts are as they are found. In order to concentrate attention on the most basic processes, let us first consider the mean state, leaving the temporal fluctuations in the general circulation as a problem of much greater difficulty to be touched upon later.

If an observer equipped with suitable instruments were to measure the motions of the atmosphere from some extraterrestrial vantage point, in the same manner as we measure the motions in the sun, he would probably be impressed by the irregularities of the details, but at the same time he would discern that there is a pronounced general drift of the air from west to east relative to the earth in middle latitudes, extending from the surface to the stratosphere and even beyond. On the other hand, in the more equatorial regions (and at times near the poles, at least the North Pole) he would discern a drift from east to west from the surface to great heights. This situation immediately poses perhaps the most important problem concerning the general circulation. The specific question involved here is how the belts of westerlies can maintain their high rate of rotation in the face of the retarding effect of surface friction, flanked as they are by oppositely directed winds on at least their equatorial sides. No really satisfactory rational theory for this state of affairs has yet been given, although some deductions can easily be made concerning the nature of certain aspects of the mechanism which is necessary to maintain these existing motions.

The retarding effect of the surface frictional forces on the middle-latitude westerlies may be looked upon as a continuous abstraction of absolute angular momentum from the atmosphere in these regions. According to simple principles of Newtonian mechanics, this drain can be compensated only by an equivalent flow of angular momentum into the westerly belts. Likewise the surface frictional effect in the regions of the easterlies may be interpreted as a flow of angular momentum from the earth into the atmosphere. In order that the angular momentum so transferred into the easterly regions should not progressively accumulate and destroy these wind systems, it is necessary that an equivalent flow out of these regions should exist.

The inescapable conclusion is that the accounts are balanced by a flow of angular momentum from the easterlies in the more tropical regions poleward to the westerly belts (the polar easterlies are of relatively small importance in this connection). Such a flow of angular momentum, let us say northward at the northern border of the tropical easterlies, can be measured in terms of an equivalent tangential stress acting across a vertical surface parallel to the latitude circle. A crude estimate of the value of this stress may be made from existing information concerning the surface frictional forces on the easterlies to the south.[1] The result is of the order of 50 to 100 dynes cm^{-2}. Molecular and small-scale eddy viscosity cannot transmit stresses of this magnitude under the existing conditions, so that very large-scale nonzonal components of motion must furnish the necessary eddy-transfer of angular momentum. We thus come to the very pertinent observation due to Jeffreys [5], namely that the large nonzonal components of motion in the atmosphere are necessary for the maintenance of the average zonal components.

Most classical models for the general circulation as well as some more recent ones (see for example Rossby [9]) have followed along lines originally proposed by Hadley [4] in that they assume the existence of large, slow, convectively driven closed circulations in meridional planes. The development of the mean zonal motions is then ascribed to the effect of the earth's rotation on these primary circulations. According to this view the necessary transport of angular momentum could be achieved if, for example, the poleward branches of the meridional circulations carry more angular momentum than the returning ones at other levels.

For several reasons many modern meteorologists have come to view models of the Hadley type with skepticism. In the first place, the warmest regions of the atmosphere are not usually found in the tropics as most schemes of this kind visualize, but rather some distance away from the equator. Also, at best it is difficult to account for the great extent of the westerlies in the atmosphere on any such basis. Finally, there is a suggestion in the climatological distribution of precipitation and in the poleward flow of air in the friction layer for the existence of a slow meridional circulation with an upward branch in middle latitudes and a downward branch toward the subtropics, as originally suggested by Bergeron [2]. Such a "reverse" cell with equatorward flow aloft would, due to the action of Coriolis forces, tend to establish east winds in the

1. It is here assumed that the main transfer of momentum takes place in the troposphere.

region where the strongest westerlies are actually encountered. The writer rather inclines to the view that although very small mean meridional circulations do perhaps exist, their role in the horizontal transport of angular momentum, at least in the middle latitudes, is overshadowed by the characteristics of other horizontal motions. This is not in conflict with the views expressed by Jeffreys [5] and is reinforced by analysis of data to be quoted presently.

If emphasis is placed upon the horizontal circulations in transporting angular momentum poleward, by and large the zonal and meridional components of velocity should be correlated at each level where such transport occurs. Evidences of this correlation are then to be expected in the detailed structure of the instantaneous horizontal streamline patterns observed, as pointed out by the writer elsewhere [10]. Qualitatively these expectations are amply borne out even by casual inspection of weather maps. Except at high latitudes, closed horizontal circulations usually exhibit a northeast-southwest elongation (Northern Hemisphere) and the troughs, ridges, and shear lines show a tendency to tilt in this same sense. All these characteristics are recognized almost instinctively by the meteorologist as typical of atmospheric flow patterns. From our viewpoint they are telltale indications of a poleward flow of angular momentum. One may nevertheless ask whether an objective quantitative evaluation of the transport by this mechanism can be made from actual hemispheric data. For it is not sufficient to advance merely a qualitative substitute for the classical hypothesis without investigating the potency of the new alternative to produce the needed effect. A question involved here is whether the observations we possess are extensive enough and of sufficient accuracy. It is a simple matter to set up an integral expression for this (say) northward flow of absolute angular momentum across the vertical surface at a given latitude after the manner of Jeffreys, or to derive it from the atmospheric equations of motion as has been done by Widger [17]. The latter author proceeded to evaluate this flow of angular momentum by finite difference methods, as follows.

During the last several years sufficient observations of the free atmosphere have been made to allow the construction of daily isobaric charts through most of the troposphere. In addition it is becoming possible to obtain fairly complete direct radiowind observations extending to great heights on a circumpolar basis within restricted latitude belts. The global wind distributions may be approximated from isobaric charts according to the geostrophic wind formula or may be taken directly from actual wind observations. The use of the geostrophic estimates may introduce certain errors for the present purpose. On the other hand this use of the geostrophic winds automatically excludes the contributions to the angular momentum transport due to mean meridional circulations of the Hadley type, so that an advantage is gained if it is desired to study other modes of such transport. Several surveys of the angular momentum balance have been made in the past few years (Widger [17], Mintz,[2] Starr and White [12]) both from isobaric charts making use of the geostrophic approximation, and directly from a circumpolar network of actual wind observations. The survey of the angular momentum balance made by Widger covers the month of January 1946 and the results give the total geostrophic transfer by latitudes for various layers during this period up to the 7.5 km level. In this study estimates were made of the surface frictional torques. The investigation by Mintz covers the month of January 1949 and his results give the geostrophic flux of angular momentum at various levels up to 100 mb. Starr and White made use of a circumpolar network of actual wind observations at a mean latitude of 31°N for a period of six months from February 1949 to August 1949 up to an elevation of 50,000 ft. For various details of these investigations reference must be made to the original papers.

The computations give results which are entirely reasonable, being quite in accord with what would be expected on the basis of the foregoing discussion. Thus the total northward transport increases in magnitude from low latitudes to about 30°N as the surface easterlies are passed, then decreases progressively northward as angular momentum is removed by surface frictional torques acting in the westerly belt. Nearer to the pole the transport is reversed, indicating a flow southward from the polar easterly zone, although the magnitudes involved here are small as is to be expected from the fact that the torque arm associated with the surface frictional forces is small in the polar regions.

The main transfer across 30°N increases in intensity with elevation, reaching a pronounced maximum at about the level of the jet stream. On the basis of estimates of the surface torques during these periods it appears that sufficient angular momentum is transported into the belt of westerlies to maintain them against friction. Let it be noted, however, that the general results appear to be in harmony with the thesis that *practically all the necessary horizontal transfer of angular momentum could be accomplished without recourse to the agency of mean meridional circulations.* On the basis of what has been said, however, we cannot make any statement concerning other possible functions of meridional circulations such as, for example, the vertical transport of angular momentum.

At this point let us pause in order to take stock of what has been described and to see more clearly how it fits into the plan for research advanced in the introductory paragraphs. Have we by this study of angular-momentum considerations provided a theory for or achieved a rational solution for the problem of the distribution of zonal westerlies and easterlies in the atmosphere? Not by a long way. We did not solve the equations of motion[3] nor did we deal with radiative

2. "The Geostrophic Meridional Flux of Angular Momentum for the Month of January 1949." Presented at the 109th national meeting of the American Meteorological Society, Jan. 29–Feb. 1, 1951, New York.

3. Actually only one equation of motion, namely the one for the zonal direction, is needed to treat the balance of absolute

processes, which must ultimately be responsible for all air motions and are a determining factor for the form of the general circulation. What we have done is simply make a systematic analysis of data to portray as clearly as possible one important, but not patently apparent, attribute of the general circulation—namely, the angular-momentum balance. This study attempts to show how the actual atmosphere attains an internal consistency in one important respect.

What purpose can such information as this serve in the future of meteorological theory? The answer to this query is obvious. Once the announcement was made by Kepler that planets move in ellipses, any proposed dynamic theory of mechanics which would require them to move in significantly different paths was immediately pushed into the limbo. Likewise for the atmosphere any proposed rational theory for the general circulation which significantly contradicts the general outlines of the needed angular-momentum balance at once finds itself afflicted with a serious and perhaps even crucial drawback. On the other hand, the uses of such information need not be purely negative. The facts might well serve as one guide (among many others) for the formulation of satisfactory rational schemes.

What has been described is not a very profound concept, but merely a rather simple consequence of Newton's laws of motion applied to the atmosphere. The gist of the idea was presented many years ago, by Jeffreys in 1926. How is it then that it was permitted to lie fallow for so long a time? Aside from the lack of proper data, a contributing factor has doubtless been the preoccupation of research meteorologists with phenomena on a relatively local scale, this in turn being due to the limitation of our mental horizons to a scale commensurate with the daily weather map.

One might well venture the guess that at present we have not attained even a measure of the tasks which confront us. And this statement is in no way intended as a repetition of a common platitude. We have not fathomed the profundity of our subject, let alone solved the fundamental problems. In the years to come, when our knowledge will have increased, we may in retrospect wonder why we were so inappreciative of various gross and essential attributes of the general circulation. It would therefore be wrong and dangerous to post a sign along any plausible path of research which states "Thou shalt not enter here," seeking thereby to channel activity along certain predetermined lines. True science tolerates no prohibitions, and in principle no servile adherence to tradition. Rather we must open up and exploit all possible new channels that appear reasonable in order that we do not miss important facts.[4] It is in this spirit that the writer would recommend the further exploration of the essential properties of the global circulation *as we know it from observations* with regard to such dynamic quantities as angular momentum, vorticity, kinetic energy, heat energy, geopotential energy, and latent heat, so that we may find ourselves in a more advantageous position in formulating rational hypotheses.

For the purpose of exemplifying further the suggestions previously made, let us consider in a general way their application to questions concerning energy in the atmosphere. Much of what has been written concerning this subject has been deficient in two respects. In the first place investigators have been prone to lump together various diverse forms of energy indiscriminately, thereby losing the advantages to be gained from the fact that each form of energy is produced from and converted to other forms in its own characteristic fashion, permitting individual study. Likewise for each form there exist specific modes of transfer and redistribution. Unless these specific characteristics are subject to scrutiny in detail, only very broad generalizations can be reached, which lack a keen enough resolving power to yield much that is new concerning the mechanism of the atmosphere.

In the second place, as will be discussed presently, the modes of energy transfer within the atmosphere are so effective that no feature such as a cyclone can be treated independently without due allowance for exchanges of energy between it and the remaining atmosphere. It is therefore inappropriate to treat such a feature as in any sense a closed system. Modern trends in the literature are beginning to give proper cognizance to this circumstance, although much of the too-restricted point of view is still prevalent. Probably the only system which can be treated as a closed one (mechanically) is the atmosphere as a whole and even then it cannot be treated as closed from a thermodynamic viewpoint because of radiative exchanges of energy with the cosmic environment and otherwise.

Proceeding now to a discussion of the kinetic energy balance of the atmosphere, it is worthy of note in the first place to observe that whereas angular momentum is a quantity which is conserved in the sense that no real sources of it can exist, kinetic energy on the other hand may appear or disappear because of transformation processes involving other forms of energy. In other words, the flow and redistribution processes for kinetic energy are not source-free as is the case with absolute angular momentum. We may therefore say a priori that there is no reason to suppose that there may not exist in the atmosphere preferred source-regions for

angular momentum about the earth's polar axis. In effect this equation takes on the form of a continuity equation for absolute angular momentum.

4. When one compares the general nature of the results of research in meteorology as reported in our journals with the nature of research communications in let us say chemistry, there is a fundamental contrast to be noted. In our field there is a great diversity of views expressed on individual topics. Often these views are in direct conflict. Also one gains the impression that there is less of the orderly progressive accumulation of knowledge. This phenomenon is doubtless due to the fact that meteorology is in a far different stage of development as a science. In the better sense of the word we are alchemists still in the process of groping for a common denominator of unifying principles. We must not relax but rather continue to stumble and grope with more determination. The writer has the immutable belief that a proper foundation *does* exist and *will* be found.

atmospheric kinetic energy as well as preferred regions for its disappearance by conversion into other forms of energy through friction or otherwise.

With the aid of the examination and interpretation of the fundamental dynamical principles relating to kinetic energy given elsewhere in this volume[5] we shall now attempt to discuss this phase of the energy problem. In the reference mentioned there is presented a discussion of the process whereby kinetic energy is produced in the atmosphere. According to the views expressed there, kinetic energy of large-scale motions can be generated only through the work of expansion by pressure forces. Furthermore, kinetic energy associated with *horizontal* motions can be generated only through work done by *horizontal* pressure forces. It is thus pointed out that the rate at which horizontal kinetic energy is generated at a given point in the atmosphere is equal to the product of the pressure into the horizontal divergence of velocity. This carries the implication that regions of horizontal convergence are hydrodynamic sinks for kinetic energy which act independently of friction.

Speaking next of the transport of horizontal kinetic energy, it has been shown that this process is accomplished either through the work done by the pressure forces in virtue of the horizontal velocities across the boundary or by the advection of sensible kinetic energy. Since the latter (meridional) transport is small, it follows that the significant (meridional) transport of kinetic energy is accomplished through the work done by the pressure forces, which in turn is *proportional to the advection of internal heat energy* and occurs in addition to it. Applying these ideas to the transfer of kinetic energy across the vertical boundaries of an equatorial strip between two fixed middle latitudes $+\phi$ and $-\phi$, our general information would lead us to suppose that there exists a net transport of internal heat energy and therefore of kinetic energy poleward across these boundaries. Estimates from data tend to confirm this supposition, and indeed it also appears reasonable from common synoptic experience. We are therefore confronted by the very important conclusion that regardless of the nature of other details of the atmospheric circulations the more tropical regions appear to be preferred regions for kinetic-energy generation in that they not only produce sufficient kinetic energy to overcome frictional losses within these regions, but also provide an excess which is transferred to the polar caps.

Apparently we may also conclude that in the long run the more polar regions must serve as preferred regions for the disappearance of kinetic energy, since here the losses must be sufficiently great not only to offset whatever amounts of kinetic energy are generated locally but also to absorb the kinetic energy which is supplied from the more tropical regions. It is therefore reasonable to suppose that in the normal state of affairs in the atmosphere there are vast amounts of kinetic energy generated in the more tropical regions and vast amounts consumed in the polar caps. The existing kinetic energy is merely a small difference due to the fact that the positive generation process in the more tropical regions is slightly larger than the losses in regions of negative generation in the polar caps.

It is a matter of interest to examine the import of the views expressed above for the nature of the secondary circulations of the middle latitudes. It would thus be suggested that the kinetic energy supply for the main cyclone belt of each hemisphere is perhaps provided by the poleward flow of such energy from more tropical regions. Furthermore this flow is measured by (but exists in addition to) the poleward flow of internal heat energy. Although this flux of energy has some average mean value, there can be no doubt that in its details the process is a sporadic one with irregularities both in time and in longitude. Each cyclone may thus be considered as producing a spurt or poleward surge in this transport locally during its period of development.

According to the classical view, the kinetic energy for a developing cyclone is obtained from the sinking of dense masses of cold air in the immediate vicinity, so that by and large the decrease in potential and internal energy represents a corresponding increase in kinetic energy. From the present viewpoint the kinetic energy increase could equally well be derived from a pronounced local poleward flow from the more tropical regions, since a developing cyclone is accompanied by a marked increase in the net local poleward transfer of internal heat energy.

As a matter of fact it is not too difficult to visualize the general outlines of a certain type of instability associated with a developing cyclone when viewed in this way. For if it is granted that there exists a supply of warm and cold air in the vicinity, it may be that the pronounced local surge of kinetic energy once started serves to increase the intensity of the cyclone which in turn further increases the flow of kinetic energy and the intensity of the cyclone. The intensification finally ceases when complete occlusion takes place and the net heat transport disappears.

In a recent discussion the writer [11] has made an endeavor to elaborate further this picture of the kinetic energy balance of the atmosphere, making use of the assumption that mean meridional circulations do not play a significant role in these processes. The plausibility of this supposition is supported by the study of the angular momentum balance outlined earlier. Under such circumstances, since data show that near the surface poleward-moving individual air masses possess a higher specific volume than the equatorward-moving ones, it follows that there should exist a *mean horizontal velocity divergence* in the more tropical regions and a *mean horizontal velocity convergence* in the polar caps. This situation should thus contribute to a net production of kinetic energy, in view of the normal mean meridional distribution of pressure along horizontal

5. Consult "Applications of Energy Principles to the General Circulation" by Victor P. Starr, pp. 568–574.

surfaces.[6] Such estimates as can be made indicate that this gross average action may be of very great effectiveness.

In view of the fact that the correlation of density with meridional motion is most marked near the surface, it follows that the mean meridional distribution of this divergence and convergence shows greatest contrasts at low levels. This at once suggests that the mean meridional distribution of *net external heating and cooling* of the atmosphere is linked with the process, since the heating of the atmosphere is most intense near the surface and much of the cooling is accomplished through surface effects also. Thus it would be logical to suppose that the net external heating in the more equatorial latitudes causes a net horizontal expansion, while the net cooling in the polar caps brings about a net horizontal contraction, accounts then being balanced, as far as mass continuity is concerned, by differential advection across middle latitudes.

If one agrees to argue on this basis, a far-reaching observation may be made, subject of course to the correctness of the premises involved. Earlier in the discourse it was pointed out that one of the most fundamental questions in meteorology relates to the presence of broad belts of westerlies in the middle latitudes of each hemisphere. Speaking now only of the lower levels we see that the presence of the westerlies coincides with the requirement that the regions of net external heating and divergence should coincide with regions of higher pressure along horizontal surfaces. Unless this situation exists a net production of kinetic energy to overcome friction would be impossible. Thus, if for a moment we were to visualize a hypothetical situation with mean easterlies in middle latitudes, the net heating and consequent divergence would take place at a relatively low pressure in the more tropical regions, while the convergence and cooling in the polar caps would take place at a relatively high pressure. Such a hypothetical situation would then be rapidly destroyed, since more kinetic energy would necessarily disappear in the polar caps through convergence than could be generated in the more tropical regions by heating and divergence.

Although the net heating and cooling of the earth is ultimately caused by radiational exchanges with space, when one isolates the atmosphere and considers the circumstances in the winter season especially, due regard has to be given to the strong heating of the atmosphere by the oceans when cold air masses flow out over relatively warm water surfaces. As has been pointed out by Sverdrup and others [13], the oceans serve in a manner of speaking as a hot-water heating system. It should thus be expected that during winter the regions of net heating for the atmosphere extend much farther into middle latitudes than might otherwise be supposed.

6. Since variations of pressure along horizontal surfaces are necessary for the generation of a net amount of kinetic energy to overcome friction, it is indeed reasonable on general grounds to suppose that the atmosphere behaves in such a way as to take advantage of the large systematic pressure differences between the polar and tropical regions.

Since we have been speaking of the meridional transfer of internal heat energy, it is desirable to mention also the effects of the meridional transfer of latent heat. Crude estimates made by the writer using such data as are available indicate that very considerable amounts of energy are transferred poleward by this means. However, whereas the transport of internal heat energy cannot take place without a proportional transport of kinetic energy, the meridional transfer of latent heat energy is not related to the kinetic energy in the same way. It is thus possible to transfer large amounts of energy in the form of latent heat without a proportional kinetic energy flow being present. Whereas the flow of kinetic energy and therefore of internal heat energy may be regulated by the distribution of pressure and of divergence and convergence, this particular limitation does not enter as far as latent heat is concerned. One could therefore find arguments to support the view that the transport of latent heat furnishes a means for balancing the radiation requirements of the general circulation which bypasses the kinetic energy balance. For if the portion of the total meridional energy flow represented by the transfer of latent heat energy had instead to be transported as additional internal heat energy, a far more intense flow of kinetic energy would also be necessary. Whether this would imply a more "vigorous" general circulation is, however, a question which cannot be readily dealt with.

The meridional transport of geopotential energy is of course immediately ruled out in the average condition discussed here, because of the assumption that mean meridional circulations are not significant.

In the discussion mentioned above [11], an endeavor was made to study the fluctuations in the kinetic energy balance of the Northern Hemisphere with the aid of 5-day mean data such as are used by the U. S. Weather Bureau in extended-period forecasting. Only data for the colder half of each of seven successive years were used, so that essentially winter conditions were studied. By approximate methods a measure was secured of the intensity of differential advection of air with varying density across 45°N. This quantity τ is therefore a measure of the net volume transport northward across 45°N and hence also a measure of the mean convergence in the polar cap and to some degree probably a measure of the mean divergence in the more tropical zones of the Northern Hemisphere. A mean for the layer between sea level and 10,000 ft was used.

It became immediately apparent that the quantity τ undergoes large fluctuations, being high during those periods which are commonly designated as "low-index" periods and low during "high-index" conditions. The practical question involved here concerns itself with the cause of these vagaries in the behavior of the general circulation, for if this were known a better insight into the long-range forecasting problem might result.

In view of the fact that τ is also a rough measure of the rate of kinetic energy transport into the polar cap, it is interesting to note its relationship to the general pressure distribution there. If these pressures are low compared to the pressures farther south, a relatively

large fraction of the kinetic energy so transferred should remain without being destroyed by the mean convergence, while if these pressures are high practically all the kinetic energy fed into the cap should disappear because of the mean convergence. For this purpose it was decided to examine τ with reference to the *area-mean pressure deficit* north of 45°N as compared to the pressure at 45°N. This pressure deficit was denoted by π and was computed from sea-level pressures. It became apparent that abnormally large values of τ are, generally speaking, encountered only when the pressure deficit is small or negative, in other words when the capacity of the polar cap to destroy kinetic energy is abnormally great.

From these general observations it would appear that large meridional transports of internal heat energy and of kinetic energy take place during those periods when the mean meridional pressure distribution at lower levels is relatively flat north of the subtropics with no marked polar low present. How can a situation such as this maintain itself over appreciable intervals of time, as is known to be the case? Is it a quasi-steady state or is it a dislocation which becomes progressively more difficult to maintain? We can only speculate about the answers, but it is rather interesting to do so.

First let us consider the fact that it is during these low-index conditions that very cold air masses are injected into southerly latitudes. This in turn means a very strong heating of the atmosphere in middle latitudes and the subtropics, especially over ocean areas, so that a rapid replenishment of heat and also a rapid generation of kinetic energy would be a normal accompaniment of this state of affairs. This consideration would argue for the possibility of quasi-steady state.

On the other hand in order to maintain a quasi-steady condition, means would have to be present to dispose of large quantities of heat in the polar cap. It is unlikely that the net heat loss through radiation could be stepped up enough to meet this requirement. Progressive melting of ice in the polar regions could absorb large quantities of heat, but probably exactly the opposite is actually the case, since very low temperatures are apt to prevail at low levels in the arctic and subarctic regions under these circumstances.

Observationally there is some qualitative evidence that during prolonged low-index conditions there is apt to be present in polar regions an accumulation of abnormally warm air a little distance above the surface in the troposphere and in the stratosphere. Also there is some evidence that there is actually a progressive depletion of heat energy from middle latitudes (in spite of the strong surface heating) with a consequent progressive southward shift of the latitude of the maximum westerlies at higher levels. What ultimately puts a stop to the trend of developments is not obvious.

Contrasted with the low-index regime, during periods of a strong polar vortex at low levels the heat and kinetic energy transport is weak. Relatively warm air streams across the continents and oceans in a more zonal manner so that no very vigorous heating takes place. So far no difficulties seem to be present as far as the more southerly regions are concerned. It would appear, however, that progressive cooling should now take place in the arctic regions since the abnormally low heat input would appear to be insufficient to maintain the *status quo*.

On the whole there may be good and sufficient physical reasons, such as those mentioned above, why the general circulation does not persist indefinitely in one or the other abnormal conditions but rather tends to shift about a general average state. There is nevertheless the important question why the average state does not persist without such pronounced departures as those just described. The writer's conjecture on the matter is that the average state can too easily be disrupted by the effects of the nonzonal continentality found especially in the Northern Hemisphere. Such effects may be partly due to the fact that the continents provide avenues for the southward penetration of intensely cold air masses which can thus arrive at relatively low latitudes without undue modification. The occasional outpourings of such cold masses over adjacent water areas at fairly low latitudes could easily result in abnormal heating of the atmosphere sufficient to disrupt the average regime.

Also it should not be forgotten that large mountain ranges extending over an appreciable spread of latitude can exert powerful mechanical effects upon the atmosphere, as pointed out by White [16]. By way of example large pressure differences across the Rockies in North America could perhaps disrupt the angular momentum balance in the belt of the prevailing westerlies.

Finally the suggestion made by Willett [18] that the seat of the variation could perhaps be found in the variations in solar output, and hence the heating of the atmosphere, deserves study. For it may be that the final answer is bound up in a combination of effects, especially if long-term aberrations of the general circulation during historic and geological periods are also considered.

From what has been said in the several preceding paragraphs it might appear that too little emphasis has been placed upon the influence of the conditions in the upper troposphere and in the stratosphere. In the final analysis it is obvious that the entire atmosphere must form an integrated system. It would seem, however, that the lower layers furnish the seat of thermodynamic activity and provide the motive force for the general circulation. The writer's colleague, Dr. H. L. Kuo, is currently engaged in a theoretical study of the effects which might result at the level of the jet stream from forced disturbances originating in the lower levels. From his preliminary work it would appear that the generation and maintenance of the jet stream itself are a necessary consequence of such impulses received from below. Likewise the characteristics of the angular momentum balance as outlined previously, including such features as the tilted (and properly curved) trough and ridge lines, as well as the development of regions of sharp shear to the north and to the south of the jet follow as consequences of the analysis.

Having made an excursion into some of the ramifi-

cations involved in the study of the balance of angular momentum and of kinetic energy in the atmosphere, let us now consider the general circulation from the standpoint of a *rational theory* for atmospheric motions. By this term we mean essentially a *solution of the equations of motion* which purports to give a picture of the circulation or some portion of it. This is sharply contrasted with the study of the balance of various quantities, because the latter involves hydrodynamical principles only to the extent of formulating several integral requirements for the atmosphere. The investigation of how the atmosphere actually fulfills these requirements is then necessarily an empirical and observational problem.

Logically, then, the empirical picture obtained from the study of the balance of various quantities and from other observational studies should give us the fundamental physical characterization of the atmosphere which is then to be explained in terms of some rational theory. But, in a larger sense, the proper physical characterization of the general circulation is a subject which up to the present has hardly been entered upon. We need quantitative estimates of the flow of heat, of kinetic energ of momentum, etc., for the mean state and also when consideration is given to synoptic and seasonal variations. The labor involved in order to supply this information is bound to be tremendous, involving the cooperation of meteorological services over the entire globe. Nevertheless progress is being made and there is reason for optimism.

In view of this state of affairs it is not surprising that the development of a rational theory for the general circulation has not made very much headway up to the present time. Thus the scope of the more successful efforts toward the solution of the hydrodynamic equations has necessarily been severely limited to certain portions of the atmospheric circulation where the introduction of drastic simplifications still leads to results of interest. As an example we might cite the several solutions of the two-dimensional barotropic vorticity equation given by Rossby [8] and recently elaborated by Charney and Eliassen [3] and others. For short-period extrapolation it appears that these results may prove to be of more and more value in forecasting, even though these solutions leave unanswered some of the fundamental questions concerning the energy balance when long-term effects are contemplated.

As a general comment it can be said that really adequate solutions which link the mechanics of the atmospheric circulation to the source of energy from solar radiation are a desideratum for future research, although the situation does not warrant undue pessimism. In this connection reference may be made to a recent publication by Lorenz [6] which is concerned with the solution of the two-dimensional equations of motion for the region near the pole, taking into account friction and external heating and cooling.

Leaving now the discussion of illustrative topics, what are the avenues for future progress which are indicated as of today? This is necessarily a subjective matter in which various investigators must follow their own inclinations and tastes, for there is no shortage of diverse problems which are worth while. The inclinations of the writer, as one of these investigators, are probably quite apparent from what has already been said. The underlying and oft-recurring motif of this essay is the need for a more complete and systematic empirical description of the basic physical processes involved in the general circulation. We are now beginning to have sufficient observational material at least in the Northern Hemisphere.

Before making concrete proposals, let us take a glance at what is being done which may serve as a beginning toward this aim. In the enumeration of activities which follows, any omission of important work is inadvertent and is due simply to the writer's lack of information. At the Massachusetts Institute of Technology Willett [18] has for a period of years gathered statistical information concerning the general circulation in the Northern Hemisphere in the form of 5-day averages. More recently he has begun gathering similar data for the Southern Hemisphere. The writer has found this material invaluable for the study of the general circulation. The Department of Meteorology of the University of California at Los Angeles has undertaken a study of the angular momentum balance of the atmosphere by means of finite difference integration procedures. This project will probably also include a similar study of the energy balance later.

Priestley [7] of Australia has proposed a program for the observational study of the momentum and energy balance of the atmosphere, utilizing radiosonde and radio-wind observations directly. Samples of his analyses already prepared by him have proven to be of great interest, but require elaboration.

Van Mieghem [15] of Belgium has proposed the study of the balance of various forms of energy, entropy, and momentum, emphasizing the importance of such studies for the progress of research concerning the general circulation.

The Extended Forecast Division of the U. S. Weather Bureau has for some years been amassing observational material in the form of 5-day averages for the Northern Hemisphere. The Weather Bureau [14] has recently also been preparing in published form very complete daily Northern Hemisphere maps for the surface and 500 millibars. These charts are most xcellent and should prove to be invaluable for researc.. purposes.

Finally mention may again be made of the studies of the angular momentum and energy balance initiated by the writer at the Massachusetts Institute of Technology.

The items listed above do not purport to cover all important research on the general circulation conducted currently, but rather only such activities as are apt to furnish statistical information concerning the gross character of the general circulation. Thus various other synoptic and theoretical activities are not included even though they may in the end furnish indispensable insight as to the interpretation of the empirical picture given by the statistics.

The general trend to be discerned is that the em-

pirical study of the general circulation is becoming a matter involving vast amounts of data properly and purposefully organized in order to be useful for the systematic evaluation of various specific and well-defined processes over long periods of time. It is being undertaken piecemeal by various institutions to an extent determined by their financial resources and facilities. In the initial stages of such exploratory activities it is probably most desirable that there should exist diversified direction of individual small-scale projects of this nature. However, once the best general patterns for such research have become apparent, a number of considerations point to the desirability of more consolidated effort. Thus some central organization could perhaps offer the advantages of a long-term program, more economical operation, better facilities such as high-speed computing equipment and better availability of computed results. Such an institute could even be of an international character under the auspices of the United Nations, since the purpose would indeed be of global importance in a very literal sense.

Beyond the broad recommendations just set forth, the writer wishes to state his belief that periodic reviews of progress such as the ones contained in this volume should be planned by appropriate organizations in an effort to stimulate comparison of differing opinions and to bring forth suggestions. Another matter of somewhat similar nature is the periodic publication of selected reprinted papers, adjudged to be of fundamental importance, in the form of collections similar to those edited by Cleveland Abbé [1] several decades ago. A practical problem here would be the selection of a capable (and so to speak disinterested) editor.

Finally a word about certain philosophical and theoretical aspects. In a science such as meteorology in which the rational explanation of the most gross features is still to be found, much attention must be given to questions regarding the general techniques for *interpretive* research in order to ascertain if possible the ones which are suitable. In any well-developed science, as for instance physics, it is often possible to arrive at new results by purely deductive means. However, it must be granted that the more fundamental scientific achievements of an interpretative nature have usually stemmed from *the deliberate introduction of new physical hypotheses*. Such hypotheses are usually the direct product of creative minds who through sufficient insight are able to recast a given problem into a new form. Although such formulation of novel hypotheses is perhaps the most difficult task confronting a scientist, we cannot afford to dispense with it in any attempt at understanding the motions of the atmosphere at the present time.

Without the use of creative imagination to give it form, our science could easily become a repetitious accumulation of bits of petty dogma, a jargon of catchwords and sophistries, a species of scholasticism lacking a firm basis or unifying principles. We need men with vision and courage who would not be content in occupying themselves with odd bits of research, but would tackle the fundamental problems. Men who have something of the spirit of daring which prompted Newton to propose the theory of universal gravitation and Einstein to suggest anew the quantum character of radiation.

Much of what has been said in the last two paragraphs has a direct bearing upon the use of mathematics in meteorological research. To regard the fundamental problems of meteorology as purely mathematical ones is hardly warranted. It is true that many of the hydrodynamical and physical principles involved are capable of mathematical statement, but it is only through the leaven of some purely physical hypothesis that we are guided to the appropriate mathematical use of these principles.

As stated previously, present-day meteorology may be thought of as being in the Keplerian era. It finds itself, however, side by side with very much more advanced physical sciences in which research is carried on with the aid of very sophisticated mathematical and theoretical tools designed for the purpose through the laborious efforts of past generations. Although much may be gained by borrowing certain of these techniques where proper analogies have been established, still there are certain philosophical pitfalls which arise when this is done merely in slavish effort at imitation without proper caution. Such superficial efforts are sometimes entered upon in order to gloss over the prerequisite of clear physical thinking. We cannot telescope the evolution of a new science by omitting essential phases of its natural development. We cannot hope for a magic carpet which would carry us directly from Kepler to Einstein, eliminating the growing pains of Newtonian mechanics.

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A STUDY OF THE GENERAL CIRCULATION AND A POSSIBLE THEORY SUGGESTED BY IT

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Summary: A recently completed analysis of two years of data from two circumpolar chains of northern hemisphere stations, and one year of data from three additional chains, indicates that horizontal eddies transport as much angular momentum, water, and energy poleward as is required by the interaction of the atmosphere with its environment. The meridional circulations as measured transport minor amounts of angular momentum and water. The eddy flux of angular momentum is against the gradient of angular velocity throughout much of the atmosphere, so that kinetic energy is transferred from the eddies to the mean flow. The eddy flux of sensible heat is primarily with the temperature gradient, but is against the gradient in the lower stratosphere.

These results form parts of a possible theory of the general circulation. Other parts are suggested by various theoretical studies and by analogies with certain experimental models. It appears that the role of heating is to maintain available potential energy, represented primarily by the poleward temperature gradient. The resulting state is unstable, so that eddies appear. The potential energy of the mean flow maintains the eddies against dissipative effects, and the eddies maintain the kinetic energy of the mean flow against dissipative effects.

An extensive study has recently been completed by the General Circulation Project at the Massachusetts Institute of Technology, under the direction of Prof. VICTOR P. STARR. The principal results of this study have been prepared for publication by Prof. STARR and Dr. ROBERT M. WHITE[1]. The first portion of this presentation is concerned with the results of that study.

The study was based upon all available upper-wind and radiosonde observations, at standard levels up to 100 millibars, at five circumpolar chains of northern-hemisphere stations, during the year 1950, together with some low-latitude observations for 1949 and 1951. The five chains of stations were located approximately at latitudes 13, 31, 42·5, 55, and 70 degrees north, and each chain contained from ten to nineteen stations, with a total of 75 stations. The data were extracted directly from the coded messages, as published in the Daily Series Synoptic Weather Maps, prepared by the United States Weather Bureau[2] (1949 *et seq.*) in co-operation with the

Army, Navy, and Air Force, and involved altogether a total of 176 thousand individual wind readings, 57 thousand humidity readings, and 77 thousand temperature readings.

Average values of certain quantities were computed from these data. The nature of the data renders short-period averages of doubtful significance, so that attention was given primarily to yearly averages and summer and winter averages. These quantities included the zonally averaged eastward and northward wind components, specific humidities and temperatures, and also the horizontal poleward fluxes of angular momentum, water, and total energy, as functions of latitude and elevation. The vertically averaged horizontal fluxes were compared with those required by considerations of continuity, as inferred from the best available estimates of the latitudinal distribution of exchanges of angular momentum, water, and total energy between the atmosphere and its environment.

The instantaneous local poleward flow of mass may be resolved into three components: the portion due to the long-period mean meridional circulation; the portion due to the additional instantaneous meridional circulation; and the portion due to large-scale horizontal eddy motion. Each of these components accomplishes a separate mode of poleward transport of any quantity. The transports of angular momentum and water were resolved into these three modes of transport, while in the case of total energy only the transport by horizontal eddies was computed.

The computations of the angular-momentum balance reveal the great importance of horizontal eddies, as compared to meridional circulations, in accomplishing the total transport across those latitudes where the transport is greatest. The transport by eddies appears sufficient to satisfy balance requirements, as determined from estimates of surface torques. The mean meridional circulation as measured possesses the familiar three-cell pattern, but most of the computed values are too small to be statistically significant in view of the nature of the data. The evidence against mean meridional circulations stronger than about one metre per second is fairly conclusive, except at low levels in the tropics, where a direct circulation stronger than two metres per second may occur in winter.

A further result is that the eddy flux of angular momentum is directed primarily toward latitudes of higher angular velocity. This counter-gradient flux results in a net conversion of the kinetic energy of the eddies into kinetic energy of the zonal flow, as has recently been pointed out by Kuo,[3] and in more detail by Van Mieghem[4].

The computations of the water balance again reveal the great importance of horizontal eddies, which transport sufficient water to satisfy the balance requirements, as estimated from evaporation and precipitation studies. The transport of latent heat of condensation which accompanies the transport of water accounts for at least half of the total necessary energy transport at subtropical latitudes.

The horizontal transport of total energy involves the transport of potential energy as well as sensible heat and latent heat. Since the potential energy per unit mass is very large at great heights, the transport of potential energy by the meridional circulation is greatly influenced by the nature of this circulation in the upper stratosphere, where sufficient data were unavailable.

Computations were therefore restricted to the transport of total energy by horizontal eddies, which do not transport potential energy. Again the eddies transport sufficient energy to satisfy the balance requirements, as inferred from radiation-balance figures. In lower latitudes the transport of sensible heat is not sufficient to satisfy the balance requirements, and the additional necessary energy is transported in the form of latent heat.

A further result is that the eddy flux of sensible heat is generally directed toward colder latitudes. However, in the lower stratosphere, the flux is against the temperature gradient, since the gradient is reversed, but the flux is still poleward.

This study makes it possible to present a comprehensive description of the processes which are responsible for maintaining the energy of the general circulation. Descriptions of certain of these processes have been appearing with increasing frequency in recent meteorological literature.

In discussing the energy of the general circulation, it is convenient to introduce the concept of available potential energy. This quantity may be defined as the difference between the total potential plus internal energy of the atmosphere, and the minimum potential plus internal energy which could result from any adiabatic redistribution of mass. This same quantity was described by MARGULES[5] in his famous paper concerning the energy of storms. The properties of available potential energy have been discussed in detail by the writer.[6]

Available potential energy may be expressed approximately in terms of a weighted vertical average of the horizontal variance of temperature. In this respect it bears a certain analogy to kinetic energy, which, aside from the contribution of the mean wind, depends upon the variances of the wind components. Just as the kinetic energy may be resolved into zonal and eddy kinetic energy, by an analysis of variance of wind, so the available potential energy may be resolved into zonal and eddy available potential energy, by an analysis of variance of temperature into the variance of zonally averaged temperature and the variance of temperature within latitude circles. Just as the rate of conversion of zonal into eddy kinetic energy depends primarily upon the eddy flux of angular momentum along the gradient of angular velocity, so the rate of conversion of zonal into eddy available potential energy depends upon the eddy flux of sensible heat along the temperature gradient. Zonal and eddy available potential energy may respectively be converted into zonal and eddy kinetic energy by meridional circulation and eddy motions.

The immediate effect of heating by the environment is the generation of zonal available potential energy, through heating in warm latitudes and cooling in cold latitudes. Crude estimates based upon the radiation-balance figures of ALBRECHT[7] indicate a generation of about 200×10^{20} ergs per second, or about two per cent of the total solar energy received at the outer limit of the atmosphere.

Since the horizontal eddy flux of sensible heat is primarily toward colder latitudes, there is a net conversion of zonal into eddy available potential energy, that is, the variance of temperature within latitude circles increases at the expense of the variance across latitudes. The rate of this conversion, estimated from the figures of the General Circulation Project, also appears to be about 200×10^{20} ergs per second.

Environmental effects probably destroy some of this eddy available potential energy instead of generating more, in view of the probable presence of heating of cold air masses and cooling of warm air masses at the same latitudes. However, some of the eddy available potential energy must be converted into eddy kinetic energy, since there is no other source for the latter, because, as we have seen, zonal kinetic energy acts as a sink for eddy kinetic energy. The rate of conversion from eddy to zonal kinetic energy is about 10×10^{20} ergs per second, according to the figures of the General Circulation Project (see STARR[8]). The rate of conversion from eddy available potential energy to eddy kinetic energy depends upon the correlation within latitude circles between temperature and vertical motion, and cannot be computed directly from available data. Presumably both forms of energy are destroyed by friction.

There remains the possible direct conversion of zonal available potential energy to zonal kinetic energy. However, the weak meridional circulation measured by Starr and White[1] leads to a conversion from zonal kinetic energy back to zonal available potential energy, at the rate of about 2×10^{20} ergs per second. The direction of this conversion results from the presence of the indirect cell in the latitudes of the strongest temperature gradient.

Now that we have established a picture of the energy cycle of the general circulation from the available observations, we may ask why the atmosphere chooses to operate in this particular fashion. In attempting to learn the answer, we shall be guided by the so-called 'dishpan' experiments being performed at the University of Chicago (see FULTZ[9], STARR and LONG[10]). In these experiments a circular cylindrical vessel containing water is rotated at a constant rate, and is subjected to uniform heating about the circumference and cooling at the centre. The flow of the water is observed by means of tracers.

Under sufficiently low rotation and sufficiently strong heating, the observed flow is nearly symmetric about the centre, and consists of a strong zonal flow with a weak superposed meridional circulation. Under faster rotation or weaker heating, the flow loses its symmetry, and exhibits meandering currents which bear a close resemblance to those found upon upper-level hemispheric weather maps.

The breakdown of the symmetric flow after a critical combination of rotation and heating is exceeded is suggestive of an instability phenomenon. Certainly the forced flow observed in the symmetric régime is dynamically stable. It seems possible that under supercritical conditions any forced flow will be dynamically unstable, in the sense that small superposed disturbances will grow until they become important features of the flow pattern. A study by the writer[11] appears to confirm this conclusion. Moreover, no tendency is revealed for the development of barotropic instability, that is, instability related to the horizontal distribution of vorticity. Instead, the instability seems to be baroclinic, that is, related to the temperature gradient and the accompanying vertical wind shear. Baroclinic instability increases with decreasing static stability (see CHARNEY[12]), and in this case seems to set in when the meridional circulation becomes too weak to maintain strong static stability.

On the other hand, the problem has been studied much more extensively

by Kuo[13, 14], who has regarded the symmetric régime with its accompanying meridional overturning as a large-scale convective phenomenon. Heating tends to generate such convection, while rotation tends to suppress it. Thus Kuo finds that for sufficiently high rotation or weak heating, large-scale convection is impossible, that is, the heating cannot force any symmetric flow.

These studies may seem to represent different points of view, but the results are actually compatible, and together allow for one régime where symmetric flow will be observed, another régime where symmetric flow is mathematically possible, in the sense that it satisfies the appropriate equations and boundary conditions, but will not be observed because it is dynamically unstable, and still another régime where symmetric flow is not mathematically possible.

If the atmosphere is really analogous to the model experiments, the existence of eddies may be attributed to instability. A similar point of view was expressed by EADY[15], at the Centenary of the Royal Meteorological Society. Various studies of baroclinic instability have suggested that typical atmosphere zonal flow patterns are generally unstable.

The atmosphere thus falls into the unsymmetric régime because the particular combination of rotation and heating which characterizes the atmosphere is one of those which is incapable of maintaining large-scale symmetric convection, or stable symmetric flow. The existence of this combination of rotation and heating may be ascribed to chance; possibly a more nearly symmetric régime would prevail if the earth rotated more slowly.

The eddies in the atmosphere and the experiments are finite rather than infinitesimal, but it seems plausible that these eddies are maintained by the same processes which cause small eddies to grow when superposed upon an unstable flow. Likewise, any suppression of finite eddies may be due to the same processes which prevent small eddies from growing when superposed upon a stable flow.

If the zonal flow in the atmosphere is in general barotropically stable, the processes which tend to suppress the eddies convert their kinetic energy directly into kinetic energy of the zonal flow, since potential energy is not involved. If in addition the flow is baroclinically unstable, the maintenance of the eddies must result from a transfer of available potential energy, rather than kinetic energy, from the zonal circulation, and the kinetic energy of the eddies is obtained from the available potential energy which the eddies have acquired. We thus arrive at the picture of the energy cycle of the general circulation which we have already established from the observations.

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GEOPHYSICAL RESEARCH PAPERS

No. 35

BALANCE REQUIREMENTS OF THE GENERAL CIRCULATION

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ABSTRACT

The balances of angular momentum, energy and water are examined on the basis of hemispherically distributed observations of wind, temperature and moisture for the entire year 1950. The function of various kinds of organized atmospheric circulations in maintaining these balances is brought out through the analysis of the data. The results lead to a clearer understanding of the physical processes which are important in the maintenance of the general circulation.

The angular momentum balance is examined first. The poleward flux of eastward angular momentum is evaluated from actual wind data extending from the surface to 100 mb and from the tropics to the polar regions. It is found that the flux of such momentum between tropical and middle latitudes is principally accomplished through the agency of the large scale hori zontal eddies in the atmosphere. The maximum flux occurs in the vicinity of latitude 30° N at the boundary between surface easterlies and westerlies. The flux is found to be sufficient to account for the drain of such momentum in the region of the westerlies by surface torques. It is further pointed out that the kinetic energy of the westerlies results from the conversion from the kinetic energy of the disturbances and not from the direct action of the mean meridional circulations.

The energy and water balances for the northern hemisphere are presented and discussed in the light of observational determinations of the flux of both of these properties by various

kinds of atmospheric circulations. Comparisons with requirements estimated by other investigators show that the large scale horizontal eddies transport sufficient energy and water to account for most of the balance requirements.

BALANCE REQUIREMENTS OF THE GENERAL CIRCULATION

1. Introduction

One approach to the problem of describing and explaining the general circulation of the earth's atmosphere consists in examining certain integral requirements deduced from broad dynamic principles governing atmospheric motions. These integral requirements may be formulated in terms of the balances of angular momentum, energy, water, mass and other properties. Such an approach has been used with increasing frequency in recent years. An analysis of these balances for the entire northern hemisphere based upon actual wind, temperature and moisture observations covering the entire year 1950 are presented in this paper.

The properties, angular momentum, energy, water, and mass are each conservative in that they cannot be generated or destroyed within the atmosphere, but can only be redistributed or in the case of energy converted from one form to another. Over sufficiently long periods of time there can be no significant net flow of these properties into the atmosphere nor outflow from it. However, for each property there is a net inflow at certain latitudes compensated by a net outflow at others. To complete the balances there must be a flux of such properties within the atmosphere across certain latitudes. By and large the flux of a property across a latitude is accomplished by the exchange across the latitude of equal air masses containing

different amounts of the property. Part of the mass exchange may consist of a northward mass flow at certain elevations accompanied by a southward mass flow at different elevations. Such flows may be said to form meridional circulations. In addition mass exchange across a latitude circle may occur if there is a net northward mass flow at certain elevations, times, and longitudes, compensated by southward flow at the same elevations, but at different times and/or longitudes. Such flows may be said to constitute horizontal eddies.

The necessity for a flux of angular momentum across a given latitude may be described as a continuity requirement for angular momentum. Necessities for the flux of other properties may be similarly described. The necessity for the fluxes may be expressed as a set of three relations. A fourth relation results from the necessity for a zero transport of mass. Thus for each latitude we may with sufficient accuracy write that:

$$\frac{1}{gt_1} \int_0^{P_0} \int_0^{t_1} \int_0^{L} v dx dt dp = 0, \qquad (1.0)$$

$$\frac{1}{gt_1} \int_0^{P_0} \int_0^{t_1} \int_0^{L} Mv dx dt dp = K_1, \qquad (1.1)$$

$$\frac{1}{gt_1} \int_0^{P_0} \int_0^{t_1} \int_0^{L} Wv dx dt dp = K_2, \qquad (1.2)$$

$$\frac{1}{gt_1} \int_0^{P_0} \int_0^{t_1} \int_0^{L} Ev dx dt dp = K_3. \qquad (1.3)$$

In these integrals t_1 is the length of time interval over which the integration is to be performed, P_o is the surface pressure, L is the length of the latitude circle, v is the northward velocity component, M is the absolute angular momentum per unit mass, $E = E' + \frac{P}{\rho}$ where E' is the total energy per unit mass, and P/ρ is a measure of the work done by pressure forces,*
W is the total water content per unit mass, dx is an element of linear distance eastward, dt is an element of time, dp is an element of pressure in the vertical, g is the acceleration of gravity, and K_1, K_2, K_3 represent the required flux of the various properties across the latitude circles.**

Estimates of the constants K_1, K_2, and K_3 may be made from independent considerations. It is of great interest, among other things, to attempt to evaluate the left hand sides of the expressions from atmospheric observations to determine the measure of agreement which can be obtained. Of even greater importance, however, is the determination of how the details of the atmospheric circulations are organized in order to satisfy

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* It is to be noted that the pressure-work transfer occurs in a constant ratio R/Cv to the internal energy transfer. The sum of the transfer of internal energy and the pressure-work transfer is called the sensible heat or enthalpy transfer.

** In the four expressions (1.0)-(1.3) it has been assumed that the atmosphere is in hydrostatic equilibrium, that the acceleration of gravity, g, is everywhere constant, and the variation of the length of the latitude circle, L, with p is negligible. In addition it should be recognized that P_o is variable in both horizontal space and time. In the first equation (1.0) the net transport of mass due to the net meridional flux of water vapor has been neglected.

such balance requirements. In the sections that follow an attempt will be made to answer these questions.

2. Procedure

There are certain general procedures concerning data collection and analysis which are common to the individual investigations of the atmospheric angular momentum, energy, and water balances. The basic data include observations of the northward and eastward wind velocity components, the temperature and moisture content at all levels in the atmosphere at numerous points along the various latitudes which are to be studied. Because the location of upper-air observing stations is determined largely by consideration of economics and geography, the selection of stations is somewhat restricted. Attempts were made therefore to select strings of key stations in the vicinity of certain latitude circles, which were strategically located in order to provide the required information. The distribution of these key stations over the northern hemisphere is shown in Fig. 1. The mean latitudes of the several strings are 13°N, 31°N, 42.5°N, 55°N and 70°N. For the most part the same observing stations were used for the study of all three balances. However at latitude 13°N evaluations of the angular momentum flux only were made. The listing of all key stations together with certain alternate stations which were used when data from key stations were missing are given in Tables 1-5. Wherever possible attempts were

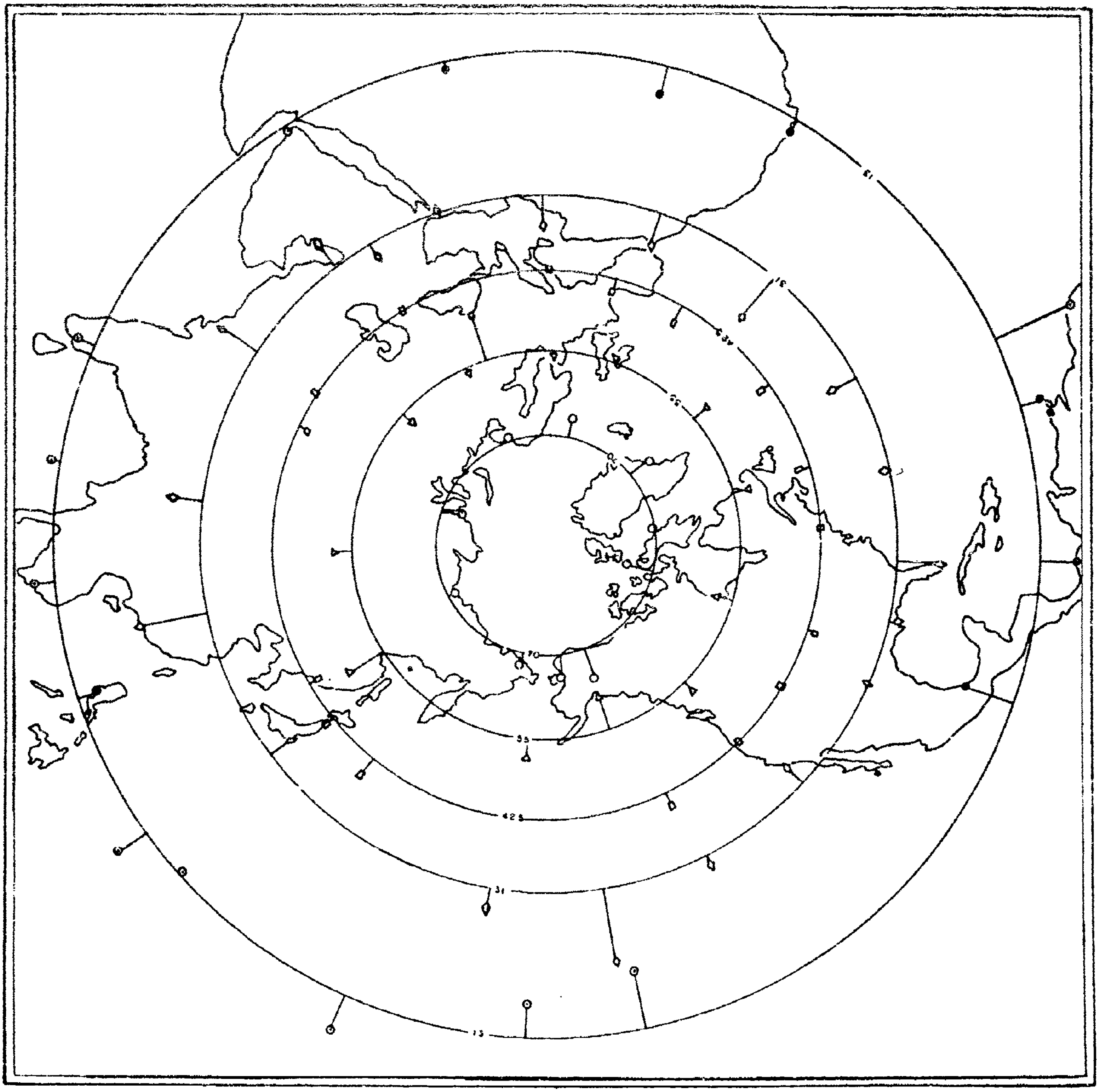

Fig. 1. The distribution of key stations over the northern hemisphere used in the investigations of the angular momentum, energy, and water balances for the year 1950. The mean latitudes of the various strings of key stations are located as indicated at 13°, 31°, 42.5°, 55°, and 70°N.

made to use radio-wind observations. However in certain areas where these were not available pilot balloon observations were used.

To give some idea of the availability of observations, the frequency of occurrence of the wind observations for the entire year is given in Tables 6-10. The frequency of occurrence of temperature and moisture observations which were usable (that is accompanied by wind observations) is so closely similar to that for the winds that no good purpose would be served by also tabulating them.

It will be noted that the wind observations at 13°N were abstracted at constant height levels, whereas those at other latitudes were abstracted only at constant pressure levels. This was done in connection with another study of the angular momentum flux in tropical regions in which it was desirable to have a more detailed description of the angular momentum exchange processes than is required here. The constant height levels used for comparison with similar data at other latitudes at constant pressures are those which are starred in Table 6.

Wind, temperature and moisture reports for the various stations, levels, and latitudes were taken from the data tabulations of the Daily Series Synoptic Weather Maps prepared by the U. S. Weather Bureau in cooperation with the Army, Navy and Air Force for the year 1950. All observations are

for 0300 G.M.T. for each day. No attempt was made to correct or exclude original data where doubtful. This course was followed in order to maintain objectivity in the computations.

2.1 Evaluation of the flux of atmospheric properties

The flux of various atmospheric properties on the scale of the general circulation has been studied in the recent past by a large number of investigators: Starr,[18] Bjerknes,[5] Priestley,[15,16] Widger,[31] Starr and White,[22-24] White,[28,29] Mintz,[13] Benton,[4] and others.

More recently Lorenz[12] and Benton[4] have discussed a generalized method whereby the flux of atmospheric properties can be studied. These generalized methods admit certain specialized formulations of the problem which while being less general, possess the distinct advantage of being more physically meaningful. Several alternative specialized methods of treating the flux problem have been used in the past notably by Priestley[15] and by Starr and White.[21] Each of these formulations is entirely legitimate and may be used to answer appropriately phrased questions concerning the nature of the exchange processes. In this paper the analysis of the transfer processes will be made in accordance with the formulation previously used by Starr and White [21] and the notations will be entirely analogous to that used previously by the authors.

In the investigations which follow we shall be interested in the flux of certain atmospheric properties due to various kinds of organized atmospheric circulations across several latitude circles. We shall thus be interested in evaluating and interpreting expressions of the type:

$$\frac{1}{g} \int_{0}^{p_0} \int_{0}^{t_1} \int_{0}^{L} a v dx dt dp = \frac{p_0 t_1 L}{g} (\overline{[\alpha v]}), \tag{2.0}$$

where α is the amount of a given property per unit mass, the brackets represent a mean with respect to longitude, the bar a mean with respect to time and the parenthesis a mean with respect to pressure in the vertical. The expression $(\overline{[\alpha v]})$ may then be expanded in terms of the means of the various quantities entering this product and the appropriately defined deviations from these means. Thus it has been shown (Starr and White [23] that it is possible to expand such a term as follows:

$$(\overline{[\alpha v]}) = ([\bar{\alpha}])([\bar{v}]) + ([\bar{\alpha}]'[\bar{v}]') + (\overline{[\alpha]'[v]'}) + (\overline{[\acute{\alpha}\acute{v}]}), \tag{2.1}$$

or alternatively as:

$$(\overline{[\alpha v]}) = ([\bar{\alpha}][\bar{v}]) + ([\bar{\alpha}]'[\bar{v}]') + ([\bar{\alpha}'\, \bar{v}']) + ([\overline{\alpha' v'}]). \tag{2.2}$$

The Eq. (2.1) is the one which has been used by Starr and White [21,22,24] and which will be used throughout this paper while expansion (2.2) has been used by Priestley.[15] The physical interpretation of each of the terms in Eqs. (2.1) and (2.2) have been discussed by Starr and White.[23] In brief the terms in Eq. (2.1) may be interpreted as being associated with the following kinds of circulations:

a. The term $(\overline{[\alpha]})(\overline{[v]})$ is a measure of the flux of a quantity α which results when there is a net shift of mass across a latitude circle. Over long periods of time this term must vanish although it may be of some importance over short periods. In actual evaluation from data, even over long periods of time, this term may not vanish exactly because of the inadequacies of the data.

b. The term $([\bar{\alpha}]'[\bar{v}]')$ is a measure of the flux of α due to the existence of mean meridional circulations.

c. The term $(\overline{[\alpha]'[v]'})$ is a measure of the flux of α associated with instantaneous meridional circulations.

d. The term $(\overline{[\alpha' v']})$ is a measure of the flux of α associated with large scalc horizontal eddies.

The integrals were evaluated from data in accordance with the following procedures:

$$[\alpha] = \frac{1}{n'} \sum_{1}^{n'} \alpha_{n'}, \qquad (2.3)$$

where n' is the number of observations on a given day at a given level and latitude:

$$[\alpha v] = \frac{1}{n'} \sum_{1}^{n'} \alpha_{n'} \, v_{n'}, \qquad (2.4)$$

$$[\alpha' v'] = [\alpha v] - [\alpha][v], \qquad (2.5)$$

$$\overline{[\alpha]} = \frac{1}{n} \sum_{1}^{n} [\alpha]_n , \qquad (2.6)$$

where n is the number of days on which two or more observations are reported at a given level and latitude:

$$[\overline{\alpha v}] = \frac{1}{n}\sum_{1}^{n} [\alpha v]_n , \tag{2.7}$$

$$[\overline{\alpha' v'}] = \frac{1}{n}\sum_{1}^{n} [\alpha' v']_n , \tag{2.8}$$

$$\overline{[\alpha]'[v]'} = [\overline{\alpha v}] - [\overline{\alpha}][\overline{v}] , \tag{2.9}$$

$$([\overline{\alpha}]) = \frac{1}{P_{1000} - P_{100}} \sum_{i=1}^{i} [\overline{\alpha}]_i \, \Delta P_i , \tag{2.10}$$

where P_{1000} is the sea level pressure considered to be constant and equal to 1000 mb, P_{100} is the pressure at 100 mb, ΔP is the increment of pressure for a given layer for which $[\overline{\alpha}]$ is evaluated:

$$([\overline{\alpha v}]) = \frac{1}{P_{1000} - P_{100}} \sum_{i=1}^{i} [\overline{\alpha v}]_i \, \Delta P_i , \tag{2.11}$$

$$([\overline{\alpha}]'[\overline{v}]') = ([\overline{\alpha}][\overline{v}]) - ([\overline{\alpha}])([\overline{v}]). \tag{2.12}$$

In order to secure some measure of the statistical reliability of some of the time averages, confidence limits were calculated and are included in some of the tables. These limits are defined as twice the standard error and indicate approximately the 95 percent confidence level.

The space-time means $[\overline{\alpha}]$ represent averages over the number of days involved in each case of the daily space averages $[\alpha]$. As such these space-time averages are formed by an equal weighting of the daily values. Owing to the fact that the daily values $[\alpha]$ are based upon a number of observations which may vary considerably, an equal weighting introduces a certain amount of artificiality in the procedure. For purposes of comparison therefore the space-time averages were

evaluated by an alternative method and computed as:

$$\{\alpha\} = \frac{1}{N}\sum_{1}^{N}\alpha_N \,, \tag{2.13}$$

where the braces indicate a space-time average of this second kind, and N represents the total number of available observations at a given level and latitude for the entire year. This method allows the formation of a quantity:

$$\{\alpha'v'\} = \{\alpha v\} - \{\alpha\}\{v\} \,, \tag{2.14}$$

which in the case of an equal number of observations on each day would become identical to the sum of the transports of a given property by circulations of the kind described in c) and d) of the preceding section.

According to the definitions of the quantities involved it follows that we may define a correlation coefficient between α and v as:

$$r(\alpha,v) = \frac{\{\alpha v\}-\{\alpha\}\{v\}}{\sigma(\alpha)\sigma(v)} \,, \tag{2.15}$$

where $\sigma(\alpha)$ and $\sigma(v)$ are the standard deviations of α and v respectively and defined as:

$$\sigma(\alpha) = \sqrt{\{\alpha^2\}-\{\alpha\}^2}\;;\; \sigma(v) = \sqrt{\{v^2\}-\{v\}^2}\,. \tag{2.16}$$

In the various tables of the succeeding sections the quantities for which tabulations are recorded have been defined and evaluated as outlined in this section.

3. The Angular Momentum Balance

On several occasions in the past, the authors have reported on the progress of their investigations of the angular momentum balance of the atmosphere (Starr and White,[21-24] and

Starr.[20] Previous reports ha discussed only conditions prevailing at individual latitudes in restricted regions of the hemisphere for non-homogeneous time periods. The results presented in this paper describe conditions over the entire northern hemisphere for the full year 1950, and represent a distillation of findings based upon some three years of investigation into this problem.

3.1 Procedure

In studies of the angular momentum balance of the atmosphere it is of fundamental importance to arrive at an understanding of the manner in which the atmospheric circulations are organized to bring about a flux of this momentum from low to high latitudes. The formulation of the problem has been given by Starr,[18] Bjerknes,[5] Widger,[31] and Mintz.[13]

The total meridional transport of absolute angular momentum across a latitude circle may be thought of as the sum of the transport of relative angular momentum and the transport of angular momentum due to the earth's rotation. Since there is no net meridional transport of mass in the atmosphere in the long run, the transport of angular momentum must be brought about by exchange processes. Furthermore since the angular momentum due to the earth's rotation per unit mass is essentially constant with respect to elevation in the atmosphere at a given latitude, the net effect of all exchange processes must be negligible as far as this component is concerned. It therefore follows that for the purposes at

hand we may omit this contribution and deal only with the transport of relative angular momentum. Also, owing to the fact that the distance from the earth's axis varies percentually only very slightly with elevation in the atmosphere at a given latitude, the transport of relative angular momentum is given by the transport of relative linear momentum except for a constant factor. For these reasons the investigation of the balance requirement as outlined in Eq. (1.1) is equivalent to the problem of investigating the nature of the term $(\overline{[uv]})$ where u is the relative eastward component of velocity. According to Eq. (2.1) the problem is to determine the magnitudes of the various terms in the relation:

$$(\overline{[uv]}) = (\overline{[u]})(\overline{[v]}) + (\overline{[u]}'\,\overline{[v]}') + (\overline{[u]'[v]'}) + (\overline{[u'v']}) \qquad (3.0)$$

3.2 Results

The results of evaluation of the terms in expansion (3.0) are given in Tables 11-15 which are of exactly the same form as those previously presented by Starr and White.[21,22,24] In addition certain supplementary Tables 16-18 showing the meridional profiles of various quantities are given. One final compilation, Table 19, represents the distillation from this vast array of data of the magnitudes of the flux of angular momentum associated with each of the terms in Eq. (3.0).

3.3 Meteorological implications

Assuming the representativeness of the data in Tables [16-18] it is possible to make the following important inferences:

a. In Table 19, the mean rate of flux of angular momentum across various latitude circles by the physical processes set forth in Eq. (3.0) is given. The outstanding feature here, as it was in previous data compilations of this type, is the overwhelming importance of the flux of eastward angular momentum of the atmosphere by the large scale horizontal eddies, as compared with the flux by the mean meridional circulations. This is the principal result of these investigations and is in accord with the concepts originally proposed by Starr.[18]

The magnitude of the flux by the horizontal eddies is again demonstrated by this sample of data to be sufficient to account for the drain of such momentum by surface torques in the westerlies according to the best estimates which can at present be made of K_1, (Priestley).[16]

The necessary conclusion is that other agencies of transportmust be either of negligible importance or by fortunate circumstance strangely compensating. The evidence in Table 19 indicates that the first of these possibilities must be proper.

Over long periods of time the momentum flux due to the net-shift of mass across latitude circles must vanish. It is only because of the inadequacies of the data that a measurable flux is obtained. A more realistic measure of the total flux of momentum would perhaps be represented by the difference between the third and fourth rows of Table 19. This "adjusted total" is given in the last column.

The features of the distribution of the intensity of the hori-zontal eddy flux, with elevation and latitude, are given in Table 18 and they are substantially the same as those pointed out by Starr and White,[21,22,24] for tropical and subtropical latitudes. In middle and high latitudes the distribution is not unexpected and conforms to previously stated ideas (Starr).[18] The principal point concerns the equatorward flux of momentum across 70°N which is associated with the surface polar easterlies.

b. The intensity of the mean meridional circulations in the atmosphere is usually inferred indirectly by consideration of the effects which they are thought to produce. Direct measures of such cellular circulations have never been obtained. It would be an achievement to state that the measurements of the mean net meridional component of motion shown in Table 17 and in Tables 11-15, column 3, do indeed represent direct measures of such circulations. Unfortunately these measures cannot be taken at their face value in view of the possible biases existing within the data and because of the ratio of the magnitude of the standard deviation in the northward component of the wind to the observed mean values, as indicated in Table 17.

However, these measurements do offer important indirect evidence concerning the mean meridional circulations which are none the less of fundamental importance. The most outstanding

feature of the measurements is that in general, the values of $[\bar{v}]$ are quite small. In a statistical sample of this size one would expect that any sizable mean meridional circulation would be reflected in the data. The strong suggestion therefore is that any mean meridional circulations which exist probably possess an order of magnitude less than 1 m sec^{-1}.

c. The zonal wind velocity profile as evaluated from these data are shown in Table 16. Since these evaluations are based upon actual wind measurments rather than geostrophic approximations they possess an intrinsic value. Aside from this point, however, one of the major conclusions drawn from this table concerns the vertically averaged values of the zonal winds. The indications show that the mass of the atmosphere between the surface and 100 mb and poleward of 13°N, possesses a mean drift eastward relative to the surface of the earth. This mean relative rotation, in the face of the drag of the earth's surface, raises a question of fundamental importance concerning the maintenance of this westerly drift. Angular momentum considerations cannot answer this question for the only information which such considerations will yield is the fact that there must exist a balance of surface torques, if we are to consider the hemisphere to be a mechanically closed system.

The answer to this question must be sought in a consideration of the balance of the kinetic energy of this component of motion. It is not difficult to derive an appropriate mechanical

energy equation; this has been done by many authors and recently, for example, by Van Mieghem.[27] In this form of the mechanical energy equation one of the physical processes which must be considered among others, concerns the work done by the large scale horizontal eddies in generating or dissipating this type of energy. The rate of generation of this type of energy is measured by the volume integral over the entire hemisphere of the product of the large scale horizontal eddy stress and the meridional gradient of the mean relative angular velocity. As pointed out by Starr[20] from evaluations of this term from several independent data sources, enough mechanical energy of mean rotation is generated by the horizontal eddies to account for the estimated dissipation by surface friction. In these data the reflection of this condition lies in the comparison of the distribution of the horizontal eddy flux of momentum and of the zonal winds, Tables 16 and 18. It will be noted that as far poleward as 42°N the flux of momentum proceeds in the direction of stronger westerly winds. This being so, it appears that we must revise our fundamental concepts about the maintenance of the general circulation of the atmosphere, at least insofar as we desire to seek an explanation of the mean westerly drift.

The suggestion here is that the kinetic energy of the mean westerly rotation is maintained by a conversion from the kinetic energy of the large scale disturbances. The further implication is that the primary thermal drive of the atmosphere manifests

itself in the generation of individual disturbances and not in the generation of simple convective mean meridional circulations.

3.4 Critical remarks

The deficiencies inherent in compilations of this type have been explored by Starr and White,[21] and for the most part the present data contain exactly the same drawbacks. A simple enumeration of these difficulties will suffice, the reader being referred to the references for a detailed discussion:

a. The geographic sampling with respect to longitude is determined by the location of stations. The representativeness of the data for the entire hemisphere is questionable at some latitudes. Coverage over Asia and the Pacific is particularly bad, especially in the upper troposphere.

b. The data are presented as representations at certain mean latitudes. The stations used for any given latitude cover an appreciable zonal belt as can be seen from Tables 1-5 and Fig. 1.

c. The randomness with respect to the mean troughs and ridges in the streamline pattern may be affected by the location of the observing stations. The general smallness of the net mean meridional motions suggests that this effect is not too pronounced.

d. Although radio wind soundings have been used wherever available to eliminate bias with respect to weather conditions, such observations are biased in favor of light winds.

e. The contributions above 100 mb and in the layer from the surface to 1000 mb in middle latitudes have not been evaluated.

In addition, because of the irregularities of the earth's surface, a certain fictitious component is introduced in the lowest levels where the surface of the earth rises above the 1000 mb level. At 13°N a similar difficulty arises since evaluations have been made only between 2,000 and 55,000 ft.

f. At 13°N where evaluations were made at constant levels, a constant standard atmosphere density was used in vertical integrations.

g. In general the availability of data at the highest levels is considerably poorer than that at the lower levels and hence less reliable.

4. <u>The Water Balance of the Atmosphere</u>

In a manner analogous to the investigation of the angular momentum balance we may study the atmospheric water balance. In the same sense that the angular momentum considerations represent a constraint on the general circulation, the water balance represents a similar consistency requirement for the general circulation. The necessity for a transport of water in the atmosphere arises from the fact that in certain latitude belts, there exists an excess of precipitation over evaporation with reverse conditions prevailing at other latitude belts. For the most part those regions with an excess of evaporation over precipitation lie in the subtropics while those regions with an excess of precipitation over evaporation lie in mid-latitude and polar areas. Over long periods of time the amount of precipitable water in the atmosphere does

not change and the deficits and excesses of precipitation over evaporation must be made up by the transport of water by the atmospheric circulations. How such transports are accomplished, and the magnitude of such transports, will be discussed in section 4.1.

4.1 Procedure

The evaluation of the left hand side of expression (1.2) proceeds in a manner exactly analogous to that for the angular momentum balance except for the following considerations:

a. The transport of water by the atmospheric circulations may occur in either solid or liquid form and also in vapor form. For the purposes of the current study only the transport in vapor form will be considered, it being highly probable that the transport in liquid and solid form is small in comparison. To this degree of approximation, expression (1.2) may be written as:

$$\frac{1}{g}\int_{0}^{P_o}\int_{0}^{t_1}\int_{0}^{L} qvdxdtdp \approx \frac{P_o t_1 L}{g}\left(\overline{[qv]}\right), \tag{4.0}$$

where q is the specific humidity. The problem of evaluating the transport of water in the atmosphere then reduces to the problem of investigating the relative magnitudes of the terms in the expression:

$$(\overline{[qv]})=(\overline{[q]})(\overline{[v]})+(\overline{[q]}'\,\overline{[v]}')+(\overline{[q]'[v]'})+(\overline{[q'v']}). \tag{4.1}$$

b. The measurements of specific humidity were obtained from observations of temperature and dewpoint at the various stations and levels. In the radiosonde observations, at certain combinations of temperature and humidity, a phenomenon called "motorboating"

occurs. Below these limits the evaluation of the dewpoint is not possible and hence the specific humidity is unobtainable. However when "motorboating" occurs the approximate maximum values of the dewpoint for specified temperatures is given. (See Radiosonde and Rawinsonde code).[26] In this investigation therefore whenever "motorboating" occurred, the dewpoint was assumed to be the maximum for the given air temperature as specified.

c. Because of the low moisture content above 500 mb, and also because "motorboating is most frequent above this level, the computations were restricted to the standard pressure levels up to and including the 500 mb level. The effects of this procedure on the results will be discussed in a subsequent section

d. The computations were carried out at the four northerly latitudes 31°N, 42.5°N, 55°N, and 70°N.

e. The estimates of K_2 were derived from data covering the evaporation and precipitation over the northern hemisphere originally given by Wust [32] and modified by Conrad [7] and again by Benton.[4] The process of estimating K_2 is accomplished by determining the difference between the precipitation and evaporation for given latitude belts and assuming that this difference is balanced by a flux of water in the atmosphere.

4.2 Results

The basic results are presented in Tables 20-23 for the various latitudes. These tables are of the same form as the tables summarizing the angular momentum data in the previous section.

In addition a summary Table 24 is presented in which the flux of moisture by the various kinds of atmospheric circulations are compared with estimated requirements.

4.3 Meteorological Implications

Certain general descriptive comments regarding the water vapor transport processes in the atmosphere are as follows:

a. Unlike the angular momentum the maximum flux of water vapor occurs close to the ground falling off rapidly with height.

b. These transport processes appear to be dominated by the horizontal eddies in the atmosphere at the latitudes which have been selected for study.

c. The latitudinal variation in the total water vapor flux appears to show a maximum around 42.5°N although that part due to the horizontal eddies appears to increase to the equatorward limits of the data.

d. The values of both the total and horizontal eddy flux possess 95 percent confidence limits which exclude zero in all cases.

e. The degree to which these data satisfy estimated balance requirements can be determined from Table 24. Columns 1 and 2 represent two different estimates of the magnitudes of the required fluxes. The remaining columns represent the vertically integrated flux of water across various latitude circles by atmospheric ircu-lations of various kinds. The comparison with estimated require-ments indicates very favorable agreement at most latitudes. The

last column of Table 24 is included for the purpose of comparison and represents the results of an investigation of the water vapor flux over the North American continent by Benton.[4]

f. If the transports of water vapor given in Table 24 are expressed in units of energy by multiplying each value by the latent heat of water vapor (approx. 600 cals/gm) one may obtain a measure of the energy amount that is transported poleward in latent form by the various processes. Since this point will be discussed in greater detail in the next section suffice to point out here that the fractions of the total required energy flux represented by these data is considerable and at 31°N represents more than 50 percent of the required total energy flux. The efficiency of this form of energy transport is therefore quite remarkable and one might speculate on the intensity of the circulation which would be required to transport the latent energy represented here in the form of sensible heat.

4.4 Critical remarks

For the most part, those critical remarks made in reference to the study of the angular momentum balance are also applicable here. A few supplementary remarks should be made however.

a. Since the evaluations were made for the lower half of the atmosphere the magnitudes of the water vapor fluxes are probably slightly underestimated. However because of the rapid decrease in the water vapor content with height this factor is not thought to be too serious.

b. Because most observations are located over land areas, the ocean areas where there exists a higher moisture content may have been insufficiently sampled. It is difficult to estimate the effects of this condition.

c. One fortunate consideration present in this study as contrasted with the angular momentum study is that the data for the lower atmosphere where the principal flux occurs are relatively more plentiful than they are for the higher atmosphere where the maximum flux of momentum occurs.

d. The effect of assuming that the dewpoint takes on the maximum possible value when "motorboating" occurs is probably to underestimate the flux. This is probably true since motorboating occurs with low humidities and such humidities will tend to be found more often in northerly than in southerly currents, thereby reducing the correlation between wind direction and humidity.

e. Because the surface pressure has been assumed to be constant at 1000 mb a certain fictitious component enters the computation where surface topography extends above this level. This is a more serious matter than in consideration of the angular momentum flux since the maximum flux occurs near the ground level. It is difficult to estimate the effects of this consideration on the computations, except that it tends to cause an overestimation of the flux.

5. The Energy Balance of the Atmosphere

In the past several years the second author of this paper has reported on various studies of the energy flux in the atmosphere (White).[28-30] For the most part these investigations were restricted

in their geographical extent, length of time, and did not extend through the entire vertical depth of the atmosphere. The investigation described in section 5.1 undertakes to examine the energy flux for the entire northern hemisphere for one entire year, 1950. This investigation represents a completed phase of a much broader investigation into energy transfer processes.

5.1 Procedure

We wish to evaluate the magnitudes of the various terms appearing in the expanded form of Eq. (1.3). The investigation of the energy balance is considerably more complicated than that of either the angular momentum or water balance. This is due to the fact that the energy may be transferred in many forms including a transfer by means of pressure forces. It is therefore necessary to elaborate the nature of the energy transfer processes further.

The flux of total energy in the atmosphere has been discussed by Starr.[19] He has shown that a close approximation is given by the expression:

$$\frac{1}{g} \int_0^{P_0} \int_0^{t_1} \int_0^{L} E v\,dx\,dt\,dp \approx \frac{1}{g} \int_0^{P_0} \int_0^{t_1} \int_0^{L} (C_p T + qL + gz) v\,dx\,dt\,dp, \tag{5.0}$$

which states that the total energy flux may occur in the form of sensible heat (C_pT), latent heat (qL) and potential energy (gz). In Eq. (5.0) the transfer of the kinetic energy of existing motions has been neglected as being of negligible magnitude compared with the other modes of energy transfer. The discussion of the transfer of latent energy which is proportional to the water vapor transfer

has been presented in the previous section and will be elaborated further in a subsequent part of this section. The evaluation of the remaining terms is simplified since the geopotential is constant at a given elevation and hence there can be no transfer of potential energy by processes which involve correlations between the geopotential and any horizontal component of the wind at that level. It is sufficient therefore to attempt the evaluation of the remaining terms which reduces to the problem of evaluating the expression:

$$(\overline{[T]})(\overline{[v]})+(\overline{[T]}'\,\overline{[v]}')+(\overline{[T]'[v]'})+(\overline{[T'v']})+(\overline{[Z]})(\overline{[v]})+(\overline{[Z]}'\,\overline{[v]}'). \tag{5.1}$$

the problem introduced by the necessity for the evaluation of the last two terms in (5.1) is almost insurmountable and renders the true evaluation of the total energy flux extremely difficult. These last two terms can be regarded as transfers of "height". Because this property of the atmosphere goes to infinity with elevation, the transfer of energy in this form becomes at best indeterminate. Actually the next to the last term offers little difficult,y since over long periods of time there can be no net mass shift and the transfer of potential energy by this process must vanish. This leaves only the meridional cell term which involves the correlations of net meridional velocity at given levels and "height" as a major difficulty. It is apparent that the entire character of the energy flux is dependent upon the actual magnitude of this term, and without complete and accurate data to the top of the atmosphere, evaluation of this form of energy transfer is meaningless.

As a conseauence of this feature, the evaluations of tne sensible heat flux by mean celluar circulations likewise becomes meaningless since the mean cellular flux of potential energy is always accompanied by an oppositely directed flux of sensible heat. Indeed it can be shown that the mean cellular flux of potential energy is exactly cancelled by the oppositely directed flux of sensible heat in an adiabatic atmosphere.

It appears therefore that our data are sufficient only to evaluate the flux of energy by horizontal eddy exchange processes. We may then inquire about the magnitude of these horizontal eddy exchange processes and the degree to which they account for required balances.

One additional feature complicates this picture. If the balance requirements to be satisfied are taken to be those prescribed by radiational balance studies, then the transport of energy within the oceans also enters into consideration. No attempt is made here to evaluate this ocean energy flux, but note should be taken of a recent study by Jung [10] on this matter.

The estimates of the total required energy flux, K_3, on the basis of radiation requirements have been made by many investigators. Their results are widely varying and for the sake of completeness four such estimates are given here, made by Bjerknes [6] based upon an investigation by Albrecht,[1] Baur and Phillips,[2] Gabites,[9] and London,[11] the reader being referred to the original papers for details of such estimates.

5.2 Results

The results of the computations of the sensible heat flux within the atmosphere are presented in Tables 25-28. The results of the computations of the latent heat flux which is proportional to the flux of water vapor, have been presented in Tables 20-24. A supplementary table (29) presents energy flux requirements and the measured eddy energy flux within the atmosphere by the horizontal eddy processes.

5.3 Meteorological implications

It has been pointed out that the problem of the energy balance of the atmosphere is considerably more complicated than the balances of the other quantities considered in the previous sections. The meteorological implications which one may draw from this array of data are therefore conditioned by the limitations of the data and the complexities of the problem:

a. The data in Tables 25-28 and Table 29 reveal certain prominent characteristics of the poleward eddy flux of sensible heat which have been noted in previous investigations (White,[28,29] Mintz.[14] One of these concerns the location of the latitude of maximum poleward eddy flux of sensible heat which is found to occur in this investigation slightly north of 50°N. Radiation flux requirements as indicated in Table 29 show that the maximum poleward flux of total energy occurs closer to latitude 40°N.

If the estimates of the required fluxes are correct, at least insofar as the location of the latitude of maximum total energy

flux is concerned, then mechanisms other than the eddy sensible heat flux of energy transport must be active to the south of this latitude. According to the data of Table 29, the transport of energy in latent form by eddy processes can account for this transport. Other modes of energy transfer which may be of some importance, concern the flux due to mean meridional circulations (for which the data at hand are inadequate) and the flux within the oceans.

b. The percentages of the total eddy energy flux accomplished in latent and sensible heat form may be deduced from the data of Table 29. Although the latent energy flux comprises a large fraction of the total at all latitudes it is greatest at latitude 31°N where it accounts for more than 50 percent of the evaluated total.

c. The variation of the eddy flux intensity of sensible heat with elevation, as indicated in Tables 25-28, at almost all latitudes, reveals a feature first noticed by Priestley [15] and again by Mintz,[14] but which was not indicated in a study by White [28] for the North American continent. If we exclude the 1,000 mb level from consideration, it being too easily rendered unrepresentative because of local conditions, it is noted that the horizontal eddy flux of sensible heat has a maximum at the lowest levels, decreasing through midtroposphere and reaching a secondary maximum at about the 200 mb level. The existence of a poleward eddy flux of sensible heat at levels of 200 and 100 mb, when considered in conjunction with the meridional gradient of temperature (see column 2, Tables 25-28) which is reversed at these levels, reveals a condition in

which eddy processes act to build up rather than dissipate the mean meridional temperature gradient. A similar condition with regard to the angular momentum balance was noted in a previous section. In view of these conditions we must begin to realize that generative as well as dissipative large scale eddies are common atmospheric phenomena:

d. The comparison between various balance requirement estimates and the computed fluxes of energy are given in Table 29. It is to be noted that estimates of the flux requirements vary considerably. It is not possible to specify which is most reliable. The last column of this table gives the total eddy flux of energy in the form of latent and sensible heat. It appears that at all latitudes studied the poleward energy flux as evaluated from wind, temperature, and moisture data appears to satisfy balance requirements, at least as far as the accuracy of the requirement estimates permit. It follows on the basis of these data that other modes of poleward energy transfer must either be small in comparison or compensate one another.

5.4 Critical remarks

The critical remarks of section 3. in reference to the angular momentum balance are also applicable here.

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Table 1. List of key stations (numbered) and alternates. Mean latitude of key stations 13°N.

	Station	Latitude		Longitude		Altitude (ft)	Type
1.	Gao	16°	16'N	00°	03'W	899	pilot balloon
	Agadez	16	59	07	59 E	1706	"
	Niamey	13	31	02	26 E	755	"
	Birni N'Koni	13	48	05	15 E	968	"
	Zinder	13	48	09	00 E	1604	"
	Nema	16	36	07	16 W	892	"
	Mopti	14	30	04	12 W	906	"
	Segou	13	24	06	09 W	961	"
	Kano	12	02	08	32 E	1561	"
	Minna	09	37	06	32 E	853	"
	Kandi	11	08	02	56 E	961	"
	Mango	10	21	00	30 E	525	"
	Tamale	09	25	00	53 W	604	"
	Ouagadougou	12	22	01	31 W	984	"
	Bobo-Dioulasso	11	10	04	18 W	1404	"
2.	El Fasher	13	37	25	20 E	2395	"
	Karima	18	33	31	51 E	820	"
	Atbara	17	42	33	58 E	1142	"
	Khartoum	15	36	32	33 E	1263	"
	Geneina	13	29	22	27 E	2641	"
	El Obeid	13	10	30	14 E	1887	"
	Kosti	13	10	32	40 E	1253	"
	Malakal	09	33	31	39 E	1276	"
	Wau	07	42	28	01 E	1440	"
3.	Aden (Khormaksar)	12	50	45	01 E	3	radio wind
	Aden (Sheikh Othman)	12	53	44	58 E	33	pilot balloon
	Riyan	14	39	49	23 E	46	"
	Kamaran Island	15	20	42	37 E	20	"
	Asmara	15	17	38	55 E	--	"
	Massawa	15	36	39	28 E	--	"
	Assab	13	01	42	43 E	--	"
	Djibouti	11	36	43	09 E	23	"
	Berbera	10	26	45	01 E	--	"
	Hargeisa	09	30	44	05 E	--	"
4.	Trichinopoly	10	49	78	42 E	256	"
	Madras	13	04	80	15 E	52	"
	Bangalore	12	58	77	35 E	3022	"
	Mangalore	12	52	74	51 E	72	"
	Anantapur	14	41	77	37 E	1148	"
	Trivandrum	08	29	76	57 E	200	"
	Cochin	09	58	76	14 E	10	"
	Vengurla	15	55	73	40 E	--	"
	Masulipatam	16	11	81	08 E	10	"
	Nagercoil	08	11	77	26 E	112	"
	Minicoy	08	18	73	00 E	10	"

Table 1. (Continued)

Station	Latitude	Longitude	Altitude (ft)	Type
5. Port Blair	11° 40'N	992° 43'E	262	pilot balloon
Sandoway	18 28	94 21 E	30	"
Bassein	16 46	94 46 E	13	"
Mingaladon	16 54	96 11 E	92	"
Tavoy	14 06	98 13 E	112	"
Mergui	12 26	98 36 E	66	"
Victoria Point	09 58	98 35 E	122	"
Penang	05 18	100 16 E	12	"
6. Saigon	10 49	106 40 E	30	"
Phnom-penh	11 33	104 55 E	39	"
Tourane	16 02	108 12 E	26	"
Pattle	16 33	111 37 E	20	"
Seno	16 40	105 00 E	603	"
7. Clark Field	15 10	120 34 E	644	radio wind
Laoag	18 11	120 32 E	12	pilot balloon
Baguio	16 25	120 36 E	4859	"
Sangley Point	14 30	120 55 E	8	"
Legaspi	13 08	123 44 E	57	"
Puerto Princesa	09 45	118 44 E	47	"
Cuyo	10 51	121 02 E	9	"
Cebu	10 20	123 54 E	138	"
Surigao	09 48	125 30 E	67	"
8. Yap	09 29	138 08 E	--	radio wind
Peleliu Island	07 00	134 15 E	--	pilot balloon
9. Harmon Field	13 31	144 49 E	176	radio wind
North Field	13 34	144 55 E	526	pilot balloon
Saipan	15 19	145 44 E	--	"
Truk	07 15	151 50 E	--	"
Tinian	14 59	145 36 E	--	"
10. Kwajalein	08 43	167 44 E	10	"
Eniwetok Atoll	11 21	162 21 E	21	pilot balloon
Ponape	06 50	158 12 E	--	"
Wake Island	19 18	166 37 E	--	radio wind
Majuro	07 06	171 24 E	--	pilot balloon
Roi	09 24	167 26 E	--	"
11. Johnson Island	16 44	169 31 W	20	radio wind
12. Hilo	19 44	155 04 W	33	"
13. Veracruz	19 12	96 08 W	10	pilot balloon
Tacubaya	19 24	99 12 W	7579	"
Mexico City	19 26	99 08 W	7340	"
Ciudad del Carmer	18 39	91 49 W	5	"
Swan Island	17 24	83 56 W	35	"
14. Albrook Field	08 58	79 33 W	21	radio wind
Plato Magdalena	09 48	74 48 W	190	"

Table 1. (Continued)

Station	Latitude	Longitude	Altitude (ft)	Type
15. Waller Field	10° 36'N	61° 12'W	--	radio wind
Port of Spain	10 36	61 21 W	30	pilot balloon
Chaguaramas Bay	10 41	61 37 W	--	"
Pearls Air Field	12 09	61 37 W	--	"
Beane Field	13 45	60 57 W	26	"
Fort-de-France	14 37	61 04 W	479	"
Raizel Airdrome	16 16	61 31 W	--	"
Pointe-a-Pitre	16 14	61 33 W	33	"
Gustavia	17 54	62 52 W	7	"
Coolidge Field	17 07	61 47 W	34	"
Ramey Air Force Base	18 30	67 08 W	228	"
Santa Isabel	17 58	66 24 W	28	"
San Juan	18 28	66 06 W	62	radio wind
Roosevelt Roads	18 15	65 38 W	--	pilot balloon
Kingshill	17 42	64 48 W	53	"
16. Cayenne-Rochambeau	04 50	52 22 W	--	"
17. Dakar-Ouakam	14 40	17 26 W	131	radio wind
Dakar-Yoff	14 41	17 25 W	62	pilot balloon
Sal	16 44	22 57 W	180	"
Nouakchott	18 07	15 36 W	66	"
Saint-Louis	16 01	16 30 W	10	"
Tambacounda	13 46	13 41 W	144	"

Table 2. List of key stations (numbered) and alternates. Mean latitude of key stations 31°N.

Station	Latitude	Longitude	Altitude (ft)	Type
1. Qrendi	35° 50'	14° 27'E	442	radio wind
Luqa	35 51	14 29 E	253	pilot balloon
2. Farouk	30 08	31 24 E	223	radio wind
3. Habbaniya	33 22	43 34 E	144	"
4. Bahrein	26 16	50 37 E	3	"
5. Hyderabad	25 23	68 25 E	95	pilot balloon
Fort Sandeman	31 21	69 27 E	4613	"
Peshawar	34 01	71 35 E	1165	"
Ambala	30 23	76 46 E	892	"
Bikaner	28 00	73 18 E	735	"
Jodhpur	26 18	73 01 E	735	"
Bhuj	23 15	69 48 E	344	"
Ahmedabad	23 02	72 35 E	164	"

Table 2. (Continued)

	Station	Latitude	Longitude	Altitude (ft)	Type
6.	Dibrugarh	27° 28'N	94° 55'E	348	pilot balloon
	Tezpur	26 37	92 47 E	259	"
	Cooch Behar	26 20	89 27 E	157	"
	Asansol	23 41	86 59 E	413	"
	Gaya	24 45	84 57 E	364	"
	Calcutta	22 33	88 20 E	20	"
	Gorakhpur	26 45	83 22 E	253	"
7.	Hong Kong	22 18	114 10 E	--	radio wind
8.	Tokyo	35 33	139 46 E	37	"
	Tateno	36 03	140 08 E	89	"
	Shionomisaki	33 27	135 46 E	243	"
	Yonago	35 26	133 21 E	26	"
	Itazuke	33 35	130 42 E	122	"
9.	Midway Island	28 13	177 21 W	0	"
10.	Honolulu	21 20	157 55 W	15	"
11.	Ship	30 00	140 00 W	--	"
12.	Santa Maria	34 56	120 25 W	238	"
	Oakland	37 44	122 12 W	7	"
13.	Big Spring	32 14	101 30 W	2537	"
	El Paso	31 48	106 24 W	3916	"
14.	New Orleans	30 00	90 16 W	30	"
	Lake Charles	30 13	93 09 W	32	"
15.	Miami	25 49	80 17 W	12	"
	Tampa	27 58	82 32 W	11	"
16.	Kindley Field	32 22	64 40 W	16	"
	Nassua	25 03	77 23 W	5	"
17.	Ship	35 00	48 00 W	--	"
18.	Lagens	38 45	27 05 W	171	"
19.	North Front	36 09	05 21 W	8	"
	Maison-Blanche	36 43	03 14 E	92	"

Table 3. List of key stations (numbered) and alternates. Mean latitude of key stations 42.5°N.

	Station	Latitude	Longitude	Altitude (ft)	Type
1.	Bordeaux	44° 50'N	00° 43'W	157	radio wind
	Lyon	45 43	04 55 E	659	"
	Portela	38 46	09 09 W	338	pilot balloon
2.	Rome	41 48	12 35 E	400	radio wind
	Milano-Lirate	45 28	09 17 E	397	pilot balloon
	Cagliari-Elmas	39 15	09 03 E	39	"

Table 3. (Continued)

	Station	Latitude		Longitude		Altitude (ft)	Type
3.	Odessa	46°	29'N	30°	38'E	141	radio wind
	Bucuresti-Bancasa	44	30	26	05 E	302	pilot balloon
4.	Tbilisa	41	43	44	48 E	1325	"
	Erevan	40	08	44	28 E	2976	"
5.	Tashkent	41	20	69	18 E	1572	"
	Stalinabad	38	35	68	48 E	2625	"
6.	Alma-Ata	43	15	76	35 E	2763	"
7.	Vladivostok	43	07	131	55 E	420	"
8.	Misawa	40	43	141	22 E	120	radio wind
	Wakkanai	45	25	141	41 E	6	pilot balloon
	Sapporo	43	04	141	20 E	54	"
9.	Ship	38	24	153	12 E	--	radio wind
10.	Ship	40	00	142	00 W	--	radio wind
11.	Medford	42	23	122	52 W	1329	"
	Boise	43	34	116	13 W	2858	"
12.	Lander	42	48	108	43 W	5558	"
	Grand Junction	39	06	108	32 W	4839	"
	Rapid City	44	09	103	06 W	3218	"
13.	Omaha	41	18	95	54 W	982	"
	Columbia	38	58	92	22 W	785	"
	St. Cloud	45	35	94	11 W	1043	"
14.	Albany	42	45	73	48 W	292	"
	Nantucket	41	15	70	04 W	12	"
	Buffalo	42	56	78	44 W	706	"
15.	Sable Island	43	56	60	02 W	25	pilot balloon
16.	Ship	44	00	41	00 W	--	radio wind
17.	Ship	45	00	16	00 W	--	"

Table 4. List of key stations (numbered) and alternates. Mean latitude of key stations 55°N.

	Station	Latitude		Longitude		Altitude (ft)	Type
1.	Kobenhaven Kastrup	55°	38'N	12°	40'E	7	radio wind
	Berlin-Tempelhof	52	28	13	23 E	157	"
	Hannover/Langen Lagen	52	28	09	42 E	167	"
2.	Moscow (moskva)	55	47	37	38 E	528	"
	Kaunas	54	55	23	56 E	272	pilot balloon
3.	Sverdlovsk	56	44	60	38 E	945	radio wind
4.	Irkutsk	52	20	104	13 E	1434	pilot balloon
5.	Khabarovsk	48	28	135	03 E	151	"
6.	Massacre Bay	52	50	173	11 E	--	radio wind
	Longview	51	53	176	40 W	10	pilot balloon

Table 4. (Continued)

Station	Latitude	Longitude	Altitude (ft)	Type
7. Anchorage	61° 13'N	149° 50'W	132	radio wind
Ship	50 00	145 00 W	--	"
8. Prince George	53 54	122 40 W	2218	"
Annette Island	55 04	131 33 W	113	"
9. Churchill	58 47	94 11 W	44	"
10. Goose	53 20	60 25 W	144	"
Mingan Light Point	51 --	64 -- W	--	pilot balloon
11. Ship	52 42	35 30 W	--	radio wind
12. Aldergrove	54 39	06 13 W	220	"
Camborne	50 13	05 19 W	288	"
Larkhill	51 11	01 48 W	436	"
Downham Market	52 37	00 24 E	123	"

Table 5. List of key stations (numbered) and alternates. Mean latitude of key stations 70°N.

Station	Latitude	Longitude	Altitude (ft)	Type
1. Ship	66° 00'N	02° 00'E	--	radio wind
2. Murmansk	68 57	33 03 E	151	pilot balloon
Arkhangelsk	64 34	40 31 E	33	"
3. Dikson	73 30	80 24 E	66	"
Khatanga	71 59	102 28 E	230	"
Amderma	69 46	61 41 E	171	"
4. Bukhta Tiksi	71 35	129 02 E	23	"
Zhigansk	66 45	123 24 E	194	"
Verkhoiansk	67 33	133 24 E	400	"
5. Mys Shmidta	68 55	179 29 E	30	"
Ost Chetyrek	70 38	162 24 E	--	"
6. Kotzebue	66 52	162 38 W	16	radio wind
Nome	64 31	165 26 W	46	"
7. Fairbanks	64 48	147 49 W	454	"
Aklavik	68 14	135 00 W	30	"
8. Resolute Bay	74 41	94 54 W	56	"
Arctic Bay	73 00	85 18 W	36	"
Cambridge Bay	69 07	105 01 W	45	"
9. Clyde River	70 25	68 33 W	26	"
Egedesminde	68 42	52 52 W	157	"
10. Angmagssalik	65 36	37 34 W	118	"
Cap Tobin	70 25	21 58 W	138	"

Table 6. Percentage of total possible observations at each level for each key station at latitude 13°N.

	Elevation in thousands of feet											
Station	2*	6*	10*	14	20*	25	30*	35	40*	45	50	55*
Gao	64	65	62	44	11	0	0	0	0	0	0	0
El Fasher	92	93	92	91	84	33	27	14	10	4	1	1
Aden	75	85	73	12	71	21	64	10	58	3	3	41
Trichinopoly	100	0	100	0	88	84	75	7	0	0	0	0
Port Blair	97	86	91	41	52	24	13	2	0	0	0	0
Saigon	89	86	88	76	63	13	5	3	2	1	0	0
Clark Field	97	97	96	92	85	53	46	34	38	29	28	30
Yap	75	84	78	58	64	43	46	30	32	20	10	8
Harmon Field	88	94	93	78	89	72	86	71	83	65	54	48
Kwajalein	99	100	99	98	99	95	98	90	96	80	62	57
Johnson Island	75	91	90	70	79	57	63	45	49	31	27	16
Hilo	90	99	99	90	97	84	91	76	82	57	37	30
Veracruz	98	94	89	79	62	10	2	0	0	0	0	0
Albrook Fie d	82	93	92	77	83	61	65	47	49	24	6	1
Waller Field	100	100	100	96	97	84	91	77	84	64	47	48
Cayenne Roch	56	52	39	30	23	18	16	17	14	13	7	4
Dakar	92	91	78	41	35	18	28	16	24	12	8	8

Table 7. Percentage of total possible observations at each level for each key station at latitude 31°N.

	Pressure levels in millibars						
Station	1000	850	700	500	300	200	100
Qrendi	4	58	58	56	52	38	22
Farouk	60	88	88	86	62	28	4
Habbaniya	38	72	72	74	72	62	44
Bahrein	18	68	66	68	62	50	32
Hyderabad	1	0	90	70	50	0	0
Dibrugarh	63	0	86	38	16	0	0
Hong Kong	70	67	64	58	39	34	22
Tokyo	98	98	99	97	88	72	38
Midway Island	80	76	73	68	68	60	34
Honolulu	92	82	67	57	46	34	16
Ship	84	80	81	72	58	42	14
Santa Maria	99	99	100	96	87	74	39
Big Spring	0	100	99	94	76	60	22
New Oreans	98	98	98	87	66	52	18
Miami	100	100	99	97	92	80	37

Table 7. (Continued)

Station	1000	850	700	500	300	200	100
	Pressure levels in millibars						
Kindley Field	90	91	88	79	44	16	2
Ship	77	82	85	82	74	61	22
Lagens	86	96	94	86	75	49	12
North Front	80	82	76	66	54	36	8

Table 8. Percentage of total possible observations at each level for each key station at latitude 42.5°N.

Station	1000	850	700	500	300	200	100
	Pressure levels in millibars						
Bordeaux	95	93	91	78	72	58	18
Rome	78	89	84	76	69	45	0
Odessa	12	78	74	64	31	14	0
Tbilisa	3	75	73	58	9	0	0
Tashkent	1	88	84	55	5	1	0
Alma-Ata	1	66	66	27	4	1	0
Vladivostok	17	13	8	3	0	0	0
Misawa	91	92	96	95	80	51	21
Ship	39	32	24	13	6	2	1
Ship	52	50	50	44	33	23	4
Medford	0	100	99	95	74	59	35
Lander	0	98	100	100	87	65	25
Omaha	5	100	100	97	87	74	30
Albany	96	95	86	70	41	21	3
Sable Island	52	0	0	0	0	0	0
Ship	76	81	83	81	69	48	13
Ship	70	67	68	42	6	1	0

Table 9. Percentage of total possible observations at each level for each key stations at latitude 55°N.

Station	1000	850	700	500	300	200	100
	Pressure levels in millibars						
Kobenhaven	98	100	100	100	99	91	48
Moscow	0	97	95	97	62	0	0

Table 9. (Continued)

Station	Pressure levels in millibars 1000	850	700	500	300	200	100
Sverdlovsk	0	73	68	66	8	5	0
Irkutsk	0	57	48	34	3	0	0
Kharbarovsk	5	36	35	10	6	4	0
Massacre Bay	67	75	68	60	39	25	11
Anchorage	85	98	98	97	87	67	26
Prince George	79	99	99	97	91	66	27
Churchill	86	89	89	82	54	28	6
Goose	80	97	93	67	27	12	2
Ship	74	81	86	85	72	56	27
Aldergrove	63	98	99	99	98	97	74

Table 10. Percentage of total possible observations at each level for each key station at latitude 70°N.

Station	Pressure levels in millibars 1000	850	700	500	300	200	100
Ship	89	81	81	82	75	63	17
Murmansk	97	77	77	68	44	0	0
Dikson	93	34	23	16	9	3	0
Bukhta Tiksi	93	45	26	17	5	2	0
Mys Shmidta	82	21	16	11	5	0	0
Kotzebue	100	93	87	77	54	39	15
Fairbanks	100	99	96	94	80	64	26
Resolute Bay	100	96	93	88	71	55	25
Clyde River	98	75	73	63	24	19	19
Angmagssalik	95	81	80	78	55	34	8

Table 11. Numerical analysis of momentum balance data for entire year 1950 at latitude 13°N. The levels are given in thousands of feet. All velocities are in m sec^{-1}. Internal consistency of figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\bar{u}]$	$[\bar{v}]$	$\overline{[u][v]}$	$\overline{[uv]}$	$\overline{[u]}\,\overline{[v]}$	$\overline{[u]'[v]'}$	$[\overline{u'v'}]$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	γ	N
55,000	-1.0	-0.51	-1	- 1	+1	-1	0	280	-1.3	-0.48	0	0	+0.00	808
		±0.48	±5	±10			±6							
50,000	+4.9	-0.99	-5	- 5	-5	0	0	293	+4.7	-1.13	- 5	0	+0.00	879
		±0.51	±6	± 8			±5							
45,000	+8.4	-0.57	+1	+ 8	-5	+6	+7	348	+8.1	-0.75	+ 8	+14	+0.11	1248
		±0.51	±6	± 9			±6							
40,000	+7.6	-0.23	+3	+11	-2	+5	+8	364	+7.4	-0.27	+13	+15	+0.11	1892
		±0.41	±5	± 8			±6							
35,000	+6.5	+0.19	+7	+15	+1	+6	+8	362	+6.3	+0.18	+14	+13	+0.13	1652
		±0.40	±5	± 7			±4							
30,000	+3.0	-0.04	+1	+ 6	0	+1	+6	365	+3.0	-0.09	+ 7	+ 7	+0.10	2493
		±0.27	±2	± 4			±3							
25,000	+0.4	-0.48	0	+ 3	0	0	+3	364	+0.3	-0.45	+ 3	+ 3	+0.07	2364
		±0.23	±1	± 2			±2							
20,000	-1.6	-0.06	0	+ 3	0	0	+3	365	-1.6	-0.09	+ 3	+ 3	+0.10	3758
		±0.21	±0	± 1			±1							
14,000	-2.5	+0.19	0	+ 2	0	0	+2	365	-2.6	+0.17	+ 2	+ 2	+0.09	3505
		±0.15	±0	± 1			±1							
10,000	-2.6	-0.12	+1	+ 2	0	0	+2	365	-2.7	-0.12	+ 2	+ 2	+0.09	4712
		±0.12	±0	± 1			±1							
6,000	-3.4	-0.29	+2	+ 3	+1	+1	+1	365	-3.2	-0.23	+ 3	+ 3	+0.11	4521
		±0.14	±1	± 1			±1							
2,000	-2.5	-0.89	+6	+ 7	+2	+4	+1	365	-2.3	-0.77	+ 7	+ 5	+0.16	4762
		±0.22	±1	± 1			±1							
Integral (10^7 CGS units)				+4.0	-0.1	+1.3	+2.7				+4.0	+4.2	Sum	32,594

Table 12. Numerical analysis of momentum balance data for entire year 1950 at latitude 31°N. The levels are given in hundreds of millibars. All velocities are in m sec^{-1}. Internal consistency of figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\bar{u}]$	$[\bar{v}]$	$\overline{[u][v]}$	$[\overline{uv}]$	$[\bar{u}][\bar{v}]$	$\overline{[u]'[v]'}$	$[u'v']$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
100	+ 9.5	+0.20 ±0.51	− 2 ± 8	+15 ±11	+2	−4	+18 ± 6	316	+ 8.3	+0.37	+16	+13	+0.12	1396
200	+17.7	−0.50 ±0.47	−12 ±11	+31 ±15	−9	−3	+43 ±10	365	+16.6	−0.29	+35	+40	+0.18	3077
300	+15.1	+0.17 ±0.32	+ 4 ± 7	+36 ± 9	+3	+1	+32 ± 6	365	+14.5	+0.18	+35	+32	+0.17	4325
500	+ 9.0	+0.03 ±0.19	0 ± 2	+13 ± 4	0	0	+13 ± 3	365	+ 8.9	+0.03	+13	+13	+0.14	5213
700	+ 4.4	−0.05 ±0.14	0 ± 1	+ 5 ± 2	0	0	+ 5 ± 1	365	+ 4.5	−0.06	+ 5	+ 5	+0.11	5775
850	+ 1.5	+0.08 ±0.12	+ 1 ± 0	+ 4 ± 1	0	+1	+ 3 ± 1	365	+ 1.4	+0.06	+ 4	+ 3	+0.09	5242
1000	− 0.8	−0.28 ±0.12	+ 1 ± 0	+ 3 ± 1	0	0	+ 3 ± 1	365	− 0.8	−0.33	+ 3	+ 3	+0.16	4518
Integral (10^7 CGS units)				+13.9	−0.4	−0.1	+14.3				+14.1	+14.0	Sum 29,546	

Table 13. Numerical analysis of momentum balance data for entire year 1950 at latitude 42.5°N. The levels are given in hundreds of millibars. All velocities are in m sec^{-1}. Internal consistency of figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\bar{u}]$	$[\bar{v}]$	$\overline{[u][v]}$	$[\overline{uv}]$	$[\bar{u}][\bar{v}]$	$\overline{[u]'[v]'}$	$[\overline{u'v'}]$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
100	+13.1	-1.00 ±0.87	-13 ±15	- 6 ±19	-13	0	+ 6 ± 7	202	+12.6	-0.92	- 5	+ 6	+0.06	546
200	+18.0	-1.15 ±0.70	-13 ±14	+13 ±18	-21	+8	+26 ±10	345	+17.8	-0.91	+16	+32	+0.17	1684
300	+16.8	-0.22 ±0.59	- 4 ±11	+17 ±14	- 4	0	+21 ± 9	365	+16.3	-0.08	+21	+22	+0.11	2463
500	+12.2	-0.20 ±0.29	- 3 ± 4	+ 9 ± 6	- 2	-1	+12 ± 4	365	+12.0	-0.16	+ 9	+11	+0.09	3637
700	+ 7.1	+0.12 ±0.19	+ 1 ± 2	+ 5 ± 3	+ 1	0	+ 4 ± 2	365	+ 6.9	+0.13	+ 5	+ 4	+0.07	4335
850	+ 3.9	+0.19 ±0.19	+ 1 ± 1	+ 2 ± 2	+ 1	0	+ 1 ± 2	365	+ 3.9	+0.17	+ 2	+ 2	+0.04	4436
1000	+ 1.8	+0.50 ±0.20	+ 1 ± 1	+ 2 ± 1	+ 1	0	+ 1 ± 1	365	+ 1.7	+0.53	+ 2	+ 1	+0.03	2508
Integral (10^7 CGS units)				+ 6.7	- 3.5	+0.5	+ 9.7				+ 7.8	+10.3	Sum 19,609	

Table 14. Numerical analysis of momentum balance data for entire year 1950 at latitude 55°N. The levels are given in hundreds of millibars. All velocities are in m sec^{-1}. Internal consistency of figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\overline{u}]$	$[\overline{v}]$	$[\overline{u}][\overline{v}]$	$[\overline{uv}]$	$[\overline{u}\,\overline{v}]$	$[\overline{u}^*\overline{v}^*]$	$[\overline{u'v'}]$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
100	+ 8.3	+0.66 ±0.55	+ 9 ± 6	+19 ±11	+ 6	+3	+10 ± 6	275	+ 8.2	+0.75	+20	+14	+0.21	763
200	+13.2	+0.59 ±0.72	+12 ±11	+17 ±15	+ 8	+4	+ 5 ± 9	361	+13.2	+0.68	+19	+10	+0.05	1601
300	+12.4	+0.79 ±0.67	+12 ± 9	+17 ±15	+10	+2	+ 6 ±12	365	+12.2	+0.75	+15	+ 6	+0.02	2361
500	+ 9.2	+0.18 ±0.37	+ 2 ± 4	+ 7 ± 7	+ 2	0	+ 5 ± 6	365	+ 9.2	+0.17	+ 8	+ 7	+0.04	3309
700	+ 6.0	+0.25 ±0.25	+ 2 ± 2	+ 1 ± 3	+ 2	+1	- 2 ± 3	365	+ 6.0	+0.24	+ 1	- 1	-0.01	3615
850	+ 3.3	+0.18 ±0.21	+ 1 ± 1	0 ± 3	+ 1	+1	- 1 ± 2	365	+ 3.8	+0.17	0	- 1	-0.01	3702
1000	+ 1.9	+0.17 ±0.22	+ 1 ± 1	- 2 ± 2	0	0	- 2 ± 1	364	+ 1.6	+0.16	- 1	- 1	-0.05	2219
Integral (10^7 CGS units)				+ 6.8	+ 3.3	+1.1	+ 2.4				+ 6.9	+ 3.6	Sum	17,570

Table 15. Numerical analysis of momentum balance data for entire year 1950 at latitude 70°N. The levels are given in hundreds of millibars. All velocities are in m sec^{-1}. Internal consistency of figures given is limited by rounding-off approximations.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\bar{u}]$	$[\bar{v}]$	$\overline{[u][v]}$	$[\overline{uv}]$	$[\bar{u}][\bar{v}]$	$\overline{[u]'[v]'}$	$[\overline{u'v'}]$	n	$\{u\}$	$\{v\}$	$\{uv\}$	$\{uv\}-\{u\}\{v\}$	r	N
100	+4.1	+0.61 ±0.72	+4 ±7	+ 4 ±10	+2	+ 2	- 1 ± 4	141	+3.7	+0.54	+3	+ 1	+0.02	349
200	+5.5	+1.16 ±0.69	+4 ±7	+ 1 ±12	+6	- 3	- 2 ± 8	311	+5.3	+1.24	+5	- 1	-0.01	1008
300	+4.8	+1.58 ±0.82	-2 ±8	-10 ±14	+8	-10	- 8 ±10	350	+4.6	+1.68	-8	-16	-0.08	1397
500	+3.3	-0.62 ±0.49	-1 ±3	- 6 ± 7	+2	- 3	- 5 ± 6	363	+3.3	+0.74	-4	- 7	-0.05	2164
700	+1.4	+0.38 ±0.33	-1 ±1	0 ± 3	+1	- 1	+ 1 ± 3	364	+1.4	+0.44	+1	0	0.00	2387
850	+0.4	+0.09 ±0.26	0 ±1	+ 1 ± 2	0	0	+ 2 ± 2	365	+0.4	+0.12	+2	+ 2	+0.04	2556
1000	+0.2	-0.11 ±0.15	0 ±0	- 2 ± 1	0	0	- 2 ± 1	365	+0.2	-0.12	-2	- 2	-0.08	3455
Integral (10^7 CGS units)				- 2.3	+2.4	- 2.3	- 2.4				-1.1	- 3.7	Sum	13,316

Table 16. The distribution of $[\bar{u}]$ and $\sigma(u)$ with elevation and latitude for the year 1950. All velocities in m sec^{-1}. Levels in millibars.

Level		70.0°N	55.5°N	42.5°N	31.0°N	13.0°N
100	$[\bar{u}]$	+ 4.1	+ 8.3	+13.1	+ 9.5	- 1.0
	$\sigma(u)$	6.8	8.3	11.0	13.6	11.7
200	$[\bar{u}]$	+ 5.5	+13.2	+18.0	+17.7	+ 7.6
	$\sigma(u)$	10.7	13.6	13.9	17.7	13.9
300	$[\bar{u}]$	+ 4.8	+12.4	+16.8	+15.1	+ 3.1
	$\sigma(u)$	14.0	16.4	14.2	16.2	10.3
500	$[\bar{u}]$	+ 3.3	+ 9.2	+12.2	+ 9.0	- 1.6
	$\sigma(u)$	10.6	12.2	11.2	10.7	6.7
700	$[\bar{u}]$	+ 1.4	+ 6.0	+ 7.1	+ 4.4	- 2.6
	$\sigma(u)$	7.9	9.0	8.3	7.8	6.0
850	$[\bar{u}]$	+ 0.4	+ 3.3	+ 3.9	+ 1.5	- 3.4
	$\sigma(u)$	7.0	8.0	6.9	6.4	5.6
1000	$[\bar{u}]$	+ 0.2	+ 1.9	+ 1.8	- 0.8	- 2.5
	$\sigma(u)$	4.6	5.5	5.2	4.6	6.4
	$([\bar{u}])$	+ 2.7	+ 7.9	+10.4	+ 8.0	+ 0.1

Table 17. The distribution of $[\bar{v}]$ and $\sigma(v)$ with elevation and latitude for the year 1950. All velocities in m sec^{-1}. Levels in millibars.

Level		70.0°N	55.0°N	42.5°N	31.0°N	13.0°N
100	$[\bar{v}]$	+ 0.6	+ 0.7	- 1.0	+ 0.2	- 0.5
	$\sigma(v)$	6.4	8.1	9.6	8.2	7.2
200	$[\bar{v}]$	+ 1.2	+ 0.6	- 1.2	- 0.5	- 0.2
	$\sigma(v)$	10.8	13.8	13.9	13.0	8.5
300	$[\bar{v}]$	+ 1.6	+ 0.8	- 0.2	+ 0.2	0.0
	$\sigma(v)$	14.5	16.8	14.3	11.5	6.7
500	$[\bar{v}]$	+ 0.6	+ 0.2	- 0.2	0.0	- 0.1
	$\sigma(v)$	11.7	12.7	10.6	8.3	4.8
700	$[\bar{v}]$	+ 0.4	+ 0.2	+ 0.2	0.0	- 0.1
	$\sigma(v)$	8.1	9.3	7.5	6.4	4.0
850	$[\bar{v}]$	+ 0.1	+ 0.2	+ 0.2	+ 0.1	- 0.3
	$\sigma(v)$	6.5	7.9	6.7	6.1	4.2
1000	$[\bar{v}]$	- 0.1	+ 0.2	+ 0.5	- 0.3	- 0.9
	$\sigma(v)$	4.6	5.2	5.3	4.4	5.0
	$([\bar{v}])$	+0.64	+0.37	-0.17	-0.03	-0.22

Table 18. The distribution of $\overline{[u'v']}$ with elevation and latitude for the year 1950. All velocities in m sec^{-1}. Vertical integrals at foot of table are in 10^7 CGS units.

Level	70.0°N	55.0°N	42.5°N	31.0°N	13.0°N
100	- 1	+10	+ 6	+18	0
200	- 2	+ 5	+26	+43	+ 8
300	- 8	+ 6	+21	+32	+ 6
500	- 5	+ 5	+12	+13	+ 3
700	+ 1	- 2	+ 4	+ 5	+ 2
850	+ 2	- 1	+ 1	+ 3	+ 1
1000	- 2	- 2	+ 1	+ 3	+ 1
Integral	-2.4	+ 2.4	+ 9.7	+14.3	+ 2.7

Table 19. The flux of angular momentum in units of 10^{25} gm cm^2 sec^{-2} across the specified latitude circles, due to organized circulations as expressed by the terms in expansion 3.0. The required flux κ_1 taken after Priestley (1951).

1	2	3	4	5	6	7	8
Latitude	κ_1	Unadjusted total	Mass shift	Mean cell	Inst. cell	Horiz. eddy	Adjusted total
Investigators	Priestley 1951	Starr and White 1953					
70.0°N	—	- 0.7	+ 0.5	+ 0.3	- 0.7	- 0.7	- 1.2
55.0°N	+ 4	+ 5.7	+ 2.3	+ 0.5	+ 1.0	+ 2.0	+ 3.4
42.5°N	+15	+ 9.3	- 2.3	- 2.6	+ 0.7	+13.4	+11.6
31.0°N	+21	+25.9	- 0.5	- 0.3	- 0.1	+26.7	+26.4
13.0°N	+ 6	+ 9.6	0.0	+ 0.2	+ 3.1	+ 6.6	+ 9.6

Table 20. Numerical analysis of water balance data for entire year 1950 at latitude 31°N. All velocities are in m sec^{-1} and humidities in gm kg^{-1}.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[q]}$	$\overline{[v]}$	$\overline{[q][v]}$	$\overline{[qv]}$	$\overline{[q]}\overline{[v]}$	$\overline{[q]'[v]'}$	$\overline{[q'v']}$	n	$\{q\}$	$\{v\}$	$\{qv\}$	$\{qv\}-\{q\}\{v\}$	r	N
500	+ 1.0	+0.03	+0.1	+0.9	+0.0	+0.1	+0.8	365	+ 1.0	+0.03	+0.9	+0.8	+0.12	5213
		±0.19	±0.2	±0.2			±0.1							
700	+ 3.2	-0.05	-0.1	+1.9	-0.2	+0.1	+2.0	365	+ 3.2	-0.06	+1.9	+2.1	+0.14	5775
		±0.14	±0.4	±0.6			±0.3							
850	+ 6.1	+0.08	+0.2	+4.3	+0.5	-0.3	+4.1	365	+ 6.1	+0.06	+4.2	+3.8	+0.22	5242
		±0.12	±0.7	±0.8			±0.5							
1000	+10.6	-0.28	-2.6	+2.0	-3.0	+0.4	+4.6	365	+10.6	-0.33	+1.4	+4.9	+0.27	4518
		±0.12	±1.2	±1.3			±0.4							
Integral (10^2 CGS units)				+1.3	-0.2	+0.0	+1.5				+1.3	+1.5	Sum	20748

Table 21. Numerical analysis of water balance data for entire year 1950 at latitude 42.5°N. All velocities are in m sec^{-1} and humidities in gm kg^{-1}.

1.	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[q]}$	$\overline{[v]}$	$\overline{[q][v]}$	$\overline{[qv]}$	$\overline{[q]}\overline{[v]}$	$\overline{[q]'[v]'}$	$\overline{[q'v']}$	n	$\{q\}$	$\{v\}$	$\{qv\}$	$\{qv\}-\{q\}\{v\}$	r	N
500	+ 0.8	-0.20	+0.1	+0.8	-0.2	+0.2	+0.7	365	+ 0.8	-0.16	+0.8	+1.0	+0.14	3637
		±0.30	±0.2	±0.3			±0.2							
700	+ 2.5	+0.12	+0.6	+2.9	+0.3	+0.3	+2.3	365	+ 2.6	+0.13	+2.9	+2.y	+0.20	4335
		±0.19	±0.5	±0.6			±0.3							
850	+ 4.4	+0.19	+0.9	+4.4	+0.8	+0.1	+3.5	365	+ 4.5	+0.17	+4.3	+3.6	+0.21	4436
		±0.19	±0.8	±0.9			±0.4							
1000	+ 6.5	+0.50	+4.5	+7.4	+3.2	+1.2	+2.9	365	+ 6.6	+0.53	+7.6	+4.1	+0.22	2508
		±0.20	±1.4	±1.5			±0.6							
Integral (10^2 CGS units)				+1.9	+0.4	+0.2	+1.3				+1.9	+1.5	Sum	14916

Table 22. Numerical analysis of water balance data for entire year 1950 at latitude 55°N. All velocities are in m sec^{-1} and humidities in gm kg^{-1}.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\bar{q}]$	$[\bar{v}]$	$\overline{[q][v]}$	$[\overline{qv}]$	$[\bar{q}][\bar{v}]$	$\overline{[q]'[v]'}$	$[\overline{q'v'}]$	n	$\{q\}$	$\{v\}$	$\{qv\}$	$\{qv\}-\{q\}\{v\}$	r	N
500	+0.5	+0.18	+0.2	+1.1	+0.1	+0.1	+0.9	365	+0.5	+0.17	+1.1	+1.0	+0.15	3309
		±0.37	±0.2	±0.2			±0.2							
700	+1.8	+0.25	+0.7	+3.5	+0.4	+0.3	+2.8	365	+1.8	+0.24	+3.5	+3.1	+0.23	3615
		±0.25	±0.4	±0.6			±0.4							
850	+3.2	+0.18	+0.9	+4.5	+0.6	+0.3	+3.6	365	+3.2	+0.17	+4.6	+4.0	+0.24	3702
		±0.21	±0.7	±0.8			±0.4							
1000	+4.3	+0.17	+1.8	+4.8	+0.7	+1.1	+2.9	364	+4.4	+0.16	+4.8	+4.1	+0.31	2219
		±0.22	±1.0	±1.1			±0.4							
Integral (10^2 CGS units)				+1.9	+0.2	+0.2	+1.5				+1.9	+1.7	Sum	12845

Table 23. Numerical analysis of water balance data for entire year 1950 at latitude 70°N. All Velocities are in m sec^{-1} and humidities in gm kg^{-1}.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\bar{q}]$	$[\bar{v}]$	$\overline{[q][v]}$	$[\overline{qv}]$	$[\bar{q}][\bar{v}]$	$\overline{[q]'[v]'}$	$[\overline{q'v'}]$	n	$\{q\}$	$\{v\}$	$\{qv\}$	$\{qv\}-\{q\}\{v\}$	r	N
500	+0.3	+0.44	+0.3	+0.7	+0.2	+0.2	+0.3	362	+0.4	+0.62	+0.7	+0.5	+0.14	2067
		±0.51	±0.2	±0.2			±0.1							
700	+1.1	+0.27	+0.9	+2.2	+0.3	+0.6	+1.3	363	+1.2	+0.35	+2.3	+1.9	+0.23	2280
		±0.34	±0.4	±0.5			±0.3							
850	+2.0	+0.09	+1.0	+2.6	+0.2	+0.8	+1.6	364	+2.0	+0.12	+2.6	+2.3	+0.22	2392
		±0.27	±0.6	±0.7			±0.3							
1000	+2.5	-0.43	-0.3	+1.0	-1.1	+0.8	+1.3	363	+2.5	-0.47	+0.8	+2.0	+0.21	1724
		±0.21	±0.3	±0.8			±0.4							
Integral (10^2 CGS units)				+1.0	+0.0	+0.3	+0.7				+1.0	+0.9	Sum	8463

Table 24. The flux of water in units of 10^{11} gm sec^{-1} across the specified latitude circles, due to organized circulations as expressed by the terms in expansion 4.1. The required flux estimates k_a after Wust (1922) as modified by Conrad (1936) and by Benton (1953).

1	2	3	4	5	6	7	8	9	10
Latitude	k_a	k_a	Unadjusted total	Mass shift	Mean cell	Inst. cell	Horiz. eddy	Adjusted total	Horiz. eddy
Investigators	Wust-Conrad 1936	Wust-Benton 1950	Starr and White 1953						Benton N. Am. 1953
70.0°N	+0.8	+0.5	+1.4	+0.6	-0.5	+0.5	+0.9	+0.8	+1.0
55.0°N	+3.6	+3.2	+4.4	+1.1	-0.5	+0.4	+3.4	+3.3	+2.3
42.5°N	+7.5	+5.5	+5.6	-0.9	+2.1	+0.6	+3.8	+6.5	+5.4
31.0°N	+6.2	+3.3	+4.6	-0.5	-0.2	+0.0	+5.2	+5.1	+3.3

Table 25. Numerical analysis of sensible heat flux data for entire year 1950 at latitude 31°N. All velocities are in m sec^{-1} and temperatures in °A.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$\overline{[T]}$	$\overline{[v]}$	$\overline{[T][v]}$	$[\overline{Tv}]$	$[\overline{T}][\overline{v}]$	$\overline{[T]'[v]'}$	$[T'v']$	n	$\{T\}$	$\{v\}$	$\{Tv\}$	$\frac{\{Tv\}}{\{T\}\{v\}}$	r	N
100	+205.8	+0.25 ±0.53	+ 49.9 ±109.4	+ 53.1 ±109.7	+51.4	-1.6	+3.2 ±2.8	302	+205.5	+0.42	+89.2	-2.9	-0.06	1253
200	+218.1	-0.42 ±0.53	- 91.4 ±115.8	- 84.0 ±115.7	-91.6	+0.2	+7.4 ±2.7	365	+218.3	-0.34	-66.0	+8.2	+0.11	2859
300	+235.8	-0.01 ±0.33	- 0.9 ± 24.1	+ 2.1 ± 75.8	- 2.4	+1.5	+3.0 ±1.8	365	+236.1	+0.01	+ 8.4	+6.1	+0.08	4003
500	+261.8	+0.03 ±0.19	+ 8.7 ± 50.6	+ 10.2 ± 50.3	+ 7.8	+0.8	+1.5 ±1.1	365	+261.9	+0.03	+ 9.0	+1.2	+0.02	5181
700	+277.8	-0.06 ±0.14	- 17.8 ± 38.2	- 15.6 ± 38.1	-16.7	-1.2	+2.3 ±0.9	365	+277.8	-0.06	-17.0	-0.4	-0.01	5756
850	+285.4	+0.08 ±0.12	+ 20.7 ± 35.0	+ 25.7 ± 35.3	+22.8	-2.1	+5.0 ±1.1	365	+285.4	+0.06	+21.4	+4.3	+0.11	5218
1000	+291.6	-0.30 ±0.12	- 80.5 ± 34.6	- 77.7 ± 34.6	-87.5	+7.0	+2.8 ±0.7	365	+292.3	-0.33	-93.6	+2.9	+0.12	4507
Integral (10^7 CGS units)				- 8.3	-11.7	+0.0	+3.1				- 6.0	+2.3	Sum 28,777	

Table 26. Numerical analysis of sensible heat flux data for entire year 1950 at latitude 42.5°N. All velocities are in m sec^{-1} and temperatures in °A.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\overline{T}]$	$[\overline{v}]$	$[\overline{T}][\overline{v}]$	$[\overline{Tv}]$	$[\overline{T}][\overline{v}]$	$[\overline{T}][\overline{v}]$	$[\overline{T'v'}]$	n	$\{T\}$	$\{v\}$	$\{Tv\}$	$\{Tv\}-\{T\}\{v\}$	r	N
100	+215.3	-1.00 ±0.87	-216.3 ±187.2	-211.6 ±187.6	-215.3	-1.0	+ 4.8 ± 3.8	202	+215.2	-0.92	-192.5	+ 5.5	+.12	546
200	+218.7	-1.15 ±0.70	-247.2 ±153.3	-238.9 ±153.1	-251.5	+4.3	+ 8.3 ± 4.5	345	+219.0	-0.91	-188.5	+10.7	+.11	1684
300	+231.3	-0.22 ±0.59	- 45.8 ±135.6	- 42.8 ±135.5	- 50.9	+5.1	+ 3.0 ± 3.2	365	+231.9	-0.08	- 11.7	+ 6.9	+.06	2463
500	+256.0	-0.19 ±0.30	- 45.0 ± 75.1	- 43.1 ± 74.8	- 48.6	+3.6	+ 1.9 ± 1.9	365	+256.6	-0.16	- 33.7	+ 7.4	+.10	3625
700	+271.7	+0.12 ±0.19	+ 34.6 ± 51.4	+ 41.3 ± 51.2	+ 32.6	+2.0	+ 6.6 ± 1.3	365	+272.0	+0.13	+ 44.2	+ 8.8	+.15	4319
850	+279.3	+0.19 ±0.19	+ 54.4 ± 52.0	+ 65.2 ± 52.1	+ 53.1	+1.3	+10.8 ± 1.4	365	+279.6	+0.17	+ 64.1	+16.6	+.28	4425
1000	+284.0	+0.50 ±0.20	+144.9 ± 57.2	+150.4 ± 57.5	+142.0	+2.9	+ 5.5 ± 1.2	365	+284.2	+0.53	+161.5	+10.8	+.27	2494
Integral (10^8 CGS units)				- 27.7	- 29.6	+2.7	+ 5.2				- 7.7	+ 8.9	Sum	19,556

Table 27. Numerical analysis of sensible heat flux data for entire year 1950 at latitude 55°N. All velocities are in m sec^{-1} and temperatures in °A.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\bar{T}]$	$[\bar{v}]$	$\overline{[\bar{T}][\bar{v}]}$	$\overline{[Tv]}$	$\overline{[\bar{T}]}\,\overline{[\bar{v}]}$	$\overline{[\bar{T}]'[\bar{v}]'}$	$\overline{[T^*v^*]}$	n	$\{T\}$	$\{v\}$	$\{Tv\}$	$\{Tv\}-\{T\}\{v\}$	r	N
100	+220.2	+0.66 ±0.55	+149.1 ±120.7	+154.7 ±120.3	+145.4	+3.7	+ 5.6 ± 2.7	275	+220.5	+0.75	+174.5	+ 9.2	+0.21	763
200	+220.5	+0.59 ±0.72	+135.0 ±157.1	+140.2 ±156.6	+130.1	+4.9	+ 5.2 ± 4.0	361	+220.6	+0.68	+163.1	+13.1	+0.14	1601
300	+225.5	+0.79 ±0.67	+177.0 ±150.5	+179.4 ±150.8	+178.1	-1.2	+ 2.5 ± 2.7	365	+225.5	+0.75	+172.5	+ 3.4	+0.04	2361
500	+249.2	+0.19 ±0.37	+ 48.9 ± 92.3	+ 57.5 ± 92.6	+ 47.4	+1.6	+ 8.5 ± 2.4	365	+249.4	+0.17	+ 56.2	+13.8	+0.13	3295
700	+264.3	+0.26 ±0.25	+ 70.7 ± 66.4	+ 83.4 ± 66.4	+ 68.7	+2.0	+12.7 ± 2.0	365	+264.2	+0.24	+ 81.7	+18.3	+0.23	3599
850	+271.4	+0.17 ±0.21	+ 48.3 ± 57.0	+ 65.8 ± 56.9	+ 46.1	+2.2	+17.5 ± 2.0	365	+271.3	+0.17	+ 82.3	+16.2	+0.21	3684
1000	+275.6	+0.16 ±0.22	+ 47.5 ± 59.8	+ 59.3 ± 60.0	+ 44.1	+3.4	+11.8 ± 2.0	364	+276.1	+0.16	+ 59.8	+15.7	+0.28	2207
Integral (10^5 CGS units)				+ 90.1	+ 80.3	+1.2	+ 8.8				+ 92.1	+12.1	Sum	17,510

Table 28. Numerical analysis of sensible heat flux data for entire year 1950 at latitude 70°N. All velocities are in m sec^{-1} and temperature in °A.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Level	$[\overline{T}]$	$[\overline{v}]$	$\overline{[T][v]}$	$[\overline{Tv}]$	$[\overline{T}][\overline{v}]$	$\overline{[T'][v']}$	$[\overline{T'v'}]$	n	$\{T\}$	$\{v\}$	$\{Tv\}$	$\{Tv\}-\{T\}\{v\}$	r	N
100	+225.1	+0.57	+133.0	+133.3	+128.3	+4.7	+0.4	120	+225.2	+0.51	+120.5	+ 5.6	+0.03	288
		±0.75	±167.1	±167.4			±2.9							
200	+223.0	+1.22	+275.0	+276.4	+272.0	+3.0	+1.4	297	+223.1	+1.31	+297.1	+ 4.8	+0.02	919
		±0.71	±158.2	±157.9			±4.0							
300	+223.3	+1.48	+333.4	+333.4	+329.0	+4.4	+0.0	345	+222.4	+1.68	+376.6	+ 3.0	+0.01	1302
		±0.85	±188.3	±188.6			±3.2							
500	+244.4	+0.42	+110.0	+115.2	+102.7	+7.3	+5.2	362	+244.8	+0.61	+161.2	+11.9	+0.03	2047
		±0.52	±126.2	±126.5			±3.1							
700	+259.0	+0.29	+ 82.7	+ 91.9	+ 75.1	+7.6	+9.2	363	+259.4	+0.38	+113.6	+15.1	+0.10	2266
		±0.34	± 87.3	± 87.4			±2.1							
850	+265.8	+0.08	+ 26.9	+ 36.5	+ 21.3	+5.6	+9.6	364	+266.1	+0.10	+ 41.7	+15.1	+0.1.	2374
		±0.27	± 70.6	± 71.0			±2.0							
1000	+267.0	-0.43	-111.7	-104.1	-114.8	+3.1	+7.6	363	+267.9	-0.47	-113.9	+11.6	+0.11	1716
		±0.21	± 57.4	± 57.8			±2.3							
Integral (10^5 CGS units)				+123.6	+113.5	+5.2	+4.9				+145.0	+ 9.6	Sum	10,912

Table 29. The flux of energy in units of 10^{14} cal sec^{-1} across the specified latitudes, due to organized circulations as expressed by the terms in expansions 4.1 and 5.1. The required flux K_3 after investigators indicated.

1	2	3	4	5	6	7	8	9	10	11	12
					SENSIBLE HEAT			LATENT HEAT			SUM
Latitude	K_3	K_3	K_3	K_3	Inst. cell	Horiz. eddy	Total eddy	Inst. cell	Horiz. eddy	Total eddy	Total eddy
Investigators	London 1953	Gabites 1950	Saur & Philipps 1935	Albrecht Bjerknes 1933	Starr and White 1953						
70.0°N	+ 1.9	+ 3.7	---	+ 2.0	+ 1.7	+ 1.6	+ 3.3	+ 0.3	+ 0.5	+ 0.8	+ 4.1
55.0°N	+ 4.7	- 8.1	+ 7.5	+ 4.8	+ 1.0	+ 4.8	+ 5.8	+ 0.3	+ 2.0	+ 2.3	+ 8.1
42.5°N	+ 6.4	+10.3	+10.2	+ 6.3	+ 1.9	+ 3.7	+ 5.6	+ 0.4	+ 2.3	+ 2.7	+ 8.3
31.0°N	+ 6.2	+ 9.3	+ 7.4	+ 6.2	+ 0.3	+ 2.5	+ 2.8	+ 0.0	+ 3.1	+ 3.1	+ 5.9

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VICTOR P. STARR, *Massachusetts Institute of Technology, Cambridge, Massachusetts:* **Modern developments in the study of the general circulation of the atmosphere.** The purpose of this discussion is to present a general account of the research program concerning the study of the global circulation of the atmosphere, pursued during the past few years by my colleagues and myself at the Massachusetts Institute of Technology. In a space as brief as is now at my disposal, it is possible to give only a short sketch of this work, the most fundamental aim of which has been to discover the mode of operation of the atmosphere as a heat engine.

The source of energy for all atmospheric motions resides in the heating by solar radiation, all other energy sources being quite minor. Furthermore, the action of solar radiation depends upon the fact that it provides a differential

heating between more polar and more equatorial latitudes in each hemisphere. Uniform heating of the atmosphere would, of course, not result in the differences in pressure along horizontzl surfaces which are necessary for the creation of kinetic energy (see, for example, Starr 1948). In fluid mechanics, it is customary to refer to the process of generation of motion by differential heating as convection. It is therefore, a process of convection, which one must study in attempting to understand the workings of the general circulation.

In these attempts, it is convenient to make reference to the classic investigations of M. Margules, early in this century. Margules (1903) pointed out that in a closed atmospheric system generation of kinetic energy takes place in proportion to the disappearance or release of potential energy and internal heat energy. Owing to the fact that in an air column extending upwards in an unlimited fashion and in hydrostatic equilibrium the latter form of energy is proportional to the potential energy, it therefore suffices for reason of brevity to speak only of potential energy. The researches of Margules also pointed out that the release of potential energy is accomplished by the rising of warm air and sinking of colder air, which, in a closed system, would represent a sinking of the center of gravity.

In the actual atmosphere, the radiational heating continuously replenishes the supply of available potential energy (see Lorenz, 1955), while the process of transformation into kinetic energy proceeds as outlined by Margules at some average rate sufficient to overcome frictional losses. In these terms, we may now state that one of the most fundamental questions which general circulation studies must include is the specification of the components of motion in the atmosphere which are actually responsible for the releasing of potential energy. Various other weighty and important questions also arise, of course, but this one may be looked upon as a starting point. Thus, another question relates to the manner in which the kinetic energy of the zonally-averaged motions is maintained, since although these motions are known to be present they are not an immediate consequence of the convective process. Logically then, there must exist a connecting link whereby the kinetic energy generated by the convection process becomes in part funneled into these components of motion.

In order to try to arrive at suitable answers to these and many other related questions, our program of general circulation research at M.I.T. has involved three more or less distinct approaches. The first and perhaps most important branch of our work has been an extremely extensive study of meteorological atmospheric data for the northern hemisphere. The second approach has been through the study of model experiments, which in recent years, thanks to the work of Prof. Fultz at the University of Chicago, have yielded analogues to the hemispheric circulation of atmosphere. In the third place, we have pursued extensive theoretical analyses, both from the standpoint of securing analytical solutions and also more recently through the application of high-speed digital computers. It is my intention to give a bare thumb-nail sketch of the results obtained in each of these lines of endeavour.

In the past, as mentioned by Dr. Charney, the basic convective motions in the atmosphere have been looked upon as being a general rising of air at lower latitudes and sinking at higher latitudes in the form of toroidal overturnings,

similar to those involved in Hadley's (1735) theory of the trade winds, published shortly after the time of Newton. A number of variations of this scheme were put forth during the course of time up to the present (for example, Ferrel 1885, Oberbeck 1888). All, however, ascribe the motions of the general circulation as being due to the release of potential energy by this type of overturning process (Rossby, 1947, is an exception). Any verification of these theories from direct observations is dependent upon the accumulation and reduction of a vast amount of data. This was actually one of the primary objectives of our observational studies. The results are that the sum total effect of toroidal overturnings as measured from the data for the northern hemisphere is, if anything, in such a direction as to convert kinetic energy back into potential energy (Starr 1954). This arises from the indication that the most vigorous toroidal overturning observable is a reverse one in middle latitudes, involving descent of warm air to the south and ascent of cold air to the north.

In view of this situation, it follows that there must be other fundamental convective motions in the atmosphere which liberate potential energy and convert it into kinetic energy. What are then the observational indications for this alternative process? The results show that the convection proceeds in terms of many smaller cells, these in fact being the cyclones and anticyclones of middle latitudes which involve the rising of warm air masses and sinking of cold ones (White and Saltzman, 1956). It would thus appear that purely from measurements of atmospheric motions in relation to air temperature the atmosphere shuns extremely large convective cell sizes.

Speaking now of experimental studies, it is manifestly true that laboratory models which duplicate hemispheric conditions of the atmosphere in all detail are impossible. However, if one sets one's sights merely upon the reproduction of the largest scale phenomena and more specifically upon those processes involved in these phenomena which turn out to be relatively insensible to the presence of details, success may be achieved, as has been so well demonstrated by the studies of Prof. Fultz. The particular experiments which we have studied are those in which motions relative to a rotating cylindrical vessel are generated in water, purely as a consequence of heating the (plane) bottom at the rim and cooling nearer the center. A variety of regimes may be obtained in such experiments as a result of changes principally in the intensity of differential heating and rate of rotation. Under proper conditions, flow patterns are obtained bearing an unmistakable resemblance to those found in the hemispheric circulation in the atmosphere (for example, Fultz and Corn 1954, Hide 1953). This resemblance is not confined to one level, but exhibits similarity in vertical structure as well, reproducing such features as occluding cyclones and fronts near the bottom in proper relation to the cyclonic and anticyclonic troughs and ridges at higher levels (Faller, 1956).

The so-to-speak meteorological regime is distinguished from a regime of symmetrical convection which occurs at a sufficiently slower rate of rotation (or stronger heating with the same rotation). These axially-symmetrical motions are characterized by a radial inflow of the fluid near the top and a divergence near the bottom, which results from the ascent of the fluid over the heat source and a

sinking above the cold source at the center. This regime, therefore, exemplifies the action presupposed in the classical theories of the general circulation. Significantly, however, it is not the regime which has meteorological similarity. The one discussed earlier, which has such similarity, is moreover characterized if anything by a radial outflow of fluid near the top over practically all radii, much as is found in the atmosphere over a wide range of latitudes.

In other experimental examples of convection, which need not be direct analogues of atmospheric processes, such as convective motions of the Bénard type, it is simple to demonstrate that one effect of the presence of rotation is to diminish the characteristic cell sizes in the plane normal to the axis of rotation. One may thus, for example, obtain Bénard convection cells a foot or two in vertical extent but only an inch or two in horizontal cross-section. It is useful to regard the convective motions in the cylindrical vessel described above as an extension of this action of rotation. In other words, we are again confronted by a preference of the fluid to resort to smaller convective cell sizes in the case of strong rotation, other things being unchanged.

Since the hemispheric air motion, and also the circulations in the experiment possessing meteorological similarity, are characterized by strongly developed zonal motions, one may ask how kinetic energy resulting from the convective overturnings becomes so organized. One superficially attractive feature of the classical theories for the general circulation is the simplicity with which this connecting mechanism is visualized. It is there supposed that the toroidal convection generates kinetic energy, which in the first instance appears as kinetic energy of meridional motions, but which immediately becomes converted to kinetic energy of mean zonal motions by an essentially quasistatic process, through the action of Coriolis forces (this presumably is the actual mechanism in the heated cylinder when the symmetrical regime is present). If, however, the toroidal circulations actually present in the atmosphere (or in the meteorological regime in the heated cylinder) operate to increase the potential energy at the expense of kinetic energy, this process cannot be used to maintain the mean zonal motions. From the physical equations of motion, it follows that only one other alternative mechanism of any significance can perform this function. The alternative mechanism involves the flow of kinetic energy from the larger scale non-zonal disturbances into the mean zonal flow (for example, Kuo, 1951). This is in direct opposition to the state of affairs in the case of turbulent phenomena, where the kinetic energy becomes degraded into smaller and smaller eddies. From the standpoint of direct measurements in the atmosphere, the process envisaged here involves a flow or transport of angular momentum from zones of low angular velocity about the polar axis into regions of high angular velocity (Starr, 1953). It is to be noted that the observational evidence which has by now been obtained for the presence of this action can scarcely be disputed (see also, for example, Starr and White 1954, Mintz 1951).

From the standpoint of philosophical considerations, one may with profit observe that the non-zonal disturbances in the atmosphere differ in an essential manner from the disturbances visualized in most turbulence studies. The former ones, as we have seen, are the seat for the release of potential energy which becomes

converted most directly into the kinetic energy of the disturbances. The system thus involves a systematic insertion of energy into one scale of eddies. One is therefore not confronted by any fundamental inconsistency, if some of this energy is then transferred systematically to maintain the mean motion. No doubt, other examples of fluid motion in which eddy kinetic energy is so transferred into the mean motion will be found in the future.

As might be supposed, the third branch of our work has consisted of efforts to deduce theoretically the essential nature of the general circulation, and also of its experimental analogues, as a consequence of the hydrodynamical equations. In the attempts to do this by analytical means, one may be guided by various theoretical studies of other instances of convection, such as the theoretical analysis of the Bénard problem by Rayleigh (1916), Jeffreys (1928), and others. More recently, the analyses of certain convective problems due to Chandrasekhar for astrophysical applications have been of great use to us. The procedures involved consist of the use of linearization and the determination of the modes of convection which have a positive and appreciable rate of development. As to the results, it is desirable at the outset to remark that in other instances of convection such as those studied by Chandrasekhar (1953) and others, the theoretical effect of the presence of rotation is to diminish the cell size of convection units, as was stated in connection with the experimental results. In the case of the atmosphere and also for the meteorological regime in the rotating experiments, this same theoretical result is obtained (Kuo 1954, 1955, 1956; Lorenz 1953). Thus, it is confirmed also theoretically that in a system such as the atmosphere convection cannot proceed in the form of large axially symmetrical toroidal components of motion, but rather is constrained by the rotation to break up into a number of smaller cells, which in actuality are the cyclonic and anticyclonic disturbances of middle latitudes.

Through the use of analytical techniques, it is also possible to investigate the conditions under which kinetic energy may be exchanged between the mean motion and the eddies. Such studies have much in common with classical stability studies discussed by Helmholtz (1868), Rayleigh (1913), Heisenberg (1924), Tollmien (1929), and more recently by Lin (1945), to name but a few. When a suitable arrangement of circumstances is made, the result is gained that disturbances having the character of cyclones and anticyclones in the atmosphere would transfer their kinetic energy into the mean zonal flow, thus underlining the results from observational studies and the inferences from the laboratory models (Kuo 1949, 1953).

As discussed by Dr. Charney, a numerical solution for the gross features of the general circulation has been obtained by Phillips (1955) at the Institute for Advanced Study, through the use of high-speed electronic computation. These results are in very good agreement with our observational and theoretical results as here presented, so that the desirability of very intensive further work along such lines is strongly suggested. Motivated by similar aims, several of my colleagues have for some time been engaged in the numerical integration of the equations for a two-layer model of the atmosphere in the northern hemisphere and also for a corresponding three-dimensional model. In order to eliminate possible sources of misconception, it may be well to point out that in such work the effects of radia-

tional heating and cooling are taken into account, as is also frictional dissipation. An answer is then sought from the equations as to the regime of motion which ultimately develops. At the present moment, it is still too early to predict the outcome of our own computations, although it is reasonable to suppose that results comparable in essence to those of Phillips will follow.

As a general conclusion, it would appear that meteorologists now have for the first time a correct framework for the discussion and further study of the mechanics of the general circulation. In the course of future years, this cannot help having a profound influence on almost every branch of the science. As an immediate prospect, it is to be hoped that numerical solutions will be designed which incorporate more and more detailed conditions, such as the effects of orography, land and sea contrasts, seasonal effects, and possibly various other variations in the incoming solar radiation, both real and hypothetical. One hardly needs to enumerate here the variety of scientific questions which would have light thrown upon them from such investigations.

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What Constitutes our New Outlook on the General Circulation?

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Abstract

In this article the essential physical characteristics of the new outlook upon the actual operation of the general circulation are presented in a concise manner, mainly as a matter of philosophical interest. Certain general attitudes of the writer, pertaining to this subject are expounded for purposes of record.

1 Philosophical preface

In the progressive development of a scientific subject of any complexity, much depends upon the clear delineation of its status at any given time. For particular reasons this is ordinarily not an easy task. Almost always the really major conceptual mutations succeed each other at a pace such that the life-span of an individual can measure only a small phase of the historical process, or only a particular set of events which if he is lucky, may constitute some striking *dénouement*—some burst of inflorescence from a soil long cultivated and enriched through patient labor by others during more quiescent periods. Contrasted with this organically dictated slowness in the evolution of underlying outlook and ideas, the present moment confronts the individual with a welter of voices and activities which generally have the tendency to distract him from the long-term viewpoint, and tempt him into a mistaken assessment which gives undue weight to the present. However, the seeming magic gilding of the present is notoriously unenduring, and soon all current activities must be picked over along with the entire past record for those simple "irreducible and stubborn facts" which, together with the generalizations that might be drawn from them form the basis of science, according to Alfred North Whitehead.

In what follows I make no pretense to have achieved any completeness in regard to portraying the current status of knowledge relating to the general circulation. Moreover it is my belief that, on the contrary, any aspiration to completeness in this effort would, of itself, militate against the success of my plan, which is to select only certain salient facts and to present them simply and unadorned. In this way it may perhaps prove possible not merely to outline a wealth of detail but, I hope, rather to achieve some degree of perspective. It must also be pointed out that my remarks are directed first of all to those who have at least in some measure studied the subject, and only in a secondary fashion to those who wish to obtain an initial orientation in it. Consistent with this statement, background material will not be repeated here, except as specially required.

Let us now direct our attention, by way of suitable—or even necessary—preparation, to what might be understood by a given outlook upon the general circulation. Meteorology is a subject which includes the study of the motions of a mobile fluid. These motions take place in three-dimensional space and in time, in accordance with the mechanics of Newton, to sufficient accuracy for our purposes. Under these circumstances, there should exist no impassable barrier to the formulation of our knowledge concerning the *motions* of the atmosphere in terms of simple actual or mental pictures. Not only this, but it is my definite suspicion that no theory, idea or other claim to insight concerning the motions of the atmosphere has reached a final degree of crystallization until such direct physical picturization is reached, in order to illustrate its

content. This is not to say that any plausible method of deduction or other form of argument may not be employed for the purpose of discovering new characteristics of behavior of the atmosphere: it only means that the final result concerning our physical system should give us physical pictures. Ordinarily then, however laborious and time consuming the various historic stages of the evolution of our subject may have been, the essence of the concepts is traceable in terms of fairly simple pictures—perhaps I should say deceivingly simple pictures, when one thinks of the work represented by them.

The need for underlining the last-mentioned state of affairs concerning general circulation studies is to be regarded as symptomatic, *inter alia*, of the fact that, whether we like it or not, meteorologists have been struggling with problems of the most primitive kind concerning the motions of the atmosphere—that is if our knowledge is to be appraised from an absolute viewpoint. The questions at issue have not been such as relate to some fine points concerning a general scheme that is accepted as sound. Rather the questions have had more of the nature as to whether some crucial portion of the system operates forward or in reverse—in other words the gross shape of things has been the subject of conjecture. I say "has been" advisedly, because, as will be more apparent later, my belief is that we have now at last come to final grips with many of these problems.

Because of several possible ambiguities which are inherent in the term, I have thus far avoided use of the word *theory* of the general circulation. These ambiguities have been touched upon by Lorenz (1954) in his discussion of energy transformations. What is of more direct concern to us here is the question of what is the *de facto* operation of the gross air motions and related effects. A theory, let us say having the form of a mathematical model of some physical system, may propose a certain set of processes, but is subject to appraisal of its merits according to two criteria. Are the proposed processes the same as the *de facto* ones? Let us say that they are; then there is the further question whether the postulates of the theory are sound and the deductions rigorous and free from error.

Generally speaking, in hydrodynamics all problems dealing with the flow of real fluids (except a few trivial ones) tend to be of vast complexity. The result is that the second necessary criterion for a successful theory can seldom if ever be rigorously fulfilled *à priori* for any flow even approaching the involved structure of the entire atmosphere. Under these circumstances the first duty of research is to make known, in terms as penetrating and scientifically useful as can be the *de facto* processes. One has merely to reflect upon the array of actual hydrodynamical phenomena to be convinced of this heuristic nature of the theories which have been built to represent them. The more theoretical work I see done concerning the general circulation, the more I am convinced that it represents perhaps a prime example of how only through a thoroughgoing combination of empirical knowledge and abstract principles can progress be made toward a unified mathematical model for the motions of the atmosphere. It is only for an unexamined credulity to suppose that the present position in this subject has been reached through some deductive stroke of genius and not through the elaborate use of observational pictures at practically every step, consciously or otherwise.

Once this unity of progress is properly understood and appreciated, there arises a new level of awareness that enables us to place historic contributions to our subject in their proper settings. We see that the limiting factor always has been, and will continue to be, the completeness and accuracy of the *de facto* pictures of the general circulation, although some words of qualification are needed. What counts are not mere tabulations of data; it is their intelligent organization according to physical laws so as to lead to physical depiction of relevant processes and schemes of motion. These are then to be juxtaposed in order to compare them with previous pictures. The formalization of new knowledge according to a theoretical presentation then follows almost as a matter of course, even though the developments may require considerable time and effort.

Why is there this ever present drive for theoretical presentation of material which has been empirically exhibited, as though the intent were to rediscover it more properly by seeming selfsufficient deductive reasoning? Aside from practical applications which often follow directly from theory (these I shall not speak further about here), the intrinsic reason is the same as in other physical sciences, namely a deep craving of the human mind for completeness and perfection of logical structure. This of course is extremely valuable because, being practically always incapable of absolute fulfillment (especially in meteorology), the residual inconsistencies always point to further *de facto* features of the real world to be elucidated empirically. Ideally, a theory of the general circulation should predict (not now in the sense of a weather forecast) certain processes in the atmosphere which might be subsequently detected empirically. Unfortunately, due no doubt to the complexity of the system considered, correct processes have thus far found their incorporation into theoretical models, almost without exception, only *after* their empirical discovery. In another sense, any logical system of this kind, even if complete and self consistent, represents but an abstraction taken from the real world. At any time empirical facts may confront it with phenomena not encompassed by its self-sealing structure. This has happened often in physical science in the past, and its continuance is to be anticipated as one attribute of progress and growth.

2. The concept of convection in the general circulation

At the root of all organized thinking concerning the origin of the large atmospheric motions there has always been the notion of a convective process with upward and downward motions of air of contrasting temperature in different regions. Any doubt as to the correctness of this approach was in essence removed by the classic studies of Margules. A good part of the history of meteorology can be traced in terms of efforts to determine in what manner this basic driving action proceeds. Since vertical velocities in the large atmospheric structures are so small that their direct measurement is not practical, the delineation of the convective units has been—and still is—a matter of inference from other measurements. In the past some of these indirect attempts have been little better than sheer guesses. Correspondingly, the mathematical models evolved to fit these (as we now know) less correct arrangements, while interesting and ingenious technically, are henceforth mainly of historical importance.

Chiefly because of convenience of reproduction, I have selected a picture due to Ferrel (1889) as typical of the classical scheme of the large convective overturnings in the atmosphere (Fig. 1). The all-important feature contained in the model is that it provided for the descent of cold air at high latitudes and a rise of warm air at low latitudes, which implies a conversion of potential and internal energy into kinetic energy *à la* Margules by *zonally symmetric* overturnings. The release of energy so brought about is supposedly then fed into large average zonal components of motion, as will be further touched upon in the following discussions.

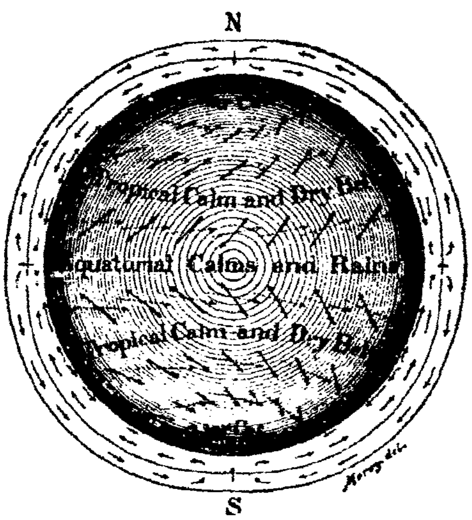

Fig. 1. General circulation scheme according to Ferrel.

Fig. 1 shows the so-called Ferrel cell in middle latitudes. It is important to observe the nature of this feature as it was originally intended according to Ferrel's later and more definitive conceptions. Clearly its contribution to the energy converting process spoken of

above is negative, i.e., to change kinetic energy back into potential and internal. However, due to its small vertical and meridional dimensions, this negative effect in Ferrel's scheme was insufficient to interfere materially with the *dominant* positive effect mentioned earlier. Much that has been written in recent years serves to blurr this essential point rather than to keep it in sharp relief, as clear thinking on the subject demands. A net positive energy-release by the mean meridional circulations in the classic schemes, exemplified by this picture, is an absolute necessity because no other means are included to provide a proper substitute. Let us compare this with modern views.

One feature of the new outlook upon the general circulation is that:

(A) *The net energy release by the mean meridional circulation is in all probability slightly negative.* This implies at once that other components of motion must act so as to provide convective overturnings which involve smaller cell sizes, and release enough energy not only to drive the global circulation against friction, but also to offset the braking action of the mean meridional circulations.

It should specifically be spelled out, moreover, that this situation results because *the mean meridional circulations contemplated in the classical schemes apparently do not exist in the actual atmosphere.* If they did, their effects would inescapably be associated with a mechanics of the general circulation of the classic type also in other respects, a proposition by now known to be diametrically opposed to the facts on almost all important counts (see Starr 1955, Pfeffer 1958). Furthermore it serves but to confuse the issue, so as to lose sight of the really important considerations, simply to insist that after all there are forced meridional circulations, when they actually act to produce, if anything, a small net negative energy transformation. When such isolated arguments are made in a spirit prejudicial to the new outlook here spoken of, the true situation is beclouded only for the undiscriminating or the uninitiated.

Another feature of the new outlook now follows rather directly from what was said:

(B) *The effective convective units for the release of potential and internal energy are are systems of a size corresponding to the nonzonal inequalities in the atmospheric motions. The output of these units in part serves to drive the average zonal easterlies and westerlies.*

The notion that disturbances of the general proportions of cyclones and anticyclones release potential and internal energy gained considerable ground in the past generation or so in connection with the studies of polar front phenomena. The realization that this energy could serve also to drive the zonal easterlies and westerlies, without an added net release by mean meridional circulations, is of more recent origin, however. In its nature this process is of turbulent character, irreducible to a steady-state motion through any reasonable transformation of coordinates. Even cursory regard of the day-to-day vagaries of the systems on weather maps is sufficient to convince one on this score.

By this time more detailed approximations to the instantaneous vertical velocity field over the northern hemisphere are beginning to be available. An examination of such maps reveals at once the pronounced nonzonal character of the distributions, as well as the predominance of high wave numbers (perhaps around ten).

3. The concept of angular momentum balance in the general circulation

There is some cause to speculate that at a rather early date certain scientists as, e.g., Sir John Herschel and von Helmholtz were concerned in some detail with the balance of absolute angular momentum in the atmosphere. Nevertheless the effective study of the subject from actual data had to await more recent times. In our deliberations we thus come to the next characteristic of the new outlook:

(C) *The zonally averaged easterlies and westerlies do not receive their supply of absolute angular momentum to offset ground friction through the action of the mean meridional circulations as necessitated by the classic schemes. Rather, this action is performed by the eddies, i.e., the nonzonal inequalities of air motions.*

The entire development of the modern view of the general circulation has revolved around the study of the angular momentum problem. This was true of the contributions of Jeffreys (e.g., 1926). Application of mixing length concepts to the absolute angular momentum distribution led Arakawa (1941) to propose a scheme of general circulation mechanics departing from the classic one and more like the modern view. (Apparently without at least direct mention of Arakawa's work, Lighthill (1954) repeated somewhat the same contention.) Rossby (1947, 1948), although he did not treat the angular momentum question *per se*, nevertheless sought to explain the distribution of mean zonal winds by a mixing theory applied to vorticity, obtaining a scheme which again departed from the classic picture in the correct sense. Once the true character of the angular momentum balance, as stated above, was determined directly from wind observations, by the writer and others some ten years ago, it became evident to numerous people that the most basic philosophy of atmospheric motions was in imperative need of major revision.

Further analysis of the angular momentum picture from observations, notably by Saltzman has concerned itself chiefly with the spectral resolution of the eddy process with respect to wave number around latitude circles. His finding some two years ago that wave number three is of special importance, immediately was an aid to the formulation of mathematical models of the general circulation devised by Bryan (1957).

4. Kinetic energy exchange between the eddies and the zonally average easterlies and westerlies

The interaction between the eddies and the mean zonal flow can be examined not only from the standpoint of the flow of angular momentum, but also from the standpoint of the kinetic energy of the wind field resolved into these two parts. This view involves concepts relating to the theory of turbulence as discussed by Osborne Reynolds; however due to the circumstances of our problem (to wit, the use of averages with respect to the length of closed latitude circles), certain of the ambiguities of the Reynolds formulation automatically drop out. The contribution to the new outlook in this context is as follows:

(D) *The transfer of angular momentum across latitute circles by the eddies is dominantly from zones of low angular velocity of the mean zonal winds to zones of high angular velocity; thus the effect is precisely the reverse of that corresponding to a frictional dissipation of kinetic energy, and is contrary to all classic concepts concerning the subject.*

The concept of a viscosity of one sort or another is of common occurrence among classical general circulation schemes. In order to include the effect here spoken of by means of an eddy viscosity, the appropriate coefficient would not only need to vary with latitude and perhaps otherwise, but, most important, it would have to be dominantly negative—something assuredly not contemplated classsically.

It needs to be expressed explicitly at this point that the probable negative conversion of kinetic energy back into potential and internal, stated under (A), is directly at the expense of the kinetic energy of the mean zonal winds. Since the *only* effective alternative manner in which the latter energy can be maintained against friction is through some type of eddy action, it is virtually certain that this action is the one spoken of here.

The work of Saltzman (1957a, 1957b, 1958) gives the result that wave number three is most important in feeding energy into the mean flow. It does not of necessity follow, however, that in the atmosphere the scale of disturbances responsible for the bulk of this energy transfer is the same as that representing the size of convective units. Many considerations suggest the contrary, so that there probably is flow of kinetic energy from some rather high wave numbers to certain lower ones. Efforts are being made to investigate this point.

5. Closing Commentaries

The foregoing material comprises one kind of summary which may be made of the new outlook upon the grossest problem posed by the science of meteorology. The sum and substance of these ideas is capable of formulation in other fashions, perhaps more formal

and sophisticated. My own propensity has however always been to give perference to those versions which appear to me to be the most simple and show historical continuity, except for specific technical purposes where special forms are often expedient.

It is a rather obvious proposition that the several ideas put forward above lend themselves to the synthesis of new over-all schemes into which they might enter without much mutual incompatibility. In this work there has been of late a noticeable quickening of pace which has been most refreshing to me, and has served to confirm estimates which I had made many years ago of the importance and necessity of investigating the *de facto* processes enumerated here as a basis for the proper prosecution of such projects. It is now my prediction that the role here played by these inquiries will be even better understood as time goes by. To those of my colleagues and friends who are occupied in these enterprises, I wish to extend my sincere best wishes and express my hopes for a rewarding future. Indeed I trust that we can go rather far with our present momentum. However, jointly with these efforts, let us not in the future, through some unwitting narrowness of approach, again become remiss in what I might here call simply the interpretative branch of observational meteorology, so essential for the unity of progress.

The historical vicissitudes attendant upon the promulgation of almost every classical scientific idea of consequence are so singular that one is tempted to think a moral is pointed up. Superficialities aside, this circumstance remains unchanged in regard to basic ideas. A development, doubtless inevitable, which has accompanied the exposition of the new trends concerning general circulation concepts, has been the opposition with which ideas, such as those that form the burden of this discourse, were greeted from the outset. I now speak not alone of my personal experience. In spite of professions to the contrary, the first devotion of the human mind is to established moorings. Conservatism has been the more instinctive posture for people whether in science or otherwise, and nonconformity was the first treason. Although this trait probably stems from some profound historic and prehistoric lessons of experience, my own personal rationalization of it would be but partially set forth without, at the very least, further reference to the ingredient perhaps best expressed in a quotation from our New England philosopher R. W. Emerson: "It is only low merits that can be enumerated. Fear, when your friends say to you what you have done well, and say it through; but when they stand with uncertain timid looks of respect and half-dislike, and must suspend their judgement for years to come, you may begin to hope."

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TRENDS OF THOUGHT CONCERNING METEOROLOGICAL RESEARCH

by Victor P. Starr (*)

Summary — Certain problems of synoptic meteorology relating to the nature of the large quasi-stationary cyclonic and anticyclonic centers aloft are examined from the standpoint of an hypothesis which seeks to connect their maintenance with that of the smaller transient disturbances. The suggested link between these two scales of eddies is in many ways similar to that operating in the general circulation in order to account for the sustained existence of the circumpolar mean zonal motions, according to the more modern concepts. Newer views concerning the dynamics of large convective processes, and the essential nonlinearity of the laws governing motions in the atmosphere, figure importantly in the arguments advanced.

Zusammenfassung — Gewisse Probleme der synoptischen Meteorologie im Zusammenhang mit der Natur von grossen halb-stationären zyklonischen und antizyklonischen Zentren in der höheren Atmosphäre werden von einem hypothetischen Standpunkt aus untersucht: und es wird versucht, deren Bestehen in Zusammenhang zu bringen mit dem Vorhandensein von kleineren, vorübergehenden Störungen. Der vorgeschlagene Zusammenhang zwischen diesen beiden Arten von Wirbel - Systemen ähnelt in vielen Beziehungen der allgemeinen Zirkulation, die verantwortlich ist für das fortwährende Bestehen der zirkumpolaren durchschnittlichen zonalen Strömungen, wie sie zufolge modernerer Auffassungen erklärt werden. Neuere Standpunkte, die die Dynamik von grossen konvektiven Vorgängen in der Atmosphäre und die nicht-linearen Gesetze im Hinblick auf die atmosphärischen Strömungs-Gesetze betreffen, werden hauptsächlich in der folgenden Abhandlung untersucht.

1. *Introduction* — In this discourse our attention will be limited to the older and more standard division of atmospheric studies, namely that dealing directly with the synoptic occurrences which appear as normal features on weather maps of various types. It is this branch of meteorology which has, through the years, formed the preoccupation of the weather forecasters and other people engaged in various related practical endeavors. We shall not, on the other hand, consider the many newer catagories of inquiry which have come to the fore in recent years, to a large degree through the impetus given to them by research results in branches of science other than meteorology proper. We thus have as our aim the discussion of perhaps the most traditional aspects of atmospheric studies.

If, for a moment, we should try to forget our scientific sophistication and

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look upon that which our synoptic maps show through the eyes of an intelligent neophyte, what we see must appear as something indeed not to be expected on *a priori* grounds. For most of us engaged in the subject, the feeling of shock of this first surprising apprehension has been dulled through prolonged familiarity — we must stop to think in order to regain the full sense of mystery posed by the so commonly observed happenings. And a mystery they are still, however much respect we may feel for all the studies that have been made of them and all the light that has thereby been thrown upon their workings. What makes the atmosphere choose a mode of response to solar heating seemingly so sownright fantastic?

Let us examine the progress that has so far been made along the path to be followed in order to solve this riddle, or if not to solve it, at least to make it somewhat less of an enigma. This is, of course, the problem of the general circulation, because all modern developments point to the idea that the general circulation can be conceived of adequately only as the sum total of all the motions of the atmosphere.

Although there are many people who no doubt are still unconvinced and would perhaps wish to take issue with me, I believe that the old orthodoxy regarding the mode of operation of the general circulation has effectively been laid to rest, and is replaced by a set of ideas much more closely in accord with the synoptic facts of life.

We shall not here enter into various extended repetitions of materials much elaborated by myself and by others elsewhere (see, e.g., STARR 1958), except for those portions specifically needed.

The essential characteristics of the new outlook which we shall utilize for the remainder of our present task are two in number. First, it appears in pertinent testimony from many sources, that one effect of the rotation of the earth upon the primary convective components of motion in the general circulation is to limit the horizontal dimensions of the convection cells (see, e.g., STARR 1955, 1959; KUO 1956). In its most striking application this notion is exemplified by the modern conception that cells as large as the so-called mean meridional circulations are not primary convective motions in the atmosphere, in direct contradiction to the classic expositions of the subject. On the whole they produce a contrary result, that is, a rising of colder air and sinking of warmer air, and must therefore be indirectly forced components of motion, as stressed by CHARNEY (1951, 1956) and by others (e.g., KUO 1956-a). Actually, speaking now in terms of harmonic analysis around complete latitude circles, the effective cell size of the primary convective motions is associated with higher wave numbers rather than zero.

In the second place, the new outlook has come to terms with the undeniable proposition that the motions of the atmosphere, once generated, are subject to laws which are pronouncedly and inherently nonlinear. This at once precludes any *a priori* simplification of the interactions between various components of motion. Such interactions can be specified only through their evaluation from a detailed nonlinear solution — in t[illegible] d from the true solution furnished by nature. Thus, speaking again in terms of a Fourier resolution of the field of motion around latitude circles, the interactions of various harmonics with each other cannot be stated without extensive investigations such as only modern efforts have begun to produce.

In the operation of the general circulation the principle of nonlinear interaction has found a striking illustration in the maintenance of the kinetic energy of the mean zonal winds. The action is to supply energy to the zero harmonic (the zonal mean) from the energy associated with higher wave numbers. It is therefore apparent that the two principles spoken of stand in a complimentary relation to each other, since it is the higher wave numbers that are associated with the primary convective actions and consequently with the appearance of kinetic energy in these same scales.

2. *Application of the first principle to synoptic systems* — It would be rather singular if the earth's rotation were to interfere with the convective action of the mean meridional circulations, and leave unhampered *all other* large atmospheric units so that they might act as primary convective systems. Instead, it is much more natural to expect that various sufficiently large synoptic circulations, especially at more polar locations should by virtue of this inhibition be rendered incapable of serving such a function.

Through the years I have kept this contention in my mind. Together with many co-workers, I have made it a practice of observing with some regularity the large cyclonic centers at upper levels over Canada and other northerly locations, especially during winter. The general impression is easily gotten that these disturbances have relatively cold cores, while at the same time near-saturation with respect to moisture seems to preclude the notion that this cold air is sinking and spreading. Rather the opposite is suggested, namely that these systems appear not to behave in the manner of energy releasing convective units. This aspect of the subject has been pointed out by REED (1956).

Dut to the presence of only extremely small amounts of moisture in this very cold air, no dense clouds appear to form in accompaniment to the slow upward motions. One may nevertheless legitimately raise the question whether enough condensation products in the form of fine ice particles may not be formed in order to ascribe to this cause the common incidence of atmospheric optical phenomena at higher latitudes.

On the other hand, it has long been standard meteorological information that the subtropical anticyclones have in them at upper levels comparatively warm conditions with a tendency to downward vertical motion — again not in accordance with a primary convective action.

We may next consider where indeed do the direct convective actions actually take place. Although charts of (instantaneous) vertical motion have been difficult to secure during the years past, we are beginning to find ourselves in a more fortunate situation in this regard, thanks to the developments in numerical analysis on a hemispheric scale. A sample chart of this kind is given in Fig. 1. It is seen that the wave numbers around the hemisphere characterizing this chart are rather high, as has been already pointed out. It is apparent that the cell sizes corresponding to an instantaneous picture are more fine-grained as compared with the large cyclonic and anticyclonic disturbances spoken of previously.

It would thus appear that the large quasi-stationary patterns at upper levels in the troposphere are subjected to a sort of parasitic behavior on the part of smaller more transient disturbances. One is lead to suspect that we have here an analogy *in parvo* of the circumstances studied so extensively in recent years relative to the general circulation. That is to say, the available potential and internal

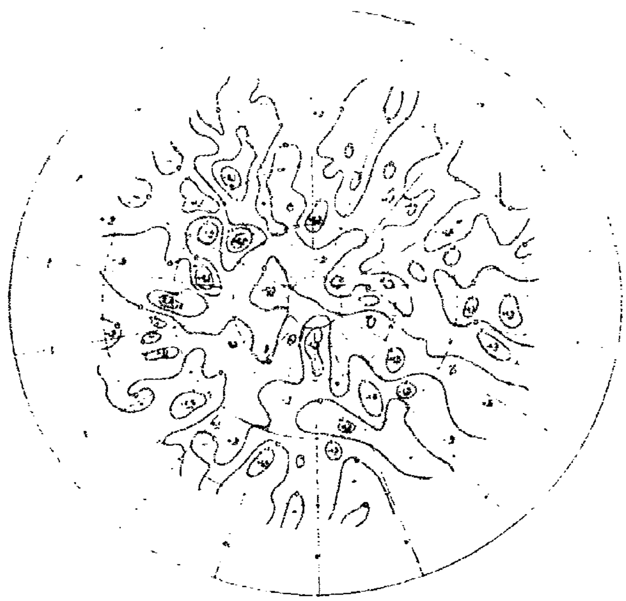

Fig. 1 - *Distribution of the vertical velocity* (cm sec^{-1}) *over the northern hemisphere at* 600 mb *for October* 27, 1958. Computed from instantaneous actual data by the Joint Numerical Weather Prediction Unit, Suitland, Maryland, using a two parameter model.

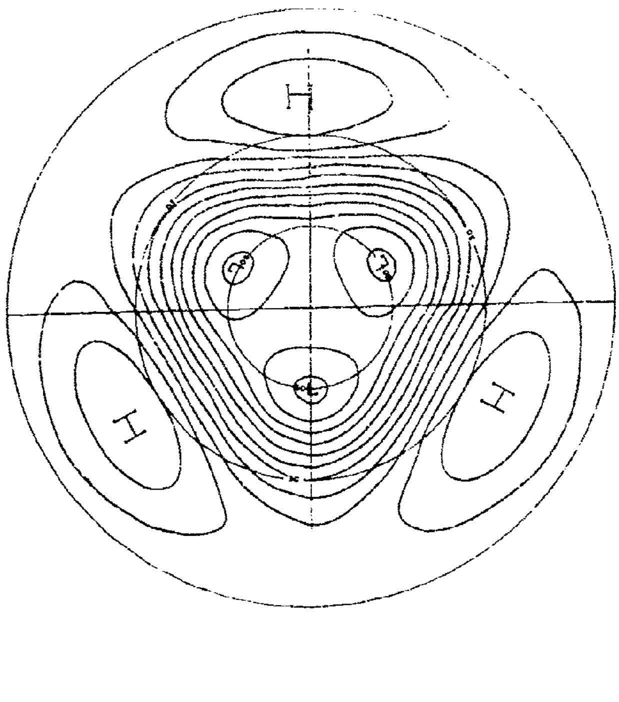

Fig. 2 - *Three-dimensional baroclinic solution of the hydrodynamical equations for a hemisphere, devised by* Dr. H. L. Kuo. The pattern may be either stationary or it may rotate with a uniform and constant angular velocity. The Lows are cold and the Highs are warm.

energy stored in the large scale quasi-stationary baroclinic systems in the atmosphere, cannot be released through primary convective processes on this same scale, but requires the intervention of a still smaller scale of disturbances — chiefly the traveling wave cyclones — to accomplish this end result. We could here again suppose that this available potential plus internal energy is first converted into an eddy available potential plus internal energy on a smaller scale prior to its relaease. The comparable concepts, without this finer resolution, for the general circulation, have been discussed by STARR (1954) and LORENZ (1955). In this manner the baroclinic energy could again be released without the necessity of having the large scale components themselves act as convective units. We may infer that the store of potential and internal energy of the gross pattern would be linked to radiative exchanges and the processes of the general circulation.

It may be asked whether or not there exists more direct evidence that the pattern of flow corresponding to quasi-stationary large disturbances is of such a dynamic character as to enable it to exist without a convective release of its own solenoidal energy. According to our suppositions such a persistence should be possible as a rather stationary flow, if the small scale parasitic disturbances are absent — and if friction is assumed to be dispensed with for the moment.

The evidence in this regard is fortunately most encouraging. For precisely these conditions, my friend and colleague, Dr. H. L. KUO, has demonstrated that there exists a solution of the hydrodynamical equations in analytical form for the atmosphere over the hemisphere which, moreover, is of the desired type also in other respects. In actuality there are a variety of such solutions, which may be thought of as threedimensional baroclinic counterparts of the celebrated hemispheric solutions of NEAMTAN (1946) for the case of a barotropic atmosphere. The motions are assumed to be adiabatic. Dr. KUO has kindly granted me permission to reproduce a map showing one of these theoretical flows in Fig. 2, from his manuscript on this general subject (KUO 1959). Without a doubt, solutions of this kind will be of the utmost significance for the further understanding of the general circulation in its further detail.

3. *Application of the second principle to synoptic systems* — The large quasi-stationary pattern, exemplified if one wishes by KUO's solution, that is, devoid of the finer structure, could not persist under attrition by friction. In order to prevent the disappearance of the pattern in the actual atmosphere, it therefore is necessary that there should exist some method of maintaining these circulations through the utilization of the kinetic energy developed in the finer scale disturbances by primary convective activity.

This consideration at once brings us face to face with the proposition of applying the principle of nonlinear interaction, between the motions of the large pattern on the one hand, and the convectively generated motions of the fine structure on the other. What is required are proper eddy transfers of momentum to the large cyclones and anticyclones, and also an associated flow of kinetic energy from the fine structure to these entities. The analogous processes for the circumpolar vortex have been announced, for example, by STARR (1953-a). If comparable actions exist here also, then the more complete interrelationship between the two scales of eddies now discussed is revealed in the aspect of a symbiosis, rather than a simple parasitic feeding of the smaller on the larger. A number of my associates working in synoptic research have, without solicitation, suggested to me that some such process seems to be a real manifestation.

Apart from plausibility and apparent necessity, what further guarantees or encouragements do we have that such a view is dynamically reasonable and represents *de facto* reality? The application of the angular momentum principle to actual observational data relating to systems smaller than the circumpolar vortex, and centered at arbitrary geographical locations, has been dealt with by several writers in recent years. Among these are STARR (1953), LORENZ (1953), SALTZMAN & PFEFFER (1955), SALTZMAN (1955), and PFEFFER (1958). The one all-important fact for the present discourse which emerges from these investigations is that, with the exception of the inner cores of hurricanes, the angular momentum transports take place almost entirely by horizontal eddy exchange processes. Beyond this point the studies mentioned are not too well adapted for our present uses, and we shall have to look to some others.

For some years now my colleague, Dr. B. SALTZMAN, has been engaged in studies of the hemispheric circumpolar vortex through the application of the technique of spectral analysis to the momentum and energy processes involved in the general circulation. The general plan of this type of investigation from extensive observational data, together with some partial and preliminary results, was presented by SALTZMAN (1957, 1958) in published form. Since then he has collaborated with Dr. A. FLEISHER, also of our staff, in an effort to treat massive amounts of data in this fashion through the use of intricate programs for a high speed computer. The preparation of these programs has taken over a year, but by now some results are beginning to be available. As yet much cannot be said, but it does already appear from the SALTZMAN and FLEISHER statistics that some energy is transfered from wave numbers around ten to successively lower ones and ultimately to wave number zero. This would, of course, be an action of the general

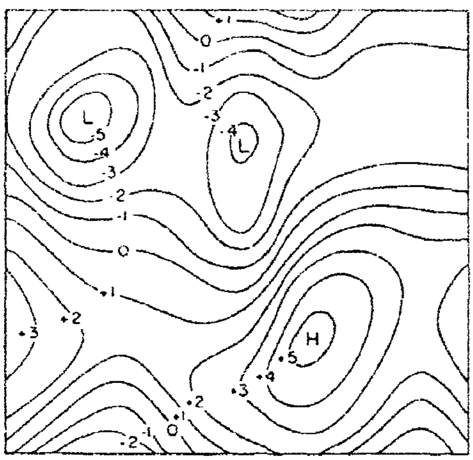

Fig. 3 - *Streamline pattern construct by* Dr. B. SALTZMAN *from an analytical expression consisting of a few double Fourier components.* Computations from the barotropic vorticity equation illustrate the nonlinear interaction of the harmonics used.

nature spoken of above. It is hoped that a more definitive outcome will be forthcoming shortly.

In a recent paper SALTZMAN (1959) has endeavored to examine the dynamic consistency of the general notions described here by devising a simple model of barotropic frictionless flow in order to illustrated the essential points. The model

is defined by three sets of double Fourier components for a cyclic rectangular region of a plane surface. These represent (*a*) a mean zonal flow, (*b*) a large quasi-stationary disturbance, and (*c*) a fine structure. Kinetic energy was found to drain from the fine structure into the large disturbance, while a smaller flow takes place from the large disturbance to the mean flow. It goes without saying that in such a barotropic scheme the fine structure would suffer a net kinetic energy loss. The probability seems good that, as a first approximation, the main character of the nonlinear interactions here considered can be represented by suitable barotropic processes. With the kind permission of the author, the composite stream function picture consisting of the sum of the three components is shown in Fig. 3, in order to display its general similarity to charts of upper-air flow.

Another extremely interesting barotropic study has been published by MARTIN (1958). From a statistical study of barotropic forecasts, for the northern hemisphere, this investigator has found that during the iteration process the kinetic energy of north-south motions in what would correspond to our fine structure decreases, as it does also in SALTZMAN's model. Due perhaps to the omission of the kinetic energy of west-east motions, no corresponding increase in the (north-south) kinetic energy of the large scale disturbances is found by MARTIN. On the whole this study is most suggestive, and work of this kind should be encouraged to the fullest extent possible.

4. *Circumscription and critique* — In the preceding material certain separations of scale were outlined which are somewhat too distinct. This was done in order to permit the presentation of a more accessible set of ideas dealing with an idealized scheme. In this way an appeal can be made with better advantage to the general experience of persons engaged in synoptic meteorology, and others who are interested in making a more meaningful interpretation of the evolution normally displayed by weather patterns. In actual fact the separation of scales, with the exception of the zero (hemispheric) wave number perhaps, can be effected most satisfactorily through some type of spectral statistics. There are any number of modes for doing this. As one good beginning we may look forward with anticipation to the SALTZMAN and FLEISHER statistics mentioned before, although it is unclear whether their formulation is the one best suited for this purpose.

There is of course ample evidence in what has been said that the genesis of the concepts under discussion proceeds from the much discussed newer outlook concerning the general circulation. Likewise, the further development of them unfolds under the aegis of momentum and energy considerations, just as in the latter topic. This perhaps should not be a cause for perplexity to the philosophically minded person, in view of the fundamental nature of these two quantities, both in classical as well as in modern physical theory.

It must be realized that the treatment here given of the subject discussed, constitutes but an outline of a field of investigation which could become extensive in respect to many of its possible ramifications. Observational studies are at once indicated both from the statistical and synoptic sides — provided, of course, that studies be properly grounded in dynamic principles. Moreover, various theoretical investigations could be initiated in order to round out and complete the picture. On the one hand, these could aim at the more precise and complete portrayal of individual actions inherent in the general scheme, one at a time. On the other, more comprehensive composite theoretical structures could perhaps be devised, if necessary, reducing the complexity of each action to its barest essential minimum.

5. *A personal memorandum* — For such as may be willing to listen, I wish to place before them a few of my subjective thoughts, reactions and feelings concerning the state of development which has been reached in those branches of meteorology touched upon in the foregoing material. No one could be more keenly aware than I am of the fact that synoptic meteorology has been the cutting edge of the tool with which we have, by-and-large, laid open the main mass of information concerning the nature of the motions of the atmosphere. The program of synoptic meteorology, forecasting problems aside, has been one of exploration and exposal to view in convenient form, of the *prima facie* evidence about the distribution of the main meteorological variables in space and in time. The program is not, and likely never will be, completed. There are newer and newer regions of the atmosphere to be convassed — there is finer and finer detail to be exhibited. Also, additional variables from time to time are found to be of enough importance to be charted.

Once, however, returns of this kind are in, synoptic meteorology must secure the aid of hydrodynamical theory in order to digest its findings further — synoptic meteorology cannot proceed beyond in its own purity. Neglect of, or at best improper attention to, this almost platitudinous statement has resulted in a form of stagnation that has come to be accepted even in a sense as a standard situation. Hence the imposing mass of banalities in the literature of meteorology *comme il faut*. Our dynamicist shares to a large degree, the onus for such conditions.

What has been lacking during the past — with worthy and notable exceptions, let it be said — is the *proper kind of rapprochement* between synoptic and dynamic meteorology. This should be a union based on mutual compatibility, not a shotgun wedding. On the one side, the synoptic meteorologist has tried to extort from is theoretical colleague answers to those specific problems whose effective treatment, if achieved now, would pre-empt in advance all the slow progress we might realistically expect to make in a hundred years to come. What desperations have been the consequences. On the other side, the all-too-often ethereal and esoterically vapid pronouncements served up by the dynamicist, as insistent offerings to the more practical man, have induced in the latter something vaguely like intellectual disorganization, not to mention the grotesque deterioration of his vocabulary.

But I do not wish to exaggerate. It may be that my appraisals are formed from experience of a dated vintage, and that a certain amelioration has stolen in. Not that there is little left still to be tended to, but a closer look does suggest the emergence of a changing attitude. There are signs of the stirrings of a new *Zeitgeist* in meteorology, which, let us hope, will replace the old and habitual frustrations by a new spirit of enterprize and adventure. For my part, it is a faith that there is room for enterprise and novelty not of a cheaply spectacular kind, which alone furnishes me with an adequate motive to continue my research.

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*Dr. Ward is a former student of ours working for AFCRL, USAF. We have continued to collaborate with him, and have therefore included five of his published papers because of their pertinence.

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13. ABSTRACT

Observational research results dealing with the general circulation of the atmosphere, obtained from investigations carried on at M.I.T. during the past eighteen years are summarized for more convenient reference, primarily with a view to the study of analogous behavior in certain respects of other cosmical fluid systems such as the solar atmosphere in particular. A total of seventy six selected papers are included in the volume, together with a preface and foreword by the compilers. A list of project reports issued during the period is also appended.

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